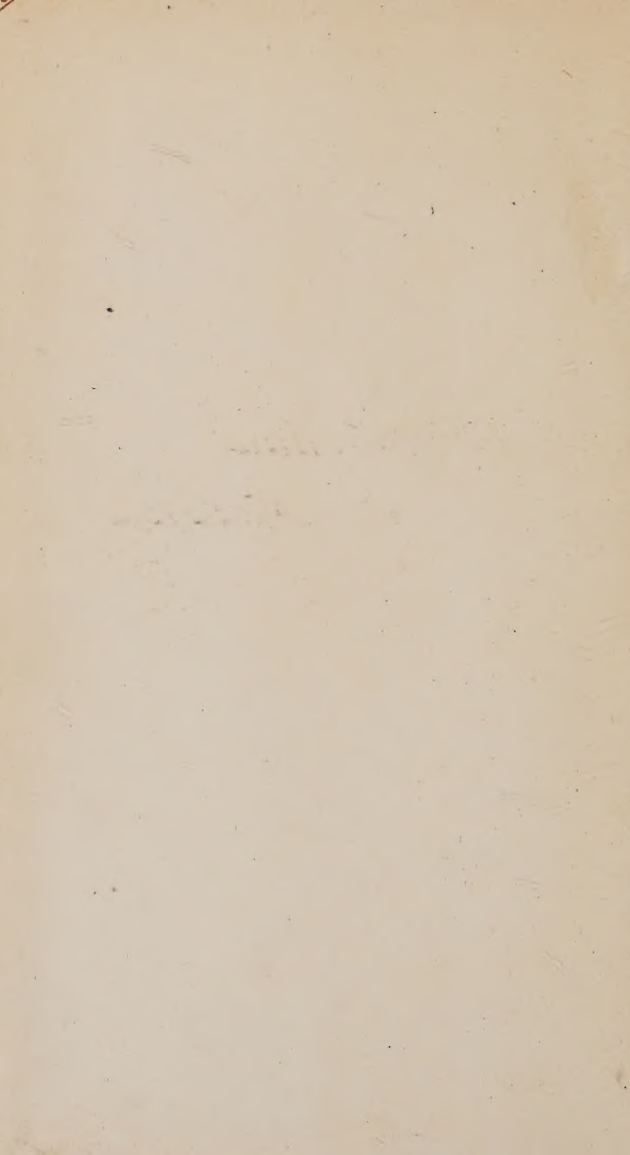




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**WORKS OF  
PROF. WALTER L. WEBB**

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# RAILROAD CONSTRUCTION.

THEORY AND PRACTICE.

*A TEXT-BOOK FOR THE USE OF STUDENTS  
IN COLLEGES AND TECHNICAL SCHOOLS.*

BY

WALTER LORING WEBB, C.E.,

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## PREFACE TO FIRST EDITION.

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THE preparation of this book was begun several years ago, when much of the subject-matter treated was not to be found in print, or was scattered through many books and pamphlets, and was hence unavailable for student use. Portions of the book have already been printed by the mimeograph process or have been used as lecture-notes, and hence have been subjected to the refining process of class-room use.

The author would call special attention to the following features:

a. Transition curves; the multiform-compound-curve method is used, which has been followed by many railroads in this country; the particular curves here developed have the great advantage of being exceedingly simple, and although the method is not theoretically exact, it is demonstrable that the differences are so small that they may safely be neglected.

b. A system of earthwork computations by means of a slide-rule (which accompanies the volume) which enables one to compute readily the volume of the most complicated earthwork forms with an accuracy only limited by the precision of the cross-sectioning.

c. The "mass curve" in earthwork; the theory and use of this very valuable process.

d. Tables I, II, III, and IV have been computed *ab novo*. Tables I and II were checked (after computation) with other tables, which are generally considered as standard, and all discrepancies were further examined. They are believed to be perfect.

e. Tables V, VI, VII, and IX have been borrowed, by permission, from "Ludlow's Mathematical Tables." It is believed that five-place tables give as accurate results as actual field

practice requires. Tables VIII and X have been compiled to conform with Ludlow's system.

The author wishes to acknowledge his indebtedness to Mr. Chas. A. Sims, civil engineer and railroad contractor, for reading and revising the portions relating to the cost of earthwork.

Since the book is written primarily for students of railroad engineering in technical institutions, the author has assumed the usual previous preparation in algebra, geometry, and trigonometry.

WALTER LORING WEBB.

UNIVERSITY OF PENNSYLVANIA,  
PHILADELPHIA,  
Jan. 1, 1900.

## PREFACE TO SECOND EDITION.

---

SINCE the issue of the first edition the author has conferred with many noted educators in civil engineering, among them the late Professors E. A. Fuertes and J. B. Johnson, regarding the most desirable size of page for this book. The inconvenience of the octavo edition for field-work was found to be limiting its use. It was therefore decided to recast the whole work and reduce the page from "octavo" to "pocket-book" size. Advantage was then taken of the opportunity to revise freely and to add new matter. The original text has now been almost doubled by the addition of several chapters on structures, train resistance, rolling stock, etc., and also several chapters giving the fundamental principles of the economics of railroad location. Those who are familiar with the late Mr. Wellington's masterpiece, "The Economic Theory of Railway Location," will readily appreciate the author's indebtedness to that work. But while the same general method has been followed, the author has taken advantage of the classification of operating expenses adopted by the Interstate Commerce Commission, has used the figures published by them (which were unavailable when Mr. Wellington wrote), and has developed the theory on an independent basis, with the exception of a few minor details. Those who deny the utility of such methods of computation are referred to §§ 367, 426, and elsewhere for a practical discussion of that subject.

The author's primary aim has been to produce a "text-book for students," and the subject-matter has therefore been cut down to that which may properly be required of students in

the time usually allotted to railroad work in a civil-engineering curriculum. On this account no extended discussion has been given to the multitudinous forms of various railroad devices in the chapters on structures. The aim has been to teach the principles and to guide the students into proper methods of investigation.

January, 1903.

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# RAILROAD CONSTRUCTION.

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## CHAPTER I.

### RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail.

### RECONNOISSANCE SURVEYS.

**1. Character of a reconnoissance survey.** A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

**2. Selection of a general route.** The general question of running a railroad between two towns is usually a financial rather

than an engineering question. Financial considerations usually determine that a road *must* pass through certain more or less important towns between its termini. When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alignment, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it) will be taken up in Chap. XIX, *et seq.*

3. **Valley route.** This is perhaps the simplest problem. If the two towns to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alignment is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade" \* for the whole road is as great as or greater than the steepest natural valley slope, more freedom may be used in adopting that alignment which has the least cost — regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river banks should be examined for suitable locations for abutments

---

\* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

and piers. If the soil is soft and treacherous, much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

**4. Cross-country route.** A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnaissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnaissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

**5. Mountain route.** The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by "development" accompanied perhaps by tunneling. "Development" consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired. The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between *A* and *B* was surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley and continued the climb along the opposite slope. (b) *Switch-*

back. On the steep side-hill *BCD* (Fig. 1) a very considerable gain in elevation was accomplished by the switchback *CD*. The gain in elevation from *B* to *D* is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from *C* to *D*. (c) *Bridge spiral*. When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the

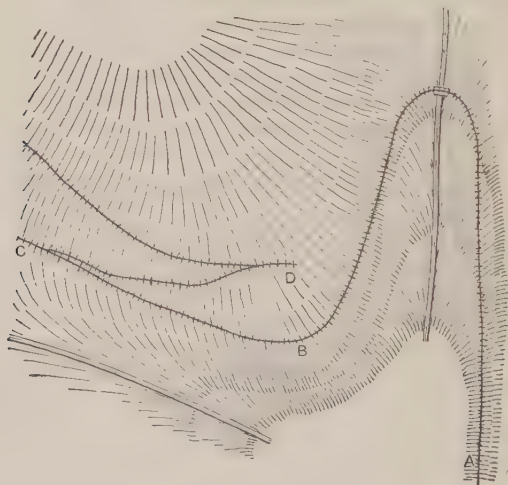


FIG. 1.

bottom of the valley, a bridge spiral may be desirable. In Fig. 2 the line ascends the stream valley past *A*, crosses the stream at *B*, works back to the narrow place at *C*, and there crosses itself, having gained perhaps 100 feet in elevation. (d) *Tunnel spiral* (Fig. 3). This is the reverse of the previous plan. It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long. Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.

On Plate I are shown three separate ways (as actually constructed) of running a railroad between two points a little over three miles apart and having a difference of elevation of nearly

1100 feet. At *A* the Central R. R. of New Jersey runs *under* the Lehigh Valley R. R. and soon turns off to the northeast for about six miles, then doubles back, reaching *D*, a fall of about 1050 feet with a track distance of about 12.7 miles. The L. V. R. R. at *A* runs to the westward for six to seven miles,

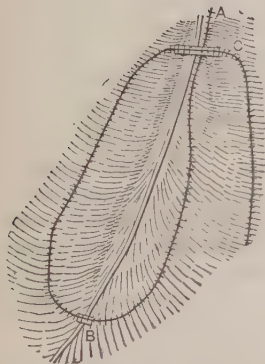


FIG. 2.

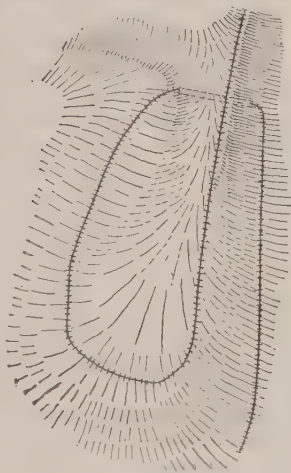


FIG. 3.

then turns back until the roads are again close together at *D*. The track distance is about 14 miles and the drop a little greater, since at *A* the L. V. R. R. crosses *over* the other, while at *D* they are at practically the same level. From *B* to *C* the distance is over eleven miles. From *A* directly down to *D* the C. R. R. of N. J. runs a "gravity" road, used exclusively for freight, on which cars alone are hauled by cable. The main-line routes are remarkable examples of sheer "development." Even as constructed the L. V. R. R. has a grade of about 95 feet per mile, and this grade has proved so excessive for freight work that the company has constructed a cut-off (not shown on the map) which leaves the main line at *A*, nearly parallels the C. R. R. to *C*, and then running in a northeasterly direction again joins the main line beyond Wilkesbarre. The grade is thereby cut down to 65 feet per mile.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.



**6. Existing maps.** The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

**7. Determination of relative elevations.** A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the "exploratory" survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at "headquarters," and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be "reduced to 32° F." The form of notes for the mercurial barometer readings should be as follows:

| Time.      | Merc.<br>Barom. | Attached<br>Therm. | Reduction<br>to 32° F. | External<br>Therm. | Corrected<br>reading. |
|------------|-----------------|--------------------|------------------------|--------------------|-----------------------|
| 7:00 A. M. | 29.872          | 72°                | — .117                 | 73°                | 29.755                |
| :15        | .866            | 73.5               | .121                   | 75                 | .745                  |
| :30        | .858            | 75                 | .125                   | 76                 | .733                  |
| :45        | .850            | 76                 | .127                   | 77                 | .723                  |



The corrections in column 4 are derived from Table XI by interpolation.

Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. If the difference of elevation is excessive (as when climbing a high mountain) even the best aneroid will "lag" and not recover its normal reading for several hours, but this does not apply to such differences of elevation as are met with in railroad work. The best aneroids read directly to  $\frac{1}{100}$  of an inch of mercury and may be estimated to  $\frac{1}{1000}$  of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read, at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. The field notes for the aneroid should be taken as shown in the first four columns of the tabular form. The "corrected aneroid" readings of column 5 are found by correcting the readings of column 3 by the mean difference between the mercurial and aneroid when compared at morning and night. Column 6 is a copy of the "corrected readings" from the office notes, interpolated when necessary for the proper time. Column 7 is similarly obtained. Col. 8 is obtained from cols. 4 and 5, and col. 9 from cols. 6 and 7, with the aid of Table XII. The correction for temperature (col. 11), which is generally small unless the difference of elevation is large, is obtained with the aid of Table XIII. The elevations in Table XII are elevations above an assumed datum plane, where under the given atmospheric conditions the mercurial reading would be 30". Of course the position of this assumed plane changes with varying atmospheric conditions and so the elevations are to be considered as *relative* and their difference taken. [See the author's "Problems in the Use and Adjustment of Engineering In-

(Left-hand page of Notes.)

| Time. | Place.       | Aneroid. | Therm. | Corr.<br>Aner. | Corr.<br>Merc. |
|-------|--------------|----------|--------|----------------|----------------|
| 7:00  | Office       | 29.628   | 73°    | .....          | 29.755         |
| 7:10  | 40           | 29.662   | 72°    | 29.789         | 29.748         |
| 7:30  | saddle-back  | 29.374   | 63°    | 29.501         | 29.733         |
| 7:50  | river cross. | 29.548   | 70°    | 29.675         | 29.720         |

struments," Prob. 22.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary, the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass *B* is 260 feet higher than a determined bridge crossing at *A* which is six miles distant, and that another pass *C* is 310 feet higher than *A* and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for reconnaissance purposes* the added accuracy of the spirit-leveling method is hardly worth its cost.

§. Horizontal measurements, bearings, etc. When there is no map which may be depended on, or when only a skeleton map is obtainable, a rapid survey, sufficiently accurate for the purpose, may be made by using a pocket compass for bearings and a telemeter, odometer, or pedometer for distances. The telemeter [stadia] is more accurate, but it requires a definite clear

(Right-hand page of Notes.)

| Temp. at<br>headqu. | Approx.<br>field read. | Approx.<br>headq. read. | Diff. | Corr. for<br>temp. | Diff.<br>elev. |
|---------------------|------------------------|-------------------------|-------|--------------------|----------------|
| —                   | —                      | —                       | —     | —                  | —              |
| 75°                 | 192                    | 230                     | — 38  | —(+ 2)             | — 40           |
| 76                  | 457                    | 244                     | +213  | +(+10)             | +223           |
| 77                  | 297                    | 256                     | + 41  | +(+ 2)             | + 43           |

sight from station to station, which may be difficult through a wooded country. The odometer, which records the revolutions of a wheel of known circumference, may be used even in rough and wooded country, and the results may be depended on to a small percentage. The pedometer (or pace-measurer) depends for its accuracy on the actual movement of the mechanism for each pace and on the uniformity of the pacing. Its results are necessarily rough and approximate, but it may be used to fill in some intermediate points in a large skeleton map. A hand-level is also useful in determining the relative elevation of various topographical features which may have some bearing on the proper location of the road.

**9. Importance of a good reconnoissance.** The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to competition, no amount of perfection in detailed alignment or roadbed construction can make the road a profitable investment.

#### PRELIMINARY SURVEYS.

**10. Character of survey.** A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad is to follow a river having very steep banks the choice of location is sometimes limited at places to a very few feet of width

and the belt to be surveyed may be correspondingly narrowed. In very flat country the desired width may be only limited by the ability to survey points with sufficient accuracy at a considerable distance from what may be called the "backbone line" of the survey.

**11. Cross-section method.** This is the only feasible method in a wooded country, and is employed by many for all kinds of country. The *backbone line* is surveyed either by observing magnetic bearings with a compass or by carrying forward

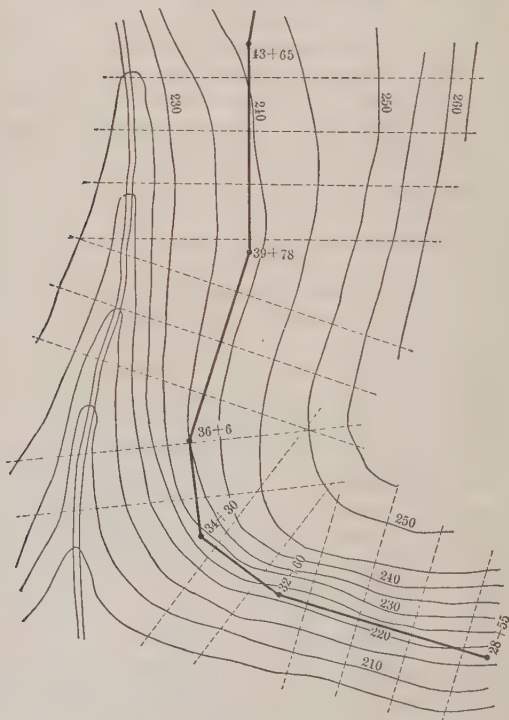


FIG. 4.

absolute azimuths with a transit. The compass method has the disadvantages of limited accuracy and the possibility of considerable local error owing to local attraction. On the other

hand there are the advantages of greater simplicity, no necessity for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth with a needle reading. It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun or Polaris, and to check the azimuths every few miles by azimuth observations. Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet. When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course. The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

**12. Cross-sectioning.** It is usually desirable to obtain contours at five-foot intervals. This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths. The method of use may perhaps be best explained by an example. Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4. *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum. When the Locke level on its five-foot rod is placed at *C*, the level has an elevation of 163.3. Therefore when a point is found (as at *a*) where the level will read 3.3 on the rod, that point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point (*a*), carry the level to some point (*b*) such that the level will sight at the *top* of the rod. *b* is then on the 165-

foot contour, and the *horizontal* distance *ab* added to the horizontal distance *ac* gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour.

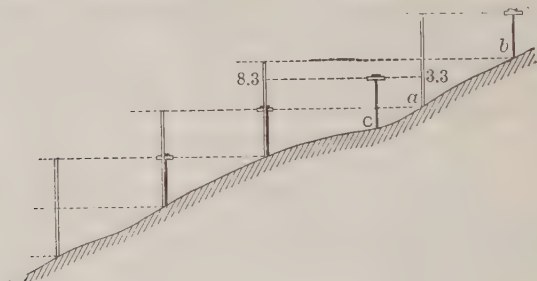


FIG. 5.

Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions. Plot the work up the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses

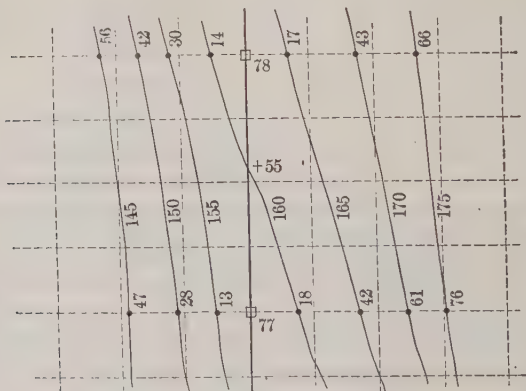


FIG. 6.

the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fifty-



foot distances from the center should be determined. The method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party.

**13. Stadia method.** This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be easily neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

Since the students taking this work are assumed to be familiar with the methods of stadia topographical surveys, this part of the subject will not be further elaborated.

**14. "First" and "Second" preliminary surveys.** Some engineers advocate two preliminary surveys. When this is done, the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of lines but of areas, that their aim is to survey only those topographical features which would have a deter-

mining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical sections* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

#### LOCATION SURVEYS.

15. "**Paper location.**" When the preliminary survey has been plotted to a scale of 200 feet per inch and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves the alignment may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connecting them. The determining points should first be considered. Such points are the termini of the road, the lowest practicable point over a summit, a river-crossing, etc. So far as is possible, having due regard to other considerations, the road should be a "surface" road, i.e., the cut and fill should be made as small as possible. The maximum permissible grade must also have been determined and duly considered. The method of location differs radically according as the lines joining the determining points have a very low grade or have a grade that approaches the maximum permissible. With very low natural grades it is only necessary to strike a proper balance between the requirements for easy alignment and the avoidance of excessive earthwork. When the grade between two determined points approaches the maximum, a study of the location may be begun by finding a strictly surface line which will connect those



points with a line at the given grade. For example, suppose the required grade is 1.6% and that the contours are drawn at 5-foot intervals. It will require 312 feet of 1.6% grade to rise 5 feet. Set a pair of dividers at 312 feet and step off this interval on successive contours. This line will in general be very irregular, but in an easy country it may be fairly close to the proper location line, and even in difficult country such a surface line will assist greatly in selecting a suitable location. When the larger part of the line will evidently consist of tangents, the tangents should be first located and should then be connected by suitable curves. When the curves predominate, as they generally will in mountainous country, and particularly when the line is purposely lengthened in order to reduce the grade, the curves should be plotted first and the tangents may then be drawn connecting them. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the *comparative* amount of earthwork required. After deciding on the paper location, the length of each tangent, the central angle (see § 21), and the radius of each curve should be measured as accurately as possible. Since a slight error made in such measurements, taken from a map with a scale of 200 feet per inch, would by accumulation cause serious discrepancies between the plotted location and the location as afterward surveyed in the field, frequent tie lines and angles should be determined between the plotted location line and the preliminary line, and the location should be altered, as may prove necessary, by changing the length of a tangent or changing the central angle or radius of a curve, so that the agreement of the check-points will be sufficiently close. The errors of an inaccurate preliminary survey may thus be easily neutralized (see § 33). When the preliminary line has been properly run, its "backbone" line will be very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

**16. Surveying methods.** A transit should be used for alignment, and only precise work is allowable. The transit stations

should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earth-work, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that  $\frac{1}{1000}$  of a foot has an angular value of only 7

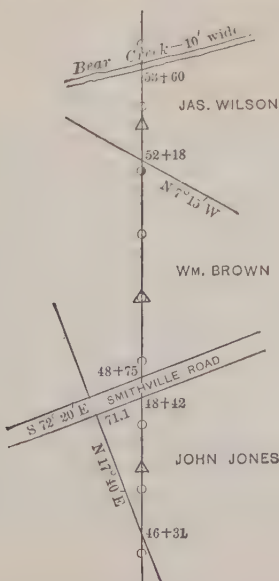
## FORM OF NOTES.

[Left-hand page.]

| Sta.           | Align-<br>ment.                                              | Vernier. | Tangential<br>Deflection. | Calculated<br>Bearing. | Needle.     |
|----------------|--------------------------------------------------------------|----------|---------------------------|------------------------|-------------|
| 54             |                                                              |          |                           |                        |             |
| ⊙ 53<br>+ 72.2 | P.T.                                                         | 9° 11'   | 18° 22'                   | N 54° 48' E            | N 62° 15' E |
| 52             |                                                              | 7 57     |                           |                        |             |
| 51             |                                                              | 6 15     |                           |                        |             |
| ⊙ 50           | [ 3° 24' curve to right for<br>18° 22'; tang. dist., 272.5 ] | 4 33     |                           |                        |             |
| 49             |                                                              | 2 51     |                           |                        |             |
| 48             |                                                              | 1 09     |                           |                        |             |
| ⊙ + 32<br>47   | P.C.                                                         | 0°       |                           |                        |             |
| 46             |                                                              |          |                           | N 36° 26' E            | N 44° 0' E  |

seconds at a distance of 300 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will in-

[Right-hand page.]



clude the position and elevation of all streams, and even dry gullies, which are crossed

**Measurements** should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137 + 69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4° curve" which turns to the *right*.

**Alignment.** The alignment is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

**17. Form of Notes.** Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read up the page, so that the sketch will be properly oriented when looking ahead along the line. The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the "calculated bearings" are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

## CHAPTER II.

### ALIGNMENT.

IN this chapter the alignment of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alignment may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

### SIMPLE CURVES.

**18. Designation of curves.** A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the "degree of curve" and is indicated by  $D$ . Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If  $AB$  in Fig. 4 represents a unit chord ( $C$ ) of a curve of radius  $R$ , then by the above definition the angle  $AOB$  equals  $D$ . Then

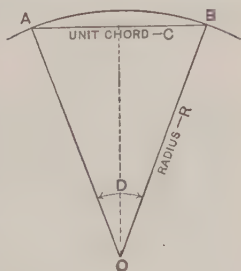


FIG. 7.

$$AO \sin \frac{1}{2}D = \frac{1}{2}AB = \frac{1}{2}C.$$

$$\therefore R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

or, by inversion,

$$\sin \frac{1}{2}D = \frac{C}{2R}. \quad . . . . . (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a  $0^{\circ} 01'$  curve up to a  $10^{\circ}$  curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of  $R$  may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since  $R$  does not strictly vary inversely as  $D$ , yet the resulting value is within a tenth of one per cent for all commonly used values of  $R$ , and is sufficiently close for many purposes, as will be shown later.

**19. Length of a subchord.** Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length) will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calculation by noting that two equal subchords, each subtending the angle  $\frac{1}{2}D$ , will evidently be slightly longer

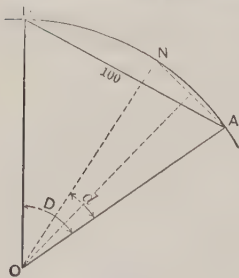


FIG. 8.

than 50 feet each. If  $c$  be the length of a subchord subtending the angle  $d$ , then, as in Eq. 2,

$$\sin \frac{1}{2}d = \frac{c}{2R},$$

or, by inversion,

$$c = 2R \sin \frac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord  $= 100 \frac{d}{D}$  For example, a *nominal* subchord of 40 feet will subtend an angle of  $\frac{40}{100}$  of  $D^\circ$ ; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a  $10^\circ$  curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

**20. Length of a curve.** The length of a curve is always indicated by the quotient of  $100L \div D$ . If the quotient of  $L \div D$  is a whole number, the length as thus indicated is the true length *measured in 100-foot chord lengths*. If it is an odd number or if the curve begins and ends with a subchord (even though  $L \div D$  is a whole number), theoretical accuracy requires that the *true* subchord lengths shall be used, although the difference may prove insignificant. The length of the arc (or the mean length of the two rails) is therefore always in excess of the length as given above. Ordinarily the amount of this excess is of no practical importance. It simply adds an insignificant amount to the length of rail required.

*Example.* Required the nominal and true lengths of a  $3^\circ 45'$  curve having a central angle of  $17^\circ 25'$ . First reduce the degrees and minutes to decimals of a degree.  $(100 \times 17^\circ 25') \div 3^\circ 45' = 1741.667 \div 3.75 = 464.444$ . The curve has four 100-foot chords and a nominal chord of 64.444. The true chord should be 64.451. The actual arc is

$$17^\circ.4167 \times \frac{\pi}{180^\circ} \times R = 464.527$$

The excess is therefore  $464.527 - 464.451 = 0.076$  foot.

**21. Elements of a curve.** Considering the line as running from *A* toward *B*, the beginning of the curve, at *A*, is called the *point of curve* (*PC*). The other end of the *curve*, at *B*, is





central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded.

In Table II is given the value of the tangent distances, external distances, and long chords for a  $1^\circ$  curve for various central angles. The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and by the approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

**24. Exercises.** (a) What is the tangent distance of a  $4^\circ 20'$  curve having a central angle of  $18^\circ 24'$ ?

(b) Given a  $3^\circ 30'$  curve and a central angle of  $16^\circ 20'$ , how far will the curve pass from the vertex? [Use Eq. 7]

(c) An  $18^\circ$  curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords?

(d) Given two tangents making a central angle of  $15^\circ 24'$ . It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq. 4 inverted.)

**25. Curve location by deflections.** The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 10), if the first chord is to be a full chord we may deflect an angle  $VAa$  ( $=\frac{1}{2}D$ ), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle  $bAa$  ( $=\frac{1}{2}D$ ) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ( $\frac{1}{2}d$ ) is something less than  $\frac{1}{2}D$ . The last deflection ( $BAV$ ) is

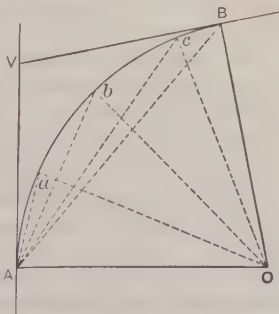


FIG. 10.

of course  $\frac{1}{2}A$ . It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with  $\frac{1}{2}A$ .

*Example.* Given a  $3^{\circ} 24'$  curve having a central angle of  $18^{\circ} 22'$  and beginning at sta.  $47+32$ , to compute the deflections. The nominal length of curve is  $18^{\circ} 22' \div 3^{\circ} 24' = 18.367 \div 3.40 = 5.402$  stations or 540.2 feet. The curve therefore ends at sta.  $52+72.2$ . The deflection for sta. 48 is  $\frac{68}{100} \times \frac{1}{2}(3^{\circ} 24') = 0.68 \times 1^{\circ}.7 = 1^{\circ}.156 = 1^{\circ} 09'$  nearly. For each additional 10 feet it is  $1^{\circ} 42'$  additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \frac{1}{2}(3^{\circ} 24') = 1^{\circ}.2274 = 1^{\circ} 14' \text{ nearly.}$$

The deflections are

|                                      |                          |
|--------------------------------------|--------------------------|
| P. C. . . . . Sta. 47+32 . . . . .   | 0°                       |
| 48 . . . . .                         | 0° + 1° 09' = 1° 09'     |
| 49 . . . . .                         | 1° 09' + 1° 42' = 2° 51' |
| 50 . . . . .                         | 2° 51' + 1° 42' = 4° 33' |
| 51 . . . . .                         | 4° 33' + 1° 42' = 6° 15' |
| 52 . . . . .                         | 6° 15' + 1° 42' = 7° 57' |
| P. T. . . . . Sta. 52+72.2 . . . . . | 7° 57' + 1° 14' = 9° 11' |

As a check  $9^{\circ} 11' = \frac{1}{2}(18^{\circ} 22') = \frac{1}{2}A$ . (See the Form of Notes in § 17.)

26. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of  $0^{\circ}$ , so that when the telescope is turned to  $0^{\circ}$  it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied

This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (*a*) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (*b*) is just as simple as method (*a*) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(*b*) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *PC*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual.

**RULE.** When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station. When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for any station, forward or back, is that originally computed for it from the *PC*. In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at any visible station and noting whether



FIG. 11.

its deflection agrees with that originally computed. As a numerical illustration, assume a  $4^\circ$  curve, with  $28^\circ$  curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is  $0^\circ$ . The reading on sta. 1 is  $2^\circ$ ; when the reading is  $4^\circ$  the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be  $6^\circ$  and  $8^\circ$ . Occupy 4; sight to 2 with a reading of  $4^\circ$ . When the reading is  $8^\circ$  the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally computed deflections of  $10^\circ$ ,  $12^\circ$ , and  $14^\circ$ . When occupying 7 a backsight may be taken to any visible station with the plates reading the deflection for that station; then when

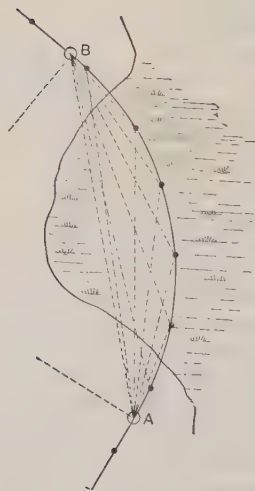


FIG. 12.

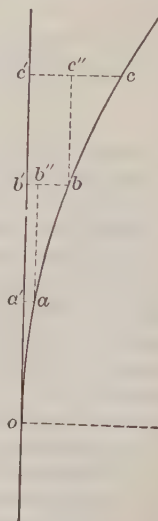


FIG. 13.

the plates read  $14^\circ$  the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

**27. Curve location by two transits.** A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord  $AB$  (Fig. 12) may be determined by triangulation or otherwise, and the elements of the curve computed, including (possibly) subchords at each end. The deflection from  $A$  and  $B$  to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

**28. Curve location by tangential offsets.** When a curve is very flat and no transit is at hand the following method may be used (see Fig. 13): Produce the back tangent as far forward as necessary. Compute the ordinates  $Oa'$ ,  $Ob'$ ,  $Oc'$ , etc., and the abscissæ  $a'a$ ,  $b'b$ ,  $c'c$ , etc. If  $Oa$  is a full station (100 feet), then

$$\left. \begin{aligned} Oa' &= Oa' &= 100 \cos \frac{1}{2}D, \text{ also } = R \sin D; \\ Ob' &= Oa' + a'b' &= 100 \cos \frac{1}{2}D + 100 \cos \frac{3}{2}D, \\ & &\text{also } = R \sin 2D; \\ Oc' &= Oa' + a'b' + b'c' = 100(\cos \frac{1}{2}D + \cos \frac{3}{2}D + \cos \frac{5}{2}D), \\ & &\text{also } = R \sin 3D; \end{aligned} \right\} \quad (9)$$

etc.

$$\left. \begin{aligned} a'a &= &100 \sin \frac{1}{2}D, \text{ also } = R \text{ vers } D; \\ b'b &= a'a + b''b &= 100 \sin \frac{1}{2}D + 100 \sin \frac{3}{2}D, \\ & &\text{also } = R \text{ vers } 2D; \\ c'c &= b'b + c''c &= 100(\sin \frac{1}{2}D + \sin \frac{3}{2}D + \sin \frac{5}{2}D), \\ & &\text{also } = R \text{ vers } 3D; \end{aligned} \right\} \quad (10)$$

etc.

The functions  $\frac{1}{2}D$ ,  $\frac{3}{2}D$ , etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that  $Ob'$  (for example) is one half of the long chord for four stations; also that  $b'b$  is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and terms will be correspondingly altered. The modifications may

be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

In Table II are given the long chords for a  $1^\circ$  curve for various values of  $L$ . Dividing the value as given by the degree of the curve, we have an approximate value which is amply close for low degrees of curvature, especially for laying out curves with out a transit. For example, given a  $4^\circ 30'$  curve, required the ordinate  $Oc'$ . This is evidently one half of a chord of six stations, with  $L=27^\circ$ . Dividing 2675.1 (which is the long chord of a  $1^\circ$  curve with  $L=27^\circ$ ) by 4.5 we have 594.47; one half of this is the required ordinate,  $Oc'=297.23$ . The exact value is 297.31, an excess of .08, or less than .03 of 1%. The true values are always slightly in excess of the value as computed from Table II.

*Exercise.* A  $3^\circ 40'$  curve begins at sta. 18+70 and runs to sta. 23+60. Required the tangential offsets and their corresponding ordinates. The first ordinate  $= 30 \cos \frac{1}{2}(\frac{3.0}{100} \times 3^\circ 40') = 30 \times .99995 = 29.9985$ ; the offset  $= 30 \sin 0^\circ 33' = 30 \times .0096 = 0.288$ . For the second full station (sta. 20) the ordinate  $= \frac{1}{2}$  long chord for  $L=2(1^\circ 06' + 3^\circ 40')$  with  $D=3^\circ 40'$ . Dividing 476.12, from Table II, by  $3\frac{2}{3}$ , we have 129.85. Otherwise, by Eq. 9, the ordinate  $= 30 \times \cos 0^\circ 33' + 100 \cos (1^\circ 06' + 1^\circ 50') = 30.00 + 99.87 = 129.87$ . The offset for sta. 20  $= 30 \sin 0^\circ 33' + 100 \sin (1^\circ 06' + 1^\circ 50') = 0.288 + 5.12 = 5.41$ . Work out similarly the ordinates and offsets for sta. 21, 22, 23, and 23+60.

**29. Curve location by middle ordinates.** Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to  $z$ , the chord  $za = 2 \times 100 \cos \frac{1}{2}D$ ,  $A'a = 100 \cos \frac{1}{2}D$ , and  $A'A = am = zn = 100 \sin \frac{1}{2}D$ . Set off  $A'A'$  perpendicular to the tangent and  $A'a$  parallel to the tangent.  $AA' = aa' = bb' = cc'$ , etc.  $= 100 \sin \frac{1}{2}D$ . Set off  $aa'$  perpendicular to  $a'A$ . Produce  $Aa'$  until  $a'b = A'a$ , thus determining  $b$ . Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before  $ra = Am' = c' \cos \frac{1}{2}d'$ , and  $rA = am' = c' \sin \frac{1}{2}d'$ . Also  $sz = An' = c'' \cos \frac{1}{2}d''$ , and  $sA = zn' = c'' \sin \frac{1}{2}d''$ , in which  $(d' + d'') = D$ . The points  $z$  and  $a$  being determined on the ground,  $aa'$  may be computed and set off as before and the curve continued in



full stations. A subchord at the end of the curve may be located by a similar process.

30. Curve location by offsets from the long chord. (Fig. 16.) Consider at once the general case in which the curve commences with a subchord (curvature,  $d'$ ), continues with one or more full



FIG. 14.

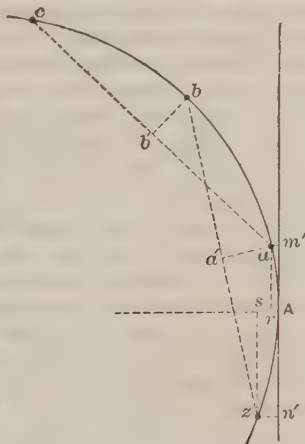


FIG. 15.

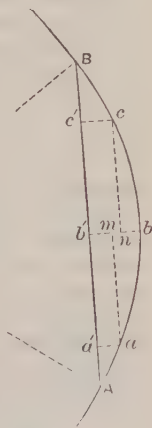


FIG. 16.

chords (curvature of each,  $D$ ), and ends with a subchord with curvature  $d''$ . The numerical work consists in computing first  $AB$ , then the various abscissæ and ordinates.  $AB = 2R \sin \frac{1}{2}A$ .

$$\left. \begin{aligned} Aa' &= Aa' &= c' \cos \frac{1}{2}(A - d'); \\ Ab' &= Aa' + a'b' &= c' \cos \frac{1}{2}(A - d') + 100 \cos \frac{1}{2}(A - 2d' - D); \\ Ac' &= Aa' + a'b' + b'c' &= c' \cos \frac{1}{2}(A - d') + 100 \cos \frac{1}{2}(A - 2d' - D) \\ & &+ 100 \cos \frac{1}{2}(A - 2d'' - D); \end{aligned} \right\} \quad (11)$$

also

$$= AB - Bc' \quad \therefore = 2R \sin \frac{1}{2}A - c'' \cos \frac{1}{2}(A - d'').$$

$$\left. \begin{aligned} a'a &= a'a &= c' \sin \frac{1}{2}(A - d'); \\ b'b &= a'a + mb &= c' \sin \frac{1}{2}(A - d') + 100 \sin \frac{1}{2}(A - 2d' - D); \\ c'c &= b'b - nb &= c' \sin \frac{1}{2}(A - d') + 100 \sin \frac{1}{2}(A - 2d' - D) \\ & &- 100 \sin \frac{1}{2}(A - 2d'' - D); \end{aligned} \right\} \quad (12)$$

also

$$= c'' \sin \frac{1}{2}(A - d'').$$

The above formulæ are considerably simplified when the

curve begins and ends at even stations. When the curve is very long a regular law becomes very apparent in the formation of all terms between the first and last. There are too few terms in the above equations to show the law.

**31. Use and value of the above methods.** The chief value of the above methods lies in the possibility of doing the work without a transit. The same principles are sometimes employed, even when a transit is used, when obstacles prevent the use of the normal method (see § 32, c). If the terminal tangents have already been accurately determined, these methods are useful to locate points of the curve when rigid accuracy is not essential. Track foremen frequently use such methods to lay out unimportant sidings, especially when the engineer and his transit are not at hand. Location by tangential offsets (or by offsets from the long chord) is to be preferred when the curve is flat (i.e., has a small central angle  $J$ ) and there is no obstruction along the tangent, or long chord. Location by middle ordinates may be employed regardless of the length of the curve, and in cases when both the tangents and the long chord are obstructed. The above methods are but samples of a large number of similar methods which have been devised. The choice of the particular method to be adopted must be determined by the local conditions.

**32. Obstacles to location.** In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide, in each case, which is the best method to use. It is frequently advisable to devise a special solution for some particular case.

**a. When the vertex is inaccessible.** As shown in § 26, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Some-

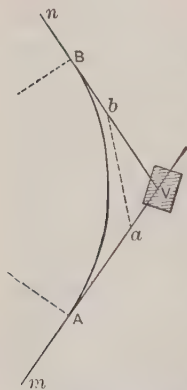


FIG. 17.

positions of the transit and comparatively short sights. Some-

times the location of the tangents is already determined on the ground (as by  $bn$  and  $am$ , Fig. 17), and it is required to join the tangents by a curve of given radius. *Method.* Measure  $ab$  and the angles  $Vba$  and  $baV$ .  $A$  is the sum of these angles. The distances  $bV$  and  $aV$  are computable from the above data. Given  $A$  and  $R$ , the tangent distances are computable, and then  $Bb$  and  $aA$  are found by subtracting  $bV$  and  $aV$  from the tangent distances. The curve may then be run from  $A$ , and the work may be checked by noting whether the curve as run ends at  $B$ —previously located from  $b$ .

*Example.* Assume  $ab=546.82$ ; angle  $a=15^\circ 18'$ ; angle  $b=18^\circ 22'$ ;  $D=3^\circ 40'$ ; required  $aA$  and  $bB$ .

$$A=15^\circ 18' + 18^\circ 22' = 33^\circ 40'$$

|           |                                               |                                    |         |
|-----------|-----------------------------------------------|------------------------------------|---------|
| Eq. (4)   | $R$                                           | $(3^\circ 40')$                    | 3.19392 |
|           | $\tan \frac{1}{2}A$                           | $= \tan 16^\circ 50'$              | 9.48080 |
|           | $T$                                           | $= 472.85$                         | 2.67472 |
| <hr/>     |                                               |                                    |         |
| $aV = ab$ | $\frac{\sin 18^\circ 22'}{\sin 33^\circ 40'}$ | $ab$                               | 2.73784 |
|           |                                               | $\log \sin 18^\circ 22'$           | 9.49844 |
|           |                                               | $\text{co-log } \sin 33^\circ 40'$ | 0.25621 |
|           |                                               | $aV = 310.81$                      | 2.49250 |
|           |                                               | $AV = 472.85$                      |         |
|           |                                               | $aA = 162.04$                      |         |
| <hr/>     |                                               |                                    |         |
| $bV = ab$ | $\frac{\sin 15^\circ 18'}{\sin 33^\circ 40'}$ | $ab$                               | 2.73784 |
|           |                                               | $\log \sin 15^\circ 18'$           | 9.42139 |
|           |                                               | $\text{co-log } \sin 33^\circ 40'$ | 0.25621 |
|           |                                               | $bV = 260.29$                      | 2.41545 |
|           |                                               | $BV = 472.85$                      |         |
|           |                                               | $bB = 212.56$                      |         |

**b.** When the point of curve (or point of tangency) is inaccessible. At some distance ( $As$ , Fig. 18) an unobstructed line  $pn$  may be run parallel with  $AV$ .  $nv = py = As = R$  vers  $a$ .

$$\therefore \text{vers } a = As \div R.$$

$$ns = ps = R \sin a.$$



also frequently used in locating new parallel tracks and modifying old tracks.

a. To move the forward tangent parallel to itself a distance  $x$ , the point of curve ( $A$ ) remaining fixed. (Fig. 20.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x}{\sin \Delta} \cdot \cdot \cdot \cdot \cdot (13)$$

$$AV' = AV + VV'.$$

The triangle  $BmB'$  is isosceles and  $Bm = B'm$ .

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \Delta}$$

$$\therefore R' = R + \frac{x'}{\text{vers } \Delta} \cdot \cdot \cdot \cdot \cdot (14)$$

The solution is very similar in case the tangent is moved inward to  $V''B''$ . Note that this method necessarily changes the

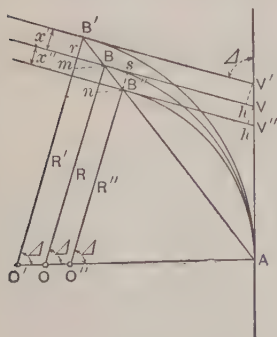


FIG. 20.

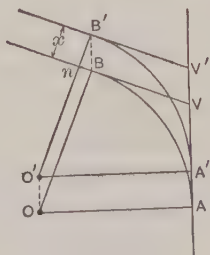


FIG. 21.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance  $x$ , the radius being unchanged. (Fig. 21.) In this case the whole

curve is moved bodily a distance  $OO' = AA' = VV' = BB'$ , and moved parallel to the first tangent  $AV$

$$BB' = \frac{B'n}{\sin nBB'} = \frac{x}{\sin \Delta} = AA'. \quad . . . (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 22.) This problem involves a change ( $\alpha$ ) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

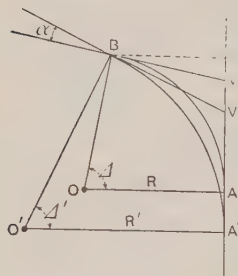


FIG. 22.

$R, \Delta, \alpha, AV$ , and  $BV$  are known.

$$\Delta' = \Delta - \alpha.$$

$$Bs = R \text{ vers } \Delta. \quad Bs = R' \text{ vers } \Delta'.$$

$$\therefore R' = R \frac{\text{vers } \Delta}{\text{vers } (\Delta - \alpha)}. \quad . . . (16)$$

$$As = R \sin \Delta. \quad A's = R' \sin \Delta'.$$

$$\therefore AA' = A's - As = R' \sin \Delta' - R \sin \Delta. \quad . . . (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary problems can be solved by the application of elementary geometry and trigonometry.

34. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point. The point ( $P$ , Fig. 23) is assumed to be determined by its distance ( $VP$ ) from the vertex and by the angle  $AVP = \beta$ .

It is required to determine the radius ( $R$ ) and the tangent distance ( $AV$ ).  $\Delta$  is known.

$$PVG = \frac{1}{2}(180^\circ - \Delta) - \beta.$$

$$= 90^\circ - (\frac{1}{2}\Delta + \beta).$$

$$PP' = 2VP \sin PVG$$

$$= 2VP \cos (\frac{1}{2}\Delta + \beta).$$

$$PSV = \frac{1}{2}\Delta.$$

$$\therefore SP = VP \frac{\sin \beta}{\sin \frac{1}{2}\Delta}.$$

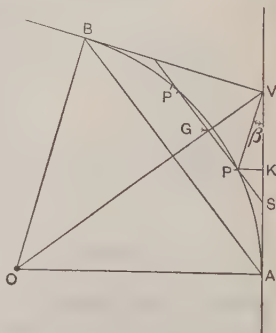


FIG. 23.

$$AS = \sqrt{SP \times SP'} = \sqrt{SP(SP + PP')}$$

$$= \sqrt{VP \frac{\sin \beta}{\sin \frac{1}{2}A} \left[ VP \frac{\sin \beta}{\sin \frac{1}{2}A} + 2VP \cos(\frac{1}{2}A + \beta) \right]}$$

$$= VP \sqrt{\frac{\sin^2 \beta}{\sin^2 \frac{1}{2}A} + \frac{2 \sin \beta \cos(\frac{1}{2}A + \beta)}{\sin \frac{1}{2}A}}.$$

$$SV = VP \frac{\sin(\frac{1}{2}A + \beta)}{\sin \frac{1}{2}A}.$$

$$AV = AS + SV$$

$$= \frac{VP}{\sin \frac{1}{2}A} [\sin(\frac{1}{2}A + \beta) + \sqrt{\sin^2 \beta + 2 \sin \beta \sin \frac{1}{2}A \cos(\frac{1}{2}A + \beta)}]. \quad (18)$$

$$R = AV \cot \frac{1}{2}A.$$

In the special case in which  $P$  is on the median line  $OV$ ,  $\beta = 90^\circ - \frac{1}{2}A$ , and  $(\frac{1}{2}A + \beta) = 90^\circ$ . Eq. 18 then reduces to

$$AV = \frac{VP}{\sin \frac{1}{2}A} (1 + \cos \frac{1}{2}A) = VP \cot \frac{1}{4}A,$$

as might have been immediately derived from Eq. 8.

In case the point  $P$  is given by the offset  $PK$  and by the distance  $VK$ , the triangle  $PKV$  may be readily solved, giving the distance  $VP$  and the angle  $\beta$ , and the remainder of the solution will be as above.

**35. Determination of the curvature of existing track.** (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at  $0^\circ$ . Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate ( $x$ ) between the middle of the string and the head of the rail. Then

$$R = \frac{\text{chord}^2}{8x} \text{ (very nearly).} \quad \dots \quad (19)$$

For, in Fig. 24, since the triangles  $AOE$  and  $ADC$  are similar,



$AO : AE :: AD : DC$  or  $R = \frac{1}{2} \overline{AD}^2 \div x$ . When, as is usual, the arc is very short compared with the radius,  $AD = \frac{1}{2} AB$ , very nearly. Making this substitution we have Eq. 19. With a chord of 50 feet and a  $10^\circ$  curve, the resulting difference in  $x$  is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail.

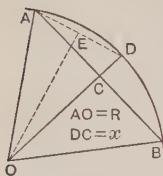


FIG. 24.

It should be diminished by  $\frac{1}{2}g$  for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a  $6^\circ$  curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches.}$$

Another much used rule is to require the foreman to have a string, knotted at the center, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in Eq. 19)  $5730 \div D$  for  $R$  and  $D \div 12$  for  $x$ . Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

**36. Problems.** A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a  $3^\circ$  curve beginning at Sta. 27+60 and running to Sta. 32+45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 17  $ab$  is measured as 217.6 feet, the

angle  $abV = 17^\circ 42'$ , and the angle  $baV = 21^\circ 14'$ . Join the tangents by a  $4^\circ 30'$  curve. Determine  $bB$  and  $aA$ .

*d.* Assume that in a case similar to Fig. 18 it was noted that a distance ( $As$ ) equal to 12 feet would clear the building. Assume that  $J = 38^\circ 20'$  and that  $D = 4^\circ 40'$ . Required the value of  $a$  and the position of  $n$ . *Solution:*

|                      |                                     |                                                |
|----------------------|-------------------------------------|------------------------------------------------|
| vers $a = As \div R$ | $As = 12$                           | log = 1.07918                                  |
|                      | $R$ (for $4^\circ 40'$ curve)       | log = 3.08923                                  |
|                      | <u><math>a = 8^\circ 01'</math></u> | log vers $a = \underline{\underline{7.98994}}$ |
| $ns = R \sin a$      |                                     | log sin $a = 9.14445$                          |
|                      |                                     | log $R = 3.08923$                              |
|                      | <u><math>ns = 171.27</math></u>     | log = <u><u>2.23369</u></u>                    |

*e.* Assume that the forward tangent of a  $3^\circ 20'$  curve having a central angle of  $16^\circ 50'$  must be moved 3.62 feet *inward*, without altering the *P.C.* Required the change in radius.

*f.* Given two tangents making an angle of  $36^\circ 18'$ . It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of  $42^\circ 21'$  with the tangent. Required the radius and tangent distance. *Solution:* Applying Eq. 18, we have

|                                                               |                                               |
|---------------------------------------------------------------|-----------------------------------------------|
| 2                                                             | log = 0.30103                                 |
| $\beta = 42^\circ 21'$                                        | log sin = 9.82844                             |
| $\frac{1}{2}J = 18^\circ 09'$                                 | log sin = 9.49346                             |
| $(\frac{1}{2}J + \beta) = 60^\circ 30'$                       | log cos = <u>9.69234</u>                      |
| .20667                                                        | <u><u>9.31527</u></u>                         |
| log sin <sup>2</sup> $\beta = 9.65688 \dots$                  | .45382                                        |
| 2   9.81987 $\dots$                                           | <u>.66049</u>                                 |
| 9.90993 $\dots$                                               | <u>.81271</u>                                 |
| nat. sin $60^\circ 30' \dots$                                 | <u>.8703</u>                                  |
| 1.6830 $\dots \dots \dots$                                    | log = <u>0.22610</u>                          |
| $VP = 93.2 \dots \dots \dots$                                 | log = <u>1.96941</u>                          |
|                                                               | 2.19551                                       |
|                                                               | log sin $\frac{1}{2}J = \underline{9.49346}$  |
| <u>Tang. dist. <math>AV = 503.36 \dots \dots \dots</math></u> | log = <u>2.70205</u>                          |
|                                                               | log cot $\frac{1}{2}J = \underline{10.48437}$ |
| $R = 1536.1 \dots \dots \dots$                                | log = <u>3.18642</u>                          |
| <u><math>D = 3^\circ 44'</math></u>                           |                                               |

## COMPOUND CURVES.

**37. Nature and use.** Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two simple curves has special properties which are worth investigating and utilizing. In the following demonstrations  $R_2$  always represents the *longer* radius and  $R_1$  the *shorter*, no matter which succeeds the other.  $T_1$  is the tangent adjacent to the curve of shorter radius ( $R_1$ ), and is invariably the shorter tangent.  $\angle_1$  is the central angle of the curve of radius  $R_1$ , but it may be greater or less than  $\angle_2$ .

**38. Mutual relations of the parts of a compound curve having two branches.** In Fig. 25,  $AC$  and  $CB$  are the two branches of

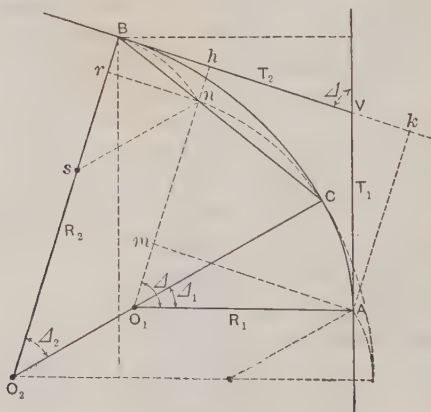


FIG. 25.

the compound curve having radii of  $R_1$  and  $R_2$  and central angles of  $\angle_1$  and  $\angle_2$ . Produce the arc  $AC$  to  $n$  so that  $AO_1n = \angle_1$ . The chord  $Cn$  produced *must* intersect  $B$ . The line  $ns$ , parallel to  $CO_2$ , will intersect  $BO_2$  so that  $Bs = sn = O_2O_1 = R_2 - R_1$ . Draw  $Am$  perpendicular to  $O_1n$ . It will be parallel to  $hk$ .

$$Br = sn \text{ vers } Bsn = (R_2 - R_1) \text{ vers } \Delta_2;$$

$$mn = AO_1 \text{ vers } AO_1n = R_1 \text{ vers } \Delta;$$

$$Ak = AV \sin AVk = T_1 \sin \Delta;$$

$$Ak = hm = mn + nh = mn + Br.$$

$$\therefore T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1. \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed ( $\Delta$  therefore known) and that a curve of given radius  $R_1$  shall start from a given point at a distance  $T_1$  from the vertex, and that the curve shall continue through a given angle  $\Delta_1$ . Required the other parts of the curve. From Eq. 20 we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}.$$

$$\therefore R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)}. \quad (22)$$

$T_2$  may then be obtained from Eq. 21.

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the  $PC$  and  $PT$ ), and the central angle of each curve; required the two radii. Solving Eq. 20 for  $R_1$ , we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \text{ vers } \Delta_2}{\text{vers } \Delta - \text{vers } \Delta_2}.$$

Similarly from Eq. 21 we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2 (\text{vers } \Delta - \text{vers } \Delta_1)}{\text{vers } \Delta_1}.$$

Equating these, reducing, and solving for  $R_2$ , we have

$$R_2 = \frac{T_1 \sin \Delta \text{ vers } \Delta_1 - T_2 \sin \Delta (\text{vers } \Delta - \text{vers } \Delta_2)}{\text{vers } \Delta_2 \text{ vers } \Delta_1 - (\text{vers } \Delta - \text{vers } \Delta_1)(\text{vers } \Delta - \text{vers } \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. 22, since  $R_2$  is always greater than  $R_1$ , the term to be added to  $R_1$  must be essentially positive—i.e.,  $T_1 \sin \Delta$  must be greater than  $R_1 \text{ vers } \Delta$ . This means that  $T_1 > R_1 \frac{\text{vers } \Delta}{\sin \Delta}$ , or that

$T_1 > R_1 \tan \frac{1}{2} \Delta$ , or that  $T_1$  is greater than the corresponding tangent on a simple curve. Similarly it may be shown that  $T_2$  is less than  $R_2 \tan \frac{1}{2} \Delta$  or less than the corresponding tangent on a simple curve. Nevertheless  $T_2$  is always greater than  $T_1$ . In the limiting case when  $R_2 = R_1$ ,  $T_2 = T_1$ , and  $\Delta_2 = \Delta_1$ .

39. **Modifications of location.** Some of these modifications may be solved by the methods used for simple curves. For example:

a. It is desired to move the tangent  $VB$ , Fig. 26, parallel to itself to  $V'B'$ . Run a new curve from the *P.C.C.* which shall reach the new tangent at  $B'$ , where the chord of the old curve

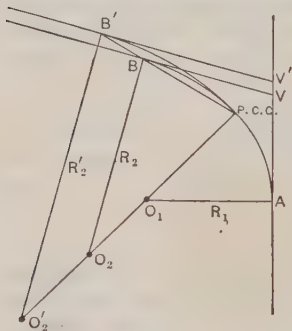


FIG. 26.

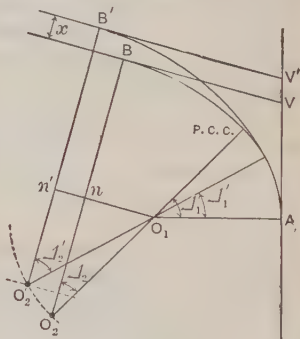


FIG. 27.

intersects the new tangent. The solution is almost identical with that in § 33, a.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 27

$$(R_2 - R_1) \cos \Delta_2 = O_2 n;$$

$$(R_2 - R_1) \cos \Delta_2' = O_2' n';$$

$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1} \cdot \cdot \cdot \cdot (24)$$

The *P.C.C.* is moved backward along the sharper curve an angular distance of  $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$ .

In case the tangent is moved inward rather than outward, the solution will apply by transposing  $\Delta_2$  and  $\Delta_2'$ . Then we shall have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1} \cdot \cdot \cdot \cdot (25)$$

The *P.C.C.* is then moved *forward*.

c. Assume the same case as (b) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 28

$$(R_2 - R_1) \cos \Delta_1 = O_1 n;$$

$$(R_2 - R_1) \cos \Delta_1' = O_1' n'.$$

$$x = O_1' n' - O_1 n$$

$$= (R_2 - R_1)(\cos \Delta_1' - \cos \Delta_1).$$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2 - R_1}. \quad (26)$$

The *P.C.C.* is moved *forward* along the easier curve an angular distance of  $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$ .

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2 - R_1} \dots \dots \dots (27)$$

The *P.C.C.* is moved *backward*.

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 29.

For the diagrammatic solution assume that  $R_2$  is to be increased by  $O_2 S$ . Then, since  $R_2'$  must pass through  $O_1$  and extend beyond  $O_1$  a distance  $O_1 S$ , the locus of the new center must lie on the arc drawn about  $O_1$  as center and with  $OS$  as radius. The locus of  $O_2'$  is also given by a line  $O_2' p$  parallel to  $BV$  and at a distance of  $R_2'$  (equal to  $S \dots P.C.C.$ ) from it. The new center is therefore at the intersection  $O_2'$ . An arc with radius  $R_2'$  will therefore be tangent at  $B'$  and tangent to the old curve produced at *NEW P.C.C.* Draw  $O_1 n'$  perpendicular to  $O_2 B$ .

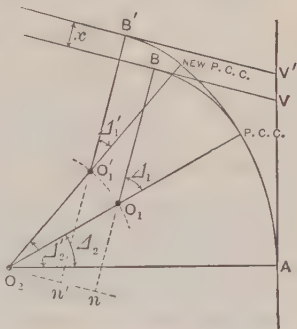


FIG. 28.

FIG. 29.

With  $O_2$  as center draw the arc  $O_1m$ , and with  $O_2'$  as center draw the arc  $O_1m'$ .  $mB = m'B' = R_1$ .

$$\therefore mn = m'n' = (R_2' - R_1) \text{ vers } \Delta_2' = (R_2 - R_1) \text{ vers } \Delta_2.$$

$$\therefore \text{vers } \Delta_2' = \frac{(R_2 - R_1)}{(R_2' - R_1)} \text{ vers } \Delta_2. \quad . \quad . \quad . \quad (28)$$

$$O_1n = (R_2 - R_1) \sin \Delta_2;$$

$$O_1n' = (R_2' - R_1) \sin \Delta_2'.$$

$$BB' = O_1n' - O_1n = (R_2' - R_1) \sin \Delta_2' - (R_2 - R_1) \sin \Delta_2. \quad (29)$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius  $R_2$ , a given change  $BB'$  is to be made.  $\Delta_2'$  and  $R_2'$  are required. Eliminate  $R_2'$  from Eqs. 28 and 29 and solve the resulting equation for  $\Delta_2'$ . Then determine  $R_2'$  by a suitable inversion of either Eq. 28 or 29.

As in §§ 32 and 33, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

**40. Problems.** *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that  $\Delta_1 = 22^\circ 16'$  and  $\Delta_2 = 28^\circ 20'$ . Required the radii.

$$[Ans. \quad R_1 = 326.92; \quad R_2 = 1574.85.]$$

*b.* A line crosses a valley by a compound curve which is first a  $6^\circ$  curve for  $46^\circ 30'$  and then a  $9^\circ 30'$  curve for  $84^\circ 16'$ . It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [*Note.* The second tangent is evidently moved *outward*. The solution corresponds to that in the first part of § 39, *c*. The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of  $BB'$ , Fig. 28, on  $V'B'$ ), it may be found by observing that it is equal to  $mn' = (R_2 - R_1)(\sin \Delta_1 - \sin \Delta_1')$ . In this case it equals 0.65 foot, which is very small because  $\Delta_1$  is nearly  $90^\circ$ . The value of  $l$ , ( $46^\circ 30'$ ) is not used, since the solution is independent of the value of  $\Delta_2$ . The student should learn to recognize



which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

#### TRANSITION CURVES.

**41. Superelevation of the outer rail on curves.** When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals  $Gv^2 \div gR$ , in which  $G$  is the weight,  $v$  the velocity in feet per second,  $g$  the acceleration of gravity in feet per second in a second, and  $R$  the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 30, if  $ob$  represents the reaction,  $oc$  will represent the weight  $G$ , and  $ao$  will represent the required centripetal force. From similar triangles we may write  $sn : sm :: ao : oc$ . Call  $g = 32.17$ . Call  $R = 5730 \div D$ , which is sufficiently accurate for this purpose (see § 19). Call  $v = 5280V \div 3600$ , in which  $V$  is the velocity in miles per hour.  $mn$  is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet.  $sm$  is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is  $4\frac{3}{4}$  inches. Calling  $sn = e$ , we have

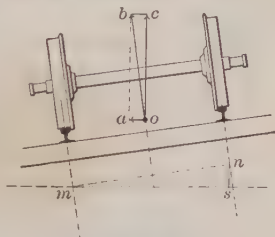


FIG. 30.

$$e = sm \frac{ao}{oc} = 4.9 \frac{Gv^2}{gR} \frac{1}{G} = \frac{4.9 \times 5280^2 V^2 D}{32.17 \times 3600^2 \times 5730}$$

$$e = .0000572 V^2 D. \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt

any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that  $R = 5730 \div D$ . In the extreme case of a  $10^\circ$  curve the error involved would be about 1%. A change of about  $\frac{1}{2}$  of 1% in the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in  $e$  due to the assumed constant value of  $sm$  is never more than a very small fraction of 1%. The rail-laying is not done closer than this. The following tabular form is based on Eq. (30):

SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

| Velocity in<br>Miles per<br>Hour. | Degree of Curve. |     |     |     |     |     |     |     |     |     |
|-----------------------------------|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|                                   | 1°               | 2°  | 3°  | 4°  | 5°  | 6°  | 7°  | 8°  | 9°  | 10° |
| 30                                | .05              | .10 | .15 | .20 | .26 | .31 | .36 | .41 | .46 | .51 |
| 40                                | .09              | .18 | .27 | .37 | .46 | .55 | .64 | .73 | .82 |     |
| 50                                | .14              | .29 | .43 | .57 | .71 | .86 |     |     |     |     |
| 60                                | .20              | .41 | .62 | .82 |     |     |     |     |     |     |

**42. Practical rules for superelevation.** A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that  $e$  in Eq. 30 varies directly as  $D$ . The above rule therefore agrees with Eq. 30 when  $V$  is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now. The rule to elevate one inch for each degree of curvature is also used and is precisely similar in its nature to the above rule. It agrees with Eq. 30 when the velocity is about 38 miles per hour, which is more nearly the average speed of trains.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation

that the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in the tabular form (§ 41) shows the six-inch limitation.

Some roads furnish their track foremen with a list of the super-elevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R. \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Putting  $x$  equal to  $e$  in Eq. 30 and solving for “*chord*,” we have

$$\begin{aligned} chord^2 &= .0000572V^2D\&R \\ &= 2.621V^2. \\ chord &= 1.62V. \quad . \quad . \quad . \quad . \quad . \quad . \quad (32) \end{aligned}$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then  $1.62V = 1.62 \times 50 = 81$  feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the inside head of the outer rail of the outer head of the inner rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

|                               |      |      |      |      |      |      |      |      |      |
|-------------------------------|------|------|------|------|------|------|------|------|------|
| Velocity in miles per hour... | 20   | 25   | 30   | 35   | 40   | 45   | 50   | 55   | 60   |
| Chord length in feet.....     | 32.4 | 40.5 | 48.6 | 56.7 | 64.8 | 72.9 | 81.0 | 89.1 | 97.2 |

The following tabular form shows the standard (at one time) on the N. Y., N. H. & H. R. R. It should be noted that the elevations do not increase proportionately with the radius, and that they are higher for descending grades than for level or

ascending grades. This is on the basis that the velocity on curves and on ascending grades will be less than on descending grades. For example, the superelevation for a  $0^{\circ} 30'$  curve on a descending grade corresponds to a velocity of about 54 miles per hour, while for a  $4^{\circ}$  curve on a level or ascending grade the superelevation corresponds to a velocity of only about 38 miles per hour.

TABLE OF THE SUPERELEVATION OF THE OUTER RAIL ON CURVES.

N. Y., N. H. &amp; H. R. R.

| Degree of curve. | Level or ascending grade. | Descending grade. |
|------------------|---------------------------|-------------------|
|                  | inches.                   | inches.           |
| $0^{\circ} 30'$  | $0\frac{1}{4}$            | 1                 |
| 1 00             | $1\frac{1}{4}$            | $1\frac{1}{2}$    |
| 1 15             | $1\frac{1}{2}$            | 2                 |
| 1 30             | 2                         | $2\frac{1}{4}$    |
| 1 45             | $2\frac{1}{4}$            | $2\frac{1}{2}$    |
| 2 00             | $2\frac{3}{8}$            | $2\frac{3}{4}$    |
| 2 15             | $2\frac{1}{2}$            | 3                 |
| 2 30             | $2\frac{1}{2}$            | $3\frac{1}{4}$    |
| 2 45             | 3                         | $3\frac{3}{8}$    |
| 3 00             | $3\frac{1}{4}$            | $3\frac{1}{2}$    |
| 3 15             | $3\frac{3}{8}$            | $3\frac{7}{8}$    |
| 3 30             | $3\frac{1}{2}$            | 4                 |
| 3 45             | $3\frac{1}{2}$            | $4\frac{1}{8}$    |
| 4 00             | 4                         | $4\frac{1}{2}$    |

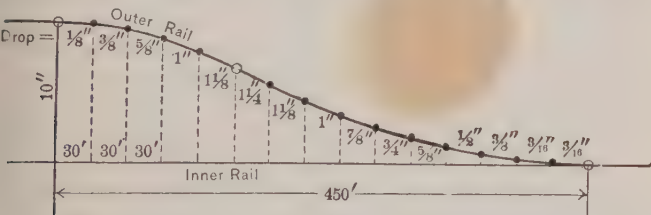
**43. Transition from level to inclined track.** On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradually. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 400 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

On the Lehigh Valley R. R. the run-off is made in the form of a reversed vertical curve, as shown in the accompanying figure. According to this system the length of run-off varies

from 120 feet, for a superelevation of one inch, to 450 feet, for a superelevation of ten inches. Such a superelevation as ten inches is very unusual practice, but is successfully operated on that road. The curve is concave upward for two-thirds of its length and then reverses so that it is convex upward.

TABLE FOR RUN-OFF OF ELEVATION OF OUTER RAIL OF CURVES.  
Drop in inches for each 30-foot rail commencing at theoretical point of curve.

| Eleva-<br>tion. | $\frac{1}{8}$ " | $\frac{1}{4}$ " | $\frac{3}{8}$ " | $\frac{1}{2}$ " | $\frac{5}{8}$ " | $\frac{3}{4}$ " | $\frac{7}{8}$ " | 1" | $1\frac{1}{8}$ " | $1\frac{1}{4}$ " | $1\frac{3}{8}$ " | 1" | $\frac{7}{8}$ " | $\frac{3}{4}$ " | $\frac{5}{8}$ " | $\frac{1}{2}$ " | $\frac{3}{8}$ " | $\frac{1}{4}$ " | $\frac{3}{16}$ " | $\frac{1}{8}$ " | $\frac{1}{16}$ " | Total. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----|------------------|------------------|------------------|----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|-----------------|------------------|--------|
| 1"              |                 | 30              | 30              |                 |                 |                 |                 |    |                  |                  |                  |    |                 |                 |                 |                 |                 | 30              |                  | 30              |                  | 120    |
| 2"              |                 | 30              |                 |                 |                 | 30              |                 |    |                  |                  |                  |    |                 |                 |                 |                 | 30              | 30              |                  | 30              |                  | 150    |
| 3"              |                 | 30              |                 |                 |                 | 30              |                 |    |                  |                  |                  |    | 30              |                 | 30              |                 | 30              |                 |                  | 30              |                  | 180    |
| 4"              |                 | 30              |                 | 30              |                 |                 | 30              |    |                  |                  |                  |    | 30              |                 | 30              | 30              | 30              |                 | 30               |                 | 30               | 240    |
| 5"              |                 | 30              |                 | 30              |                 |                 | 30              | 30 |                  |                  |                  | 30 |                 | 30              | 30              | 30              |                 | 30              |                  | 30              |                  | 270    |
| 6"              |                 | 30              |                 | 30              |                 |                 | 30              |    | 30               |                  |                  | 30 |                 | 30              | 30              | 30              | 30              |                 | 30               |                 | 30               | 300    |
| 7"              |                 | 30              |                 | 30              |                 |                 | 30              |    | 30               | 30               |                  |    | 30              |                 | 30              | 30              | 30              | 30              |                  | 30              |                  | 330    |
| 8"              |                 | 30              |                 | 30              |                 | 30              |                 |    | 30               | 30               | 30               |    |                 | 30              | 30              | 30              |                 | 30              |                  | 30              | 30               | 360    |
| 9"              | 30              |                 |                 | 30              |                 | 30              |                 | 30 | 30               |                  | 30               | 30 | 30              | 30              | 30              | 30              | 30              |                 | 30               |                 | 30               | 420    |
| 10"             | 30              |                 | 30              |                 | 30              |                 |                 | 30 | 30               | 30               |                  | 30 | 30              | 30              | 30              | 30              | 30              |                 | 30               |                 | 30               | 450    |



The figure (and also the lower line of the tabulated form) shows the drop for each thirty-foot rail length. For shorter lengths of run-off, the drop for each 30 feet is shown by the corresponding lines in the tabular form. Note in each horizontal line that the sum of the drops, under which 30 is found, equals the total superelevation as found in the first column. For example, for 4 inches superelevation, length of curve 240 feet, the successive drops are  $\frac{1}{4}$ ",  $\frac{1}{2}$ ",  $\frac{7}{8}$ ",  $\frac{7}{8}$ ",  $\frac{5}{8}$ ",  $\frac{1}{2}$ ",  $\frac{1}{4}$ ", and  $\frac{1}{8}$ " whose sum is 4 inches. Possibly the more convenient form would be to indicate for each 30-foot point the actual superelevation of the outer rail, which would be for the above case (running from the tangent to the curve)  $\frac{1}{8}$ ",  $\frac{3}{8}$ ",  $\frac{7}{8}$ ",  $1\frac{1}{2}$ ",  $2\frac{3}{8}$ ",  $3\frac{1}{4}$ ",  $3\frac{3}{4}$ ", 4".

44. Fundamental principle of transition curves. If a curve

has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30)  $e$  is directly proportional to  $D$ , the required curve must be one in which the degree of curve increases directly as the distance along the curve. The mathematical development of such a curve is quite complicated. It has, however, been developed, and tables have been computed for its use, by Prof. C. L. Crandall. The following method has the advantage of great simplicity, while its agreement with the true transition curve is as close as need be, as will be shown.

**45. Multiform compound curves.** If the transition curve commences with a very flat curve and at regular even chord lengths compounds into a curve of sharper curvature until the desired curvature is reached, the increase in curvature at each chord point being uniform, it is plain that such a curve is a close approximation to the true spiral, especially since the rails as laid will *gradually* change their curvature rather than maintain a uniform curvature throughout each chord length and then abruptly change the curvature at the chord points. Such a curve, *as actually laid*, will be a much closer approximation to the true curve than the multiform compound curve by which it is set out. There will actually be a *gradual* increase in curvature which increases directly as the length of the curve.

**46. Required length of spiral.** The required length of spiral evidently depends on the amount of superelevation to be gained, and also depends somewhat on the speed. If the spiral is laid off in 25-foot chord lengths, with the first chord subtending a  $1^\circ$  curve, the second a  $2^\circ$  curve, etc., the fifth chord will subtend a  $5^\circ$  curve, and the increase from this last chord to a  $6^\circ$  curve is the same as the uniform increase of curvature between the chords. The same spiral extended would run on to a  $12^\circ$  curve in  $(12-1)25=275$  feet. The last chord of a spiral should have a smaller degree of curvature than the simple curve to which it is joined. If the curves are very sharp, such as are used in street work and even in suburban trolley work, an increase in degree of curvature of  $1^\circ$  per 25 feet will not be sufficiently rapid, as

such a rate would require too long curves.  $2^\circ$ ,  $10^\circ$ , or even  $20^\circ$  increase per 25 feet may be necessary, but then the chords should be reduced to 5 feet. Such a rapid rate of increase is justified by the necessary reduction in speed. On the other hand, very high speed will make a lower rate of increase desirable, and therefore a spiral whose degree of curvature increases only  $0^\circ 30'$  per 25 feet may be used. Such a spiral would require a length of 375 feet to run on to an  $8^\circ$  curve, which is inconveniently long, but it might be used to run on to a  $4^\circ$  curve, where its length would be only 175 feet. Three spirals have been developed in Table IV, each with chords of 25 feet, the rate of increase in the degree of curvature being  $0^\circ 30'$ ,  $1^\circ$  and  $2^\circ$  per chord. One of these will be suitable for any curvature found on ordinary steam-railroads.

**47. To find the ordinates of a  $1^\circ$ -per-25-feet spiral.** Since the first chord subtends a  $1^\circ$  curve, its central angle is  $0^\circ 15'$  and the angle  $aQV$  (Fig. 31) is  $7' 30''$ . The tangent at  $a$  makes an angle of  $15'$  with  $VQ$ . The angle between the chord  $ba$  and the tangent at  $a$  is  $\frac{1}{2}(30') = 15'$ , and the angle  $bab'' = \frac{1}{2}(30') + 15' = 30'$ . Similarly

the angle  $abc'' = \frac{1}{2}(45') + 30' + 15' = 67' 30'' = 1^\circ 07' 30''$ ,  
and the angle  $dcd'' = 2^\circ 0'$ .

The ordinate  $aa' = 25 \sin 7' 30''$ , and

$$Qa' = 25 \cos 7' 30''.$$

$$Qb' = Qa' + a'b'$$

$$= Qa' + ab''$$

$$= 25 (\cos 7' 30'' + \cos 30').$$

$$bb' = b'b'' + bb''$$

$$= 25 (\sin 7' 30'' + \sin 30').$$

Similarly the ordinates of  $c$ ,  $d$ , etc., may be obtained.

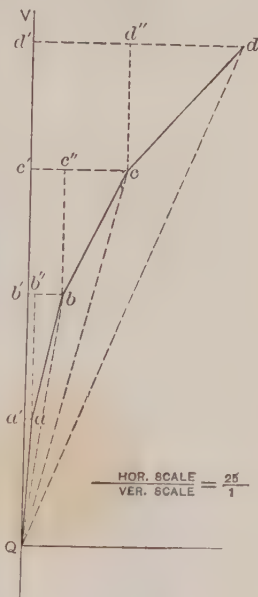


FIG. 31.







by a  $D^\circ$  curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve  $AMB$ . Introducing the spiral has the effect of throwing the curve away from the vertex a distance  $MM'$  and reducing the central angle of the  $D^\circ$  curve by  $2\phi$ . Continuing the curve beyond  $Z$  and  $Z'$  to  $A'$  and  $B'$ , we will have  $AA' = BB' = MM'$ .  $ZK$  = the  $x$  ordinate and is therefore known. Call  $MM' = m$ .  $A'N = x - R \text{ vers } \phi$ . Then

$$m = MM' = AA' = \frac{A'N}{\cos \frac{1}{2}A} = \frac{x - R \text{ vers } \phi}{\cos \frac{1}{2}A}. \quad (33)$$

$$NA = AA' \sin \frac{1}{2}A = (x - R \text{ vers } \phi) \tan \frac{1}{2}A.$$

$$VQ = QK - KN + NA + AV$$

$$= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \frac{1}{2}A + R \tan \frac{1}{2}A$$

$$= y - R \sin \phi + x \tan \frac{1}{2}A + R \cos \phi \tan \frac{1}{2}A. \quad (34)$$

When  $A'N$  has already been computed, it may be more convenient to write

$$VQ = y + R (\tan \frac{1}{2}A - \sin \phi) + A'N \tan \frac{1}{2}A. \quad (35)$$

$$VM' = VM + MM'$$

$$= R \text{ exsec } \frac{1}{2}A + \frac{x}{\cos \frac{1}{2}A} - \frac{R \text{ vers } \phi}{\cos \frac{1}{2}A}. \quad (36)$$

$$AQ = VQ - AV$$

$$= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \frac{1}{2}A. \quad (37)$$

*Example.* To join two tangents making an angle of  $34^\circ 20'$  by a  $5^\circ 40'$  curve and suitable spirals. Use  $1^\circ$ -per-25-feet spirals with five chords. Then  $\phi = 3^\circ 45'$ ,  $x = 2.999$ ,  $\frac{1}{2}A = 17^\circ 10'$ , and  $y = 124.942$ .

[Eq. 33]

|  |                         |               |
|--|-------------------------|---------------|
|  | $R$                     | 3.00497       |
|  | vers $\phi$             | 7.33063       |
|  |                         | <hr/> 0.33560 |
|  | 2.166                   |               |
|  | $x = 2.999$             |               |
|  | $A'N = 0.833$           | 9.92064       |
|  |                         | <hr/> 9.98021 |
|  | $\cos \frac{1}{2}A$     |               |
|  | $m = MM' = AA' = 0.872$ | 9.94043       |

[Eq. 36]

|  |                      |               |
|--|----------------------|---------------|
|  | $R$                  | 3.00497       |
|  | exsec $\frac{1}{2}A$ | 8.66863       |
|  |                      | <hr/> 1.67360 |
|  | $VM = 47.164$        |               |
|  | $m = 0.872$          |               |
|  | $VM' = 48.036$       |               |

[Eq. 35]

|                |                                  |                             |
|----------------|----------------------------------|-----------------------------|
| $y = 124.942$  | nat. tan $\frac{1}{2}A = .30891$ |                             |
|                | nat. sin $\phi = .06540$         |                             |
|                |                                  | <hr/> .24351                |
|                | $R$                              | 9.38651                     |
|                |                                  | <hr/> 3.00497               |
| 246.314        |                                  | 2.39148                     |
|                |                                  | <hr/>                       |
|                | [See above]                      | $A'N$ 9.92064               |
|                |                                  | $\tan \frac{1}{2}A$ 9.48984 |
|                |                                  | <hr/> 9.41048               |
| 0.257          | $AN$                             |                             |
| $VQ = 371.513$ |                                  |                             |

[Eq. 37]

|               |                     |               |
|---------------|---------------------|---------------|
|               | $R$                 | 3.00497       |
|               | $\tan \frac{1}{2}A$ | 9.48984       |
|               |                     | <hr/> 2.49481 |
| 312.471       | $AV$                |               |
| $AQ = 59.042$ |                     |               |

50. **Field-work.** When the spiral is designed during the original location, the tangent distance  $VQ$  should be computed and the point  $Q$  located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be one and perhaps two full station points on the spiral, these should also be located.  $Z$  may be located by setting off  $QK = y$  and  $KZ = x$ , or else by the tabular deflection for  $Z$  from  $Q$  and the distance  $ZQ$ , which is the long chord. Setting up the instrument at  $Z$  and sighting back at  $Q$  with the proper deflection, the tangent at  $Z$  may be found and the circular curve located as usual, its central angle being  $A - 2\phi$ . A similar operation will locate  $Q'$  from  $Z'$ .

To locate points on the spiral. Set up at  $Q$ , with the plates



checking the back points and locating all forward points up to  $Z$  if possible.

After the center curve has been located and  $Z'$  is reached, the other spiral must be located but *in reverse order*, i.e., the sharp curvature of the spiral is at  $Z'$  and the curvature decreases toward  $Q'$ .

**51. To replace a simple curve by a curve with spirals.** This may be done by the method of § 49, but it involves shifting the whole track a distance  $m$ , which in the given example equals 0.87 foot. Besides this the track is appreciably shortened, which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 1% to 5% will answer the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 34.

$$O'N = R' \cos \phi + x.$$

$$O'V = O'N \sec \frac{1}{2}A$$

$$= R' \cos \phi \sec \frac{1}{2}A + x \sec \frac{1}{2}A.$$

$$m = MM' = MV - M'V$$

$$= R \operatorname{exsec} \frac{1}{2}A - (O'V - R')$$

$$= R \operatorname{exsec} \frac{1}{2}A - R' \cos \phi \sec \frac{1}{2}A - x \sec \frac{1}{2}A + R'. \quad (38)$$

$$AQ = QK - KN + NV - VA$$

$$= y - R' \sin \phi + (R' \cos \phi + x) \tan \frac{1}{2}A - R \tan \frac{1}{2}A$$

$$= y - R' \sin \phi + R' \cos \phi \tan \frac{1}{2}A - (R - x) \tan \frac{1}{2}A. \quad (39)$$

The length of the old curve from  $Q$  to  $Q' = 2AQ + 100 \frac{A}{D}$ .

The length of the new curve from  $Q$  to  $Q' = 2L + 100 \frac{A - 2\phi}{D'}$ ,

in which  $L$  is the length of each spiral.

**Example.** Suppose the old curve is a  $7^\circ 30'$  curve with a central angle of  $38^\circ 40'$ . As a trial, compute the relative length of a new  $8^\circ$  curve with spirals of seven chords.  $\phi = 7^\circ 0'$ ;  $\frac{1}{2}A = 19^\circ 20'$ ;  $R$  (for the  $7^\circ 30'$  curve) = 764.489;  $R'$  (for the  $8^\circ$  curve) = 716.779;  $x = 7.628$ .

[Eq. 38]

|        |         |                                    |         |
|--------|---------|------------------------------------|---------|
|        |         | $R$                                | 2.88337 |
|        |         | $\text{exsec } \frac{1}{2} \Delta$ | 8.77642 |
|        | 45.687  |                                    | 1.65979 |
| $R' =$ | 716.779 |                                    |         |
|        | 762.466 | $R'$                               | 2.85538 |
|        |         | $\cos \phi$                        | 9.99675 |
|        |         | $\sec \frac{1}{2} \Delta$          | 0.02521 |
|        | 753.953 |                                    | 2.87734 |
|        |         | $x$                                | 0.88241 |
|        |         | $\sec \frac{1}{2} \Delta$          | 0.02521 |
|        | 8.084   |                                    | 0.90762 |
|        | 762.037 |                                    |         |
| $m =$  | 0.429   |                                    |         |
| $y =$  | 174.722 |                                    |         |

[Eq. 39]

|        |         |                           |         |
|--------|---------|---------------------------|---------|
|        |         | $R'$                      | 2.85538 |
|        |         | $\sin \phi$               | 9.08589 |
|        | 87.353  |                           | 1.94128 |
|        |         | $R'$                      | 2.85538 |
|        |         | $\cos \phi$               | 9.99675 |
|        |         | $\tan \frac{1}{2} \Delta$ | 9.54512 |
|        | 249.606 |                           | 2.39725 |
|        |         | $R = 764.489$             |         |
|        |         | $x = 7.628$               |         |
|        |         | 756.861                   | 2.87901 |
|        |         | $\tan \frac{1}{2} \Delta$ | 9.54512 |
|        | 265.543 |                           | 2.42413 |
|        | 352.896 |                           |         |
|        | 424.328 |                           |         |
|        | 352.896 |                           |         |
| $AQ =$ | 71.432  |                           |         |

The length of the old curve from  $Q$  to  $Q'$  is

$$100 \frac{\Delta}{D} = 100 \frac{38.667}{7.5} = 515.556$$

$$2AQ = 2 \times 71.432 = 142.864$$

$$658.420$$

New curve:  $100 \frac{\Delta - 2\phi}{D'} = 100 \frac{38.667 - 14.000}{8.0} = 308.333$

$$2L = 2 \times 175 = 350.000$$

$$658.333$$

$$\text{Difference in length} = 658.333 - 658.420 = 0.087$$

Considering that this difference may be divided among 22 joints (using 30-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius  $R'$  will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

**52. Application of transition curves to compound curves.** Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 38 and 39) regardless of the



transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is

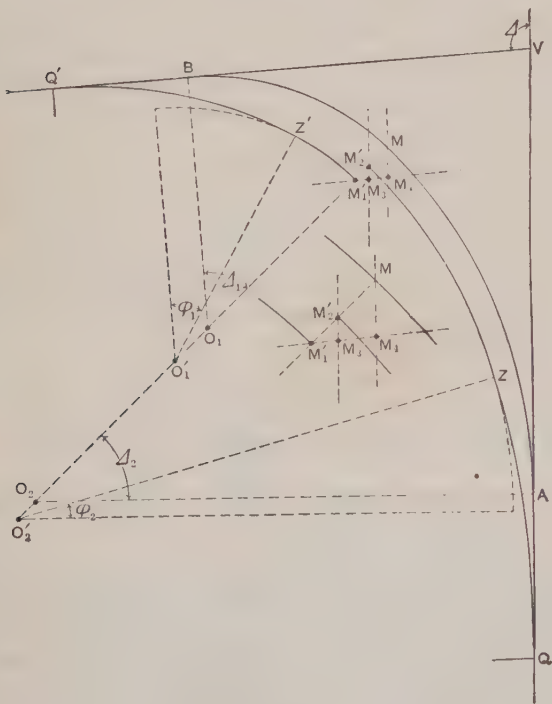


FIG. 35.

complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than  $3^\circ$  or  $4^\circ$ , there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 49, calling  $\Delta_1 = \frac{1}{2}\Delta$ , we may compute  $m_1 = MM_1'$ . Similarly, calling  $\Delta_2 = \frac{1}{2}\Delta$ , we may compute  $m_2 = MM_2'$ . But  $M_1'$  and  $M_2'$  must be made to coincide. This may be done by moving the curve  $Z'M_1'$  and its transition curve parallel to  $Q'V$  a distance  $M_1'M_3$ , and the other curve parallel to  $QV$  a distance  $M_2'M_3$ . In the triangle  $M_1'M_3M_2'$ , the angle at  $M_1' = 90^\circ - \Delta_1$ , the angle at  $M_2' = 90^\circ - \Delta_2$ , and the angle at  $M_3 = \Delta$ .

$$\left. \begin{aligned} \text{Then } M_1'M_3 &= M_1'M_2' \frac{\sin(90^\circ - \Delta_2)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_2}{\sin \Delta} \\ \text{Similarly } M_2'M_3 &= M_1'M_2' \frac{\sin(90^\circ - \Delta_1)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_1}{\sin \Delta} \end{aligned} \right\} \quad (40)$$

b. *With a transition curve on the sharper curve only.* Compute  $m_1 = MM_1'$  as before; then move the curve  $Z_1M_1'$  parallel to  $Q'V$  a distance of

$$M_1'M_4 = m_1 \frac{\cos \Delta_2}{\sin \Delta} \cdot \cdot \cdot \cdot \cdot \quad (41)$$

The simple curve  $MA$  is moved parallel to  $VA$  a distance of

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta} \cdot \cdot \cdot \cdot \cdot \quad (42)$$

If  $\Delta_1$  and  $\Delta_2$  are both small,  $M_1'M_4$  and  $MM_4$  may be more than  $m_1$ , but the lateral deviation of the new curve from the old will always be less than  $m_1$ .

53. To replace a compound curve by a curve with spirals. The solution is somewhat analogous to that of § 51. Compute  $m_1$  for the sharper branch of the curve, placing  $\Delta_1 = \frac{1}{2}\Delta$  in Eq. 38. Since  $m_1$  and  $m_2$  for the two branches of the curve must be identical, a value for  $R_2'$  must be found which will satisfy the determined value of  $m_2 = m_1$ . Solving Eq. 38 for  $R'$ , we obtain

$$R' = \frac{R \text{ vers } \frac{1}{2}\Delta - m \cos \frac{1}{2}\Delta - x}{\cos \phi - \cos \frac{1}{2}\Delta} \cdot \cdot \cdot \cdot \quad (43)$$

Substituting in this equation the known value of  $m_1 (=m_2)$  and calling  $R'=R_2'$ ,  $R=R_2$ , and  $J_2=\frac{1}{2}J$ , solve for  $R_2'$ . Obtain the value of  $AQ$  for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

*Example.* Assume a compound curve with  $D_1=8^\circ$ ,  $D_2=4^\circ$ ,  $A_1=36^\circ$ , and  $J_2=32^\circ$ . Use 1°-per-25-feet spirals;  $\phi_1=7^\circ 0'$ ;  $\phi_2=1^\circ 30'$ . Assume that the sharper curve is sharpened from  $8^\circ 0'$  to  $8^\circ 12'$ .

[Eq. 38]

|                   |                  |          |
|-------------------|------------------|----------|
|                   | $R_1$            | 2.85538̄ |
|                   | exsec $36^\circ$ | 9.37303̄ |
| 169.209 . . . . . |                  | 2.22842̄ |
| $R_1'=699.326$    |                  |          |
| 868.535           | $R_1'$           | 2.84468  |
|                   | cos $\phi_1$     | 9.99675  |
|                   | sec $A_1$        | 0.09204  |
| 857.970 . . . . . |                  | 2.93347̄ |
|                   |                  | 0.88241  |
|                   | $x_1$            | 0.09204  |
|                   | sec $A_1$        | 0.97445̄ |
|                   | 9.429 . . . . .  |          |
| 867.399           | 867.399          |          |
| $m_1=1.136$       |                  |          |

[Eq. 43]

|                   |                 |          |
|-------------------|-----------------|----------|
|                   | $R_2$           | 3.15615̄ |
|                   | vers $32^\circ$ | 9.18170̄ |
| 217.700 . . . . . |                 | 2.33785̄ |
|                   | $m_1=1.136$     | 0.05538  |
|                   | cos $32^\circ$  | 9.92842  |
|                   |                 | 9.98380  |
| 0.963 . . . . .   |                 |          |
| $x_2=0.763$       |                 |          |
| 1.726             | 1.726           |          |
| 215.974 . . . . . |                 | 2.33440̄ |

[Eq. 39]

|                   |                     |                                         |          |
|-------------------|---------------------|-----------------------------------------|----------|
|                   | nat. cos $\phi$     | = .99966                                |          |
|                   | nat. cos $A_2$      | = .84805̄                               |          |
|                   |                     | .15161̄                                 | 9.18073̄ |
| $R_2'=1424.54$    | $[4^\circ 1' 22'']$ |                                         | 3.15367̄ |
| $y_1=174.722$     |                     | $R_1'$                                  | 2.84468  |
|                   |                     | sin $\phi_1$                            | 9.08589̄ |
| 85.226 . . . . .  |                     |                                         | 1.93057̄ |
|                   |                     | $R_1'$                                  | 2.84468  |
|                   |                     | cos $\phi_1$                            | 9.99675  |
|                   |                     | $\tan \frac{1}{2} A$ [ $A_1=36^\circ$ ] | 9.86126̄ |
| 504.302 . . . . . |                     |                                         | 2.70269̄ |
|                   |                     | $R_1=716.779$                           |          |
|                   |                     | $x_1=7.628$                             |          |
|                   |                     | 709.151̄                                | 2.85074̄ |
|                   |                     | $\tan \frac{1}{2} A$                    | 9.86126̄ |
| 679.024           |                     |                                         |          |
| 600.461           | 515.235 . . . . .   |                                         | 2.71200̄ |
| $AQ_1=78.563$     | 600.461             |                                         |          |

[Eq. 39]

|                   |                                                 |         |
|-------------------|-------------------------------------------------|---------|
| $y_2 = 74.994$    | $R_2'$                                          | 3.15367 |
|                   | $\sin \phi_2$                                   | 8.41792 |
| 37.290 . . . . .  |                                                 | 1.57159 |
|                   | $R_2'$                                          | 3.15367 |
|                   | $\cos \phi_2$                                   | 9.99985 |
|                   | $\tan \frac{1}{2} \Delta (\Delta_2 = 32^\circ)$ | 9.79579 |
| 889.843 . . . . . |                                                 | 2.94931 |
|                   | $R_2 = 1432.69$                                 |         |
|                   | $x_2 = 0.76$                                    |         |
|                   | 1431.93                                         | 3.15592 |
|                   | $\tan \frac{1}{2} \Delta$                       | 9.79579 |
| 894.770 . . . . . |                                                 | 2.95171 |
| 932.060           |                                                 |         |
| 964.837           |                                                 |         |
| $AQ_2 = 32.777$   |                                                 |         |

For the length of the old track we have :

$$100 \frac{D_1}{D_1} = 100 \frac{36^\circ}{8^\circ} = 450.$$

$$100 \frac{D_2}{D_2} = 100 \frac{32^\circ}{4^\circ} = 800.$$

$$AQ_1 = 78.563$$

$$AQ_2 = 32.777$$

$$1361.340$$

For the length of the new track we have:

$$100 \frac{D_1 - \phi_1}{D_1'} = 100 \frac{29^\circ}{8^\circ.20} = 353.659$$

$$100 \frac{D_2 - \phi_2}{D_2'} = 100 \frac{30^\circ.5}{4^\circ.023} = 758.140$$

$$\text{Spiral on } 8^\circ 12' \text{ curve } 175.000$$

$$\text{“ “ } 4^\circ 01' 22'' \text{ “ “ } 75.$$

$$\text{Length of new track } = 1361.799$$

$$\text{“ “ old “ “ } = 1361.340$$

$$\text{Excess in length of new track } = 0.459 \text{ feet.}$$

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset  $m$  is only 1.136. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from  $Z'$  and  $Z$ , is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to  $m$  (1.136), the above figures should stand. Otherwise  $m$  may be diminished (and the above excess in length of track diminished) by *increasing*  $R_1'$  very slightly and making the necessary consequent changes.

## VERTICAL CURVES.

**54. Necessity for their use.** Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock. The necessity for vertical curves was even greater in the days when link couplers were in universal use and the "slack" in a long train was very great. Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in broken couplers or even derailments. A vertical curve will practically eliminate this danger if the curve is made long enough, but the rapidly increasing adoption of close spring couplers and air-brakes, even for freight trains, is obviating the necessity for such very long curves.

**55. Required length.** Theoretically the length should depend on the change in the rate of grade and on the length of the longest train on the road. A sharp change in the rate of grade requires a long curve; a long train requires a long curve; but since the longest trains are found on roads with light grades and small changes of grade, the required length is thus somewhat equalized. It has been claimed that a total curve length equal to one-third of the train length for each *tenth* of a per cent of change of rate of grade will *certainly* prevent the rear of the train from crowding against the cars in front, but such a length is admittedly excessive. Half of this length is probably ample and one-fourth of it is probably safe. Therefore, we may say, taking the even fraction  $\frac{1}{10}$  rather than  $\frac{1}{12}$ ,

$$\text{length of vertical curve} = (\text{length of longest train}) \times (\text{change of rate of grade in per cent}).$$

For example, assume a change of rate of grade of 2%; assume that the longest train will be about 720 feet. Then, by the



measurement. Since the angle  $CAB$  is one-half the total change of grade at  $B$ , it follows that  $Be = \frac{1}{2}l \times \frac{1}{2}r$ . Therefore

$$Bh = \frac{1}{8}lr. \quad (44a)$$

Since  $Bh$  (or  $eh$ ) are constant for any one curve, the correction  $sn$  at any point (see Eq. 44) equals a constant times  $Am^2$ .

**57. Numerical example.** Assume that  $B$  is located at Sta. 16+20; that the curve is to be 1200 feet long; that the grade of  $AB$  is  $-0.8\%$ , and of  $BC$   $+1.2\%$ ; also that the elevation of  $B$  above the datum plane is 162.6. Then the algebraic difference of the grades,  $r = 1.2 - (-0.8) = 2.0$ ;  $l = 12$ .  $Bh = \frac{1}{8}lr = \frac{1}{8} \times 12 \times 2 = 3.0$ .  $A$  is at Sta. 10+20 and its elevation is  $162.6 + (6 \times 0.8) = 167.4$ ;  $C$  is at Sta. 22+20 and its elevation is  $162.6 + (6 \times 1.2) = 169.8$ . The elevation of Sta. 11 is found by adding  $sn$  to the elevation of  $s$  on the straight grade line. The constant  $(eh \div \overline{Ae}^2)$  equals in this case  $3.0 \div 600^2 = \frac{1}{120000}$ . Therefore the curve elevations are

|                |                                                        |          |
|----------------|--------------------------------------------------------|----------|
| A, Sta. 10+20, | 162.6 + (6.00 $\times$ 0.8)                            | = 167.40 |
| 11             | 167.4 - (.80 $\times$ 0.8) + $\frac{1}{120000} 80^2$   | = 166.81 |
| 12             | 167.4 - (1.80 $\times$ 0.8) + $\frac{1}{120000} 180^2$ | = 166.23 |
| 13             | 167.4 - (2.80 $\times$ 0.8) + $\frac{1}{120000} 280^2$ | = 165.81 |
| 14             | 167.4 - (3.80 $\times$ 0.8) + $\frac{1}{120000} 380^2$ | = 165.56 |
| 15             | 167.4 - (4.80 $\times$ 0.8) + $\frac{1}{120000} 480^2$ | = 165.48 |
| 16             | 167.4 - (5.80 $\times$ 0.8) + $\frac{1}{120000} 580^2$ | = 165.56 |
| B, 16+20,      | 162.6 + 3.0                                            | = 165.60 |
| 17             | 169.8 - (5.20 $\times$ 1.2) + $\frac{1}{120000} 520^2$ | = 165.81 |
| 18             | 169.8 - (4.20 $\times$ 1.2) + $\frac{1}{120000} 420^2$ | = 166.23 |
| 19             | 169.8 - (3.20 $\times$ 1.2) + $\frac{1}{120000} 320^2$ | = 166.81 |
| 20             | 169.8 - (2.20 $\times$ 1.2) + $\frac{1}{120000} 220^2$ | = 167.56 |
| 21             | 169.8 - (1.20 $\times$ 1.2) + $\frac{1}{120000} 120^2$ | = 168.48 |
| 22             | 169.8 - (.20 $\times$ 1.2) + $\frac{1}{120000} 20^2$   | = 169.56 |
| C, 22+20,      | 162.6 + (6.00 $\times$ 1.2)                            | = 169.80 |

#### DEMONSTRATION OF EQ. 44.

The general equation of a parabola passing through the point  $n$  (Fig. 36) may be written

$$y^2 + y_n^2 = 2p(x + x_n),$$

from which

$$x_n = \frac{y^2}{2p} + \frac{y_n^2}{2p} - x.$$

When  $x = x_A$ ,  $y = y_A$ , and we have

$$x_n = \frac{y_A^2}{2p} + \frac{y_n^2}{2p} - x_A.$$



The general equation of a tangent passing through the point  $A$  may be written

$$yy_A = p(x + x_A),$$

from which

$$x = \frac{yy_A}{p} - x_A.$$

When  $x = x_s$ ,  $y = y_s [= y_n]$ , and we have

$$x_s = \frac{y_n y_A}{p} - x_A.$$

$$\overline{sn} = x_n - x_s = \frac{y_A^2 + y_n^2 - 2y_n y_A}{2p}.$$

$$= \frac{(y_A - y_n)^2}{2p} = \frac{\overline{Am}^2}{2p},$$

$$2p = \frac{y_A^2}{x_A} = \frac{\overline{Ae}^2}{eh}.$$

$$\therefore \overline{sn} = eh \frac{\overline{Am}^2}{\overline{Ae}^2}.$$

This proves the general proposition that if secants are drawn parallel to the axis of  $x$ , intersecting a parabola and a tangent to it, the intercepts between the tangent and the parabola are proportional to the square of the distances (measured parallel to  $y$ ) from the tangent point.

## CHAPTER III.

### EARTHWORK.

#### FORM OF EXCAVATIONS AND EMBANKMENTS.

58. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 37, in which  $e \dots g$  represents the natural surface of the ground, no matter

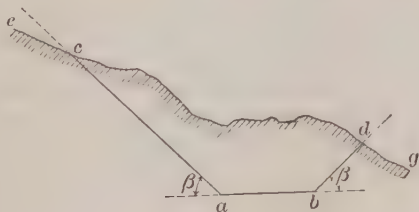


FIG. 37.

how irregular;  $ab$  represents the position and width of the required roadbed;  $ac$  and  $bd$  represent the "side slopes" which begin at  $a$  and  $b$  and which intersect the natural surface at such

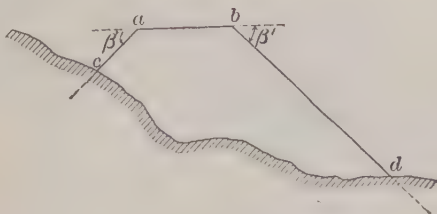


FIG. 38.

points ( $c$  and  $d$ ) as will be determined by the required slope angle ( $\beta$ ).

The normal section in fill is as shown in Fig. 38. The points  $c$  and  $d$  are likewise determined by the intersection of the re-

quired side slopes with the natural surface. In case the required roadbed ( $ab$  in Fig. 39) intersects the natural surface, both cut

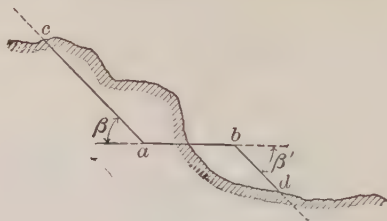


FIG. 39.

and fill are required, and the points  $c$  and  $d$  are determined as before. Note that  $\beta$  and  $\beta'$  are not necessarily equal. Their proper values will be discussed later.

59. Terminal pyramids and wedges. Fig. 40 illustrates the general form of cross-sections when there is a transition from cut to fill.  $a \dots g$  represents the grade line of the road which

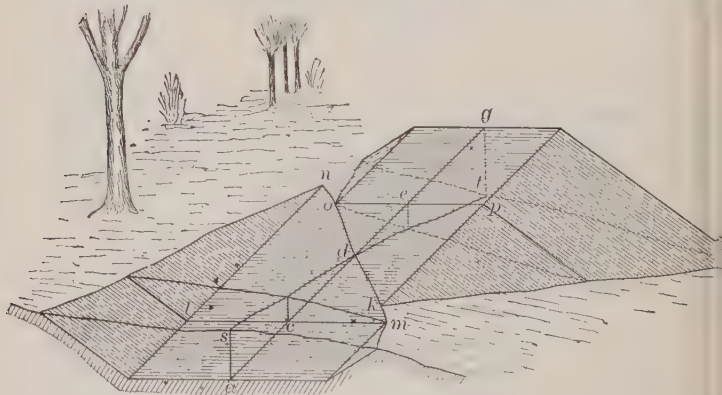


FIG. 40.

passes from cut to fill at  $d$ .  $sdt$  represents the surface profile. A cross-section taken at the point where either side of the roadbed first cuts the surface (the point  $m$  in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at  $o$ , where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates

in two pyramids. In Fig. 40 the pyramid vertices are at  $n$  and  $k$ , and the bases are  $lhm$  and  $opq$ . The roadbed is generally wider in cut than in fill, and therefore the section  $lhm$  and the altitude  $ln$  are generally greater than the section  $opq$  and the altitude  $pk$ . When the line of intersection of the roadbed and natural surface ( $nodkm$ ) becomes perpendicular to the axis of the roadbed ( $ag$ ) the pyramids become wedges whose bases are the nearest convenient cross-sections.

**60. Slopes. a. Cuttings.** The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps 4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by saturation. A slope of  $1\frac{1}{2}$  horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

**b. Embankments.** The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of  $1\frac{1}{2}$  to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite  $1\frac{1}{2}$  : 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

**61. Compound sections.** When the cut consists partly of earth and partly of rock, a compound cross-section must be made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock "runs out"—a difficult matter when it must be deter-

mined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A "berm" of about three feet is usually left on the edges of the rock cut as

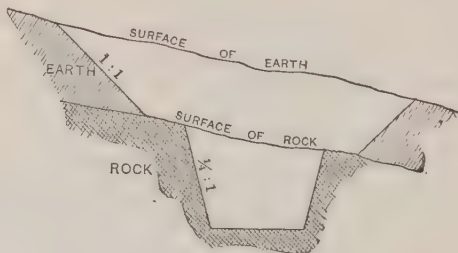


FIG. 41.

a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 89).

**62. Width of roadbed.** Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

It may be noted from the table that the average width for an *earthwork* cut, single track, is about 21.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

| Road.                               | Single Track.                            |                  | Double Track.      |              | Slope Ratios.      |         | Distance Between Track Centers. |
|-------------------------------------|------------------------------------------|------------------|--------------------|--------------|--------------------|---------|---------------------------------|
|                                     | Cut.                                     | Fill.            | Cut.               | Fill.        | Cut.               | Fill.   |                                 |
| A., T. & Santa Fé.....              | { 28' earth<br>22' rock                  | 20               | .....              | .....        | 1 : 1              | 1.5 : 1 |                                 |
| Chicago, Burlington & Quincy.....   | 14 + (2 × 5)*                            | 16               | 28 + (2 × 5)       | 30           | 1.5 : 1            | 1.5 : 1 | 14'                             |
| Chicago, Milwaukee & St. Paul.....  | 18 + (2 × 6)                             | 20 to 24         | 31 + (2 × 6)       | 33 to 37     | 1.5 : 1            | 1.5 : 1 | 13'                             |
| C., C., & St. Louis.....            | 20 + (2 × 4)                             | 20               | 33 + (2 × 4)       | 33           | 1.5 : 1            | 1.5 : 1 | 13'                             |
| Illinois Central.....               | 32' 5"                                   | 18               | .....              | .....        | 1.5 : 1            | 1.5 : 1 |                                 |
| Erie.....                           | 20' 8 1/4"                               | 20' 8 1/4"       | 33' 8 1/4"         | 33' 8 1/4"   | 1.5 : 1            | 1.5 : 1 | 13'                             |
| Lehigh Valley.....                  | 14 + (2 × 3.5)                           | 16               | 27 + (2 × 3.5)     | 30           | 1 : 1              | 1.5 : 1 | 13'                             |
| Lake Shore & Michigan Southern..... | .....                                    | .....            | 33 + (2 × 7.25)    | 32           | 1.5 : 1            | 1.5 : 1 | 13'                             |
| Louisville & Nashville.....         | 13 + (2 × 4.5)                           | 16               | .....              | .....        | 1 : 1              | 1.5 : 1 |                                 |
| Michigan Central.....               | .....                                    | .....            | (33 + 2 × 2.5)     | 33           | 1.5 : 1            | 1.5 : 1 | 13'                             |
| N. Y., N. H. & H.....               | { 21' 2" earth<br>16' rock               | 17' 2"           | 30<br>34' 2" earth | 30<br>30' 2" | 1.5 : 1<br>1.5 : 1 | 1.5 : 1 | 13'                             |
| Norfolk & Western.....              | .....                                    | .....            | 29' rock           | .....        | 1.5 : 1            | .....   | 13'                             |
| Pennsylvania.....                   | { 19' 2" light traffic<br>27' 2" heavy " | 19' 2"<br>19' 2" | 31' 4" + (2 × 4)   | 31' 4"       | 1.5 : 1            | 1.5 : 1 | 12' 2"                          |
| Union Pacific.....                  | 14 + (2 × 3.5)                           | 16               | .....              | .....        | 1 : 1              | 1.5 : 1 |                                 |

\* (2 × 5) signifies two ditches each 5 feet wide; the following cases should be interpreted similarly.

**63. Form of subgrade.** The stability of the roadbed depends largely on preventing the ballast and subsoil from becoming saturated with water. The ballast must be porous so that it will not retain water, and the subsoil must be so constructed that it will readily drain off the rain-water that soaks through the ballast. This is accomplished by giving the subsoil a curved form, convex upward, or a surface made up of two or three planes, the two outer planes having a slope of about 1:24 (sometimes more and sometimes less, depending on the soil) and the middle plane, if three are used, being level. When a circular form is used, a crowning of 6 inches in a total width of 17 or 18 feet is generally used. Occasionally the subgrade is made level, especially in rock-cuts, but if the subsoil is previously compressed by rolling, as required on the N. Y. C. & H. R. R. R., or if the subsoil is drained by tile drains laid underneath the ditches, the necessity for slopes is not so great. Rock cuts are generally required to be excavated to one foot below subgrade and then filled up again to subgrade with the same material, if it is suitable.

**64. Ditches.** "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12" to 24" wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low. The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 42.) A ditch, with a flat bottom and such



FIG. 42.

slopes as the soil requires, which approximates to the circular form will therefore be the best.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2' under the ditches, are prescribed on some roads. (See Fig. 43.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

**65. Effect of sodding the slopes, etc.** Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 43.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 43 is a copy of



designs \* presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul

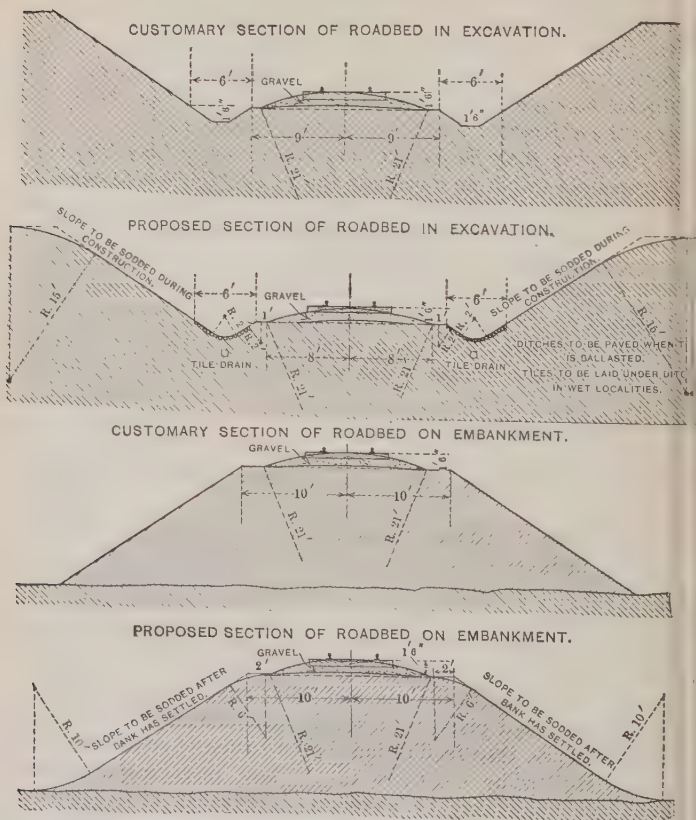


FIG. 43.—“WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS”  
Trans. Am. Soc. C. E., Sept. 1894.

R. R. The “customary sections” represent what is, with some variations of detail, the practice of many railroads. The “pro-

\* Trans. Am. Soc. Civil Eng., Sept. 1894.

posed sections" elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

#### EARTHWORK SURVEYS.

**66. Relation of actual volume to the numerical result.** It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 94.

**67. Prismoids.** To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases. These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 70 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The "two plane ends" are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of

the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (a) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (b) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

**68. Cross-sectioning.** The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the road-

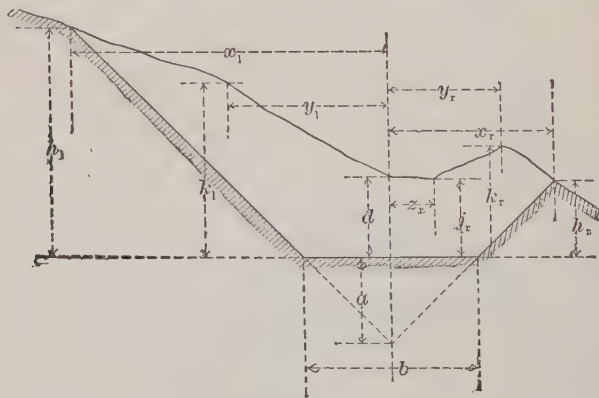


FIG. 44.

bed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are acceptable this line is assumed to be straight. According to the irreg-

ularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance ( $d$  in Fig. 44) of the roadbed below (or above) the natural surface at the center is known or determined from the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to  $d$  gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed ( $h_l, k_l, h_r$ , etc.). This is true for all cases in excavation. For fill, the rod reading at center minus  $d$  equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be sufficiently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 82.

**69. Position of slope-stakes.** The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill ( $d$ ). The distance of the slope-stake from the center for the lower side is  $x = \frac{1}{2}b + s(d + y)$ ; for the up-hill side it is  $x' = \frac{1}{2}b + s(d - y')$ .  $s$  is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both  $x$  and  $y$  are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of  $x$  for the point  $a = \frac{1}{2}b + sd$ , which is the value of  $x$  for *level* cross-sections. In the case of fills on sloping ground the value of  $x$  on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is  $s$  times the difference of elevation. Take a

numerical case corresponding with Fig. 45. The rod reading on  $c$  is 2.9;  $d=4.2$ ; therefore the telescope is  $4.2-2.9=1.3$  below grade.  $s=1.5:1$ ,  $b=16$ . Hence for the point  $a$  (or for level ground)  $x=\frac{1}{2}\times 16+1.5\times 4.2=14.3$ . At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require  $1.5\times 3=4.5$  more, but enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained.  $8.3+1.3=9.6$ , the depth of the point below grade. The point on the slope line ( $n$ ) which has this depth below grade is at a distance from the center

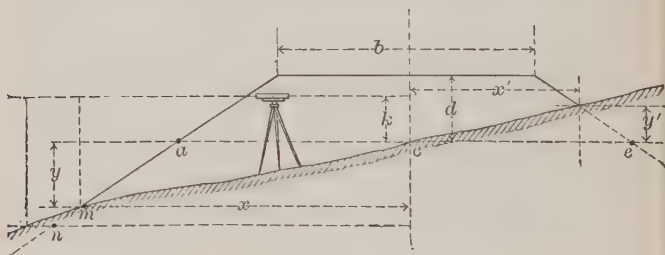


FIG. 45.

$x=8+1.5\times 9.6=22.4$ . The point on the surface ( $s$ ) having that depth is 24 feet out. Therefore the true point ( $m$ ) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by  $C$  or  $F$ . Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance

out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read up the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

**69a. Setting slope-stakes by means of "automatic" slope-stake rods.** The equipment consists of a specially graduated tape and a specially constructed rod. The tape may readily be prepared by marking on the back side of an ordinary 50-foot tape which is graduated to feet and tenths. Mark "0" at " $\frac{1}{2}b$ " from the tape-ring. The same tape may be used for several values of " $\frac{1}{2}b$ " by placing the zero at the maximum distance  $\frac{1}{2}b$  from the ring. Then graduate from the zero backward, at true scale, to the ring. When  $\frac{1}{2}b$  is less than this maximum, the tape will not be used clear to the ring. In general, the tape must be so held that the zero is always  $\frac{1}{2}b$  from the center stake. Mark off "feet" and "tenths" on a scale proportionate to the slope ratio. For example, with the usual slope ratio of 1.5 : 1 each "foot" would measure 18 inches and each "tenth" in proportion.

The rod, 10 feet long, is shod at each end and has an endless tape passing within the shoes at each end and over pulleys to reduce friction. The tape should be graduated in feet and tenths, from 0 to 20 feet—the 0 and 20 coinciding. By moving the tape so that 0 is at the bottom of the rod—or (practically) so that the 1-foot mark on the tape is one foot above the bottom of the shoe, an index mark may be placed on the back of the rod (say at 15—on the tape) and this readily indicates when the tape is "set at zero."

The method of use may best be explained from the figure and from the explicit rules as stated. The proof is given for two assumed positions of the level.

(1) Set up the level so that it is higher than the "center" and (if possible) higher than both slope-stakes, but not more than a rod-length higher. On very steep ground this may be impossible and each slope-stake must be set by separate positions of the level.



(2) Set the rod-tape at zero (i.e., so that the 15-foot mark on the *back* is at the index mark).

(3) Hold the rod at the center-stake ( $B$ ) and note the reading ( $n_1$  or  $n_2$ ). Consider  $n$  to be always plus; consider  $d$  to be plus for cut and minus for fill.

(4) Raise the tape on the *face* side of the rod ( $n+d$ ). Applied literally (and algebraically), when the level is *below* the roadbed (only possible for fill),  $(n+d) = (n_2 + (-d_f)) = n_2 - d_f$ . This being numerically negative, the tape is *lowered* ( $d_f - n_2$ ). With level at (1), for fill,  $(n+d) = (n_1 + (-d_f)) = (n_1 - d_f)$ ; this being positive, the tape is raised. With level at (1), for cut, the tape is raised ( $n_1 + d_c$ ). In every case the effect is the same as if the telescope were set at the elevation of the roadbed.

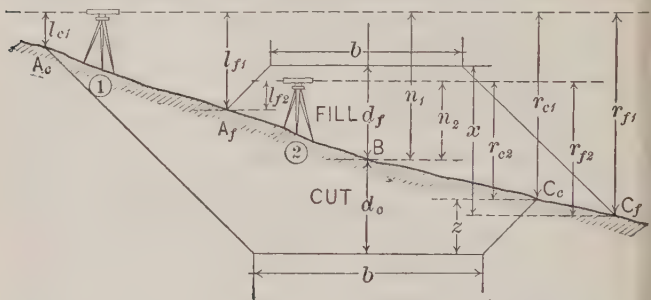


FIG. 45a.

(5) With the special distance-tape, so held that its zero is  $\frac{1}{2}b$  from the center, carry the rod out until the rod reading equals the reading indicated by the tape. Since in cut the tape is raised ( $n+d$ ), the zero of the rod-tape is always higher than the level (unless the rod is held at or below the elevation of the roadbed—which is only possible on side-hill work), and the reading at either slope-stake is necessarily *negative*. The reading for slope-stakes in fill is always positive.

(6) Record the rod-tape reading as the numerator of a fraction and the *actual distance* out (read directly from the *other* side of the distance-tape) as the denominator of the fraction.

**Proof. Fill. Level at (1).** Tape is raised ( $n_1 - d_f$ ). When rod is held at  $C_f$ , the rod reading is  $+x$ , which  $= r_{f1} - (n_1 - d_f)$ . But the reading on the back side of the distance-tape is also  $x$ .

**Fill. Level at (2).** Tape is raised ( $n_2 - d_f$ ), i.e., it is *lowered* ( $d_f - n_2$ ). When rod is held at  $C_f$ , the rod reading is  $+x$ , which

similarly  $= r_{f_2} - (n_2 - d_f) = r_{f_2} + (d_f - n_2)$ . Distance-tape as before.

**Cut Level at (1).** Tape is raised  $(n_1 + d_c)$ . When rod is held at  $C_c$  the rod reading is  $-z$ , which  $= r_{c_1} - (n_1 + d_c)$ , i.e.,  $z = (n_1 + d_c) - r_{c_1}$ . The distance-tape will read  $z$ .

**Side-hill work.** It is easily demonstrated that the method, when followed literally, may be applied to side-hill work, although there is considerable chance for confusion and error, when, as is usual,  $\frac{1}{2}b$  and the slope ratio are different for cut and for fill.

The method appears complicated at first, but it becomes mechanical and a time-saver when thoroughly learned. The advantages are especially great when the ground is fairly level transversely, but decrease when the difference of elevation of the center and the slope-stake is more than the rod length. By setting the rod-tape "at zero," the rod may always be used as an ordinary level rod and the regular method adopted, as in § 69. Many engineers who have thoroughly tested these rods are enthusiastic in their praise as a time-saver.

#### COMPUTATION OF VOLUME.

**70. Prismoidal formula.** Let Fig. 46 represent a triangular prismoid. The two triangles forming the ends lie in *parallel* planes, but since the angles of one triangle are not equal to the corresponding angles of the other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is

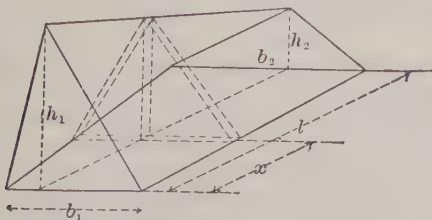


FIG. 46.

made at any point at a distance  $x$  from one end, the area of the section will evidently be

$$A_x = \frac{1}{2} b_x h_x = \frac{1}{2} \left[ b_1 + (b_2 - b_1) \frac{x}{l} \right] \left[ h_1 + (h_2 - h_1) \frac{x}{l} \right].$$



The volume of a section of infinitesimal length will be  $A_x dx$ , and the total volume of the prismoid will be \*

$$\begin{aligned}
 \int_0^l A_x dx &= \frac{1}{2} \int_0^l \left[ b_1 + (b_2 - b_1) \frac{x}{l} \right] \left[ h_1 + (h_2 - h_1) \frac{x}{l} \right] dx \\
 &= \frac{1}{2} \left[ b_1 h_1 x + (b_2 - b_1) h_1 \frac{x^2}{2l} + b_1 (h_2 - h_1) \frac{x^2}{2l} \right. \\
 &\quad \left. + (b_2 - b_1) (h_2 - h_1) \frac{x^3}{3l^2} \right]_0^l \\
 &= \frac{1}{2} \left\{ b_1 h_1 l + [(b_2 - b_1) h_1 + b_1 (h_2 - h_1)] \frac{l}{2} + (b_2 - b_1) (h_2 - h_1) \frac{l}{3} \right\} \\
 &= \frac{l}{2} \left[ \frac{1}{3} b_1 h_1 + \frac{1}{6} b_1 h_2 + \frac{1}{6} b_2 h_1 + \frac{1}{3} b_2 h_2 \right] \\
 &= \frac{l}{6} \left[ \frac{1}{2} b_1 h_1 + \frac{1}{2} b_1 (h_1 + h_2) + \frac{1}{2} b_2 (h_1 + h_2) + \frac{1}{2} b_2 h_2 \right] \\
 &= \frac{l}{6} \left[ \frac{1}{2} b_1 h_1 + 4 \left( \frac{1}{2} \cdot \frac{b_1 + b_2}{2} \cdot \frac{h_1 + h_2}{2} \right) + \frac{1}{2} b_2 h_2 \right] \\
 &= \frac{l}{6} [A_1 + 4A_m + A_2], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)
 \end{aligned}$$

in which  $A_1$ ,  $A_2$ , and  $A_m$  are the areas respectively of the two bases and of the middle section. Note that  $A_m$  is *not* the mean of  $A_1$  and  $A_2$ , although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of  $b_1$ ,  $b_2$ ,  $h_1$ , or  $h_2$ . For example,  $h_2$  may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or  $b_2$  and  $h_2$  may both vanish, the second base becoming a point and the prismoid reduces to a pyramid. Since every prismoid (as defined in § 67) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and

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\* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that  $\int dx = x$ , that  $\int x dx = \frac{1}{2} x^2$ , and that  $\int x^2 dx = \frac{1}{3} x^3$ ; also that in integrating between the limits of  $l$  and 0 (zero), the value of the integral may be found by simply substituting  $l$  for  $x$  after integration.

since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that \*

*The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.*

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It therefore becomes a simpler operation to compute volumes by approximate formulae and apply, if necessary, a correction. The most common methods are as follows:

**71. Averaging end areas.** The volume of the triangular prismoid (Fig. 46), computed by averaging end areas, is  $\frac{l}{2}[\frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2]$ . Subtracting this from the true volume (as given in the equation above Eq. 45), we obtain the correction

$$\frac{l}{12}[(b_1 - b_2)(h_2 - h_1)]. \quad \dots \quad (46)$$

This shows that if either the  $h$ 's or  $b$ 's are equal, the correction vanishes; it also shows that if the bases are roughly similar and  $b$  varies roughly with  $h$  (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

**72. Middle areas.** Sometimes the middle area is computed and the volume is assumed to be equal to the length times the middle area. This will equal  $\frac{l}{2} \times \frac{b_1 + b_2}{2} \times \frac{h_1 + h_2}{2}$ . Subtracting this from the true volume, we obtain the correction

$$\frac{l}{24}(b_1 - b_2)(h_1 - h_2). \quad \dots \quad (47)$$

As before, the form of the correction shows that if either the  $h$ 's or  $b$ 's are equal, the correction vanishes; also under the *usual* conditions, as before, the correction is *positive* and only one-half as large as by averaging end areas. Ordinarily the labor involved in the above method is no less than that of applying the exact prismoidal formula.

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\* The student should note that the derivation of equation (45) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

**73. Two-level ground.** When approximate computations of earthwork are sufficiently exact the field-work may be materially reduced by observing simply the center cut (or fill) and the natural slope  $a$ , measured with a clinometer. The area of such a section (see Fig. 48) equals

$$\frac{1}{2}(a+d)(x_l+x_r) - \frac{ab}{2}.$$

But

$$x_l \tan \beta = a + d + x_l \tan a,$$

from which

$$x_l = \frac{a+d}{\tan \beta - \tan a}.$$

Similarly,

$$x_r = \frac{a+d}{\tan \beta + \tan a}.$$

Substituting,

$$\text{Area} = (a+d)^2 \frac{\tan \beta}{\tan^2 \beta - \tan^2 a} - \frac{ab}{2}. \quad (48)$$

The values  $a$ ,  $\tan \beta$ ,  $\tan^2 \beta$  are constant for all sections, so that it requires but little work to find the area of any section.

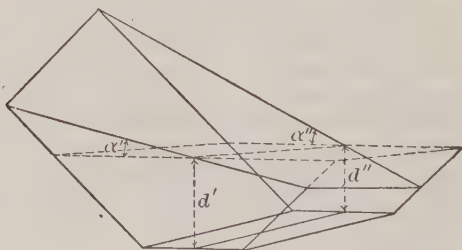


FIG. 47.

As this method of cross-sectioning implies considerable approximation, it is generally a useless refinement to attempt to com-

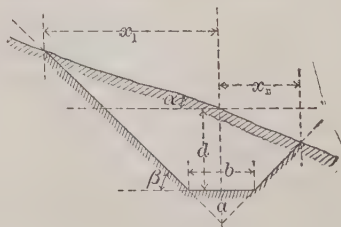


FIG. 48.

pute the volume with any greater accuracy than that obtained by averaging end areas. It may be noted that it may be easily proved that the correction to be applied is of the same form as that found in § 71 and equals

$$\frac{l}{12}[(x_l'+x_r')-(x_l''+x_r'')][(d''+a)-(d'+a)],$$

which reduces to

$$\text{Corr.} = \frac{l}{6} \left\{ \left[ (a+d') \frac{\tan \beta}{\tan^2 \beta - \tan^2 a'} - (a+d'') \frac{\tan \beta}{\tan^2 \beta - \tan^2 a''} \right] [d''-d'] \right\}. \quad (49)$$

When  $d''=d'$  the correction vanishes. This shows that when the center heights are equal there is no correction—regardless of the slope. If the slope is uniform throughout, the form of the correction is simplified and is invariably *negative*. Under the usual conditions the correction is *negative*, i.e., the method *generally gives too large results*.

**74. Level sections.** When the country is very level or when only approximate preliminary results are required, it is sometimes assumed that the cross-sections are level. The method of level sections is capable of easy and rapid computation. The area may be written as

$$(a+d)^2s-\frac{ab}{2}\dots\dots\dots (50)$$

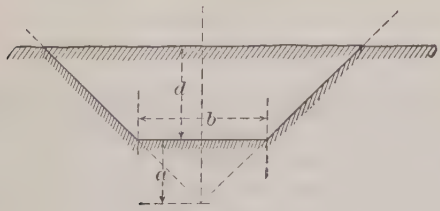


FIG. 49.

This also follows from Eq. 48 when  $a=0$  and  $\tan \beta=\frac{1}{s}$ . Here represents the “slope ratio,” i.e., the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side-slopes.

The area may also be readily determined (as illustrated in the following example) without the use of such a table: a table of squares will facilitate the work. Assuming the cross-sections at equal distances ( $=l$ ) apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0 + 2(A_1 + A_2 + \dots A_{n-1}) + A_n]. \quad (51)$$

The prismoidal correction may be directly derived from Eq. 46 as  $\frac{l}{12}[2(a+d')s - 2(a+d'')s][(a+d'') - (a+d')]$ , which reduces to

$$-\frac{ls}{6}(d' - d'')^2 \quad \text{or} \quad -\frac{b}{12} \frac{d' - d''}{a}(d' - d'')^2. \quad (52)$$

This may also be derived from Eq. 49, since  $a=0$ ,  $\tan a=0$ , and  $\tan \beta = 2a \div b$ . This correction is *always* negative, showing that the method of averaging end areas, when the sections are level, always gives too large results. The prismoidal correction for any one prismoid is therefore a constant times the *square* of a difference. The squares are always positive whether the differences are positive or negative. The correction therefore becomes

$$-\frac{l}{12} \frac{b}{a} \Sigma (d' - d'')^2. \quad (53)$$

**75. Numerical example: level sections.** Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope  $1\frac{1}{2}$  to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 79. The products should be considered as  $(a+d)(a+d) \div \frac{1}{s}$ . In this problem  $s = 1\frac{1}{2}$ ,  $\frac{1}{s} = .6667$ . To apply the rule to the first case above, place 6667 on scale *B* over 89 on scale *A*, then opposite 89 on scale *B* will be found 118.8 on scale *A*. The position of the decimal point will be evident from an approximate mental solution of the problem.

| Sta. | Center Height. | $a+d$ | $(a+d)^2$ | $(a+d)^2s$ | Areas. | $d' \sim d''$ | $(d' \sim d'')^2$ |
|------|----------------|-------|-----------|------------|--------|---------------|-------------------|
| 17   | 2.9            | 8.9   | 79.21     | 118.81     | 118.81 |               |                   |
| 18   | 4.7            | 10.7  | 114.49    | 171.74     | 343.48 | 1.8           | 3.24              |
| 19   | 6.8            | 12.8  | 163.84    | 245.76     | 491.52 | 2.1           | 4.41              |
| 20   | 11.7           | 17.7  | 313.29    | 469.93     | 939.86 | 4.9           | 24.01             |
| 21   | 4.2            | 10.2  | 104.04    | 156.06     | 312.12 | 7.5           | 56.25             |
| 22   | 1.6            | 7.6   | 57.76     | 86.64      | 86.64  | 2.6           | 6.76              |

2292.43

94.67

$$\frac{ab}{2} = \frac{6 \times 18}{2} = 54$$

$$10 \times 54 = \frac{540}{1752.43}$$

$$\frac{1752.43 \times 100}{2 \times 27} = 3245 \text{ cub. yards} = \text{approx. vol.}$$

$$\text{Corr.} = -\frac{100 \times 18}{12 \times 6 \times 27} \times 94.67 = -91 \text{ cub. yds.}$$

$$3245 - 91 = 3154 \text{ cub. yds.} = \text{exact volume.}$$

The above demonstration of the correction to be applied to the approximate volume, found by averaging end areas, is introduced mainly to give an idea of the amount of that correction. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level.

**76. Equivalent sections.** When sections are very irregular the following method may be used, especially if great accuracy is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. The *center depth* ( $d$ ) and the *slope angle* ( $a$ ) of this line can be obtained from the drawing, but it is more convenient to measure the distances ( $x_l$  and  $x_r$ ) from the center. The area may then be obtained independent of the center depth as follows: Let  $s$  = the slope ratio of the side slopes =  $\cot \beta = \frac{b}{2a}$ . (See Fig. 50.)

Then the

$$\begin{aligned} \text{Area} &= \frac{1}{2} \left( \frac{x_l + x_r}{s} \right) (x_l + x_r) - \frac{x_r}{s} \frac{x_r}{2} - \frac{x_l}{s} \frac{x_l}{2} - \frac{ab}{2} \\ &= \frac{x_l x_r}{s} - \frac{ab}{2}, \quad \dots \dots \dots (54) \end{aligned}$$

The true volume, according to the prismoidal formula, of a length of the road measured in this way will be

$$\frac{l}{6} \left[ \frac{x_l' x_r'}{s} - \frac{ab}{2} + 4 \left( \frac{x_l' + x_l''}{2} \frac{x_r' + x_r''}{2} \frac{1}{s} - \frac{ab}{2} \right) + \frac{x_l'' x_r''}{s} - \frac{ab}{2} \right].$$

If computed by averaging end areas, the approximate volume will be

$$\frac{l}{2} \left[ \frac{x_l' x_r'}{s} - \frac{ab}{2} + \frac{x_l'' x_r''}{s} - \frac{ab}{2} \right].$$

Subtracting this result from the true volume, we obtain as the correction

$$\text{Correction} = \frac{l}{6s} (x_l'' - x_l') (x_r' - x_r''). \quad . \quad . \quad (55)$$

This shows that if the side distances to either the right or left are equal at adjacent stations the correction is *zero*, and also that if the difference is small the correction is also small and very probably within the limit of accuracy obtainable by that method of cross-sectioning. In fact, as has already been shown in the latter part of § 75, it will usually be a useless

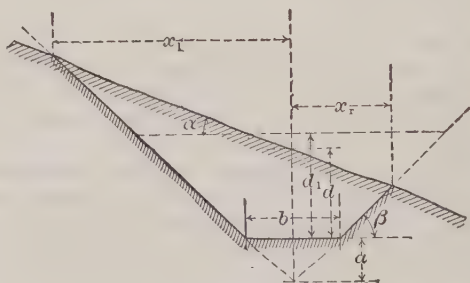


FIG. 50.

refinement to compute the prismoidal correction when the method of cross-sectioning is as rough and approximate as this method generally is.

77. Equivalent level sections. These sloping "two-level" sections are sometimes transformed into "level sections of equal



area," and the volume computed by the method of level sections (§ 74). But the true volume of a prismoid with sloping ends does not agree with that of a prismoid with equivalent bases and level ends except under special conditions, and when this method is used a correction must be applied if accuracy is desired, although, as intimated before, the assumption that the sections have uniform slopes will frequently introduce greater inaccuracies than that of this method of computation. The following demonstration is therefore given to show the scope and limitations of the errors involved in this much used method.

In Fig. 50, let  $d_1$  be the center height which gives an equivalent level section. The area will equal  $(a + d_1)^2 s - \frac{ab}{2}$ , which

must equal the area given in § 76,  $\frac{x_l x_r}{s} - \frac{ab}{2}$ .  $s = \frac{b}{2a}$ .

$$\therefore (a + d_1)^2 s = \frac{x_l x_r}{s},$$

$$\text{or} \quad a + d_1 = \frac{\sqrt{x_l x_r}}{s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

To obtain  $d_1$  directly from notes, given in terms of  $d$  and  $\alpha$ , we may substitute the values of  $x_l$  and  $x_r$  given in § 73, which gives

$$a + d_1 = (a + d) \frac{\tan \beta}{\sqrt{\tan^2 \beta - \tan^2 \alpha}} = \frac{a + d}{\sqrt{1 - s^2 \tan^2 \alpha}}. \quad . \quad (57)$$

The *true* volume of the equivalent section may be represented by

$$\frac{ls}{6} \left[ (a + d_1')^2 + 4 \left( \frac{a + d_1'}{2} + \frac{a + d_1''}{2} \right)^2 + (a + d_1'')^2 \right].$$

From this there should be subtracted the volume of the "grade prism" under the roadbed to obtain the volume of the cut that would be actually excavated, but in the following comparison, as well as in other similar comparisons elsewhere made, the volume of the grade prism invariably cancels out, and so for the sake of simplicity it will be disregarded. This expression for volume may be transposed to

$$\frac{ls}{6} \left[ \frac{x_l' x_r'}{s^2} + 4 \left( \frac{\sqrt{x_l' x_r'}}{2s} + \frac{\sqrt{x_l'' x_r''}}{2s} \right)^2 + \frac{x_l'' x_r''}{s^2} \right].$$

The true volume of the prismoid with sloping ends is (see § 76)

$$\frac{l}{6} \left[ \frac{x_l' x_r'}{s} + 4 \left( \left( \frac{x_l' + x_l''}{2} \right) \left( \frac{x_r' + x_r''}{2} \right) \frac{1}{s} \right) + \frac{x_l'' x_r''}{s} \right].$$

The difference of the two volumes

$$\begin{aligned} &= \frac{l}{6s} (x_l' x_r' + x_l'' x_r' + x_l' x_r'' + x_l'' x_r'' - x_l' x_r') \\ &\quad - 2\sqrt{x_l' x_r' x_l'' x_r'' - x_l'' x_r''}) \\ &= \frac{l}{6s} (\sqrt{x_l' x_r''} - \sqrt{x_l'' x_r'})^2. \quad \dots \quad (58) \end{aligned}$$

This shows that "equivalent level sections" do *not* in general give the true volume, there being an exception when  $x_l' x_r'' = x_l'' x_r'$ . This condition is fulfilled when the slope is uniform, i.e., when  $a' = a''$ . When this is nearly so the error is evidently not large. On the other hand, if the slopes are inclined in opposite directions the error may be very considerable, particularly if the angles of slope are also large.

**78. Three-level sections.** The next method of cross-sectioning in the order of complexity, and therefore in the order of

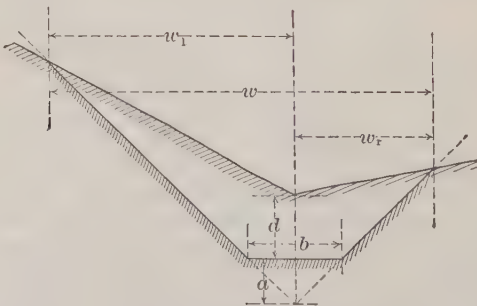


FIG. 51.

accuracy, is the method of three-level sections. The area of the section is  $\frac{1}{2}(a+d)(w_r+w_l) - \frac{ab}{2}$ , which may be written

$\frac{1}{2}(a+d)w - \frac{ab}{2}$ , in which  $w = w_r + w_l$ . If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a+d')w' - ab + (a+d'')w'' - ab]. \quad (59)$$

If we divide by 27 to reduce to cubic yards, we have, when  $l=100$ ,

$$\text{Vol} (\dots) = \frac{2}{27} \frac{5}{7} (a+d')w' - \frac{2}{27} \frac{5}{7} ab + \frac{2}{27} \frac{5}{7} (a+d'')w'' - \frac{2}{27} \frac{5}{7} ab$$

For the next section

$$\text{Vol} (\dots) = \frac{2}{27} \frac{5}{7} (a+d')w'' - \frac{2}{27} \frac{5}{7} ab + \frac{2}{27} \frac{5}{7} (a+d'')w''' - \frac{2}{27} \frac{5}{7} ab$$

For a partial station length compute as usual and multiply result by  $\frac{\text{length in feet}}{100}$ . The prismoidal correction may be obtained by applying Eq. 46 to each side in turn. For the left side we have

$$\begin{aligned} & \frac{l}{12} [(a+d') - (a+d'')](w_l'' - w_l'), \text{ which equals} \\ & \frac{l}{12} (d' - d'')(w_l'' - w_l'). \end{aligned}$$

For the right side we have, similarly,

$$\frac{l}{12} (d' - d'')(w_r'' - w_r').$$

The total correction therefore equals

$$\begin{aligned} & \frac{l}{12} (d' - d'')[(w_l'' + w_r'') - (w_l' + w_r')] \\ & = \frac{l}{12} (d' - d'')(w'' - w'). \end{aligned}$$

Reduced to cubic yards, and with  $l=100$ ,

$$\text{Pris. Corr.} = \frac{2}{81} \frac{5}{7} (d' - d'')(w'' - w'). \quad (60)$$

When this result is compared with that given in Eq. 55 there is an apparent inconsistency. If two-level ground is considered as but a special case of three-level ground, it would seem as if

the same laws should apply. If, in Eq. 55,  $x_r' = x_r''$ , and  $x_l''$  is different from  $x_l'$ , the equation reduces to zero; but in this case  $d'$  would also be different from  $d''$ ; and since  $x_l' + x_r'$  would  $= w'$ , and  $x_l'' + x_r'' = w''$  in Eq. 60,  $w'' - w'$  would not equal zero and the correction would be some finite quantity and not zero. The explanation lies in the difference in the form and volume of the prismoids, according to the method of the formation of the warped surfaces. If the surface is supposed to be generated by the locus of a line moving parallel to the ends as plane directors and along two *straight* lines lying in the side slopes, then  $x_l^{\text{mid.}}$  will equal  $\frac{1}{2}(x_l' + x_l'')$ , and  $x_r^{\text{mid.}}$  will equal  $\frac{1}{2}(x_r' + x_r'')$ , but the profile of the center line will *not* be straight and  $d^{\text{mid.}}$  will *not* equal  $\frac{1}{2}(d' + d'')$ . On the other hand, if the surfaces be generated by *two* lines moving parallel to the ends as plane directors and along a *straight* center line and straight side lines lying in the slopes, a warped surface will be generated each side of the center line, which will have uniform slopes on each side of the center at the two ends and *nowhere else*. This shows that when the upper surface of earthwork is warped (as it generally is), two-level ground should not be considered as a special case of three-level ground. This discussion, however, is only valuable to explain an apparent inconsistency and error. The method of two-level ground should only be used when such refinements as are here discussed are of no importance as affecting the accuracy.

An example is given on the opposite page to illustrate the method of three-level sections.

In the first column of yards

$$\begin{aligned} 210 &= \frac{25}{81}(a+d)w = \frac{25}{81} \times 7.3 \times 31.1; \\ 507, 734, \text{ etc.,} &\text{ are found similarly;} \\ 595 &= 210 - 61 + 507 - 61; \\ 448 &= \frac{40}{100}(507 - 61 + 734 - 61); \\ 602 &= \frac{60}{100}(734 - 61 + 392 - 61); \\ 449 &= 392 - 61 + 179 - 61. \end{aligned}$$

For the prismoidal correction,

$$\begin{aligned} -20 &= \frac{25}{81}(d' - d'')(w'' - w') = \frac{25}{81}(2.6 - 8.1)(42.8 - 31.1) \\ &= \frac{25}{81}(-5.5)(+11.7). \end{aligned}$$

For the next line,  $-3 = \frac{40}{100}[\frac{25}{81}(-2.6)(+8.7)]$ , and similarly for the rest. The "*F*" in the columns of center heights, as well

| Station.                                     | Center. | Left.                | Right.              | a + d | w    | Yards. | d' - d'' | w'' - w' | Pris. Corr. | $x_l \sim x_r$ | $\frac{V(x_l \sim x_r)}{3R}$ | Curv. Corr.* |
|----------------------------------------------|---------|----------------------|---------------------|-------|------|--------|----------|----------|-------------|----------------|------------------------------|--------------|
| 17                                           | 2.6F    | $\frac{10.6F}{22.9}$ | $\frac{0.8F}{8.2}$  | 7.3   | 31.1 | 210    |          |          |             | 14.7           | +1                           |              |
| 18                                           | 8.1F    | $\frac{15.8F}{30.7}$ | $\frac{3.4F}{12.1}$ | 12.8  | 42.8 | 507    | -5.5     | +11.7    | -20         | 18.6           | +3                           | +4           |
| + 40                                         | 10.7F   | $\frac{20.2F}{37.3}$ | $\frac{4.8F}{14.2}$ | 15.4  | 51.5 | 734    | -2.6     | + 8.7    | - 3         | 23.1           | +6                           | +4           |
| 19                                           | 6.4F    | $\frac{14.0F}{28.0}$ | $\frac{2.1F}{10.1}$ | 11.1  | 38.1 | 392    | +4.3     | -13.4    | -11         | 17.9           | +2                           | +5           |
| 20                                           | 3.7F    | $\frac{5.8F}{15.7}$  | $\frac{0.2F}{7.3}$  | 8.4   | 23.0 | 179    | +2.7     | -15.1    | -13         | 8.4            | +1                           | +3           |
| Roadbed, 14' wide in fill,<br>Slope 1½ to 1. |         |                      |                     |       |      |        |          |          |             |                |                              | +16          |
| Approx. Vol. = 2094<br>Pris. corr. = 47      |         |                      |                     |       |      |        |          |          |             |                |                              | -47          |
| True Vol. = 2047 (disregarding curv. corr.)* |         |                      |                     |       |      |        |          |          |             |                |                              |              |

Roadbed, 14' wide in fill,  
Slope 1½ to 1.

$a = \frac{b}{2s} = \frac{14}{3} = 4.7;$

$\frac{25}{2}ab = 61.$

\*For the derivation of the curvation correction, see § 93.

as in the columns of "right" and "left," are inserted to indicate *fill* for all those points. Cut would be indicated by "*C.*"

**79. Computation of products.** The quantities  $\frac{25}{27}(a+d)w$  and  $\frac{25}{27}ab$  represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for  $\frac{25}{81}(d'-d'')(w''-w')$  will assist similarly in computing the prismoidal correction. Prof. Charles J. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 in "Tables for the Computation of Railway and Other Earthwork." Another easy method of obtaining these products is by the use of a slide-rule. A slide-rule has been designed by the author to accompany this volume.\* It is designed particularly for this special work, although it may be utilized for many other purposes for which slide-rules are valuable. To illustrate its use, suppose  $(a+d)=28.2$ , and  $w=62.4$ ; then

$$\frac{25}{27}(a+d)w = \frac{28.2 \times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, is specially marked) on the sliding scale (*B*) opposite 282 on the other scale (*A*), and then opposite 624 on scale *B* will be found 1629 on scale *A*, the 162 being read directly and the 9 read by estimation. Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10

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\* The first edition of this book was octavo, and a pasteboard slide-rule, especially marked, accompanied each volume. Cutting down the size of the pages to "pocket size" prevents the incorporation of the rule with the present edition. Any slide-rule with a logarithmic unit 22½ inches long will do equally well provided that the 108 mark is specially distinguished for ready use in computing the volume and that the 324 mark is similarly distinguished for use in computing the prismoidal correction.

yards, and the tenths of a division estimated. Between 5000 and 10000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10000 cubic yards would not have been put on the scale except for the necessity of finding such products as  $\frac{2}{3}(9.1 \times 9.5)$ , for example. This product would be read off from the same part of the rule as  $\frac{2}{3}(91 \times 95)$ . In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating  $\frac{4}{10}$  of a division.

The computation for the prismoidal correction may be made similarly except that the divisor is 3.24 instead of 1.08. For example,  $\frac{2}{3}(5.5 \times 11.7) = \frac{5.5 \times 11.7}{3.24}$ . Set the 324 on scale *B* (also specially marked like 108) opposite 55 on scale *A*, and proceed as before.

**80. Five-level sections.** Sometimes the elevations over each edge of the roadbed are observed when cross-sectioning. These are distinctively termed “five-level sections.” If the center, the slope-stakes, and *one* intermediate point on each side (*not* necessarily over the edge of the roadbed) are observed, it is termed an “irregular section.” The field-work of cross-sectioning five-level sections is no less than for irregular sections with one intermediate point; the computations, although capable of peculiar treatment on account of the location of the intermediate point, are no easier, and in some respects more laborious; the cross-sections obtained will not in general represent the actual cross-sections as truly as when there is perfect freedom in locating the intermediate point; as it is generally inadvisable or unnecessary to employ five-level sections throughout the length of a road, the change from one method to another adds a possible element of inaccuracy and loses the advantage of uniformity of method, particularly in the notes and *form* of computations. On these accounts the method will not be further developed, except to note that this case, as well as any other, may be solved by dividing the whole prismoid into triangular prismoids, computing the volume by averaging end areas, and computing



the prismoidal correction by adding the computed corrections for each elementary triangular prismoid.

81. Irregular sections. In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 44) and subtracting the two external triangles. For Fig. 44 the area would be

$$\begin{aligned} & \frac{h_l + k_l}{2}(x_l - y_l) + \frac{k_l + d}{2}y_l + \frac{d + j_r}{2}z_r + \frac{j_r + k_r}{2}(y_r - z_r) \\ & + \frac{k_r + h_r}{2}(x_r - y_r) - \frac{h_l}{2}\left(x_l - \frac{b}{2}\right) - \frac{h_r}{2}\left(x_r - \frac{b}{2}\right). \end{aligned}$$

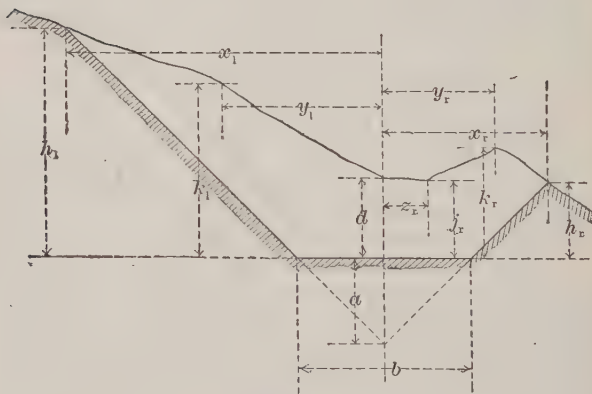


FIG. 44.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\begin{aligned} \text{AREA} = \frac{1}{2} \left[ x_l k_l + y_l (d - h_l) + x_r k_r + y_r (j_r - h_r) \right. \\ \left. + z_r (d - k_r) + \frac{b}{2} (h_l + h_r) \right]. \quad \dots \quad (61) \end{aligned}$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how

many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

*AREA equals one-half the sum of products obtained as follows:*  
*the distance to each slope-stake times the height above grade of the point next inside the slope-stake;*  
*the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;*  
*finally, one-half the width of the roadbed times the sum of the slope-stake heights.*

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The "point next inside the slope-stake" in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 78, in which one term  $\left(\frac{ab}{2}\right)$  is a constant for all sections, is preferable. In the general method, each intermediate "break" adds another term.

**82. Volume of an irregular prismoid.** If there is a break at one cross-section which is not represented at the next, the ridge (or hollow) implied by that break is supposed to "vanish" at the next section. In fact, the volume will not be correctly represented unless a cross-section is taken at the point where the ridge or hollow "vanishes" or "runs out." To obtain the true prismoidal correction it is necessary to observe on the ground the place where a break in an adjacent section, which is not represented in the section being taken, runs out. For example, in Fig. 52, the break on the left of section  $A''$ , at a distance of  $y_1''$  from the center, is observed to run out in section  $A'$  at a distance of  $y_1'$  from the center. The volume of the prismoid, computed by the prismoidal formula as in § 70, will involve the midsection, to obtain the dimension of which would require a laborious computation. A simpler process is to compute the volume by averaging end areas as in § 81 and apply a prismoidal correction. To do this write out an expression for each end area similar to that given in Eq. 61. The sum of these areas times  $\frac{l}{2}$  gives the approximate volume. As before,

for partial station lengths, multiply the result by  $\frac{\text{length in feet}}{100}$ .

There will be no constant subtractive term,  $\frac{25}{7}ab$ , as in § 78. The *true* prismoidal correction may be computed, as in § 83, or the following approximate method may be used: Consider the irregular section to be three-level ground *for the purpose of*

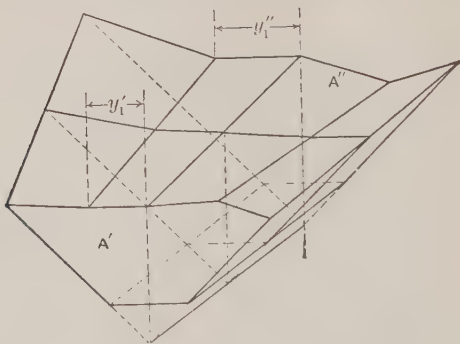


FIG. 52.

*computing the correction only.* This has the advantage of less labor in computation than the use of the true prismoidal correction, and although the error involved may be considerable in individual sections, the error is as likely to be positive as negative, and in the long run the error will not be large and generally will be much less than would result by the neglect of any prismoidal correction.

**83. True prismoidal correction for irregular prismoids.** As intimated in § 82, each cross-section should be assumed to have the same number of sides as the adjacent cross-section when computing the prismoidal correction. This being done, it permits the division of the whole prismoid into elementary triangular prismoids, the dimensions of the bases of which being given in each case by a vertical distance above grade line and by the horizontal distance between two adjacent breaks. The summation of the prismoidal corrections for each of the elementary triangular prismoids will give the true prismoidal correction. Assuming for an example the cross-section of Fig. 44, with a cross-section of the same number of sides, and with dimensions

similarly indicated, for the other end, the prismoidal correction becomes (see Eq. 46)

$$\frac{l}{12} \left[ \begin{aligned} & (h_l' - h_l'')[(x_l'' - y_l'') - (x_l' - y_l')] + (k_l' - k_l'')[(x_l'' - y_l'') - (x_l' - y_l')] \\ & + (k_l' - k_l'')(y_l'' - y_l') + (d' - d'')(y_l'' - y_l') + (d' - d'')(z_r'' - z_r') \\ & + (j_r' - j_r'')(z_r'' - z_r') + (j_r' - j_r'')[(y_r'' - z_r'') - (y_r' - z_r')] \\ & + (k_r' - k_r'')[(y_r'' - z_r'') - (y_r' - z_r')] \\ & + (k_r' - k_r'')[(x_r'' - y_r'') - (x_r' - y_r')] + (h_r' - h_r'')[(x_r'' - y_r'') - (x_r' - y_r')] \\ & - (h_l' - h_l'') \left[ \left( x_l'' - \frac{b}{2} \right) - \left( x_l' - \frac{b}{2} \right) \right] \\ & - (h_r' - h_r'') \left[ \left( x_r'' - \frac{b}{2} \right) - \left( x_r' - \frac{b}{2} \right) \right] \end{aligned} \right]$$

Expanding this and collecting terms, of which many will cancel, we obtain

$$\begin{aligned} \text{Pris. Corr} = \frac{l}{12} & \left[ (x_l'' - x_l')(k_l' - k_l'') + (y_l'' - y_l')[(d' - h_l') - (d'' - h_l'')] \right. \\ & + (x_r'' - x_r')(k_r' - k_r'') + (y_r'' - y_r')[(j_r' - h_r') - (j_r'' - h_r'')] \\ & \left. + (z_r'' - z_r')[(d' - k_r') - (d'' - k_r'')] \right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (62) \end{aligned}$$

By comparing this equation with Eq. 61 a remarkable coincidence in the law of formation may be seen, which enables this formula to be written by mere inspection and to be applied numerically with a minimum of labor from the computations for end areas, as will be shown (§ 84) by a numerical example. For each term in Eq. 61, as, for example,  $y_r(j_r - h_r)$ , there is a correction term in Eq. 62 of the form

$$(y_r'' - y_r')[(j_r' - h_r') - (j_r'' - h_r'')].$$

Each one of these terms  $[y_r'', y_r', (j_r' - h_r'), \text{ and } (j_r'' - h_r'')]$  has been previously used in finding the end areas and has its place in the computation sheet. The summation of the products of these differences times a constant gives the total true prismoidal correction in cubic yards for the whole prismoid considered.

The *constant* is the same as that computed in § 78, i.e.,  $\frac{2}{3} \frac{5}{1}$ .

**84. Numerical example; irregular sections; volume with true prismoidal correction.** (See page 98.)

Roadbed 18 feet wide in cut; slope  $1\frac{1}{2}$  to 1.

| Sta. | Center $\begin{cases} \text{cut} \\ \text{or} \\ \text{fill.} \end{cases}$ | Left.                |                                     |                                   | Right.                            |                     |
|------|----------------------------------------------------------------------------|----------------------|-------------------------------------|-----------------------------------|-----------------------------------|---------------------|
| 19   | 0.6c                                                                       | $\frac{3.6c}{14.4}$  | $\left( \frac{2.3c}{8.2} \right)$   | $\left( \frac{1.8c}{6.0} \right)$ | $\frac{0.1c}{4.2}$                | $\frac{0.4c}{9.6}$  |
| 18   | 2.3c                                                                       | $\frac{4.2c}{15.3}$  | $\frac{6.8c}{8.4}$                  | $\frac{3.2c}{5.2}$                | $\left( \frac{1.9c}{3.6} \right)$ | $\frac{1.2c}{10.8}$ |
| 17   | 7.6c                                                                       | $\frac{8.2c}{21.3}$  | $\frac{10.2c}{17.4}$                | $\frac{8.0c}{6.1}$                | $\left( \frac{5.8c}{8.0} \right)$ | $\frac{4.2c}{15.3}$ |
| +42  | 10.2c                                                                      | $\frac{12.2c}{27.3}$ | $\left( \frac{12.3c}{22.0} \right)$ | $\frac{12.6c}{8.2}$               | $\frac{6.2c}{7.5}$                | $\frac{8.4c}{21.6}$ |
| 16   | 6.8c                                                                       | $\frac{8.9c}{22.4}$  |                                     | $\frac{7.6c}{12.0}$               | $\frac{3.2c}{4.1}$                | $\frac{2.6c}{12.9}$ |

The figures in the bracket  $\left( \frac{12.3c}{22.0} \right)$  mean that it was noted in the field that the break, indicated at Sta. 17 as being 17.4 to the left, ran out at Sta. 16 +42 at 22.0 to the left. By interpolation between 8.2 and 27.3 the height of this point is *computed* as 12.3. The quantities in the other brackets are obtained similarly. These quantities are only used when the computation of the true prismoidal correction is desired. They are not needed in computing the volume by averaging end areas, nor are they used at all if the prismoidal correction is to be obtained by assuming (*for this purpose*) the ground to be *three-level* ground.

In the tabular form on page 99 the figures within the braces (—) are NOT used in computing the volume, but are only used to obtain the *differences* of widths or heights with which to compute the *true* prismoidal correction. It may be noted, as a check, that the volume, computed from these figures in the braces, is the same as that computed from the other figures. The figures within each brace (or bracket) constitute a group which must be used in connection with a group which has the same number of points, on the same side of the center, in the *next* cross-section, previous or succeeding. In the column of "Yards" under "True pris. corr.," we have, for example,  $(-5) = \frac{42}{100}(-7+0-8+3)$ .

85. Volume of irregular prismoid, with approximate prismoidal correction. If the prismoidal correction is obtained approxi-

VOLUME OF IRREGULAR PRISMOID, WITH TRUE PRISMOIDAL  
CORRECTION.

| Sta. | Width.                                                                                       | Height.                                                                                 | Yards.                                                          |     | True pris. corr.                                                   |                                                                    |                                                                |
|------|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|-----------------------------------------------------------------|-----|--------------------------------------------------------------------|--------------------------------------------------------------------|----------------------------------------------------------------|
|      |                                                                                              |                                                                                         |                                                                 |     | $w'' - w'$                                                         | $h' - h''$                                                         | Yards.                                                         |
| 16   | L $\begin{bmatrix} 22.4 \\ 12.0 \\ 12.9 \\ 4.1 \\ 9.0 \end{bmatrix}$ R                       | $\begin{matrix} 7.6 \\ -2.1 \\ 3.2 \\ 4.2 \\ 11.5 \end{matrix}$                         | $\begin{matrix} 158 \\ -23 \\ 40 \\ 16 \\ 96 \end{matrix}$      |     |                                                                    |                                                                    |                                                                |
| +42  | L $\begin{bmatrix} 27.3 \\ 8.2 \\ 27.3 \\ 22.0 \\ 8.2 \\ 21.6 \\ 7.5 \\ 9.0 \end{bmatrix}$ R | $\begin{matrix} 12.6 \\ -2.0 \\ 12.3 \\ 0.4 \\ -2.1 \\ 6.2 \\ 1.8 \\ 20.6 \end{matrix}$ | $\begin{matrix} 319 \\ -15 \\ \\ 124 \\ 13 \\ 172 \end{matrix}$ | 378 | $\begin{matrix} +4.9 \\ -3.8 \\ \\ +8.7 \\ +3.4 \end{matrix}$      | $\begin{matrix} -5.0 \\ -0.1 \\ \\ -3.0 \\ +2.4 \end{matrix}$      | $\begin{matrix} -7 \\ 0 \\ \\ -8 \\ +3 \\ (-5) \end{matrix}$   |
|      | L $\begin{bmatrix} 21.3 \\ 17.4 \\ 6.1 \\ 15.3 \\ 8.0 \\ 15.3 \\ 9.0 \end{bmatrix}$ R        | $\begin{matrix} 10.2 \\ -0.2 \\ -2.6 \\ 5.8 \\ 3.4 \\ 7.6 \\ 12.4 \end{matrix}$         | $\begin{matrix} 201 \\ -3 \\ -14 \\ \\ 107 \\ 103 \end{matrix}$ | 584 | $\begin{matrix} -6.0 \\ -4.6 \\ -2.1 \\ -6.3 \\ +0.5 \end{matrix}$ | $\begin{matrix} +2.1 \\ +0.6 \\ +0.5 \\ +0.4 \\ -1.6 \end{matrix}$ | $\begin{matrix} -4 \\ -1 \\ 0 \\ -1 \\ 0 \\ (-3) \end{matrix}$ |
|      | L $\begin{bmatrix} 15.3 \\ 8.4 \\ 5.2 \\ 10.8 \\ 10.8 \\ 3.6 \\ 9.0 \end{bmatrix}$ R         | $\begin{matrix} 6.8 \\ -1.0 \\ -4.5 \\ 2.3 \\ 1.9 \\ 1.1 \\ 5.4 \end{matrix}$           | $\begin{matrix} 95 \\ -7 \\ -22 \\ 23 \\ \\ 45 \end{matrix}$    | 528 | $\begin{matrix} -6.0 \\ -9.0 \\ -0.9 \\ -4.5 \end{matrix}$         | $\begin{matrix} +3.4 \\ +0.8 \\ +1.9 \\ +5.3 \end{matrix}$         | $\begin{matrix} -6 \\ -2 \\ -1 \\ -7 \\ (-16) \end{matrix}$    |
|      | L $\begin{bmatrix} 14.4 \\ 14.4 \\ 8.2 \\ 6.0 \\ 9.6 \\ 4.2 \\ 9.0 \end{bmatrix}$ R          | $\begin{matrix} 0.6 \\ 2.3 \\ -1.8 \\ -1.7 \\ 0.1 \\ 0.2 \\ 4.0 \end{matrix}$           | $\begin{matrix} 8 \\ \\ 1 \\ 1 \\ 33 \end{matrix}$              | 177 | $\begin{matrix} -0.9 \\ -0.2 \\ +0.8 \\ -1.2 \\ +0.6 \end{matrix}$ | $\begin{matrix} +4.5 \\ +0.8 \\ -2.8 \\ +1.8 \\ +0.9 \end{matrix}$ | $\begin{matrix} -1 \\ 0 \\ -1 \\ -1 \\ 0 \\ (-3) \end{matrix}$ |

Approx. vol. = 1667

-27

True pris. corr = -27

True volume = 1640 cubic yards

mately, by the method outlined in § 82, the process will be as shown in the tabular form on page 100. Not only is the numerical work considerably less than the exact method, but the discrepancy in cubic yards is almost insignificant.

**86. Illustration of value of approximate rules.** The tabulation on page 100 shows that when the volume of an irregular prismoid is computed by averaging end areas and is corrected

VOLUME OF IRREGULAR PRISMOID, WITH APPROXIMATE  
PRISMOIDAL CORRECTION.

| Sta. | W'th | H'ght | Yards. |     | Cen.<br>Height. | Total<br>width | $d' - d''$ | $w'' - w'$ | Approx.<br>pris. corr. |
|------|------|-------|--------|-----|-----------------|----------------|------------|------------|------------------------|
| 16   | 22.4 | 7.6   | 158    |     | +6.8            | 35.3           |            |            |                        |
|      | 12.0 | -2.1  | -23    |     |                 |                |            |            |                        |
|      | 12.9 | 3.2   | 40     |     |                 |                |            |            |                        |
|      | 4.1  | 4.2   | 16     |     |                 |                |            |            |                        |
|      | 9.0  | 11.5  | 96     |     |                 |                |            |            |                        |
| +42  | 27.3 | 12.6  | 319    |     | +10.2           | 48.9           | -3.4       | +13.6      | -14                    |
|      | 8.2  | -2.0  | -15    |     |                 |                |            |            |                        |
|      | 21.6 | 6.2   | 124    |     |                 |                |            |            |                        |
|      | 7.5  | 1.8   | 13     |     |                 |                |            |            |                        |
|      | 9.0  | 20.6  | 172    | 378 |                 |                |            |            | (-6)                   |
| 17   | 21.3 | 10.2  | 201    |     | +7.6            | 36.6           | +2.6       | -12.3      | -10                    |
|      | 17.4 | -0.2  | -3     |     |                 |                |            |            |                        |
|      | 6.1  | -2.6  | -14    |     |                 |                |            |            |                        |
|      | 15.3 | 7.6   | 107    |     |                 |                |            |            |                        |
|      | 9.0  | 12.4  | 103    | 584 |                 |                |            |            | (-6)                   |
| 18   | 15.3 | 6.8   | 95     |     | +2.3            | 26.1           | +5.3       | -10.5      | -17                    |
|      | 8.4  | -1.0  | -7     |     |                 |                |            |            |                        |
|      | 5.2  | -4.5  | -22    |     |                 |                |            |            |                        |
|      | 10.8 | 2.3   | 23     |     |                 |                |            |            |                        |
|      | 9.0  | 5.4   | 45     | 528 |                 |                |            |            | (-17)                  |
| 19   | 14.4 | 0.6   | 8      |     | +0.6            | 24.0           | +1.7       | -2.1       | -1                     |
|      | 9.6  | 0.1   | 1      |     |                 |                |            |            |                        |
|      | 4.2  | 0.2   | 1      |     |                 |                |            |            |                        |
|      | 9.0  | 4.0   | 33     | 177 |                 |                |            |            | (-1)                   |

Approx. volume = 1667

-30

Approx. pris. corr. = -30

Corrected volume = 1637 cubic yards

by considering the ground as three-level ground (*for the purposes of the correction only*), the error for the different sections

| Sections.    | True volume. | Approx. vol.<br>by averaging<br>end areas. | Difference or<br>true pris.<br>corr. | Approx. pris.<br>corr. on basis<br>of three-level<br>ground. | Error. | Approx. vol.<br>computed<br>from center<br>and side<br>heights <i>only</i> . | Error. |
|--------------|--------------|--------------------------------------------|--------------------------------------|--------------------------------------------------------------|--------|------------------------------------------------------------------------------|--------|
| 16.....16+42 | 373          | 378                                        | -5                                   | -6                                                           | -1     | 396                                                                          | -23    |
| 16+42...17   | 581          | 584                                        | -3                                   | -6                                                           | -3     | 577                                                                          | +4     |
| 17.....18    | 512          | 528                                        | -16                                  | -17                                                          | -1     | 463                                                                          | +49    |
| 18.....19    | 174          | 177                                        | -3                                   | -1                                                           | +2     | 147                                                                          | +27    |
|              | 1640         | 1667                                       | -27                                  | -30                                                          | -3     | 1583                                                                         | +57    |

is sometimes positive and sometimes negative, and in this case amounts to only 3 yards in 1640—less than  $\frac{1}{6}$  of 1%. If the





follow the uniform system. In computing the cut, as in Fig. 53, the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 61 strictly, we have two terms for the left side, one for the right, and the term involving  $\frac{1}{2}b$ , which will be  $\frac{1}{2}bh_l$  in this case, since  $h_r=0$ , and the equation becomes

$$\text{Area} = \frac{1}{2}[x_l k_l + y_l(d - h_l) + x_r d + \frac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application of Eq. 61, but for Fig. 54 all distances for the left side are zero and the elevation for the first point out is zero.  $d$  also must be

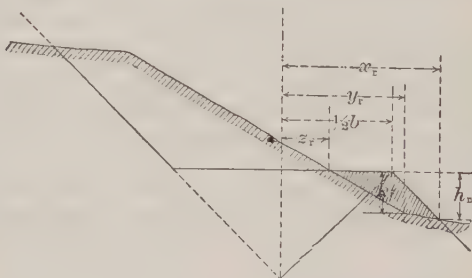


FIG. 54.

considered as zero. Following the rule, § 81, literally, the equation becomes

$$\text{Area}_{\text{Fill}} = \frac{1}{2}[x_r k_r + y_r(o - h_r) + z_r(o - k_r) + \frac{1}{2}b(o + h_r)],$$

which reduces to

$$\frac{1}{2}[x_r k_r - y_r h_r - z_r k_r + \frac{1}{2}bh_r].$$

(Note that  $x_r$ ,  $h_r$ , etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 40 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work," and a cross-section should be taken at this point.

**89. Borrow-pits.** The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

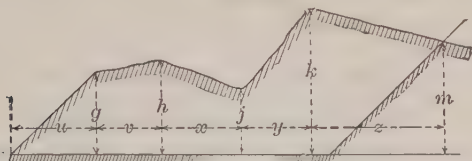


FIG. 55.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 55) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 55) is  $s$ , the area of the triangle is  $\frac{1}{2}sm^2$ . The area of the section is  $\frac{1}{2}[ug + (g+h)v + (h+j)x + (j+k)y + (k+m)z - sm^2]$ . If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 61 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correction should be computed; and since such a section as Fig. 55 does not even approximate to a three-level section, the method suggested in § 82 cannot be employed. It will then be necessary to employ the exact method, § 83, by dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 71.

**90. Correction for curvature.** The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact

solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance  $e$  from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as  $R \pm e : R$ . Therefore we have

*True vol. : nominal vol. ::  $R \pm e : R$ .  $\therefore$  True vol. =  $lA \frac{R \pm e}{R}$*  for

a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol. =  $lA' \frac{R \pm e'}{R}$* . This

shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by

multiplying the actual area by the ratio  $\frac{R \pm e}{R}$ . This being

independent of the value of  $l$ , it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway

between the two end cross-sections would be  $A_m \frac{\left(R \pm \frac{e' + e''}{2}\right)}{R}$ .

Therefore the volume of a solid which, when straight, would be  $\frac{l}{6}(A' + 4A_m + A'')$ , would then become

$$\text{True vol.} = \frac{l}{6R} \left[ A'(R \pm e') + 4A_m \left( R \pm \frac{e' + e''}{2} \right) + A''(R \pm e'') \right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$\text{Correction} = \pm \frac{l}{6R} \left[ (A' + 2A_m)e' + (2A_m + A'')e'' \right]. \quad (63)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. 63 requires that  $A_m$  be known, which requires laborious computa-

tions, but no error worth considering is involved if the equation is written approximately

$$\text{Curv. corr.} = \frac{t}{2R}(A'e' + A''e''), \quad \dots \quad (64)$$

which is the equation generally used. The approximation consists in assuming that the difference between  $A'$  and  $A_m$  equals the difference between  $A_m$  and  $A''$  but with opposite sign. The error due to the approximation is always utterly insignificant.

**91. Eccentricity of the center of gravity.** The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to consider the cross-sections as three-level ground, or, for side-hill work, to

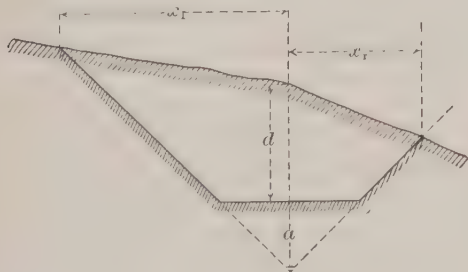


FIG. 56.

be triangular, for the purpose of this correction. The eccentricity of the cross-section of Fig. 56 (including the grade triangle) may be written

$$e = \frac{\frac{(a+d)x_l}{2} \cdot \frac{x_l}{3} - \frac{(a+d)x_r}{2} \cdot \frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3} \frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3} (x_l - x_r). \quad \dots \quad (65)$$

The side toward  $x_l$  being considered positive in the above demonstration, if  $x_r > x_l$ ,  $e$  would be negative, i.e., the center of gravity would be on the right side. Therefore, for three-level

ground, the correction for curvature (see Eq. 64) may be written

$$\text{Correction} = \frac{l}{6R} [A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which  $V'$  and  $V''$  represent the number of cubic yards corresponding to the area at each station, we may write

$$\text{Corr. in cub. yds.} = \frac{1}{3R} [V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad (66)$$

It should be noted that the value of  $e$ , derived in Eq. 65, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ( $A'e'$  of Eq. 64) equals  $\text{true area} \times e_1$ , which equals  $(\text{true area} + \frac{1}{2}ab) \times e$ . Since the value of  $e$  is very simple, while the value of  $e_1$  would, in general, be a complex quantity, it is easier to use the simple value of Eq. 65 and add  $\frac{1}{2}ab$  to the area. Therefore, in the case of three-level ground the subtractive term  $\frac{5}{24}ab$  (§ 78) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 81 and 82, which does not involve the grade triangle, a term  $\frac{5}{24}ab$  must be *added* at every station when computing the quantities  $V'$  and  $V''$  for Eq. 66.

It should be noted that the factor  $1 \div 3R$ , which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently

be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 66 shows that the correction for each station is of the form  $\frac{V(x_l - x_r)}{3R}$ .  $3R$  is generally a large quantity—for a  $6^\circ$  curve it is 2865.  $(x_l - x_r)$  is generally small. It may frequently be seen by inspection that the product  $V(x_l - x_r)$  is roughly twice or three times  $3R$ , or perhaps less than half of  $3R$ , so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as  $x_r$  is greater or less than  $x_l$ , and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that  $x_l$  is uniformly greater (or uniformly less) than  $x_r$  throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt

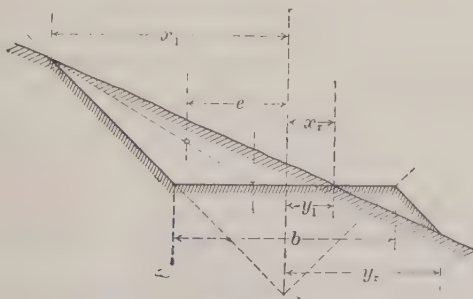


FIG. 57.

to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as  $(x_l - x_r)$  is positive or negative; if the curve is to the *left*, the correction will be positive or nega-



tive according as  $(x_r - x_l)$  is positive or negative. Therefore when computing curves to the *right* use the form  $(x_l - x_r)$  in Eqs. 66 and 68; when computing curves to the *left* use the form  $(x_r - x_l)$  in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

**92. Center of gravity of side-hill sections.** In computing the correction for side-hill work the cross-section would be treated as triangular unless the error involved would evidently be too great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at  $\frac{2}{3}$  of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus  $\frac{1}{3}$  of its horizontal projection. Therefore

$$\begin{aligned} e &= \left[ \frac{b}{2} - \frac{1}{2} \left( \frac{b}{2} + x_r \right) \right] + \frac{1}{3} \left[ x_l - \left( \frac{b}{2} - \frac{1}{2} \left( \frac{b}{2} + x_r \right) \right) \right] \\ &= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6} \\ &= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3} \\ &= \frac{1}{3} \left[ \frac{b}{2} + (x_l - x_r) \right] \dots \dots \dots (67) \end{aligned}$$

By the same process as that used in § 91 the correction equation may be written

$$\text{Corr. in cub. yds.} = \frac{1}{3R} \left[ V' \left( \frac{b}{2} + (x_l' - x_r') \right) + V'' \left( \frac{b}{2} + (x_l'' - x_r'') \right) \right]. \quad (68)$$

It should be noted that since the grade triangle is not used in this computation the volume of the grade prism is *not* involved in computing the quantities  $V'$  and  $V''$ .

The eccentricities of cross-sections in side-hill work are *never* zero, and are frequently quite large. The total volume is generally quite small. It follows that the correction for curvature is generally a vastly larger proportion of the total volume than in ordinary three-level or irregular sections.

If the triangle is wholly to one side of the center, Eq. 67 can still be used. For example, to compute the eccentricity of the triangle of fill, Fig. 57, denote the two distances to the slope-

takes by  $y_r$  and  $-y_l$  (note the minus sign). Applying Eq. 67 generally (noting that  $\frac{b}{2}$  must here be considered as negative in order to make the notation consistent) we obtain

$$e = \frac{1}{3} \left[ -\frac{b}{2} + (-y_l - y_r) \right],$$

which reduces to

$$e = -\frac{1}{3} \left[ \frac{b}{2} + y_l + y_r \right]. \quad . \quad . \quad . \quad . \quad . \quad (69)$$

As the algebraic signs tend to create confusion in these formulae, it is more simple to remember that for a triangle lying on *both* sides of the center  $e$  is always numerically equal to  $\frac{1}{3} \left[ \frac{b}{2} + (x_l \sim x_r) \right]$ , and for a triangle entirely on one side,  $e$  is numerically equal to  $\frac{1}{3} \left[ \frac{b}{2} + \text{the numerical sum of the two distances out} \right]$ . The algebraic sign of  $e$  is readily determinable as in § 91.

**93. Example of curvature correction.** Assume that the fill in Sta. 178 occurred on a  $6^\circ$  curve to the *right*.  $\frac{1}{3R} = \frac{1}{2865}$ . The quantities 210, 507, etc., represent the quantities  $V'$ ,  $V''$ , etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l \sim x_r)}{3R} = \frac{210(22.9 - 8.2)}{2865} = \frac{3101.7}{2865} = +1.$$

The sign is plus, since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is +3, and the correction for the whole section is  $1+3=4$ . For Sta. 18+40 the correction computed as 6 yards. Therefore, for the 40 feet, the correction is  $\frac{4}{100}(3+6)=3.6$ , which is called 4. Computing the others similarly we obtain a total correction of +16 cubic yards.

**94. Accuracy of earthwork computations.** The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prisms which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prisms, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the method of slope angle and center depth (§ 73), and that a cross-section, assumed as uniform, sags 0.1 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is  $\frac{1}{3}(20 \times 100) \times 0.5 = 333$  cub. ft. = 12 cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes

practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

**95. Approximate computations from profiles.** As a means of comparing the relative amounts of earthwork on two or more proposed routes which have been surveyed by preliminary surveys, it will usually be sufficiently accurate to compare the areas of cutting (assuming that the cut and fill are approximately balanced) as shown by the several profiles. The errors involved may be large in individual cases and for certain small sections, but fortunately the errors (in comparing two lines) will be largely compensated. The errors are much larger on side-hill work than when the cross-sections are comparatively level. The errors become large when the depth of cut or fill is very great. If the lines compared have the same general character as to the slope of the cross-sections, the proportion of side-hill work, and the average depth of cut or fill, the error involved in considering their relative volumes of cutting to be as the relative areas of cutting on the profiles (obtained perhaps by a planimeter) will probably be small. If the volume in each case is computed by assuming the sections as *level*, with a depth equal to the center cut, the error involved will depend only on the amount of side-hill work and the degree of the slope. If these features are about the same on the two lines compared, the error involved is still less.

#### FORMATION OF EMBANKMENTS.

**96. Shrinkage of earthwork.** The evidence on this subject as to the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently with respect to shrinkage. There has been but little uniformity in the classification of earths in the tests and experiments that have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

P. J. Flynn quotes some experiments (*Eng. News*, May 1, 1886) made in India, in which pits were dug having volumes of 400 to 600 cubic feet. The material, when piled into an embankment, measured largely in excess of the original measurement—as is the universal experience. The pits were refilled with the same material. As the rains, very heavy in India, settled the material in the pits, more was added to keep the pits full. Even after the rainy season was over, there was in every case material in excess. This would seem to indicate a permanent *expansion*, although it is possible that the observations were not continued for a sufficient time to determine the final settled volume.

On the contrary, notes made by Mr. Elwood Morris many years ago on the behavior of embankments of several thousand cubic yards, formed in layers by carts and scrapers, one winter intervening between commencement and completion, showed in each case a permanent *contraction* averaging about 10%.

All authorities agree that rockwork *expands* permanently when formed into an embankment, but the percentages of expansion given by different authorities differ even more than with earth—varying from 8 to 90%. Of course this very large range in the coefficient is due to differences in the character of the rock. The softer the rock and the closer its similarity to earth, the less will be its expansion. On account of the conflicting statements made, and particularly on account of the influence of methods of work, but little confidence can be felt in any given coefficient, especially when given to a fraction of a per cent, but the consensus of American practice seems to average about as follows:

|                                         |           |
|-----------------------------------------|-----------|
| Permanent contraction of earth. . . . . | about 10% |
| “ expansion of rock. . . . .            | 40 to 60% |

These values for rock should be materially reduced, according to judgment, when the rock is soft and liable to disintegrate. The hardest rocks, loosely piled, may occasionally give even

higher results. The following is given by several authors as the permanent contraction of several grades of earth:

|                                   |       |     |
|-----------------------------------|-------|-----|
| Gravel or sand.....               | about | 8%  |
| Clay.....                         | "     | 10% |
| Loam.....                         | "     | 12% |
| Loose vegetable surface soil..... | "     | 15% |

It may be noticed from the above table that the harder and cleaner the material the less is the contraction. Perfectly clean gravel or sand would not probably change volume appreciably. The above coefficients of shrinkage and expansion may be used to form the following convenient table.

| Material.                 | To make 1000 cubic yards of embankment will require | 1000 cubic yards measured in excavation will make |
|---------------------------|-----------------------------------------------------|---------------------------------------------------|
| Gravel or sand.....       | 1087 cubic yards                                    | 920 cubic yards                                   |
| Clay.....                 | 1111 " "                                            | 900 " "                                           |
| Loam.....                 | 1136 " "                                            | 880 " "                                           |
| Loose vegetable soil..... | 1176 " "                                            | 850 " "                                           |
| Rock, large pieces.....   | 714 " "                                             | 1400 " "                                          |
| " small ".....            | 625 " "                                             | 1600 " "                                          |
|                           | measured in excavation                              | of embankment.                                    |

97. Allowance for shrinkage. On account of the initial expansion and subsequent contraction of earth, it becomes necessary to form embankments higher than their required ultimate form in order to allow for the subsequent shrinkage. As the shrinkage appears to be all vertical (practically), the embankment must be formed as shown in Fig. 58. The effect of shrink-

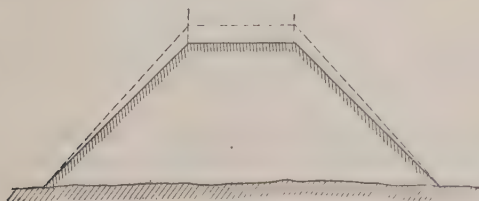


FIG. 58.

ge should not be confounded with that of slipping of the sides, which is especially apt to occur if the embankment is subjected

to heavy rains very soon after being formed, and also when the embankments are originally steep. It is often difficult to form an embankment at a slope of 1 : 1 which will not slip more or less before it hardens.

Very high embankments shrink a greater percentage than lower ones. Various rules giving the relation between shrinkage and height have been suggested, but they vary as badly as the suggested coefficients of contraction, probably for the same causes. As the fact is unquestionable, however, the extra height of the embankment must be varied somewhat as in Fig. 59, which represents a longitudinal section of an embankment.

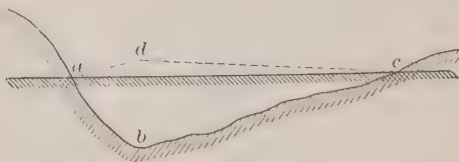


FIG. 59.

As considerable time generally elapses between the completion of the embankment and the actual running of trains, the grade *ad* will generally be nearly flattened down to its ultimate form before traffic commences, but such grades are occasionally objectionable if added to what is already a ruling grade. With some kinds of soil the time required for complete settlement may be as much as two or three years, but, even in such cases, it is probable that one-half of the settlement will take place during the first six months. The engineer should therefore require the contractor to make all fills about 8 to 15% (according to the material) higher than the profiles call for, in order that subsequent shrinkage may not reduce it to less than the required volume.

**98. Methods of forming embankments.** When the method is not otherwise objectionable, a high embankment can be formed very cheaply (assuming that carts or wheelbarrows are used) by dumping over the end and building to the full height (or even higher, to allow for shrinkage) as the embankment proceeds. This allows more time for shrinkage, saves nearly all the cost of spreading (see Item 4, § 111), and reduces the cost of roadways



(Item 5). Of course this method is especially applicable when the material comes from a place as high as or higher than grade, so that no up-hill hauling is required.

Another method is to spread it in layers two or three feet thick (see Fig. 60), which are made concave upwards to avoid

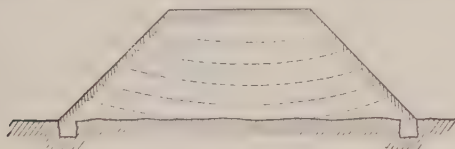


FIG. 60.

possible sliding on each other. Spreading in layers has the advantage of partially ramming each layer, so that the subsequent shrinkage is very small. Sometimes small trenches are dug along the lines of the toes of the embankment. This will frequently prevent the sliding of a large mass of the embankment, which will then require extensive and costly repairs, to say nothing of possible accidents if the sliding occurs after the road is in operation. Incidentally these trenches will be of value in draining the subsoil. When circumstances require an embankment on a hillside, it is advisable to cut out "steps" to prevent a possible sliding of the whole embankment. Merely ploughing the side-hill will often be a cheaper and sufficiently effective method.

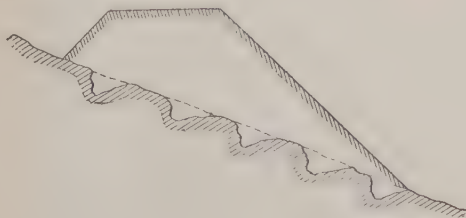


FIG. 61.

Occasionally the formation of a very high and long embankment may be most easily and cheaply accomplished by building a trestle to grade and opening the road. Earth can then be

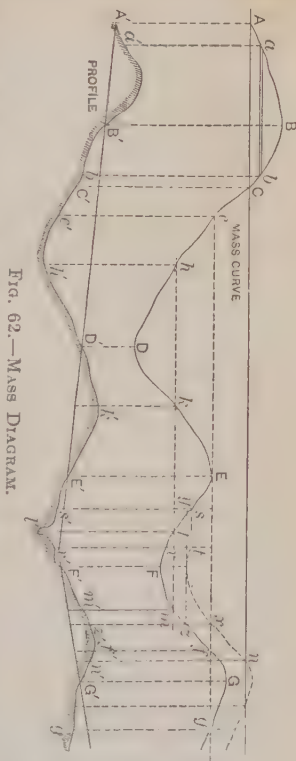
procured where most convenient, perhaps several miles away, loaded on cars with a steam-shovel, hauled by the trainload, and dumped from the cars with a patent unloader. On such a large scale, the cost per yard would be very much less than by ordinary methods—enough less sometimes to more than pay for the temporary trestle, besides allowing the road to be opened for traffic very much earlier, which is often a matter of prime financial importance. It may also obviate the necessity for extensive borrow-pits in the immediate neighborhood of the heavy fill and also utilize material which would otherwise be wasted.

#### COMPUTATION OF HAUL.

**99. Nature of subject.** As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embankment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

**100. Mass diagram.** In Fig. 62 let  $A'B' \dots G'$  represent a profile and grade line drawn to the usual scales. Assume  $A'$  to be a point past which no earthwork will be hauled. Such a point is determined by natural conditions, as, for example, a river crossing, or one end of a long level stretch along which no grading is to be done except the formation of a low embankment from the material excavated from ample drainage ditches on each side. Above the profile draw an indefinite horizontal line ( $ACu$  in Fig. 62), which may be called the "zero line." Above every station point in the profile draw an ordinate (above or be-

low the zero line) which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill -) from the point  $A'$  to the point considered. The computations of these ordinates should first be made in tabular form as shown below. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 97), will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. Place in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills -) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether



the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve  $A, B, \dots G$  may be obtained by joining the extremities of the ordinates.

| Sta.    | Yards, { cut +<br>fill - | Material.   | Shrinkage<br>factor. | Yards,<br>reduced<br>for<br>shrinkage. | Ordinate<br>in mass<br>curve. |
|---------|--------------------------|-------------|----------------------|----------------------------------------|-------------------------------|
| 46 + 70 |                          |             |                      |                                        | 0                             |
| 47      | + 195                    | Clayey soil | - 10 per cent        | + 175                                  | + 175                         |
| 48      | + 1792                   | " "         | - 10 "               | + 1613                                 | + 1788                        |
| + 60    | + 614                    | " "         | - 10 "               | + 553                                  | + 2341                        |
| 49      | - 143                    |             |                      | - 143                                  | + 2198                        |
| 50      | - 906                    |             |                      | - 906                                  | + 1292                        |
| 51      | - 1985                   |             |                      | - 1985                                 | - 693                         |
| 52      | - 1721                   |             |                      | - 1721                                 | - 2414                        |
| + 30    | - 112                    |             |                      | - 112                                  | - 2526                        |
| 53      | + 177                    | Hard rock   | + 60 per cent        | + 283                                  | - 2243                        |
| + 70    | + 180                    | " "         | + 60 "               | + 289                                  | - 1954                        |
| 54      | - 52                     |             |                      | - 52                                   | - 2006                        |
| + 42    | - 71                     |             |                      | - 71                                   | - 2077                        |
| 55      | + 276                    | Clayey soil | - 10 per cent        | + 249                                  | - 1828                        |
| 56      | + 1242                   | " "         | - 10 "               | + 1118                                 | - 710                         |
| 57      | + 1302                   | " "         | - 10 "               | + 1172                                 | + 462                         |

### 101. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at  $B, D, E, F$ , and  $G$ ) when passing from cut to fill or from fill to cut.

3. When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at  $A$  and  $C$ ) it shows (in this instance) that the cut between  $A'$  and  $B'$  will just provide the material required for the fill between  $B'$  and  $C'$ , and that no material should be hauled past  $C'$ , or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as  $ab$ ), it indicates that the cut and fill between  $a'$  and  $b'$  will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation

of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance  $dx$  apart, as at  $ab$ , the small increment of cut  $dx$  at  $a'$  will fill the corresponding increment of fill at  $b'$ , and this material must be hauled the distance  $ab$ . Therefore the product of  $ab$  and  $dx$ , which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at  $ab$ , and the total area  $ABC$  represents the summation of volume times distance for all the earth movement between  $A'$  and  $C'$ . This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at  $E$  and cutting the curve at  $e$ ,  $f$ , and  $g$ , shows that the cut and fill between  $e'$  and  $E'$  will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between  $C'$  and  $e'$ , use the material between  $D'$  and  $E'$  for the fill between  $e'$  and  $D'$ , and similarly balance cut and fill between  $E'$  and  $f'$  and also between  $f'$  and  $g'$ .

8. Similarly the horizontal line  $hklm$  may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between  $C'$  and  $h'$  would be made by borrowing; the cut and fill between  $h'$  and  $k'$  would balance; also that between  $k'$  and  $l'$  and between  $l'$  and  $m'$ . Since the area  $chDkE$  represents the measure of haul for the earth between  $e'$  and  $E'$ , and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas  $chDkE$  and  $ElFmf$ , which is the measure of haul of all the material between  $e'$  and  $f'$ , is largely in excess of the sum of the areas  $hDk$ ,  $kEl$ , and  $lFm$ , plus the somewhat uncertain measures of haul due to borrowing material for  $e'h'$  and wasting the material between  $m'$  and  $f'$ . Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount

of fill between  $e'$  and  $h'$  is represented by the *difference* of the ordinates at  $e$  and  $h$ , and similarly for  $m'$  and  $j'$ , it follows that the amount to be borrowed between  $e'$  and  $h'$  will exactly equal the amount wasted between  $m'$  and  $j'$ . By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 116).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between  $s'$  and  $r'$ , thus saving an amount in fill equal to  $tr$ . If such had been the original design, the mass curve would have been a straight horizontal line between  $s$  and  $t$  and would continue as a curve which would be at all points a distance  $tr$  above the curve  $rFmzjGig$ . If the line  $Ej$  is to be used as a zero line, its intersection with the new curve at  $x$  will show that the material between  $E'$  and  $z'$  will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line  $Ex$  and the broken line  $Estx$ . The same computed result may be obtained without drawing the auxiliary curve  $trn \dots$  by drawing the horizontal line  $zy$  at a distance  $xz (=tr)$  below  $Ex$ . The amount of the haul can then be obtained by adding the triangular area between  $Es$  and the horizontal line  $Ex$ , the rectangle between  $st$  and  $Ex$ , and the irregular area between  $rFz$  and  $y \dots z$  (which last is evidently equal to the area between  $tx$  and  $E \dots x$ ). The disposal of the material at the right of  $z'$  would then be governed by the indications of the profile and mass diagram which would be found at the right of  $g'$ . In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

102. **Area of the mass curve.** The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy

as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let  $y_0 \dots y_n$  be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 100. Let the uniform distance between ordinates ( $=100$  feet) be called 1, i.e., one *station*. Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \frac{1}{3}[y_0 + 4(y_1 + y_3 + \dots y_{n-1}) + 2(y_2 + y_4 + \dots y_{n-2}) + y_n]. \quad (70)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by  $\frac{1}{2} \times 1$ . An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 62) is shifted to  $cE$ , the drop from  $AC$  (produced) to  $E$  is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 6," § 100) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

**103. Value of the mass diagram.** The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the



extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 116. For the present it may be said that with each method of carrying material there is some limit beyond which the expense of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 62,  $eE$  or  $Ef$  exceeds the limit of profitable haul, it shows at once that some such line as  $hkln$  should be drawn and the material disposed of accordingly.

104. **Changing the grade line.** The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa*. Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be possible to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the mass

curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

**105. Limit of free haul.** It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 63 represent a pro-

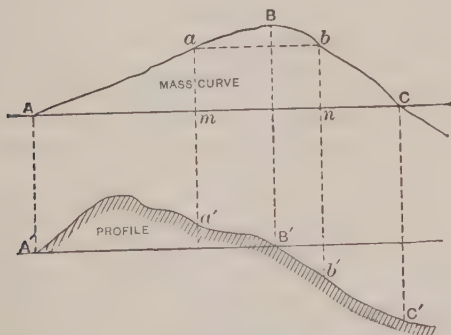


FIG. 63.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, *a* and *b*, in the mass curve which are on the same horizontal line and which are 800 feet apart. Project these points down to *a'* and *b'*. Then the cut and fill between *a'* and *b'* will just balance, and the cut between *A'* and *a'* will be needed for the fill between *b'* and *C'*. In the mass curve, the area between the horizontal line *ab* and the curve *aBb* represents the haulage of the material between *a'* and *b'*, which is all free. The rectangle *abmn* represents the haulage of the material in the cut *A'a'* across the 800 feet from *a'* to *b'*. This is also free. The sum of the two areas *Aam* and *bnC* represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point

$B'$ , the mass curve would be a symmetrical curve about the vertical line through  $B$ , and the two limiting lines of free haul would be placed symmetrically about  $B$  and  $B'$ . In general there is no such symmetry, and frequently the difference is considerable. The area  $aBbnm$  will be materially changed according as the two vertical lines  $am$  and  $bn$ , always 800 feet apart, are shifted to the right or left. It is easy to show that the area  $aBbnm$  is a *maximum* when  $ab$  is horizontal. The minimum value would be obtained either when  $m$  reached  $A$  or  $n$  reached  $C$ , depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since  $aBbnm$  is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas  $Aam$  and  $bCn$  may be obtained as in § 102. If the whole area  $AaBbC$  has been previously computed, it may be more convenient to compute the area  $aBbnm$  and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded: the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

#### ELEMENTS OF THE COST OF EARTHWORK.

(The following analysis of the cost of earthwork follows the general method given in the well-known papers published by

Ellwood Morris, C.E., in the *Journal of the Franklin Institute* in September and October, 1841. Numerous corroborative data have been obtained from various other sources, and also figures on methods not then in vogue.)

106. **General divisions of the subject.** The variations in the cost of earthwork are caused by the greatly varying conditions under which the work is done, chief among which is character of material, method of carriage, and length of haul. Any general system of computation must therefore differentiate the total cost into such elementary items that all differences due to variations in conditions may be allowed for. The variations due to character of material will be allowed for by an estimate on loose light sandy soil, and also an estimate on the heaviest soils, such as stiff clay and hard-pan. These represent the extremes (excluding rock, which will be treated separately), and the cost of intermediate grades must be estimated by interpolating between the extreme values. The general divisions of the subject will be:\*

1. Loosening.
2. Loading.
3. Hauling.
4. Spreading.
5. Keeping roadways in order.
6. Repairs, wear, depreciation, and interest on cost of plant.
7. Superintendence and incidentals.
8. Contractor's profit.

By making the estimates on the basis of \$1 per day for the cost of common labor, it is a simple matter to revise the estimates according to the local price of labor by multiplying the final estimates of cost by the price of labor in dollars per day.

107. **Item 1. LOOSENING.** (a) **Ploughs.** Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material, to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of \$5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of \$3.87 per day, being \$1 each for

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\* Trautwine.

the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

(b) **Picks.** When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate \* for a fair day's work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At \$1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated † as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) **Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay are most economically loosened by blasting. The subject of blasting will be taken up later, §§ 117-123.

(d) **Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

108. Item 2. **LOADING.** (a) **Hand-shovelling.** Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on

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\* Trautwine.

† Hurst.

account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the average of 15 to 25 cubic yards be accepted, it means, on the basis of \$1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) **Steam-shovels.\*** Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from  $\frac{1}{2}$  to  $2\frac{1}{2}$  cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The capacity of the larger sizes is about 3000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about \$5000, will average about \$1000 per month. Of this the engineer will get \$100; the

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\* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.



fireman \$50; the cranesman \$90; repairs perhaps \$250 to \$300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing \$100 per month; about five laborers and a foreman, the laborers getting \$1.25 per day and the foreman \$2.50 per day, which will amount to \$227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of \$100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

109. Item 3. **HAULING.** The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same, whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) **Carts.** The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling the load and a little faster when returning;



empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by  $s$ , the number of loads handled in 10 hours (600 minutes) would be  $600 \div (s+4)$ . The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;  
 $3\frac{1}{2}$  " " " " " level hauling; and  
 4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to  $300 + (14 \times 20) = 580$  feet on a level.

Trautwine assumes the average load for all classes of work to be  $\frac{1}{3}$  cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to  $\frac{600}{3(s+4)}$ . Dividing the cost of a cart per day by the number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver.

75 c. is allowed for the horse, which is supposed to be the total cost, including that for Sundays and rainy days. 25 c. more is allowed for the cart, harness, repairs, etc., thus making a total cost of \$1.25 per day. Some contractors employ a greater number of drivers and expect each to assist in loading. There is found to be no saving in total cost per yard, while the chances of loafing are perhaps greater. Morris instances five actual cases in which the cost of the cart (reduced to the basis of \$1 per day for labor) varied from \$1.37 to \$1.48. The items of these costs were not given.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 97), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of  $\frac{1}{3}$  cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be  $\frac{600}{5(s+6)}$ .

$$\text{Cost per yard in cents} = \frac{125 \times 5(s+6)}{600} \dots (71)$$

(b) **Wagons.** For longer leads (i.e., from  $\frac{1}{3}$  to  $\frac{2}{3}$  of a mile) wagons drawn by two horses have been found most economical. The wagons have bottoms of loose thick narrow boards and are unloaded very easily and quickly by lifting the individual boards and breaking up the continuity of the bottom, thus depositing the load directly underneath the wagon. The capacity is about one cubic yard. The cost may be estimated on the same principles as that for carts.

(c) **Wheelbarrows.** According to Trautwine, the speed of moving wheelbarrows may be considered the same as for carts, 200 feet per minute; the time spent in loading and dumping is  $1\frac{1}{2}$  minutes, and in addition about  $\frac{1}{10}$  of the time is wasted in short rests, adjusting the wheeling planks, etc. On the basis of \$1 per day for labor, an allowance of 5 c. for the barrow, and 14 loads per cubic yard, the cost of hauling per cubic yard (computed on the same principles as above) will be

$$\frac{105 \times 14(s+1.25)}{600 \times 0.9} \dots (72)$$

or rockwork the number of loads per cubic yard is estimated 24, and the time spent in loading, etc., estimated at 1.6 minutes instead of 1.25 minutes, which makes the estimate

$$\text{Cost per cubic yard} = \frac{105 \times 24(s + 1.6)}{600 \times 0.9} \quad \dots (73)$$

(d) **Scrapers.\*** Scrapers, or scoops, are especially useful in canal work, and also for railroad work when a low embankment is to be formed from borrow-pits at the sides, when the distance does not exceed 100 feet, nor the vertical height 15 feet. The slope should not exceed 1.5 to 1. Under these conditions scraper work is cheaper than any other method. Scooping may be done all in one direction, in which case two half-turns are made for each load moved; or it may be done in both directions (from both sides on to a bank, or, in canal work, from the center to each bank), in which case one load is hauled to each half-turn. The capacity of the scoops (the "drag" variety) is  $\frac{1}{10}$  cubic yard; the time lost in loading, unloading, and all other ways per load (except in turning) will average  $\frac{2}{3}$  minute; the time lost in each half-turn (semi-circle) is  $\frac{1}{3}$  minute; the speed of the horses may be estimated as 70 feet of *lead* per minute, the lead being here considered as the *sum* of the vertical and horizontal distances, and the estimate including the time of going and returning. If  $a$  represents the sum of the horizontal and vertical distances, the number of cubic yards handled per day of 10 hours by "side-scooping" will be

$$0.1 \left( \frac{600}{\frac{a}{70} + 1\frac{1}{3}} \right), \quad \text{which equals} \quad \frac{4200}{a + 93\frac{1}{3}}.$$

For "double-scooping" the formula becomes

$$0.1 \left( \frac{600}{\frac{a}{70} + 1} \right), \quad \text{which equals} \quad \frac{4200}{a + 70}.$$

Dividing the cost of a scraper per day (estimated at \$2.75) by the number of yards handled per day gives the average cost per yard.

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\* Condensed from Journ. Franklin Inst., Oct. 1841, by Morris.

Except in very loose sandy soil it is best to plough the earth first, which will cost *about* 1 c. per yard. (See § 107.) Drag-scrapers are now made chiefly of steel, and their capacity is more nearly 0.15 cubic yard. Wheeled scrapers, having a capacity of about 0.5 cubic yard, are frequently used with even greater economy and for greater distances, as they are cheaper than carts up to 250 or 300 feet of lead. Both drag- and wheel-scrapers are best operated in gangs of perhaps 10, using extra or "snap" teams to help load, and a few extra men to help in loading and unloading. The average cost of one scraper per day may thus be easily calculated and the average number of cubic yards handled per day computed as above, from which the cost per yard may be estimated.

(e) **Cars and horses.** The items of cost by this method are (a) charge for horses employed, (b) charge for men employed strictly in hauling, (c) charge for shifting rails when necessary, (d) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 6, mentioned in § 106, but it is perhaps more convenient to estimate them as follows:

The traction of a car on rails is so very small and constant that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is  $\frac{1}{120}$ , a grade of 1 : 120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1 : 120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low

borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of  $3\frac{1}{2}$  cubic yards, weighing 30 cwt. empty. Two horses took five “wagons”  $\frac{1}{3}$  of a mile on a level railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the “straight road,” another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled  $22\frac{1}{2}$  miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs., or 28.65 net tons. Allowing  $\frac{1}{120}$  as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to  $1\frac{1}{2}$  horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. The cars generally used in this country have a capacity of  $1\frac{1}{2}$  cubic yards and cost about \$65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length

of haul), costing about \$2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about \$100 each. Some cars can be obtained as low as \$70. A force of about five men and a foreman will be required to shift the tracks. The track shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal  $\frac{10}{\frac{1}{5} (\text{miles of lead}) + .15}$  or  $\frac{50}{(\text{miles of lead}) + .75}$ . Of course this quotient *must* be a whole number. Knowing the number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

- (a) Wages of engineer, who frequently fires his own engine;
  - (b) Fuel, about  $\frac{1}{4}$  to 1 ton of bituminous coal, depending on work done;
  - (c) Water, a very variable item, frequently costing \$3 to \$5 per day;
  - (d) Repairs, variable, frequently at rate of 50 to 60% per year;
  - (e) Interest on cost and depreciation, 16 to 40%.
- To these must be added, to obtain the total cost of the haul,
- (f) Wages of the gang employed in shifting track.

**110. Choice of method of haul dependent on distance.** In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the roadbed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-



scrapers and wheelbarrows are equally economical. Between 0 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrappers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this carts on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and to make the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the question of "limit of profitable haul" (§ 116) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

**III. Item 4. SPREADING.** The cost of spreading varies with the method employed in dumping the load. When the earth is dropped over the edge of an embankment there is little if any necessary work. Trautwine allows about  $\frac{1}{4}$  c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at \$1 per day attending to the unloading of 1200 two-wheeled carts each carrying  $\frac{1}{4}$  cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

**Item 5. KEEPING ROADWAYS IN ORDER.** This feature is as important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in



such matters will cost many times as much in increased labor hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of the planks is done by the wheelers, and the time for it is allowed for in the 10% allowance for "short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable, but the method is frequently used with great economy. The variations in the requirements render any general estimate of such cost impracticable.

(b) **Carts and wagons.** The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead*. The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade at some one point, are often measures of true economy. Trautwine suggests  $\frac{1}{10}$  c. per cubic yard per 100 feet of lead for earthwork and  $\frac{2}{10}$  c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

**113. Item 6. REPAIRS, WEAR, DEPRECIATION, AND INTEREST ON COST OF PLANT.** The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates  $\frac{1}{4}$  c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working.

tools, the life of all being limited to a few years, and of many tools to a few months.

**114. Item 7. SUPERINTENDENCE AND INCIDENTALS.** The incidentals include water-carriers, trimming cuts to grade, digging the side ditches, trimming up the sides of borrow-pits to prevent their becoming unsightly, etc. These last operations yield but little earth and cost far more than the price paid per cubic yard. Morris allows 1 c. per cubic yard for this item; Trautwine allows  $1\frac{1}{2}$  to 2 c. for it; while others combine items 6 and 7 and call them 5% of the total cost, which method has the merit of making the cost of items 6 and 7 a function of the character of soil and length of lead.

**115. Item 8. CONTRACTOR'S PROFIT.** This is usually estimated at from 6 to 15%, according to the sharpness of the competition and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

**116. Limit of profitable haul.** As intimated in §§ 103 and 110, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 62, that the cut and fill will exactly balance between two points, as between  $e$  and  $x$ , assuming that, as indicated in § 101 (9), a trestle has been introduced between  $s$  and  $t$ , thus altering the mass curve to *Estan*... Since there is a balance between  $A'$  and  $C'$ , the material for the fill between  $C'$  and  $e'$  must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation

between  $z'$  and  $n'$ . If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill  $C'e'$  implies a wastage of material at the cut  $z'n'$ . To compare the two methods, we may place against the plan of borrowing and wasting, (a) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill  $C'e'$ ; (b) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing  $M$  cubic yards for the fill  $C'e'$ ; (c) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut  $z'n'$  and of the spoil-bank, and the other expenses incidental to wasting  $M$  cubic yards at the cut  $z'n'$ ; (d) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure  $Cexn$ ), and the other expenses incidental to making the fill  $C'e'$  with the material from the cut  $z'n'$ , the amount of material being  $M$  cubic yards, which is represented in the figure by the vertical ordinate from  $e$  to the line  $Cn$ . The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank, *plus* the cost of loosening, loading, etc. (except hauling and roadways) of  $M$  cubic yards, *minus* the difference in cost of the excessive haul from  $Ce$  to  $xn$  and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 109, a)  $[125 \times 3(1+4)] \div 600 = 3.125$  c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume  $M=10000$  cubic yards and the area  $Cexn=180000$  yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost  $[125 \times 3(18+4)] \div 600 = 13.75$  c. per cubic yard. The cost of roadways will be  $18 \times .1$  or 1.8 c., making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be  $15.55 - (2 \times 3.225) = 9.10$  c. per yard or \$910

for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing \$895. These figures may be better compared as follows:

|                              |   |                                                    |                  |
|------------------------------|---|----------------------------------------------------|------------------|
| LONG HAUL.                   | { | Loosening, etc., 10000 yards, @ 8.95 c.            | \$ 895.          |
|                              |   | Hauling, " 10000 " @ 15.55 c.                      | 1555.            |
|                              |   |                                                    | <u>\$2450.</u>   |
| BORROWING<br>AND<br>WASTING. | { | Loosening, etc., 10000 yards (borrowed), @ 8.95 c. | \$895.           |
|                              |   | " " 10000 " (wasted), @ 8.95 c.                    | 895.             |
|                              |   | Hauling, etc., 10000 " (borrowed), @ 3.225 c.      | 322.50           |
|                              |   | " " 10000 " (wasted), @ 3.225 c.                   | 322.50           |
|                              |   |                                                    | <u>\$2435.00</u> |

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

#### BLASTING.

**117. Explosives.** The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (a) slow-burning and (b) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infusorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning

class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

**118. Drilling.** Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 64, (a) and (b). The width should flare at the bottom (a) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 64, (a). Sometimes the angle of the two faces is varied from that given, Fig. 64, (b), and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days.

For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long,  $1\frac{1}{4}$ " in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is

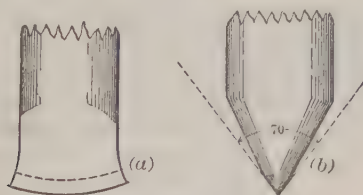


FIG. 64.

inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air for tunnel-work, thus doing the additional service of supplying fresh air to the tunnel-headings where it is most needed. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

**119. Position and direction of drill-holes.** As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so



locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the "line of least resistance." In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the

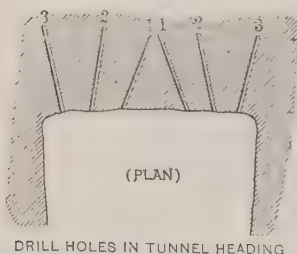


FIG. 65.

rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 65; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock, a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.

**120. Amount of explosive.** The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is  $\frac{3}{4}$  of the depth of the hole; also when the powder fills about  $\frac{1}{3}$  of the hole. For average rock the amount of powder required is as follows:

|                               |                   |        |                     |         |
|-------------------------------|-------------------|--------|---------------------|---------|
| Line of least resistance..... | 2 ft.             | 4 ft.  | 6 ft.               | 8 ft.   |
| Weight of powder.....         | $\frac{1}{4}$ lb. | 2 lbs. | $6\frac{1}{4}$ lbs. | 16 lbs. |

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1" hole, drilled 2' 8" deep, with its line of least resistance 2', and loaded with  $\frac{1}{4}$  lb. of powder, would



be filled to a depth of  $9\frac{1}{2}$ ", which is nearly  $\frac{1}{3}$  of the depth. A 3" hole, drilled 8' deep, with its line of least resistance 6', and loaded with  $6\frac{1}{4}$  lbs. of powder, would be filled to a depth of over 28", which is also nearly  $\frac{1}{3}$  of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of  $\frac{1}{4}$  to  $\frac{1}{3}$  lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

**121. Tamping.** Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best, but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

**122. Exploding the charge.** Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by paper, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping is carefully packed around it so that it will not be injured. To

produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

**123. Cost.** Trautwine estimates the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to 60 cents and even \$1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Soft tough rock frequently requires more powder than harder brittle rock.

**124. Classification of excavated material.** The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (a) earth, (b) loose rock, and (c) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

**Earth.** This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

**Loose rock.** This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

**Solid rock** includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

**125. Specifications for earthwork.** The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

**SOLID ROCK** shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

**LOOSE ROCK** shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

**HARD-PAN** shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two muckers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hard pan as above defined.

POWDER. The use of powder in cuts will not be considered as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared to the extent required by the said Engineer Maintenance of

Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half ( $2\frac{1}{2}$ ) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned hereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation

or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

## CHAPTER IV.

### TRESTLES.

**126. Extent of use.** Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about \$75,000,000. The annual charge for maintenance, estimated at  $\frac{1}{3}$  of the cost, therefore amounted to about \$9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B. M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

*a. Permanent trestles.*

1. Those of *extreme* height—then called viaducts and frequently constructed of iron or steel, as the Kinzua viaduct, 302 ft. high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

*b. Temporary trestles.*

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the



size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

**127. Trestles vs. embankments.** Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps  $\frac{1}{2}$  of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 126. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rule, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the

use of iron or steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of iron or steel trestles will be considered in this chapter.

**128. Two principal types.** There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet. They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called “bents,” and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the “bents” are all that need be considered separately.

#### PILE TRESTLES.

**129. Pile bents.** A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal “cap.” The caps are fastened to the tops of the piles by methods illustrated in Fig. 66. The method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 66 (*a* and *d*) illustrates a mortise-joint with a hardwood pin about  $1\frac{1}{4}$ " in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally an iron dowel (an iron pin about  $1\frac{1}{2}$ " in diameter and about 6" long) is inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 66 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. “Split caps,” shown in Fig. 66 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the

decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 136.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 66 (a). Up to a height

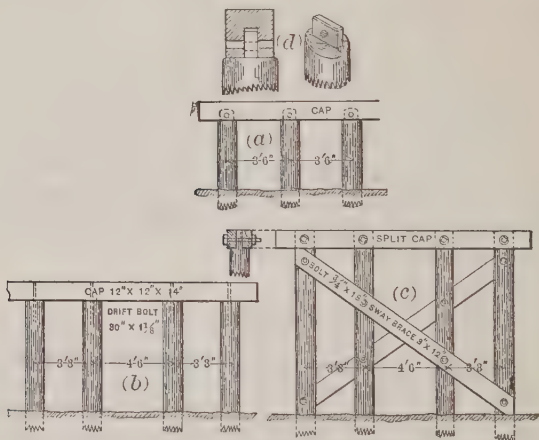


FIG. 66.

of 8 or 10 feet four piles may be used without sway-bracing, as in Fig. 66 (b), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

- |                |               |              |               |
|----------------|---------------|--------------|---------------|
| 1. Red cedar   | 5. White pine | 9. White oak | 12. Black oak |
| 2. Red cypress | 6. Redwood    | 10. Post-oak | 13. Hemlock   |
| 3. Pitch-pine  | 7. Elm        | 11. Red oak  | 14. Tamarac   |
| 4. Yellow pine | 8. Spruce     |              |               |

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather

weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

**130. Methods of driving piles.** The following are the principal methods of driving piles:

*a.* A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

*b.* The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

*c.* *Gunpowder pile-drivers*, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is attempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

*d.* *Steam pile-drivers*, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. "The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute." Such a driver would cost about \$800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water

is available, the *water-jet* is sometimes employed. A pipe is fastened along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off frequently, and especially should it be done just before the final blows which are to test its bearing-power.

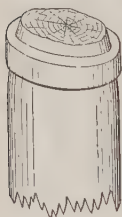


FIG. 67.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

**131. Pile-driving formulæ.** If  $R$  = the resistance of a pile, and  $s$  the set of the pile during the last blow,  $w$  the weight of the pile-hammer, and  $h$  the fall during the last blow, then we may state the approximate relation that  $Rs = wh$ , or  $R = \frac{wh}{s}$ .

This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means equal to the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow

properly for all modifying causes. As the simplest rule, specifications sometimes require that the piles shall be driven until the pile will not sink more than 5 inches under five consecutive blows of a 2000-lb. hammer falling 25 feet. The "*Engineering*

*News* formula" \* gives the safe load as  $\frac{2wh}{s+1}$ , in which  $w$  = weight of hammer,  $h$  = fall in feet,  $s$  = set of pile in inches under the last blow. This formula is derived from the above basic formula by calling the safe load  $\frac{1}{3}$  of the final resistance, and by adding (arbitrarily) 1 to the final set ( $s$ ) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a

steam pile-driver the formula becomes safe load =  $\frac{2wh}{s+0.1}$ . For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes safe load =  $\frac{4wh}{s+0.1}$ . In these last two

formulae the constant in the denominator is changed from  $s+1$  to  $s+0.1$ . The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulae have been given on account of their simplicity and their practical agreement with experience. Many other formulae have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

*Examples.* 1. A pile was driven with an ordinary hammer weighing 2500 pounds until the sinking under five consecutive blows was  $15\frac{1}{2}$  inches. The fall of the hammer during the last

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\* *Engineering News*, Nov. 17, 1892.



blows was 24 feet. What was the safe bearing power of the pile?

$$\frac{2wh}{s+1} = \frac{2 \times 2500 \times 24}{(\frac{1}{3} \times 15.5) + 1} = \frac{120000}{4.1} = 29300 \text{ pounds.}$$

2. Piles are being driven into a firm soil with a steam pile-driver until they show a *safe* bearing power of 20 tons. The hammer weighs 5500 pounds and its fall is 40 inches. What should be the sinking under the final blow?

$$40000 = \frac{2wh}{s+0.1} = \frac{2 \times 5500 \times 3.33}{s+0.1},$$

$$= \frac{36667}{40000} - 0.1 = .81 \text{ inch.}$$

**132. Pile-points and pile-shoes.** Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken



(a)



(b)

Fig. 68.

logs, or other obstructions which are liable to turn the point, it is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 68 (b). The cast-iron form shown in Fig. 68 (a) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally.

**133. Details of design.** No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The piles are generally required to be not less than 10" or 12" in diameter at the large end. The P. R. R. requires that they shall



be "not less than 14 and 7 inches in diameter at butt and small end respectively, exclusive of bark, which must be removed." The removal of the bark is generally required in good work. Soft *durable* woods, such as are mentioned in § 129, are best for the piles, but the caps are generally made of oak or yellow pine. The caps are generally 14 feet long (for single track) with a cross-section  $12'' \times 12''$  or  $12'' \times 14''$ . "Split caps" would consist of two pieces  $6'' \times 12''$ . The sway-braces, never used for less heights than 6', are made of  $3'' \times 12''$  timber, and are spiked on with  $\frac{3}{4}''$  spikes 8" long. The floor system will be the same as that described later for framed trestles.

**134. Cost of pile trestles.** The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber, the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 c. per lineal foot, and the cost of oak piles varies from 10 to 30 c. per foot according to the length, the longer piles costing more per foot. The cost of driving will average about \$2.50 per pile, or 7.5 to 10 c. per lineal foot. Since the cost of shifting the pile-driver is quite an item in the total cost, the cost of driving a long pile would be *less* per foot than for a short pile, but on the other hand the cost of the pile is *greater* per foot, which tends to make the total cost per foot constant. Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work*. The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

#### FRAMED TRESTLES.

**135. Typical design.** A typical design for a framed trestle bent is given in Fig. 69. This represents, with slight variations of detail, the plan according to which a large part of the framed trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple story construction.

**136. Joints.** (a) The mortise-and-tenon joint is illustrated in

Fig. 69 and also in Fig. 66 (a). The tenon should be about

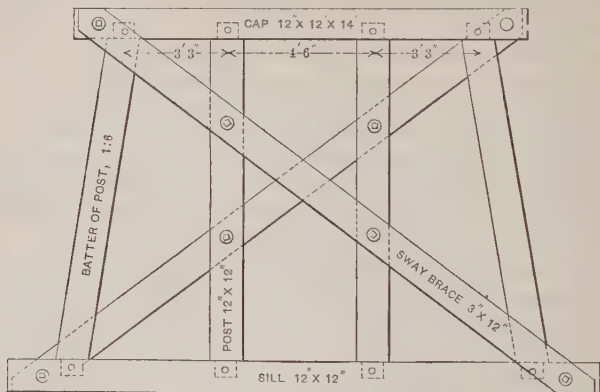


FIG. 69.

3" thick, 8" wide, and  $5\frac{1}{2}$ " long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that the mortise and tenon should be thoroughly daubed

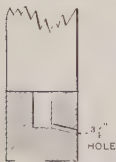


FIG. 70.

with paint before putting them together. This will tend to make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

(b) The plaster joint. This joint is made by bolting and spiking a 3" x 12" plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

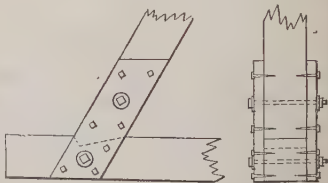


FIG. 71.

(c) Iron plates. An iron plate of the form shown in Fig. 72

(b) is bent and used as shown in Fig. 72 (a). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

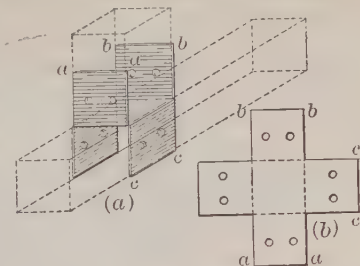


FIG. 72.

(d) **Split caps and sills.**

These are described in

§ 129. Their advantages apply with even greater force to framed trestles.

(e) **Dowels and drift-bolts.** These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt, which has been driven its full length, without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

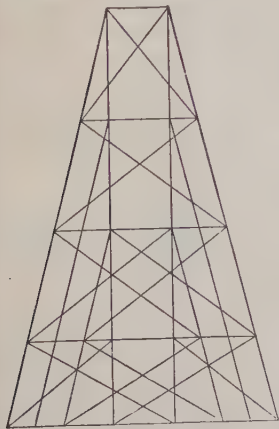


FIG. 73.

**137. Multiple-story construction.** Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 73 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly

facilitates both the original construction and subsequent repairs.

In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more difficult to repair.

Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the

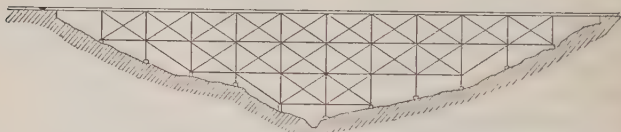


FIG. 74.

upper stories of uniform height and let the odd amount go to the lowest story, as shown in Figs. 73 and 74.

**138. Span.** The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy demands the adoption of a span that shall make the sum of these require-

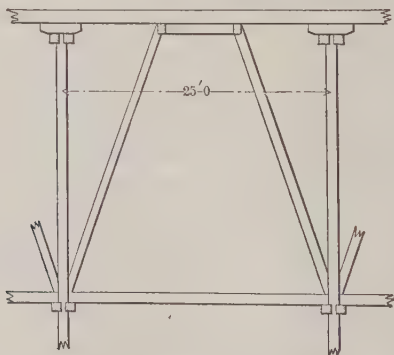


FIG. 75.

ments a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many

roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12' 6" for all single-story trestles, and a span of 25' for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

**139. Foundations.** (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 76, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 76.

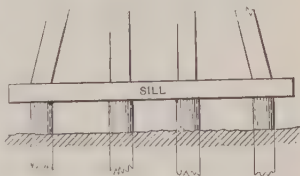


FIG. 76.

(b) **Mud-sills.** Fig. 77 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks  $12'' \times 12'' \times 6'$  are used under each bent. When the ground is very soft, two additional timbers ( $12'' \times 12'' \times$  length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

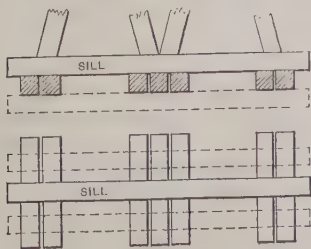


FIG. 77.

(c) **Stone foundations.** Stone foundations are the best and the most expensive. For very high trestles the Norfolk and

Western R. R. employs foundations as shown in Fig. 78, the walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72' in height a foundation-wall 39' 6" long) the foundation is made continuous. The sill



FIG. 78.

of the trestle should rest on several short lengths of 3"×12" plank laid transverse to the sill on top of the wall.

**140. Longitudinal bracing.** This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an X in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression. 3"×12" planks are often used when the design would require tensile strength only, and 8"×8" posts are often used when compression may be expected.

**141. Lateral bracing.** Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment. 6"×6" posts, forming an X and connected at the center, will answer the purpose.

**142. Abutments.** When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 139, c).

Another method is to construct a "crib" of 10"×12" timber,

laid horizontally, drift-bolted together, securely braced and embedded into the ground. Except for temporary construction such a method is generally objectionable on account of rapid decay.

Another method, used most commonly for pile trestles, and for framed trestles having pile foundations (§ 139, *a*), is to use a pile bent at such a place that the natural surface on the *up-hill* side is not far below the

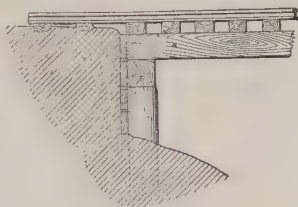


FIG. 79.

cap, and the thrust of the material, filled in to bring the surface to grade, is insignificant. 3"×12" planks are placed behind the piles, cap, and stringers to retain the filled material.

#### FLOOR SYSTEMS.

**143. Stringers.** The general practice is to use two, three, and even four stringers under each rail. Sometimes a stringer is placed under each guard-rail. Generally the stringers are made of two panel lengths and laid so that the joints alternate. A few roads use stringers of only one panel length, but this practice is strongly condemned by many engineers. The stringers should be separated to allow a circulation of air around them and prevent the decay which would occur if they were placed close together. This is sometimes done by means of 2" planks, 6' to 8' long, which are placed over each trestle bent. Several bolts, passing through all the stringers forming a group and through the separators, bind them all into one solid construction. Cast-iron "spools" or washers, varying from 4" to 3" in length (or thickness), are sometimes strung on each bolt so as to separate the stringers. Sometimes washers are used between the separating planks and the stringers, the object of the separating planks then being to bind the stringers, especially abutting stringers, and increase their stiffness.

The most common size for stringers is 8"×16". The Pennsylvania Railroad varies the width, depth, and number of stringers under each rail according to the clear span. It may be noticed that, assuming a uniform load per running foot, both the pressure per square inch at the ends of the stringers (the



caps having a width of 12") and also the stress due to transverse strain are kept *approximately* constant for the variable gross load on these varying spans.

| Clear span. | No. of pieces<br>under each rail. | Width.   | Depth.    |
|-------------|-----------------------------------|----------|-----------|
| 10 feet     | 2                                 | 8 inches | 15 inches |
| 12 "        | 2                                 | 8 "      | 16 "      |
| 14 "        | 2                                 | 10 "     | 17 "      |
| 16 "        | 3                                 | 8 "      | 17 "      |

**144. Corbels.** A corbel (in trestle-work) is a stick of timber (perhaps two placed side by side), about 3' to 6' long, placed underneath and along the stringers and resting on the cap. There are strong prejudices for and against their use, and a corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is



FIG. 80.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using corbels, may often neutralize the advantages obtained by their use.

**145. Guard-rails.** These are frequently made of 5"×8" stuff, notched 1" for each tie. The sizes vary up to 8"×8", and the depth of notch from  $\frac{3}{4}$ " to 1½". They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 81. The joints on opposite sides of the trestle should be "stag-

gered." Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught on the outer guard-rail, thus causing the truck to slew around

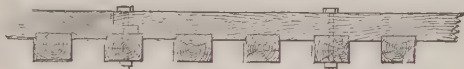


FIG. 81.

and so produce a dangerous accident. The true function of the *outside* guard-rail is thus changed to that of a tie-spacer, which keeps the ties from spreading when a derailment occurs. The inside guard-rail generally consists of an ordinary steel rail spiked about 10 inches inside of the running rail. These inner guard-rails should be bent inward to a point in the center of the track about 50 feet beyond the end of the bridge or trestle. If the inner guard-rails are placed with a clear space of 10 inches inside the running rail, the outer guard-rails should be *at least* 6' 10" apart. They are generally much farther apart than this.

**146. Ties on trestles.** If a car is derailed on a bridge or trestle, the heavily loaded wheels are apt to force their way between the ties by displacing them unless the ties are closely spaced and fastened. The clear space between ties is generally equal to or less than their width. Occasionally it is a little more than their width. 6"  $\times$  8" ties, spaced 14" to 16" from center to center, are most frequently used. The length varies from 9' to 12' for single track. They are generally notched  $\frac{1}{2}$ " deep on the under side where they rest on the stringers. Oak ties are generally required even when cheaper ties are used on the other sections of the road. Usually every third or fourth tie is bolted to the stringers. When stringers are placed underneath the guard-rails, bolts are run from the top of the guard-rail to the under side of the stringer. The guard-rails thus hold down the whole system of ties, and no direct fastening of the ties to the stringers is needed.

**147. Superelevation of the outer rail on curves.** The location of curves on trestles should be avoided if possible, especially when the trestle is high. Serious additional strains are intro-

duced especially when the curvature is sharp or the speed high. Since such curves are sometimes practically unavoidable, it is necessary to design the trestle accordingly. If a train is stopped on a curved trestle, the action of the train on the trestle is evidently vertical. If the train is moving with a considerable velocity, the resultant of the weight and the centrifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) Framing the outer posts longer than the inner posts, so

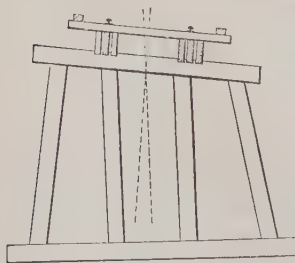


FIG. 82.

that the cap is inclined at the proper angle; axis of posts vertical. (Fig. 82.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.

(b) Notching the cap so that the stringers are at a different

**elevation.** (Fig. 83.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers, which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly increase the cost of maintenance.

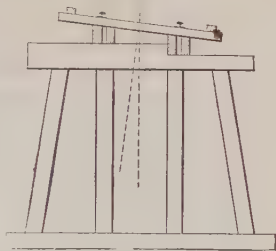


FIG. 83.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes "lag-screws" are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 144) the required inclination of the floor system may be obtained by varying the depth of the corbels.

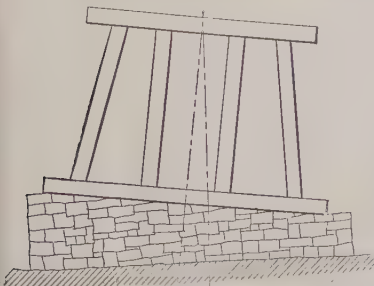


FIG. 84.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sideways,

for the slope would be considerable with a sharp curve, and the

vibration of a moving train would reduce the coefficient of friction to a comparatively small quantity.

(g) **Framing the outer posts longer.** This case is identical with case (a) except that the axis of the system of posts is inclined, as in case (f), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

**148. Protection from fire.** Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walker should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

**149. Timber.** Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one

kind of timber is used. When a very extensive trestle is to be built at a place where suitable growing timber is at hand but there is no convenient sawmill, it will pay to transport a portable sawmill and engine and cut up the timber as desired.

**150. Cost of framed timber trestles.** The cost varies widely on account of the great variation in the cost of timber. When a railroad is first penetrating a new and undeveloped region, the cost of timber is frequently small, and when it is obtainable from the company's right-of-way the only expense is felling and sawing. The work per M, B. M., is small, considering that a single stick  $12'' \times 12'' \times 25'$  contains 300 feet, B. M., and that sometimes a few hours' work, worth less than \$1, will finish all the work required on it. Smaller pieces will of course require more work per foot, B. M. Long-leaf pine can be purchased from the mills at from \$8 to \$12 per M feet, B. M., according to the dimensions. To this must be added the freight and labor of erection. The cartage from the nearest railroad to the trestle may often be a considerable item. Wrought iron will cost about 3 c. per pound and cast iron 2 c., although the prices are often lower than these. The amount of iron used depends on the detailed design, but, as an average, will amount to \$1.50 to \$2 per 1000 feet, B. M., of timber. A large part of the trestling of the country has been built at a contract price of about \$30 per 1000 feet, B. M., erected. While the cost will frequently rise to \$40 and even \$50 when timber is scarce, it will drop to \$13 (cost quoted) when timber is cheap.

#### DESIGN OF WOODEN TRESTLES.

**151. Common practice.** A great deal of trestling has been constructed without any rational design except that custom and experience have shown that certain sizes and designs are *probably* safe. This method has resulted occasionally in failures but more frequently in a very large waste of timber. Many railroads employ a uniform size for all posts, caps, and sills, and a uniform size for stringers, all regardless of the height or span of the trestle. For repair work there are practical reasons favoring this. "To attempt to run a large lot of sizes would be more wasteful in the end than to maintain a few stock sizes only. Lumber can be bought more cheaply by giving a general order for 'the run of the mill for the season,' or 'a cargo lot,' specify-



ing approximate percentages of standard stringer size, of  $12 \times 12$ -inch stuff,  $10 \times 10$ -inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The  $12 \times 12$ -inch stuff, etc., is ordered all lengths, from a certain specified length up. In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required. When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." \* For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on. In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

**152. Required elements of strength.** The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis. The strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable

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\* From "Economical Designing of Timber Trestle Bridges."



care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

**153. Strength of timber.** Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of "clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned "commercial" timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, *e.g.*, the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber. On page 172 there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

On page 173 are given the "average safe allowable working unit stresses in pounds per square inch," as recommended by the committee on "Strength of Bridge and Trestle Timbers,"

the work being done under the auspices of the Association of Railway Superintendents of Bridges and Buildings. The report was presented at their fifth annual convention, held in New Orleans, in October, 1895.

**Moduli of rupture for various timbers. [12% moisture.]**  
(Condensed from U. S. Forestry Circular, No. 15.)

| No. | Species.            | Weight per cubic foot. | Cross-bending.     |                        | Crushing end-wise. | Crushing across the grain. | Shearing along the grain. |
|-----|---------------------|------------------------|--------------------|------------------------|--------------------|----------------------------|---------------------------|
|     |                     |                        | Ultimate Strength. | Modulus of Elasticity. |                    |                            |                           |
| 1   | Long-leaf pine..... | 38                     | 12 600             | 2 070 000              | 8000               | 1180                       | 700                       |
| 2   | Cuban ".....        | 39                     | 13 600             | 2 370 000              | 8700               | 1220                       | 700                       |
| 3   | Short-leaf ".....   | 32                     | 10 100             | 1 680 000              | 6500               | 960                        | 700                       |
| 4   | Loblolly ".....     | 33                     | 11 300             | 2 050 000              | 7400               | 1150                       | 700                       |
| 5   | White ".....        | 24                     | 7 900              | 1 390 000              | 5400               | 700                        | 400                       |
| 6   | Red ".....          | 31                     | 9 100              | 1 620 000              | 6700               | 1000                       | 500                       |
| 7   | Spruce ".....       | 39                     | 10 000             | 1 640 000              | 7300               | 1200                       | 800                       |
| 8   | Bald cypress.....   | 29                     | 7 900              | 1 290 000              | 6000               | 800                        | 500                       |
| 9   | White cedar.....    | 23                     | 6 300              | 910 000                | 5200               | 700                        | 400                       |
| 10  | Douglas spruce..... | 32                     | 7 900              | 1 680 000              | 5700               | 800                        | 500                       |
| 11  | White oak.....      | 50                     | 13 100             | 2 090 000              | 8500               | 2200                       | 1000                      |
| 12  | Overcup ".....      | 46                     | 11 300             | 1 620 000              | 7300               | 1900                       | 1000                      |
| 13  | Post ".....         | 50                     | 12 300             | 2 030 000              | 7100               | 3000                       | 1100                      |
| 14  | Cow ".....          | 46                     | 11 500             | 1 610 000              | 7400               | 1900                       | 900                       |
| 15  | Red ".....          | 45                     | 11 400             | 1 970 000              | 7200               | 2300                       | 1100                      |
| 16  | Texas ".....        | 46                     | 13 100             | 1 860 000              | 8100               | 2000                       | 900                       |
| 19  | Willow ".....       | 45                     | 10 400             | 1 750 000              | 7200               | 1600                       | 900                       |
| 20  | Spanish ".....      | 46                     | 12 000             | 1 930 000              | 7700               | 1800                       | 900                       |
| 21  | Shagbark hickory..  | 51                     | 16 000             | 2 390 000              | 9500               | 2700                       | 1100                      |
| 27  | Pignut ".....       | 56                     | 18 700             | 2 730 000              | 10900              | 3200                       | 1200                      |
| 28  | White elm.....      | 34                     | 10 300             | 1 540 000              | 6500               | 1200                       | 800                       |
| 29  | Cedar ".....        | 46                     | 13 500             | 1 700 000              | 8000               | 2100                       | 1300                      |
| 30  | White ash.....      | 39                     | 10 800             | 1 640 000              | 7200               | 1900                       | 1100                      |

**154. Loading.** As shown in § 138, the span of trestles is always small, is generally 14 feet, and is never greater than 18 feet except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a consolidation locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, should be added to obtain the pressure on the caps or corbels.

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES IN POUNDS, PER SQUARE INCH. RECOMMENDED BY THE COMMITTEE ON "STRENGTH OF BRIDGE AND TRESTLE TIMBERS." (ASSOCIATION OF RAILWAY SUPERINTENDENTS OF BRIDGES AND BUILDINGS: FIFTH ANNUAL CONVENTION, NEW ORLEANS, OCTOBER, 1895.)

| Kind of timber.                                   | Tension.    |               | Compression. |               | Transverse.                      |                              | Shearing. |       |      |
|---------------------------------------------------|-------------|---------------|--------------|---------------|----------------------------------|------------------------------|-----------|-------|------|
|                                                   | With grain. | Across grain. | With grain.  | Across grain. | Ex-<br>treme<br>fiber<br>stress. | Modulus<br>of<br>elasticity. |           |       |      |
|                                                   |             |               |              |               |                                  |                              |           |       |      |
|                                                   |             |               |              |               |                                  |                              |           |       |      |
| Factor of safety.                                 |             | Ten.          | Ten.         | Five.         | Five.                            | Four.                        | Four.     | Four. |      |
| White oak.....                                    | 1000        | 200           | 1400         | 900           | 500                              | 1000                         | 550 000   | 200   | 1000 |
| White pine.....                                   | 700         | 50            | 1100         | 700           | 200                              | 700                          | 500 000   | 100   | 500  |
| Southern, long-leaf, or Georgia yellow pine.....  | 1200        | 60            | 1600         | 1000          | 350                              | 1200                         | 850 000   | 150   | 1250 |
| Douglas, Oregon, and Washington ( Yellow fir.<br> |             |               |              |               |                                  |                              |           |       |      |

Kind of timber.

This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties etc., may be estimated at 200 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard-gauge railroads may be taken as that due to four pairs of driving-axes, spaced 5' 0" apart and giving a pressure of 25000 pounds per axle. This should be increased to 40000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 25000 pounds per axle the following results have been computed:

STRESSES ON VARIOUS SPANS DUE TO MOVING LOADS OF 25000 POUNDS, SPACED 5' 0" APART.

| Span in feet. | Max. moment,<br>ft. lbs. | Max. shear. | Max. load on<br>one cap. |
|---------------|--------------------------|-------------|--------------------------|
| 10            | 65 000                   | 38 500      | 52 100                   |
| 12            | 103 600                  | 45 000      | 62 700                   |
| 14            | 142 400                  | 49 600      | 74 200                   |
| 16            | 181 400                  | 54 725      | 85 700                   |
| 18            | 220 600                  | 60 100      | 97 900                   |

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 40000 lbs. per axle, to be  $\frac{40}{25}$  of those given in the above tabulation.

**155. Factors of safety.** The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses which when excessive would result in immediate destruction such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the

neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

**156. Design of stringers.** The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12" the cost is much higher per M, B. M., and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 138, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 74200 lbs. If the stringers and cap are made of long-leaf yellow pine, which require the closely determined value of 1180 lbs. per square inch to produce a crushing amounting to 3% of the height on timber with 12% moisture, we may use 200 lbs. per square inch as a safe pressure even for green timber; this will require 371 square inches of surface. If the cap is 12" wide, this will require a width of 31 inches, or say 2 stringers under each rail, each 8 inches wide. For rectangular beams

$$\text{Moment} = \frac{1}{8} R' b h^2.$$

Using for  $R'$  the safe value 1275 lbs. per square inch, we have

$$142400 \times 12 = \frac{1}{8} \times 1275 \times 32 \times h^2,$$

from which  $h = 15''.9$ . If desired, the width may be increased to 9" and the depth correspondingly reduced, which will give similarly  $h = 14''.9$ , or say 15". This shows that two beams, 8" × 15", under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2} \frac{\text{total shear}}{\text{cross-section}} = \frac{3}{2} \frac{49600}{4 \times 9 \times 15} = 138 \text{ lbs. per sq. inch,}$$

which is a safe value, although it should preferably be less. Hence the above combination of dimensions will answer.

The deflection should be computed to see if it exceeds the somewhat arbitrary standard of  $\frac{1}{200}$  of the span. The deflection for *uniform loading* is

$$\Delta = \frac{5Wl^3}{32bh^3E},$$

in which  $l$  = length in inches;

$W$  = total load, assumed as uniform;

$E$  = modulus of elasticity, given as 2,070,000 lbs.

per sq. in. for long-leaf pine, 12% dry, and assumed to be 1,200,000 for green timber. Then

$$\Delta = \frac{5 \times 72800 \times 168^3}{32 \times 36 \times 15^3 \times 1200000} = 0''.37$$

$$\frac{1}{200} \times 168'' = 0''.84,$$

so that the calculated deflection is well within the limit. Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (40000 lbs. per axle) these stringer dimensions must be correspondingly increased.

**157. Design of posts.** Four posts are generally used for single-track work. The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased, but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12". It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks. The





post, computed as a  $7'' \times 11'$  post, would have a *safe* columnar strength of 706 lbs. per square inch. With an area of 77 square inches, this gives a working load of 54362 lbs. for *each post*, or 217448 lbs. for the four posts. Considering that 74200 lbs. is the maximum load on one cap (14 feet span), the great excess of strength is apparent.

**158. Design of caps and sills.** The stresses in caps and sills are very indefinite, except as to crushing across the grain. As the stringers are placed almost directly over the inner posts, and as the sills are supported just under the posts, the transverse stresses are almost insignificant. In the above case four posts have an area of  $4 \times 12'' \times 8'' = 384$  sq. in. The total load, 74200 lbs., will then give a pressure of 193 pounds per square inch, which is within the allowable limit. This one feature might require the use of  $8'' \times 12''$  posts rather than  $6'' \times 12''$  posts, for the smaller posts, although probably strong enough as posts, would produce an objectionably high pressure.

**159. Bracing.** Although some idea of the stresses in the bracing could be found from certain assumptions as to wind-pressure, etc., yet it would probably not be found wise to decrease, for the sake of economy, the dimensions which practice has shown to be sufficient for the work. The economy that would be possible would be too insignificant to justify any risk. Therefore the usual dimensions, given in §§ 139 and 140, should be employed.

## CHAPTER V.

### TUNNELS.

#### SURVEYING.

**160. Surface surveys.** As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 85 represents roughly a longitudinal section of the

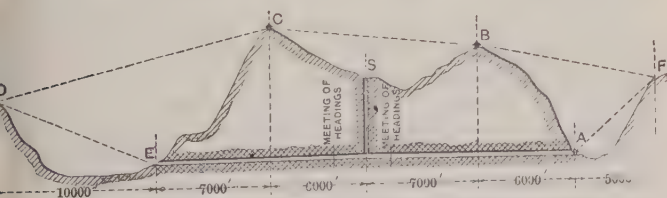


FIG. 85.—SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at A, B, C, D, E, and F, and stone houses were built at A, B, C, and D. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations D and F were necessary because E and A were invisible from C and B. The alignment at A and E having been determined with great accuracy, the true alignment was easily carried into the tunnel.

The relative elevations of *A* and *E* were determined with great accuracy. Steep slopes render necessary many settings of the level per unit of horizontal distance and require that the work be unusually accurate to obtain even fair accuracy per unit of distance. The levels are usually re-run many times until the probable error is a very small quantity.

The exact horizontal distance between the two ends of the tunnel must also be known, especially if the tunnel is on a grade. The usual steep slopes and rough topography likewise render accurate horizontal measurements very difficult. Frequently when the slope is steep the measurement is best obtained by measuring along the slope and allowing for grade. This may be very accurately done by employing two tripods (level or transit tripods serve the purpose very well), setting them up slightly less than one tape-length apart and measuring between horizontal needles set in wooden blocks inserted in the top of each tripod. The elevation of each needle is also observed. The true horizontal distance between two successive positions of the needles then equals the square root of the difference of the squares of the inclined distance and the difference of elevation. Such measurements will probably be more accurate than those made by attempting to hold the tape horizontal and plumbing down with plumb-bobs, because (1) it is practically difficult to hold both ends of the tape truly horizontal; (2) on steep slopes it is impossible to hold the down-hill end of a 100-foot tape (or even a 25-foot length) on a level with the other end, and the great increase in the number of applications of the unit of measurement very greatly increases the probable error of the whole measurement; (3) the vibrations of a plumb-bob introduce a large probability of error in transferring the measurement from the elevated end of the tape to the ground, and the increased number of such applications of the unit of measurement still further increases the probable error.

**161. Surveying down a shaft.** If a shaft is sunk, as at *S*, Fig 85, and it is desired to dig out the tunnel in both directions from the foot of the shaft so as to meet the headings from the outside, it is necessary to know, when at the bottom of the shaft, the elevation, alignment, and horizontal distance from each end of the tunnel.

The *elevation* is generally carried down a shaft by means of a steel tape. This method involves the least number of appli-

cations of the unit of measurement and greatly increases the accuracy of the final result.

The *horizontal distance from each end* may be easily transferred down the shaft by means of a plumb-bob, using some of the precautions described in the next paragraph.

To transfer the *alignment* from the surface to the bottom of a shaft requires the highest skill because the shaft is always small, and to produce a line perhaps several thousand feet long in a direction given by two points 6 or 8 feet apart requires that the two points must be determined with extreme accuracy. The eminently successful method adopted in the Hoosac Tunnel will be briefly described. Two beams were securely fastened across the top of the shaft (1030 feet deep), the beams being placed transversely to the direction of the tunnel and as far apart as possible and yet allow plumb-lines, hung from the intersection of each beam with the tunnel center line, to swing freely at the bottom of the shaft. These intersections of the beams with the center line were determined by averaging the results of a large number of careful observations for alignment. Two fine parallel wires, spaced about  $\frac{1}{16}$ " apart, were then stretched between the beams so that the center line of the tunnel bisected at all points the space between the wires. Plumb-bobs, weighing 15 pounds, were suspended by fine wires beside each cross-beam, the wires passing between the two parallel alignment wires and bisecting the space. The plumb-bobs were allowed to swing in pails of water at the bottom. Drafts of air up the shaft required the construction of boxes surrounding the wires. Even these precautions did not suffice to absolutely prevent vibration of the wire at the bottom through a very small arc. The mean point of these vibrations in each case was then located on a rigid cross-beam suitably placed at the bottom of the shaft and at about the level of the roof of the tunnel. Short plumb-lines were then suspended from these points whenever desired; a transit was set (by trial) so that its line of collimation passed through both plumb-lines and the line at the bottom could thus be prolonged.

Some recent experience in the "Tamarack" shaft, 4250 feet deep, shows that the accuracy of the results may be affected by air-currents to an unsuspected extent. Two 50-lb. cast-iron plumb-bobs were suspended with No. 24 piano-wire in this shaft. The carefully measured distances between the wires

at top and bottom were 16.32 and 16.43 feet respectively. After considerable experimenting to determine the cause of the variation, it was finally concluded that air-currents were alone responsible. The variation of the bobs from a true vertical plane passing through the wires at the top was of course an unknown quantity, but since the variation in *one* direction amounted to 0.11 foot, the accuracy in other directions was very questionable. This shows that a careful comparative measurement between the wires at top and bottom should always be made as a test of their parallelism.

**162. Underground surveys.** Survey marks are frequently placed on the timbering, but they are apt to prove unreliable on account of the shifting of the timbering due to settlement of the surrounding material. They should never be placed at the bottom of the tunnel on account of the danger of being disturbed or covered up. Frequently holes are drilled in the roof and filled with wooden plugs in which a hook is screwed exactly on line. Although this is probably the safest method, even these plugs are not always undisturbed, as the material, unless very hard, will often settle slightly as the excavation proceeds. When a tunnel is perfectly straight and not too long, alignment-points may be given as frequently as desired from

permanent stations located *outside* the tunnel where they are not liable to disturbance. This has been accomplished by running the alignment through the upper part of the cross-section, at one side of the center, where it is out of the way of the piles of masonry material, debris, etc., which are so apt to choke up the lower part of the cross-section. The position of this line relative to the cross-section being fixed, the alignment of any required point of the cross-section is readily found by means of a light frame or template with a fixed tar-

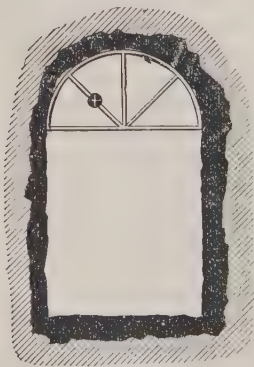


FIG. 86.

get located where this line would intersect the frame when properly placed. A level-bubble on the frame will assist in setting the frame in its proper position.

In all tunnel surveying the cross-wires must be illuminated by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

**163. Accuracy of tunnel surveying.** Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alignment, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequently, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows: The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alignment at the meeting of the headings was 0'.04, error of levels 0'.015, error of distance 0'.52. The Hoosac tunnel is over 25000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alignment was  $\frac{5}{16}$  of an inch, that of levels "a few hundredths," error of distance "trifling." The alignment, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from



the shaft. Here the error of alignment was  $\frac{9}{18}''$  and that of levels 0.134 ft.

#### DESIGN.

**164. Cross-sections.** Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The *general* form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. With very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 87 and 88.

The width of tunnels varies as greatly as the designs. Single-track tunnels generally have a width of 15 to 16 feet. Occasionally they have been built 14 feet wide, and even less, and also up to 18 feet, especially when on curves. 24 to 26 feet is the most common width for double track. Many double-track tunnels are only 22 feet wide, and some are 28 feet wide. The heights are generally 19 feet for single track and 20 to 22 feet for double track. The variations from these figures are considerable. The lower limits depend on the cross-section of the rolling stock, with an indefinite allowance for clearance and ventilation. Cross-sections which coincide too closely with what is absolutely required for clearance are objectionable, because any slight settlement of the lining which would otherwise be harmless would then become troublesome and even dangerous. Figs. 87, 88, and 89 \* show some typical cross-sections.

**165. Grade.** A grade of at least 0.2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade should be practically level, with an allowance for drainage, the

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\* Drinker's "Tunneling."





actual summit being perhaps in the center so as to drain both ways. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and

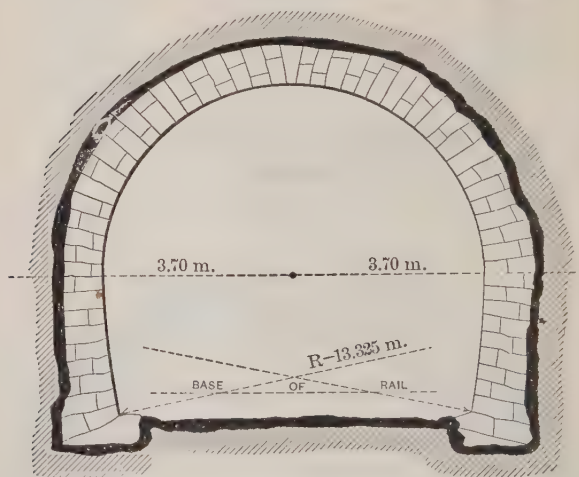


FIG. 89. - ST. CLOUD TUNNEL.

fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

**166. Lining.** It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber is cheap, it is occasionally framed as an arch and used as the *permanent* lining, but masonry is always to be preferred. Frequently the cross-section is made extra

large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

167. Shafts. Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular

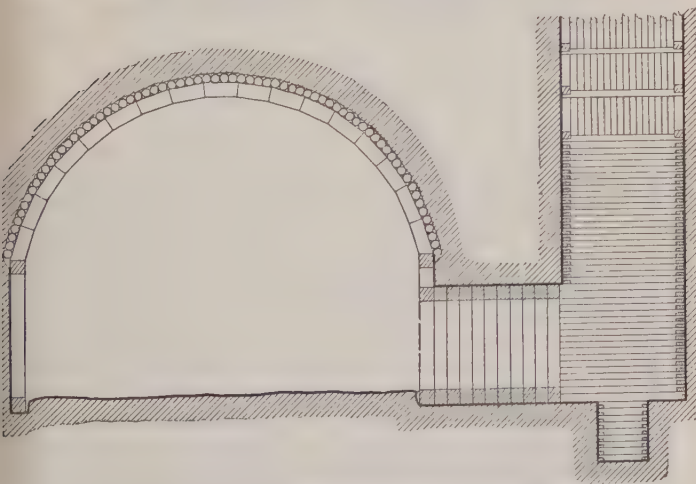


FIG. 90.—CONNECTION WITH SHAFT, CHURCH HILL TUNNEL.

cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided

such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 90.\* Fig. 91† shows a cross-section for a large main shaft. Many shafts have been built with the idea of being left open permanently for ventilation and have therefore been elaborately lined with masonry.

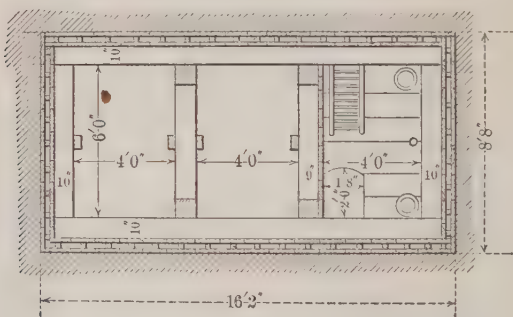


FIG. 91.—CROSS-SECTION. LARGE MAIN SHAFT.

The general consensus of opinion now appears to be that shafts are worse than useless for ventilation; that the quick passage of a train through the tunnel is the most effective ventilator; and that shafts only tend to produce cross-currents and are ineffective to clear the air. In consequence, many of these elaborately lined shafts have been permanently closed, and the more recent practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

**168. Drains.** A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as

\* Drinker's "Tunneling."

† Rziha, "Lehrbuch der Gesamten Tunnelbaukunst."

to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

#### CONSTRUCTION.

**169. Headings.** The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended and perhaps replaced, according to some regular system, so that when the full cross-section has been excavated it is supported by such timbering as is intended for it. The heading is sometimes made on the center line near the top; with other plans, on the center line near the bottom; and sometimes two simultaneous headings are run in the two lower corners. Headings near the bottom serve the purpose of draining the material above it and facilitating the excavation. The simplest case of heading timbering is that shown in Fig. 92, in which cross-timbers are placed at intervals just under the roof, set in notches cut in the side walls and supporting poling-boards which sus-

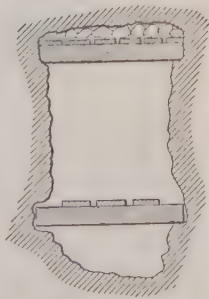


FIG. 92.

tain whatever pressure may come on them. Cross-timbers near the bottom support a flooring on which vehicles for transporting material may be run and under which the drainage may freely escape. As the necessity for timbering becomes greater, side timbers and even bottom timbers must be added, these timbers supporting poling-boards, and even the breast of the heading must be protected by boards suitably braced,

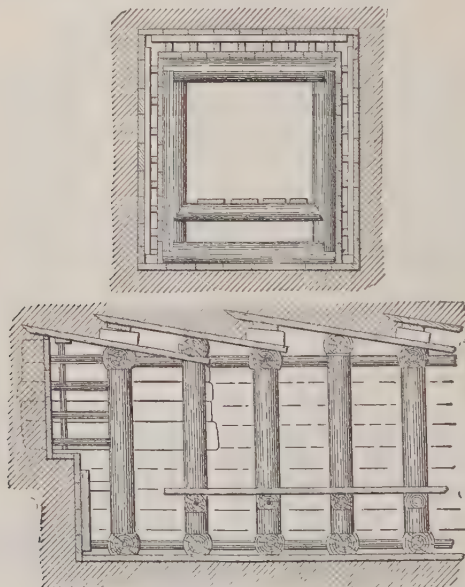


FIG. 93.—TIMBERING FOR TUNNEL HEADING.

as shown in Fig. 93. The supporting timbers are framed into collars in such a manner that added pressure only increases their rigidity.

**170. Enlargement.** Enlargement is accomplished by removing the poling-boards, one at a time, excavating a greater or less amount of material, and immediately supporting the exposed material with poling-boards suitably braced. (See Figs. 93 and 94.) This work being systematically done, space is thereby obtained in which the framing for the full cross-section may be gradually introduced. The framing is constructed with a cross-



section so large that the masonry lining may be constructed within it.

**171. Distinctive features of various methods of construction.** There are six general systems, known as the English, German, Belgian, French, Austrian, and American. They are so named

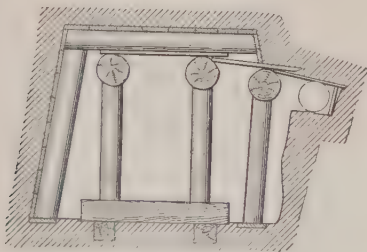
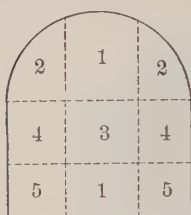


FIG. 94.

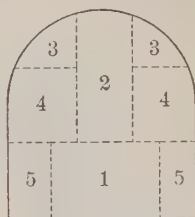
from the origin of the methods, although their use is not confined to the countries named. Fig. 95 shows by numbers (1 to 5) the order of the excavation within the cross-sections. The English, Austrian, and American systems are alike in excavating the entire cross-section before beginning the construction of the masonry lining. The German method leaves a solid core (5) until practically the whole of the lining is complete. This has the disadvantage of extremely cramped quarters for work, poor ventilation, etc. The Belgian and French methods agree in excavating the upper part of the section, building the arch at once, and supporting it temporarily until the side walls are built. The Belgian method then takes out the core (3), removes very short sections of the sides (4) immediately underpinning the arch with short sections of the side walls and thus gradually constructing the whole side wall. The French method digs out the sides (3), supporting the arch temporarily with timbers and then replacing the timbers with masonry; the core (4) is taken out last. The French method has the same disadvantage as the German—working in a cramped space. The Belgian and French systems have the disadvantage that the arch, supported temporarily on timber, is very apt to be strained and cracked by the slight settlement that so frequently occurs in soft material. The English, Austrian, and American methods differ mainly in the



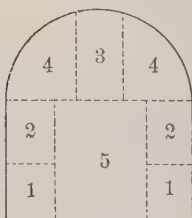
design of the timbering. The English support the roof by lines of very heavy *longitudinal* timbers which are supported at comparatively wide intervals by a heavy framework occupying the



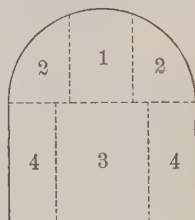
ENGLISH



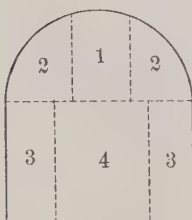
AUSTRIAN



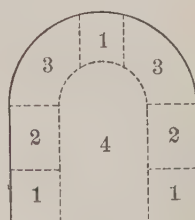
GERMAN



BELGIAN



FRENCH



AMERICAN

FIG. 95.—ORDER OF WORKING BY THE VARIOUS SYSTEMS.

whole cross-section. The Austrian system uses such frequent cross-frames of timber-work that poling-boards will suffice to support the material between the frames. The American system agrees with the Austrian in using frequent cross-frames

supporting poling-boards, but differs from it in that the "cross-frames" consist simply of arches of 3 to 15 wooden voussoirs, the voussoirs being blocks of 12"×12" timber about 2 to 8 feet long and cut with joints normal to the arch. These arches are put together on a centering which is removed as soon as the arch is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 90 and Plate III illustrate the use of the American system. Fig. 90 shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. The student is referred to Drinker's "Tunneling" and to Rziha's "Lehrbuch der Gesammten Tunnelbaukunst" for numerous illustrations of European methods of tunnel timbering.

**172. Ventilation during construction.** Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. The invention of compressed-air drills simultaneously solved two difficulties. It introduced a motive power which is unobjectionable in its application (as gas would be), and it also furnished at the same time a supply of just what is needed—pure air. If no blasting is done (and sometimes even when there is blasting), air must be supplied by direct pumping. The cooling effect of the sudden expansion of compressed air only reduces the otherwise objectionably high temperature sometimes found in tunnels. Since pure air is being continually pumped in, the foul air is thereby forced out.

**173. Excavation for the portals.** Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 96 \*

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\* Rziha, "Lehrbuch der Gesammten Tunnelbaukunst."

illustrates one method of breaking into the ground at a portal. The loose stones are piled on the framing to give stability to the framing by their weight and also to retain the earth on the

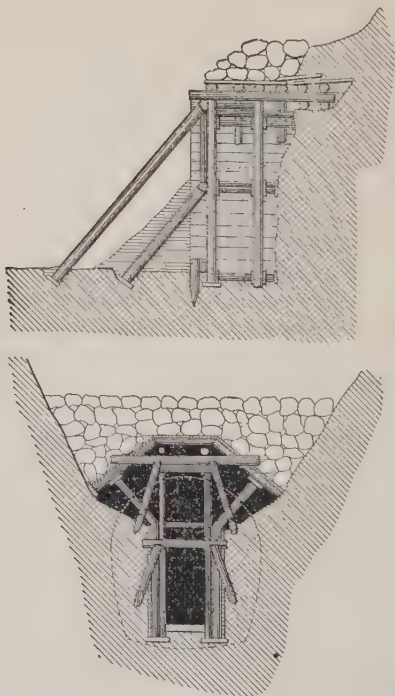


FIG. 96.—TIMBERING FOR TUNNEL PORTAL.

slope above. Another method is to sink a temporary shaft to the tunnel near the portal; immediately enlarge to the full size and build the masonry lining; then work back to the portal. This method is more costly, but is preferable in very treacherous ground, it being less liable to cause landslides of the surface material.

**174. Tunnels vs. open cuts.** In cases in which an open cut rather than a tunnel is a possibility the ultimate consideration is generally that of first cost combined with other financial con-

siderations and annual maintenance charges directly or indirectly connected with it. Even when an open cut may be constructed at the same cost as a tunnel (or perhaps a little cheaper) the tunnel may be preferable under the following conditions:

1. When the soil indicates that the open cut would be liable to landslides.

2. When the open cut would be subject to excessive snow-drifts or avalanches.

3. When land is especially costly or it is desired to run under existing costly or valuable buildings or monuments. When running through cities, tunnels are sometimes constructed as open cuts and then arched over.

These cases apply to tunnels *vs.* open cuts when the alignment is fixed by other considerations than the mere topography. The broader question of excavating tunnels to avoid excessive grades or to save distance or curvature, and similar problems, are hardly susceptible of general analysis except as questions of railway economics and must be treated individually.

**175. Cost of tunneling.** The cost of any construction which involves such uncertainties as tunneling is very variable. It depends on the material encountered, the amount and kind of timbering required, on the size of the cross-section, on the price of labor, and especially on the *reconstruction* that *may* be necessary on account of mishaps.

Headings generally cost \$4 to \$5 per cubic yard for excavation, while the remainder of the cross-section in the same tunnel may cost about half as much. The average cost of a large number of tunnels in this country may be seen from the following table:\*

| Material.       | Cost per cubic yard. |         |          |         | Cost per lineal foot. |          |
|-----------------|----------------------|---------|----------|---------|-----------------------|----------|
|                 | Excavation.          |         | Masonry. |         | Single.               | Double.  |
|                 | Single.              | Double. | Single.  | Double. |                       |          |
| Hard rock.....  | \$5.89               | \$5.45  | \$12.00  | \$8.25  | \$69.76               | \$142.82 |
| Loose rock..... | 3.12                 | 3.48    | 9.07     | 10.41   | 80.61                 | 119.26   |
| Soft ground.... | 3.62                 | 4.64    | 15.00    | 10.50   | 135.31                | 174.42   |

\* Figures derived from Drinker's "Tunneling."

A considerable variation from these figures may be found in individual cases, due sometimes to unusual skill (or the lack of it) in prosecuting the work, but the figures will generally be sufficiently accurate for preliminary estimates or for the comparison of two proposed routes.

## CHAPTER VI.

### CULVERTS AND MINOR BRIDGES.

**176. Definition and object.** Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

**177. Elements of the design.** A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i e., with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

## AREA OF THE WATERWAY.

**178. Elements involved.** The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

**a. Rainfall.** The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

**b. Area of watershed.** This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

**c. Character of soil and vegetation.** This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

**d. Shape and slope of watershed.** If the watershed is very long and narrow (other things being equal), the water from the remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

**e. Effect of design of culvert.** The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the



approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

**179. Methods of computation of area.** There are three possible methods of computation.

(a) **Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 178, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (c) is most useful. The theoretical method will not therefore be considered further.

(b) **Empirical.** As illustrated in § 180, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment. Assuming that the formulæ are sound, their use only narrows the limits of error, the final determination depending on experience and judgment.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 126, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (b) may be utilized for an approximate calculation for the required area for the tem-

porary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

**180. Empirical formulæ.** Two of the best known empirical formulæ for area of the waterway are the following:

**(a) Myer's formula:**

Area of waterway in square feet  $= C \times \sqrt{\text{drainage area in acres}}$ , where  $C$  is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

**(b) Talbot's formula:**

Area of waterway in square feet  $= C \times \sqrt[4]{(\text{drainage area in acres})^3}$ . "For steep and rocky ground  $C$  varies from  $\frac{2}{3}$  to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width,  $C$  is about  $\frac{1}{2}$ ; and if the stream is longer in proportion to the area, decrease  $C$ . In districts not affected by accumulated snow, and where the length of the valley is several times the width,  $\frac{1}{3}$  or  $\frac{1}{6}$ , or even less, may be used.  $C$  should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." \* As an illustration, if the drainage area is 100 acres the area of waterway should be  $C \times 31.6$ . The area should then vary from 5 to 31 square feet, according to the character of the country. Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

**181. Value of empirical formulæ.** The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment than as a working rule," as Prof. Talbot explicitly declares in

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\* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

commenting on his own formula. In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

**182. Results based on Observation.** As already indicated in § 179, observation of the stream in question gives the most reliable results. If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: "A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash. Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks. In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the general width of the stream, but in rapid streams among the hills the width that the stream has cut for itself through the soil should not be lessened, and in ravines carrying mountain torrents the openings must be left very much larger than the ordinary appearance of the banks of the stream would seem to make necessary." \*

As an illustration of an observation of a storm-flow through a temporary trestle, the following is quoted: "Having the flood height and velocity, it is an easy matter to determine the volume of water to be taken care of. I have one ten-bent pile trestle 135 feet long and 24 feet high over a spring branch that ordinarily runs about six cubic inches per second. Last summer during one of our heavy rainstorms (four inches in less than three hours) I visited this place and found by float observations the surface velocity at the highest stage to be 1.9 feet per second. I made a high-water mark, and after the flood-water receded found the width of stream to be 12 feet and an average depth of  $2\frac{3}{4}$  feet. This, with a surface velocity of 1.9 feet per second, would give approximately a discharge of 50

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\* J. P. Snow, Boston & Maine Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

cubic feet, or 375 gallons, per second. Having this information it is easy to determine size of opening required." \*

**183. Degree of accuracy required.** The advantages resulting from the use of standard designs for culverts (as well as other structures) have led to the adoption of a comparatively small number of designs. The practical use made of a computation of required waterway area is to determine which one of several standard designs will most nearly fulfill the requirements. For example, if a 24-inch iron pipe, having an area of 3.14 square feet, is considered to be a little small, the next size (30-inch) would be adopted; but a 30-inch pipe has an area of 4.92 square feet, which is 56% larger. A similar result, except that the percentage of difference might not be quite so marked, will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

#### PIPE CULVERTS.

**184. Advantages.** Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been

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\* A. J. Kelley, Kansas City Belt Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

temporarily lined with wood, without disturbing the roadbed or track.

**185. Construction.** Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 97), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows:

$$\text{Length} = 2s (\text{depth of embankment to top of pipe}) \div (\text{width of roadbed}),$$

in which  $s$  is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths will be used which will most nearly agree with this formula.

**186. Iron-pipe culverts.** Simple cast-iron pipes are used in sizes from 12" to 48" diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand pressure may be utilized for this work. In Fig. 97 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.





Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.

187. *Tile-pipe culverts.* The pipes used for this purpose vary from 12" to 24" in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that "double-strength" or "extra-heavy" pipe shall be used, evidently with the idea that the stresses on a culvert-pipe are greater than on a sewer-pipe. But it has been conclusively demonstrated that, no matter how deep the embankment, the pressure cannot exceed a somewhat uncertain maximum, also that the greatest danger consists in placing the pipe so near the ties that shocks may be directly transferred to the pipe without the cushioning effect of the earth and ballast. When the pipes are well bedded in *clear* earth and there is a

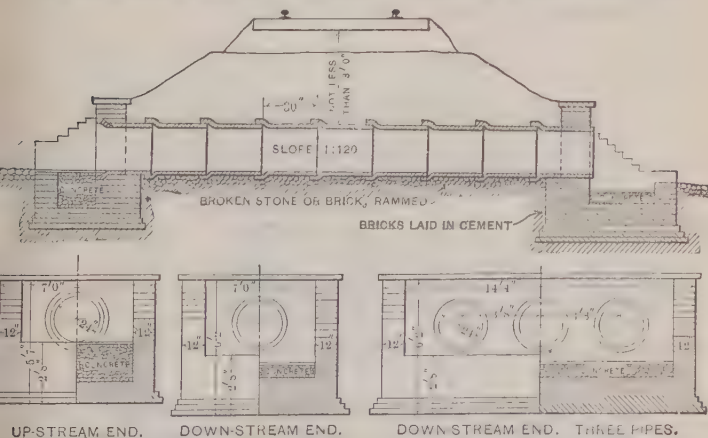


FIG. 98.—STANDARD VITRIFIED-PIPE CULVERT. PLANT SYSTEM. (1891.)

sufficient depth of earth over them to avoid direct impact (at least three feet) the ordinary sewer-pipe will be sufficiently strong. "Double-strength" pipe is frequently less perfectly burned, and the supposed extra strength is not therefore ob-



tained. In Fig. 98 are shown the standard plans for vitrified-pipe culverts as used on the "Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

#### BOX CULVERTS.

**188. Wooden box culverts.** This form serves the purpose of a cheap temporary construction which allows the use of a ballasted roadbed. As in all temporary constructions, the area should be made considerably larger than the calculated area (§§ 179-182), not only for safety but also in order that, if the smaller area is demonstrated to be sufficiently large, the permanent construction (probably pipe) may be placed inside without disturbing the embankment. All designs agree in using heavy timbers ( $12'' \times 12''$ ,  $10'' \times 12''$ , or  $8'' \times 12''$ ) for the side walls, cross-timbers for the roof, every fifth or sixth timber being notched down so as to take up the thrust of the side walls, and planks for the flooring. Fig. 99 shows some of the standard designs as used by the C., M. & St. P. Ry.

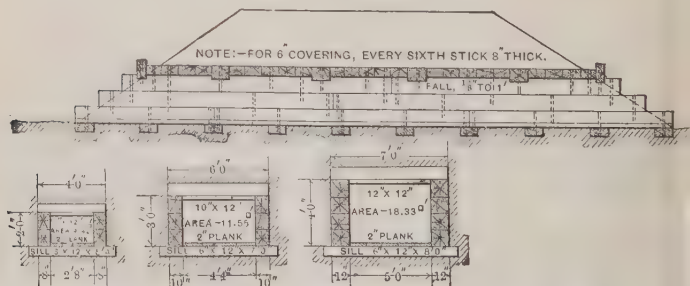


FIG. 99.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

**189. Stone box culverts.** In localities where a *good* quality of stone is cheap, stone box culverts are the cheapest form of permanent construction for culverts of medium capacity, but their use is decreasing owing to the frequent difficulty in obtaining really suitable stone within a reasonable distance of the culvert. The clear span of the cover-stones varies from 2 to 4 feet. The required thickness of the cover-stones is sometimes

calculated by the theory of transverse strains on the basis of certain assumptions of loading—as a function of the height of the embankment and the unit strength of the stone used. Such a method is simply another illustration of a class of calculations which look very precise and beautiful, but which are worse than useless (because misleading) on account of the hopeless uncer-

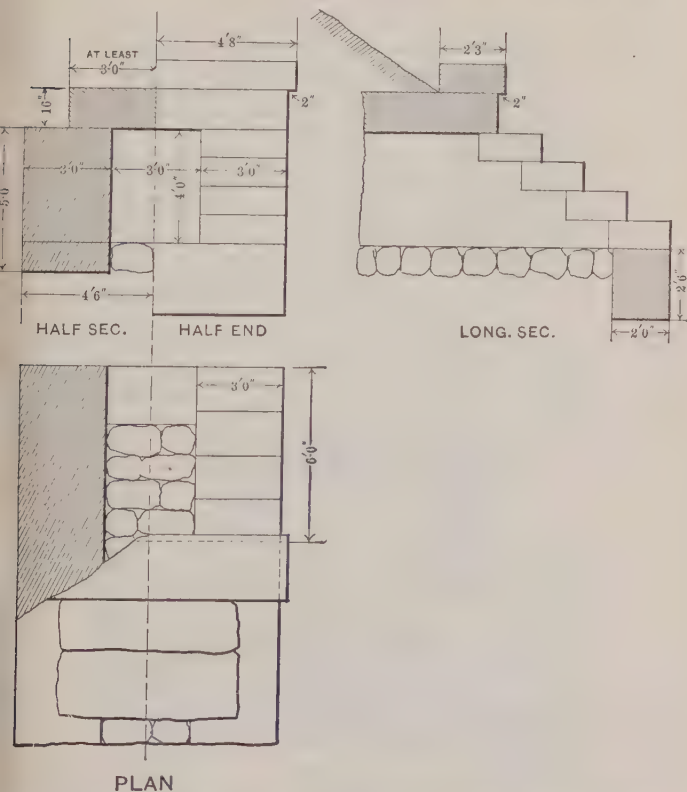


FIG. 100.—STANDARD SINGLE STONE CULVERT (3'×4'). N. & W. R.R. (1890.)

tainty as to the true value of certain quantities which must be used in the computations. In the first place the true value of the unit tensile strength of stone is such an uncertain and variable

quantity that calculations based on any assumed value for it are of small reliability. In the second place the weight of the prism of earth lying directly above the stone, plus an allowance for live load, is by no means a measure of the load on the stone nor of the forces that tend to fracture it. All earthwork will tend to

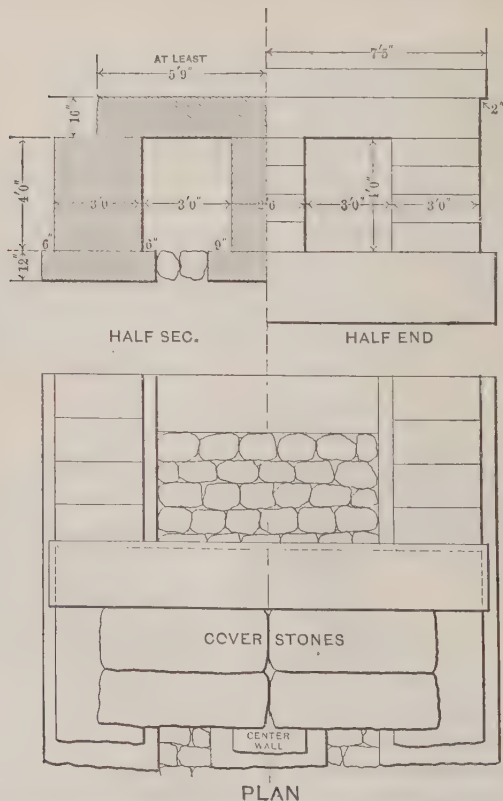


FIG. 100a.—STANDARD DOUBLE STONE CULVERT (3' X 1'). N. & W. R. R. (1890.)

form an arch above any cavity and thus relieve an uncertain and probably variable proportion of the pressure that might otherwise exist. The higher the embankment the *less* the *pro-*

*portionate* loading, until at some uncertain height an increase in height will not increase the load on the cover-stones. The effect of frost is likewise large, but uncertain and not computable. The usual practice is therefore to make the thickness such as experience has shown to be safe with a good quality of stone, i.e., about 10 or 12 inches for 2 feet span and up to 16 or 18 inches for 4 feet span. The side walls should be carried down deep enough to prevent their being undermined by scour or heaved by frost. The use of cement mortar is also an important feature of first-class work, especially when there is a rapid scouring current or a liability that the culvert will run under a head. In Figs. 100 and 100a are shown standard plans for single and double stone box culverts as used on the Norfolk and Western R. R.

**190. Old-rail culverts.** It sometimes happens (although very rarely) that it is necessary to bring the grade line within 3 or 4 feet of the bottom of a stream and yet allow an area of 10 or 12 square feet. A single large pipe of sufficient area could not be used in this case. The use of several smaller pipes side by side would be both expensive and inefficient. For similar reasons neither wooden nor stone box culverts could be used. In such cases, as well as in many others where the head-room is not so limited, the plan illustrated in Fig. 101 is a very satisfactory

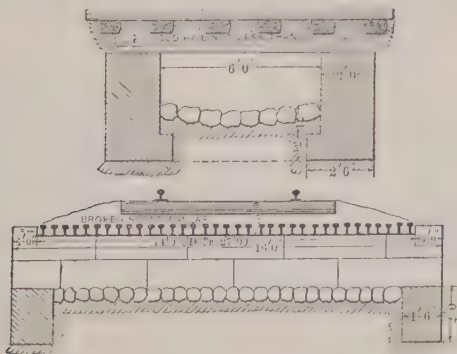


FIG. 101.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

solution of the problem. The old rails, having a length of 8 or 9 feet, are laid close together across a 6-foot opening. Sometimes the rails are held together by long bolts passing through

the webs of the rails. In the plan shown the rails are confined by low end walls on each abutment. This plan requires only 15 inches between the base of the rail and the top of the culvert channel. It also gives a continuous ballasted roadbed.

#### ARCH CULVERTS.

**191. Influence of design on flow.** The variations in the design of arch culverts have a very marked influence on the cost and efficiency. To combine the least cost with the greatest efficiency, due weight should be given to the following elements: (*a*) amount of masonry, (*b*) the simplicity of the constructive work, (*c*) the design of the wing walls, (*d*) the design of the junction of the wing walls with the barrel and faces of the arch, and (*e*) the safety and permanency of the construction. These elements are more or less antagonistic to each other, and the defects of most designs are due to a lack of proper proportion in the design of these opposing interests. The simplest construction (satisfying elements *b* and *c*) is the straight barrel arch between two parallel vertical head walls, as sketched in Fig. 102, *a*. From a hydraulic standpoint the design is poor, as the water eddies around the corners, causing a great resistance which decreases the flow. Fig. 102, *b*, shows a much better de-

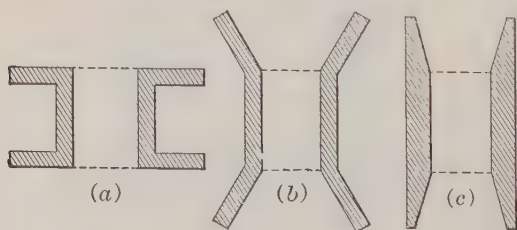


FIG. 102.—TYPES OF CULVERTS.

sign in many respects, but much depends on the details of the design as indicated in elements (*b*) and (*d*). As a general thing a good hydraulic design requires complicated and expensive masonry construction, i.e., elements (*b*) and (*d*) are opposed. Design 102, *c*, is sometimes inapplicable because the water is

liable to work in behind the masonry during floods and perhaps cause scour. This design uses less masonry than (a) or (b).

**192. Example of arch culvert design.** In Plate IV is shown the design for an 8-foot arch culvert according to the standard of the Norfolk and Western R. R. Note that the plan uses the flaring wing walls (Fig. 102, *b*) on the up-stream side (thus protecting the abutments from scour) and straight wing walls (similar to Fig. 102, *c*) on the down-stream end. This economizes masonry and also simplifies the constructive work. Note also the simplicity of the junction of the wing walls with the barrel of the arch, there being no re-entrant angles below the springing line of the arch. The design here shown is but one of a set of designs for arches varying in span from 6' to 30'.

#### MINOR OPENINGS.

**193. Cattle-guards.** (a) Pit guards. Cattle-guards will be considered under the head of minor openings, since the old-

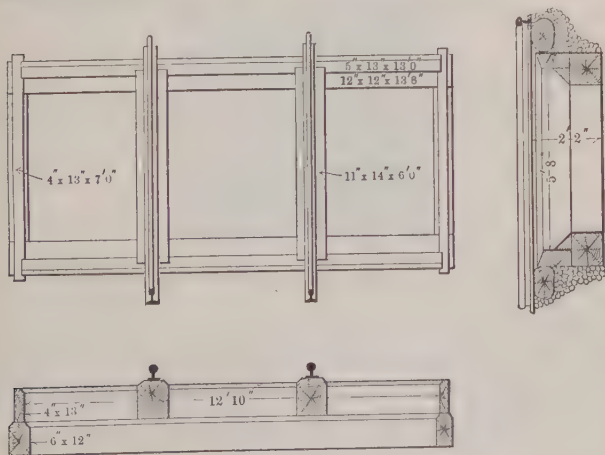


FIG. 103.—PIT CATTLE-GUARDS. P. R. R.

fashioned plan of pit guards, which are even now defended and preferred by some railroad men, requires a break in the continuity of the roadbed. A pit about three feet deep, five feet long, and as wide as the width of the roadbed, is walled up with

stone (sometimes with wood), and the rails are supported on heavy timbers laid longitudinally with the rails. The break in the continuity of the roadbed produces a disturbance in the elastic wave running through the rails, the effect of which is noticeable at high velocities. The greatest objection, however, lies in the dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless inspection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable. The (once) standard design for such a structure on the Pennsylvania R.R. is shown in Fig. 103.

(b) **Surface cattle-guards.** These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and

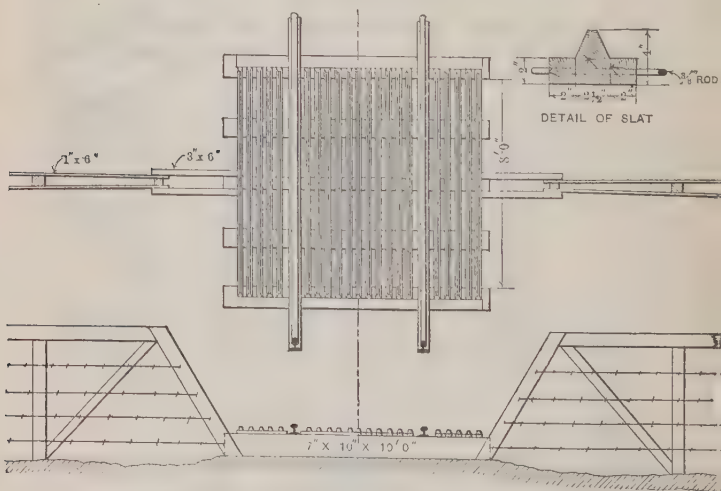


FIG. 104.—CATTLE-GUARD WITH WOODEN SLATS.

thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection,



which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars are sometimes "home-made," of wood, as shown in Fig. 104. Iron or steel bars are made as shown in Fig. 105. The general construction is the same as for the wooden bars. The metal bars have far greater durability, and it is claimed that they are more effective in discouraging cattle from attempting to cross.

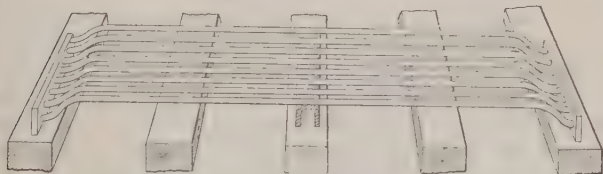


FIG. 105.—MERRILL-STEVENS STEEL CATTLE-GUARD.

**194. Cattle-passes.** Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section.

**195. Standard stringer and I-beam bridges.** The advantages of standard designs apply even to the covering of short spans

with wooden stringers or with I beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate V. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 156. When computing the required transverse strength, due allowance should be made for lateral bracing, which should be amply provided for. Note particularly the methods of bracing illustrated in Plate V. The designs calling for iron (or steel) stringers may be classed as permanent constructions, which are cheap, safe, easily inspected and maintained, and therefore a desirable method of construction.

## CHAPTER VII.

### BALLAST.

**196. Purpose and requirements.** "The object of the ballast is to transfer the applied load over a large surface; to hold the timber work in place horizontally; to carry off the rain-water from the superstructure and to prevent freezing up in winter; to afford means of keeping the ties truly up to the grade line; and to give elasticity to the roadbed." This extremely condensed statement is a description of an ideally perfect ballast. The value of any given kind of ballast is proportional to the extent to which it fulfills these requirements. The ideally perfect ballast is not necessarily the most economical ballast for all roads. Light traffic generally justifies something cheaper, but a very common error is to use a very cheap ballast when a small additional expenditure would procure a much better ballast which would be much more economical in the long run.

**197. Materials.** The materials most commonly employed are gravel and broken stone. Burnt clay, cinders, shells, and small coal are occasionally used as ballast when they are especially cheap and convenient or when better kinds are especially expensive. Although it is hardly correct to speak of the natural soil as ballast, yet many miles of cheap railways are "ballasted" with the natural soil, which is then called "mud ballast."

**Mud ballast.** When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a

gravel-bed or other source of ballast anywhere on the line of the road.

**Cinders.** The advantages consist in the excellent facilities for drainage, ease of handling, and cheapness after the road is in operation. One disadvantage is excessive dust in dry weather. Cinders are considered preferable to gravel in yards.

**Slag.** When slag is readily obtainable it furnishes an excellent ballast, free from dust and perfect in drainage qualities. Some kinds of slag are objectionable on account of their deleterious chemical effect on the ties and spikes—especially on metallic ties.

**Shells, small coal, etc.** These comparatively inferior kinds of ballast are used for light traffic when they are especially cheap and convenient. They are extremely dusty in dry weather, break up into very fine dust, and are but little better than mud.

**Gravel.** This is the most common form of ballast which may be called good ballast. In 1885, the Roadmasters Association of America voted in favor of gravel ballast as against rock ballast. Although not so stated, this action was perhaps due to a conviction of its real economy for the *average* railroad of this country, which may be called a "light traffic" road. Gravel should preferably be screened over a screen having a  $\frac{1}{2}$ " mesh, so as to screen out all dirt and the finest stones. Generally a railroad will be able to find at some point along its line a "gravel-pit" affording a suitable supply. This may be dug out with a steam-shovel, screened if necessary, and sent out over the line by the train-load at a comparatively small cost.

**Rock or broken stone.** Rock ballast is generally specified to be such as will pass through a  $1\frac{1}{2}$ " (or 2") ring. Although preferably broken by hand, machine-broken stone is much cheaper. It is most easily handled with forks. This also has the effect of screening out the dirt and fine chips which would interfere with effectual drainage. Rock ballast is more expensive in first cost, and also more troublesome to handle, than any other kind, but under heavy traffic will keep in surface better and will require less work for maintainance after the ties have become thoroughly bedded. For roads with very light traffic, running few trains, at comparatively low velocities, the advantages of rock ballast over other kinds are not so pronounced. For such roads rock ballast is an expensive luxury. The amount

of traffic which will justify the use of rock ballast will depend on the cost of obtaining ballast of the various kinds.

**198. Cross-sections.** A depth of 12" under the tie is generally required on the best roads, but for light traffic this is sometimes reduced to 6" and even less. The width is generally 1 to 2 feet less than the width of the roadbed proper—excluding ditches. If the ballast has an average width of 10 feet (12 feet at bottom and 8 feet at top) and an average depth of 15 inches (including that placed between the ties), it will require 2444 cubic yards per mile of track. The P. R. R. estimates 2500 cubic yards of gravel and 2800 cubic yards of stone ballast per mile of single track. On account of the requirements of drainage the best form of cross-section depends on the kind of ballast used.

**Mud ballast.** Since the great objection to mud ballast lies in its liability to become soft by soaking up the rain that falls, it becomes necessary that it should be drained as quickly and readily as its nature will permit. Fig. 106 shows a typical cross-

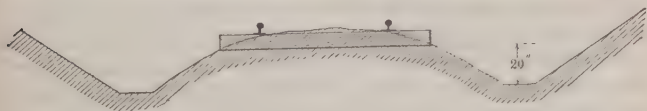


FIG. 106.—"MUD" BALLAST.

section for mud ballast. It should be crowned 2" above the top of the tie at the center, thence sloped so as to leave a slight clearance under the rail between the ties, thence sloping down to the bottom of the tie at each end and continuing to slope down to the ditch (in cut), which should be 18' or 20' below the bottom of the tie.

**Gravel, cinders, slag, etc.** The subgrade is crowned 6" or 8" in the center, as shown in Fig. 107. The ballast is crowned

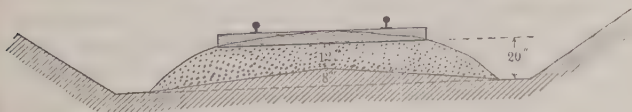


FIG. 107.—GRAVEL BALLAST.

to the top of the tie in the center, but is sloped down to the bottom of the tie at each end. This is necessary (and more

especially so with mud ballast) to prevent a possible accumulation and settlement of water at the ends of the tie, which would readily soak into the end fibers and produce decay.

**Broken stone.** Stone ballast is shouldered out beyond the ends of the ties so as to afford greater lateral binding. The space between the ties is filled up level with the tops. The

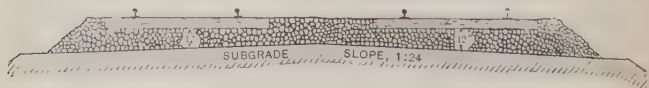


FIG. 108.—BROKEN STONE BALLAST.

perfect drainage of stone ballast permits this to be done without any danger of causing decay of the ties by the accumulation and retention of water.

**199. Methods of laying ballast.** The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications. The best method is to draw in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface. Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a “plough.” The plough has the same width as the cars and is

guided either by a ridge along the center of each car or by short posts set up at the sides of the cars. It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

**200. Cost.** The cost of ballast *in the track* is quite a variable item for different roads, since it depends (a) on the first cost of the material as it comes to the road, (b) on the distance from the source of supply to the place where it is used, and (c) on the method of handling. The first cost of cinder or slag is frequently insignificant. A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost \$1 or more per cubic yard. If suitable stone is obtainable on the company's land, the cost of blasting and breaking should be somewhat less than this. The cost of loading the ballast on to trains will be small (per cubic yard) if it is handled with steam-shovels—as in the case of gravel taken from a gravel-pit. Hand-shovelling will cost more. The cost of hauling will depend on the distance hauled, and also to a considerable extent, on the limitations on the operation of the train due to the necessity of keeping out of the way of regular trains. There is often a needless waste in this way. The "mud train" is considered a pariah and entitled to no rights whatever, regardless of the large daily cost of such a train and of the necessary gang of men. The cost of broken stone ballast *in the track* is estimated at \$1.25 per cubic yard. The cost of gravel ballast is estimated at 60 c. per cubic yard in the track. The cost of placing and tamping gravel ballast is estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to 15 c. per cubic yard. The cost of loading gravel on cars, using a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard.\*

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\* Report Roadmasters Association, 1885.



## CHAPTER VIII.

### TIES.

#### AND OTHER FORMS OF RAIL SUPPORT.

201. Various methods of supporting rails. It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) **Longitudinals.** Supporting the rails throughout their entire length. This method is very seldom used in this country except occasionally on bridges and in terminals when the longitudinals are supported on cross-ties. In § 224 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) **Cast-iron "bowls" or "pots."** These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 223).

(c) **Cross-ties of metal or wood.** These will be discussed in the following sections.

202. **Economics of ties.** The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or of hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an

opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. These last items are not so readily reducible to dollars and cents, but when it can be shown that the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore non-existent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

#### WOODEN TIES.

**203. Choice of wood.** This naturally depends, for any particular section of country, on the supply of wood which is most readily available. The woods most commonly used, especially in this country, are oak and pine, oak being the most durable and generally the most expensive. Redwood is used very extensively in California and proves to be extremely durable, so far as decay is concerned, but it is very soft and is much injured by "rail-cutting." This defect is being partly remedied by the use of tie-plates, as will be explained later. Cedar, chestnut, hemlock, and tamarack are frequently used in this country. In tropical countries very durable ties are frequently obtained from the hard woods peculiar to those countries. According to a bulletin of the U. S. Department of Agriculture issued some years ago, the proportions of the various kinds used in the United States are about as follows:

|            |     |                 |    |               |      |
|------------|-----|-----------------|----|---------------|------|
| Oak .....  | 60% | Chestnut .....  | 5% | Cypress ..... | 2%   |
| Pine.....  | 20  | Hemlock and Ta- |    | Various ..... | 1    |
| Cedar..... | 6   | marack.....     | 3  |               |      |
|            |     | Redwood .....   | 3  | Total .....   | 100% |

The limitations of timber supply have somewhat diminished the use of oak and increased the use of the softer woods in recent years.

**204. Durability.** The durability of ties depends on the climate; the drainage of the ballast; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber is grown; and, chiefly, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties.

*White oak* is credited with a life of 5 to 12 years, depending principally on the traffic. It is both hard and durable, the hardness enabling it to withstand the cutting tendency of the rail-flanges, and the durability enabling it to resist decay. *Pine* and *redwood* resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike-holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very often, the tie may need to be replaced long before any decay has set in. When the traffic is very light, the wood very durable, and the climate favorable, ties have been known to last 25 years.

**205. Dimensions.** The usual dimensions for the best roads (standard gauge) are 8' to 8' 6" long, 6" to 7" thick, and 8" to 10" wide on top and bottom (if they are hewed) or 8" to 9" wide if they are sawed. For cheap roads and light traffic the length is shortened sometimes to 7' and the cross-section also reduced. On the other hand a very few roads use ties 9' long.

Two objections are urged against sawed ties: first, that the grain is torn by the saw, leaving a woolly surface which induces decay; and secondly, that, since timber is not perfectly straight-

grained, some of the fibers are cut obliquely, exposing their ends, which are thus liable to decay. The use of a "planer-saw" obviates the first difficulty. Chemical treatment of ties obviates both of these difficulties. Sawed ties are more convenient to handle, are a necessity on bridges and trestles, and it is even claimed, although against commonly received opinion, that actual trial has demonstrated that they are more durable than hewed ties.

**206. Spacing.** The spacing is usually 14 to 16 ties to a 30-foot rail. This number is sometimes reduced to 12 and even 10, and on the other hand occasionally increased to 18 or 20 by employing narrower ties. There is no economy in reducing the number of ties very much, since for any required stiffness of track it is more economical to increase the number of supports than to increase the weight of the rail. The decreasing cost of rails and the increasing cost of ties have materially changed the relation between number of ties and weight of rail to produce a given stiffness at minimum cost, but many roads have found it economical to employ a large number of ties rather than increase the weight of the rail. On the other hand there is a practical limit to the number that may be employed, on account of the necessary space between the ties that is required for proper tamping. This width is ordinarily about twice the width of the tie. At this rate, with light ties 6" wide and with 12" clear space, there would be 20 ties per 30-foot rail, or 3520 per mile. The smaller ties can generally be bought much cheaper (proportionately) than the larger sizes, and hence the economy.

Track instructions to foremen generally require that the spacing of ties shall *not* be uniform along the length of any rail. Since the joint is generally the weakest part of the rail structure, the joint requires more support than the center of the rail. Therefore the ties are placed with but 8" or 10" clear space between them at the joints, this applying to 3 or 4 ties at each joint; the remaining ties, required for each rail length, are equally spaced along the remaining distance.

**207. Specifications.** The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) **Size.** The particular size or sizes required will be somewhat as indicated in § 205.

(b) **Kind of wood.** When the kind or kinds of wood are specified the most suitable kinds that are available in that section of country are usually required.

(c) **Method of construction.** It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months. These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least seven feet away from them, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) **Quality of timber.** The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work. The ties must be sound, reasonably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut

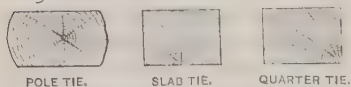


FIG. 109.—METHODS OF CUTTING TIES.

from single trees, making what is known as "pole ties" and definitely condemning those which are cut or split from larger trunks, giving two "slab

ties" or four "quarter ties" for each cross-section, as is illustrated in Fig. 109. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

208. **Regulations for laying and renewing ties.** The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 204–207. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do

not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with "wooden spikes," which are supplied to the foreman for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely. Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

**209. Cost of ties.** When railroads can obtain ties cut by farmers from woodlands in the immediate neighborhood, the price will frequently be as low as 20 c. for the smaller sizes, running up to 50 c. for the larger sizes and better qualities, especially when the timber is not very plentiful. Sometimes if a railroad cannot procure suitable ties from its immediate neighborhood, it will find that adjacent railroads control all adjacent sources of supply for their own use and that ties can only be procured from a considerable distance, with a considerable added cost for transportation. First-class oak ties cost about 75 to 80 c. and frequently much more. Hemlock ties can generally be obtained for 35 c. or less.

#### PRESERVATIVE PROCESSES FOR WOODEN TIES.

**210. General principle.** Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound. The most common methods (except one) all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled. One valuable feature of these processes lies in the fact that the softer cheaper woods



(such as hemlock and pine) are more readily treated than are the harder woods and yet will produce practically as good a tie as a treated hard-wood tie and a very much better tie than an untreated hard-wood tie. The various processes will be briefly described, taking up first the process which is fundamentally different from the others, viz., vulcanizing.

**211. Vulcanizing.** The process consists in heating the timber to a temperature of  $300^{\circ}$  to  $500^{\circ}$  F. in a cylinder, the air being under a pressure of 100 to 175 lbs. per square inch. By this process the albumen in the sap is coagulated, the water evaporated, and the pores are partially closed by the coagulation of the albumen. It is claimed that the heat sterilizes the wood and produces chemical changes in the wood which give it an antiseptic character. It has been very extensively used on the elevated lines of New York City, and it is claimed to give perfect satisfaction. The treatment has cost that road 25 c. per tie.

**212. Creosoting.** This process consists in impregnating the wood with *wood-creosote* or with *dead oil of coal-tar*. *Wood-creosote* is one of the products of the destructive distillation of wood—usually long-leaf pine. *Dead oil of coal-tar* is a product of the distillation of coal-tar at a temperature between  $480^{\circ}$  and  $760^{\circ}$  F. It would require about 35 to 50 pounds of creosote to completely fill the pores of a cubic foot of wood. But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. About 10 pounds per cubic foot, or about 35 pounds per tie, is all that is necessary. For piling placed in salt water about 18 to 20 pounds per cubic foot is used, and the timber is then perfectly protected against the ravages of the *teredo navalis*. To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are first drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. This is continued for several hours. Then, after one of the vacuum periods, the cylinder is filled with creosote oil at a temperature of about  $170^{\circ}$  F. The pumps are kept at work until the pressure is about 80 to 100 pounds per square inch, and is maintained at this pressure from one to two hours according to the size of the timber. The oil is then



withdrawn, the cylinders opened, the train pulled out and another load made up in 40 to 60 minutes. The average time required for treating a load is about 18 or 20 hours, the absorption about 10 or 11 pounds of oil per cubic foot, and the cost (1894) from \$12.50 to \$14.50 per thousand feet B. M.

**213. Burnettizing (chloride-of-zinc process).** This process is very similar to the creosoting process except that the chemical is chloride of zinc, and that the chemical is not heated before use. The preliminary treatment of the wood to alternate vacuum and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The A., T. & S. F. & R. R. has works of its own at which ties are treated by this process at a cost of about 25 c. per tie. The Southern Pacific R. R. also has works for burnettizing ties at a cost of 9.5 to 12 c. per tie. The zinc-chloride solution used in these works contains only 1.7% of zinc chloride instead of over 3% as used in the Santa Fé works, which perhaps accounts partially for the great difference in cost per tie. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection, which is more forcible with respect to timber subject to great stresses, as in trestles, than to ties, is the fact that when the solution of zinc chloride is made strong (over 3%) the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to  $\frac{1}{4}$  to  $\frac{1}{10}$  of the ultimate strength, and that the elastic limit has been reduced by about  $\frac{1}{2}$ .

**214. Kyanizing (bichloride-of-mercury or corrosive-sublimate process).** This is a process of "steeping." It requires a much longer time than the previously described processes, but does not require such an expensive plant. Wooden tanks of sufficient size for the timber are all that is necessary. The corrosive sublimate is first made into a concentrated solution of one part of chemical to six parts of *hot* water. When used in the tanks this solution is weakened to 1 part in 100 or 150. The wood will absorb about 5 to 6.5 pounds of the bichloride per 100 cubic feet, or about one pound for each 4 to 6 ties. The timber is allowed to soak in the tanks for several days, the general rule

being about one day for each inch of least thickness and one day over—which means seven days for six-inch ties, or thirteen (to fifteen) days for 12" timber (least dimension). The process is somewhat objectionable on account of the chemical being such a virulent poison, workmen sometimes being sickened by the fumes arising from the tanks. On the Baden railway (Germany) kyanized ties last 20 to 30 years. On this railway the wood is always air-dried for two weeks after impregnation and before being used, which is thought to have an important effect on its durability. The solubility of the chemical and the liability of the chemical washing out and leaving the wood unprotected is an element of weakness in the method.

**215. Wellhouse (or zinc-tannin) process.** The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution, it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. Many of them belong to one class, of which the Wellhouse process is a sample. By these processes the timber is successively subjected to the action of two chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. By the Wellhouse process, the wood is first impregnated with a solution of chloride of zinc and glue, and is then subjected to a bath of tannin under pressure. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc chloride from being washed out. It is being used by the A., T. & S. Fé R. R., their works being located at Las Vegas, New Mexico, and also by the Union Pacific R. R. at their works at Laramie, Wyo. In 1897 Mr. J. M. Meade, a resident engineer on the A., T. & S. Fé, exhibited to the Roadmasters Association of America a piece of a tie treated by this process which had been taken from the tracks after nearly 13 years' service. The tie was selected at random, was taken out for the sole purpose of having a specimen, and was still in sound condition and capable of serving many years longer. The cost of the treatment was then quoted as 13 c. per tie.

It was claimed that the treatment trebled the life of the tie besides adding to its spike-holding power.

**216. Cost of treating.** The cost of treating ties by the various methods has been estimated as follows \*—assuming that the plant was of sufficient capacity to do the work economically: creosoting, 25 c. per tie; vulcanizing, 25 c. per tie; burnettizing (chloride of zinc), 8.25 c. per tie; kyanizing (steeping in corrosive sublimate), 14.6 c. per tie; Wellhouse process (chloride of zinc and tannin), 11.25 c. per tie. These estimates are only for the net cost at the works and do not include the cost of hauling the ties to and from the works, which may mean 5 to 10 c. per tie. Some of these processes have been installed on cars which are transported over the road and operated where most convenient.

**217. Economics of treated ties.** The fact that treated ties are not universally adopted is due to the argument that the added life of the tie is not worth the extra cost. If ties can be bought for 25 c., and cost 25 c. for treatment, and the treatment only doubles their life, there is apparently but little gained except the work of placing the extra tie in the track, which is more or less offset by the interest on 25 c. for the life of the untreated tie, and the larger initial outlay makes a stronger impression on the mind than the computed ultimate economy. But when ties cost 75 c. and treatment costs only 25 c., or perhaps less, then the economy is more apparent and unquestionable. But this analysis may be made more closely. As shown in § 202, the disturbance of the roadbed on account of frequent renewals of untreated ties is a disadvantage which would justify an appreciable expenditure to avoid, although it is very difficult to closely estimate its true value. The annual cost of a system of ties may be considered as the sum of (*a*) the interest on the first cost, (*b*) the annual sinking fund that would buy a new tie at the end of its life, and (*c*) the average annual cost of maintenance for the life of the tie, which includes the cost of laying and the considerable amount of subsequent tamping that must be done until the tie is fairly settled in the roadbed, besides the regular trackwork on the tie, which is practically constant. This last item is difficult to compute, but it is easy to see that, since

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\* Bull. No. 9, U. S. Dept. of Agric., Div. of Forestry. App. No. 1, by Henry Flad.

the cost of laying the tie and the subsequent tamping to obtain proper settlement is the same for all ties (of similar form), the *average* annual charge on the longer-lived tie would be much less. In the following comparison item (c) is disregarded, simply remembering that the advantage is with the longer-lived tie.

|                                            | Untreated tie. | Treated tie. |
|--------------------------------------------|----------------|--------------|
| Original cost.....                         | 40 cents       | 65 cents     |
| Life (assumed at).....                     | 7 years        | 14 years     |
| Item (a)—interest on first cost @ 4%.....  | 1.6 cents      | 2.6 cents    |
| “ (b)—sinking fund @ 4%.....               | 5.1 “          | 3.6 “        |
| “ (c)—(considered here as balanced).....   | —              | —            |
| Average annual cost (except item (c))..... | 6.7 cents      | 6.2 cents    |

On this basis treated ties will cost 0.5 cent *less* per annum *besides* the advantage of item (c) and the still more indefinite advantages resulting from smoother running of trains, less wear and tear on rolling stock, etc., due to less disturbance of the roadbed.

In Europe, where wood is expensive, untreated ties are seldom used, as the treatment is always considered to be worth more than it costs. The rapid destruction of the forests of timber in this country is having the effect of increasing the price, so that it will not be long before treated ties (or metal ties) will be economical for a large majority of the railroads of the country.

(*Note added in 1902.*) Some modifications of the above processes have been devised in recent years, among them being the

|                                                          |                                                        |
|----------------------------------------------------------|--------------------------------------------------------|
| Creo-resinate process—creosote, resin, and formaldehyde; |                                                        |
| Water-creosote                                           | “ —emulsion of creosote and water;                     |
| Zinc-creosote                                            | “ —emulsion of creosote and zinc-chloride;             |
| Allardyce                                                | “ —injection of chloride of zinc followed by creosote; |
| Hasselmann                                               | “ —boiling in sulphates of iron, copper, etc.          |

The Atchison, Topeka and Santa Fé R. R. has compiled a record of treated pine ties removed in 1897, '98, '99, and 1900, showing that the *average* life of the ties removed had been about 11 years. On the Chicago, Rock Island and Pacific R. R., the average life of a very large number of treated hemlock and tamarack ties was found to be 10.57 years. Of one lot of 21850 ties, 12% still remained in the track after 15 years' exposure.

It has been demonstrated that much depends on the minor

details of the process—whatever it may be. As an illustration, an examination of a batch of ties, treated by the zinc-creosote process, showed 84% in service after 13 years' exposure; another batch, treated by another contractor by the same process (nomirally), showed 50% worthless after a service of six years.

#### METAL TIES.

**218. Extent of use.** In 1894 \* there were nearly 35000 miles of "metal track" in various parts of the world. Of this total, there were 3645 miles of "longitudinals" (see § 224), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of "bowls and plates" (see § 223), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are not used at all, except experimentally. In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly 35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

**219. Durability.** The durability of metal ties is still far from being a settled question, due largely to the fact that the best form for such ties is not yet determined, and that a large part of the apparent failures in metal ties have been evidently due to defective design. Those in favor of them estimate the life as from 30 to 50 years. The opponents place it at not more

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\* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

than 20 years, or perhaps as long as the best of wooden ties. Unlike the wooden tie, however, which deteriorates as much with time as with usage, the life of a metal tie is more largely a function of the traffic. The life of a well-designed metal tie has been estimated at 150000 to 200000 trains; for 20 trains per day, or say 6000 per year, this would mean from 25 to 33 years. 20 trains per day on a *single* track is a much larger number than will be found on the majority of railroads. Metal ties are found to be subject to rust, especially when in damp localities, such as tunnels; but on the other hand it is in such confined localities, where renewals are troublesome, that it is especially desirable to employ the best and longest-lived ties. Paint, tar, etc., have been tried as a protection against rust, but the efficacy of such protection is as yet uncertain, the conditions preventing any renewal of the protection such as may be done by repainting a bridge, for example. Failures in metal cross-ties have been largely due to cracks which begin at a corner of one of the *square* holes which are generally *punched* through the tie, the holes being made for the bolts by which the rails are fastened to the tie. The holes are generally *punched* because it is cheaper. Reaming the holes after punching is thought to be a safeguard against this frequent cause of failure. Another method is to round the corners of the square punch with a radius of about  $\frac{1}{8}$ ". If a crack has already started, the spread of the crack may be prevented by drilling a small hole at the end of it.

**220. Form and dimensions of metal cross-ties.** Since stability in the ballast is an essential quality for a tie, this must be accomplished either by turning down the end of the tie or by having some form of lug extending downward from one or more points of the tie. The ties are sometimes depressed in the center (see Plate VI, N. Y. C. & H. R. R. R. tie) to allow for a thick covering of ballast on top in order to increase its stability in the ballast. This form requires that the ties should be sufficiently well tamped to prevent a tendency to bend out straight, thus widening the gauge. Many designs of ties are objectionable because they cannot be placed in the track without disturbing adjacent ties. The failure of many metal cross-ties, otherwise of good design, may be ascribed to too light weight. Those weighing much less than 100 pounds have proved too light. From 100 to 130 pounds weight is being used satisfactorily on German railroads. The general outside dimensions are about



the same as for wooden ties, except as to thickness. The metal is generally from  $\frac{1}{4}$ " to  $\frac{3}{8}$ " thick. They are, of course, only made of wrought iron or steel, cast iron being used only for "bowls" or "plates" (see § 223). The details of construction of some of the most commonly used ties may be seen by a study of Plate VI.

**221. Fastenings.** The devices for fastening the rails to the ties should be such that the gauge may be widened if desired on curves, also that the gauge can be made true regardless of slight inaccuracies in the manufacture of the ties, and also that shims may be placed under the rail if necessary during cold weather when the tie is frozen into the ballast and cannot be easily disturbed. Some methods of fastening require that the base of the rail be placed against a lug which is riveted to the tie or which forms a part of it. This has the advantage of reducing the number of pieces, but is apt to have one or more of the disadvantages named above. Metal keys or wooden wedges are sometimes used, but the majority of designs employ some form of bolted clamp. The form adopted for the experimental ties used by the N. Y. C. & H. R. R. R. (see Plate VI) is especially ingenious in the method used to vary the gauge or allow for inaccuracies of manufacture. Plate VI shows some of the methods of fastening adopted on the principal types of ties.

**222. Cost.** The cost of metal cross-ties in Germany averages about 1.6 c. per pound or about \$1.60 for a 100-lb. tie. The ties manufactured for the N. Y. C. & H. R. R. R. in 1892 weighed about 100 lbs. and cost \$2.50 per tie, but if they had been made in larger quantities and with the present price of steel the cost would possibly have been much lower. The item of freight from the place of manufacture to the place where used is no inconsiderable item of cost with some roads. Metal cross-ties have been used by some street railroads in this country. Those used on the Terre Haute Street Railway weigh 60 pounds and cost about 66 c. for the tie, or 74 c. per tie with the fastenings.

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**223. Bowls or plates.** As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted "bowls" laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being



made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60% to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the failures from this cause in India being about 0.4% per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate VI.

**224. Longitudinals.\*** This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a tendency to follow along the lines of the rails. The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of

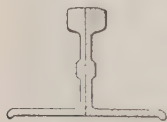


FIG. 110.

longitudinal is the Haarman compound "self-bearing" rail, having a base 12" wide and a height of 8", the alternate sections breaking joints so as to form a practically continuous rail.

Some of the other forms of longitudinals are illustrated in Plate VI.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.

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\* Although the discussion of longitudinals might be considered to be long more properly to the subject of RAILS, yet the essential idea of all designs must necessarily be the support of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here.

## CHAPTER IX.

### RAILS.

**225. Early forms.** The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1814. The "pear" section was an approach to the present form, but was very defective on account of the difficulty of designing a good form of joint. The "Stevens" section was designed in 1830 by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad; although quite defective in its proportions, according to the present knowledge of the requirements, it is essentially the present form. In 1836, Charles Vignoles invented essentially the same form in England; this form is therefore known throughout England and Europe as the Vignoles rail.

**226. Present standard forms.** The larger part of modern railroad track is laid with rails which are either "T" rails or the double-headed or "bull-headed" rails which are carried in chairs. The double-headed rail was designed with a symmetrical form with the idea that after one head had been worn out by traffic the rail could be reversed, and that its life would be practically doubled. Experience has shown that the wear of the

rail in the chairs is very great; so much so that when one head has been worn out by traffic the whole rail is generally useless.

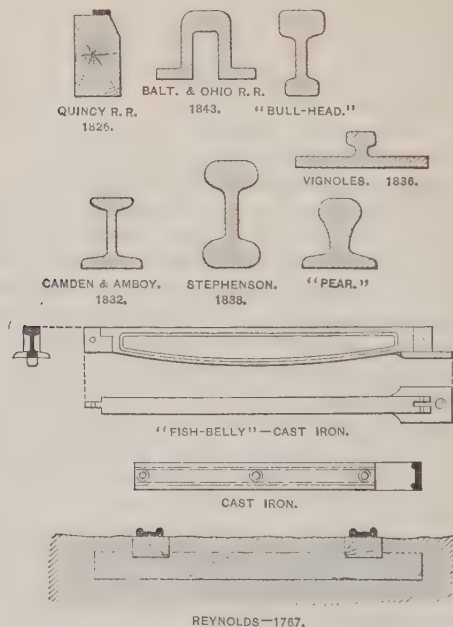


FIG. 111.—EARLY FORMS OF RAILS.

If the rail is turned over, the worn places, caused by the chairs, make a rough track and the rail appears to be more brittle and subject to fracture, possibly due to the crystallization that may have occurred during the previous usage and to the reversal of stresses in the fibers. Whatever the explanation, experience has

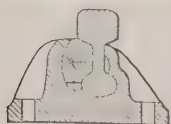


FIG. 112. — BULL-HEADED RAIL AND CHAIR.

demonstrated the *fact*. The "bull-headed" rail has the lower head only large enough to properly hold the wooden keys with which the rail is secured to the chairs (see Fig. 112) and furnish the necessary strength. The use of these rails requires the use of two cast-iron chairs for each tie. It is claimed that such track is better for heavy and fast traffic, but it is more

expensive to build and maintain. It is the standard form of track in England and some parts of Europe.

Until a few years ago there was a very great multiplicity in the designs of "T" rails as used in this country, nearly every prominent railroad having its own special design, which perhaps differed from that of some other road by only a very minute and insignificant detail, but which nevertheless would require a complete new set of rolls for rolling. This certainly must have had a very appreciable effect on the cost of rails. In 1893, the American Society of Civil Engineers, after a very exhaustive investigation of the subject, extending over several years, having obtained the opinions of the best experts of the country, adopted a series of sections which have been very extensively adopted by the railroads of this country. Instead of having the rail sections for various weights to be geometrically similar figures, certain dimensions are made constant, regardless of the weight. It was decided that the metal should be distributed through the section in the proportions of—head 42%, web 21%, and flange 37%. The top of the head should have a radius of

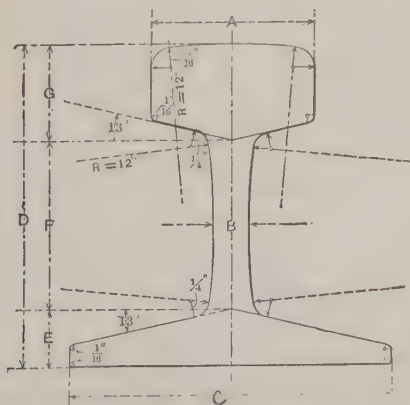


FIG. 113.—AM. SOC. C. E. STANDARD RAIL SECTION.

12"; the top corner radius of head should be  $\frac{5}{16}$ "; the lower corner radius of head should be  $\frac{1}{16}$ "; the corners of the flanges,  $\frac{1}{16}$ " radius; side radius of web, 12"; top and bottom radii of web corners,  $\frac{1}{4}$ "; and angles with the horizontal of the under side

of the head and the top of the flange, 13°. The sides of the head are vertical.

The height of the rail (*D*) and the width of the base (*C*) are always made equal to each other.

|                  | Weight per Yard.   |                 |                   |                   |                   |                   |                   |                   |                   |                   |                   |                   |                   |
|------------------|--------------------|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|                  | 40                 | 45              | 50                | 55                | 60                | 65                | 70                | 75                | 80                | 85                | 90                | 95                | 100               |
| <i>A</i>         | 17 $\frac{1}{2}$ " | 2"              | 2 $\frac{1}{8}$ " | 2 $\frac{1}{4}$ " | 2 $\frac{3}{8}$ " | 2 $\frac{1}{2}$ " | 2 $\frac{7}{8}$ " | 2 $\frac{3}{4}$ " | 2 $\frac{1}{2}$ " | 2 $\frac{1}{8}$ " | 2 $\frac{1}{4}$ " | 2 $\frac{1}{8}$ " | 2 $\frac{1}{4}$ " |
| <i>B</i>         | 2 $\frac{5}{8}$    | 2 $\frac{7}{8}$ | 3                 | 3 $\frac{1}{2}$   | 3 $\frac{3}{4}$   | 4                 | 4 $\frac{1}{2}$   | 5                 | 5 $\frac{1}{2}$   | 6                 | 6 $\frac{1}{2}$   | 7                 | 7 $\frac{1}{2}$   |
| <i>C &amp; D</i> | 3 $\frac{1}{2}$    | 3 $\frac{1}{8}$ | 3 $\frac{1}{4}$   | 3 $\frac{1}{2}$   | 4                 | 4 $\frac{1}{2}$   | 5                 | 5 $\frac{1}{2}$   | 6                 | 6 $\frac{1}{2}$   | 7                 | 7 $\frac{1}{2}$   | 8                 |
| <i>E</i>         | 8                  | 8 $\frac{1}{2}$ | 9                 | 9 $\frac{1}{2}$   | 10                | 10 $\frac{1}{2}$  | 11                | 11 $\frac{1}{2}$  | 12                | 12 $\frac{1}{2}$  | 13                | 13 $\frac{1}{2}$  | 14                |
| <i>F</i>         | 15 $\frac{1}{2}$   | 16              | 16 $\frac{1}{2}$  | 17                | 17 $\frac{1}{2}$  | 18                | 18 $\frac{1}{2}$  | 19                | 19 $\frac{1}{2}$  | 20                | 20 $\frac{1}{2}$  | 21                | 21 $\frac{1}{2}$  |
| <i>G</i>         | 16 $\frac{1}{2}$   | 17              | 17 $\frac{1}{2}$  | 18                | 18 $\frac{1}{2}$  | 19                | 19 $\frac{1}{2}$  | 20                | 20 $\frac{1}{2}$  | 21                | 21 $\frac{1}{2}$  | 22                | 22 $\frac{1}{2}$  |

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ( $\frac{5}{16}$ "") adopted for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who often use a radius of  $\frac{1}{4}$ ". On the other hand it is much less than is advocated by those

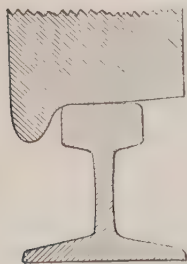


FIG. 114.—RELATION OF RAIL TO WHEEL-TREAD.

who consider that it should be nearly equal to (or even greater than) the larger radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so, as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid while the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly.

227. Weight for various kinds of traffic. The heaviest rails in regular use weigh 100 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the

N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and a few others. Probably the larger part of the mileage of the country is laid with 60- to 75-lb. rails—considering the fact that “the larger part of the mileage” consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 56-lb. rails. Roads with fairly heavy traffic generally use 75- to 85-lb. rails, especially when grades are heavy and there is much and sharp curvature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that the price of rails has been so reduced that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the **STRENGTH** and the **STIFFNESS**. If we assume that all weights of rails have *similar* cross-section, (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth powers* while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the  $\frac{3}{2}$  power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only  $6\frac{2}{3}\%$  to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

**228. Effect of stiffness on traction.** A very important but generally unconsidered feature of a stiff rail is its effect on tractive force. An extreme illustration of this principle is seen when a vehicle is drawn over a soft sandy road. The constant compression of the sand in front of the wheel has virtually the same effect on traction as drawing the wheel up a grade whose

steepness depends on the radius of the wheel and the depth of the rut. On the other hand, if a wheel, made of perfectly elastic material, is rolled over a surface which, while supported with absolute rigidity, is also perfectly elastic, there would be a forward component, caused by the expanding of the compressed metal just behind the center of contact, which would just balance the backward component. If the rail was supported throughout its length by an absolutely rigid support, the high elasticity of the wheel-tires and rails would reduce this form of resistance to an insignificant quantity, but the ballast and even the ties are comparatively inelastic. When a weak rail yields, the ballast is more or less compressed or displaced, and even though the elasticity of the rail brings it back to nearly its former place, the work done in compressing an inelastic material is wholly lost. The effect of this on the fuel account is certainly very considerable and yet is frequently entirely overlooked. It is practically impossible to compute the saving in tractive power, and therefore in cost of fuel, resulting from a given increase in the weight and stiffness of the rail, since the yielding of the rail is so dependent on the spacing of the ties, the tamping, etc. But it is not difficult to perceive in a general way that such an economy is possible and that it should not be neglected in considering the value of stiffness in rails.

**229. Length of rails.** The standard length of rails with most railroads is 30 feet. In recent years many roads have been trying 45-foot and even 60-foot rails. The argument in favor of longer rails is chiefly that of the reduction in track-joints, which are costly to construct and to maintain and are a fruitful source of accidents. Mr. Morrison of the Lehigh Valley R. R.\* declares that, as a result of extensive experience with 45-foot rails on that road, he finds that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly  $\frac{3}{4}$ " for a 60-foot rail. The Pennsylvania R. R. and the Norfolk and Western R. R. each have a considerable mileage laid with 60-foot rails.

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\* Report, Roadmasters Association, 1895.



**230. Expansion of rails.** Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about  $160^{\circ}$ , or say from  $-20^{\circ}$  F. to  $+140^{\circ}$  F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about  $\frac{3}{4}$  inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., experimented with a section over 500 feet long, which, although not a single rail, was made "continuous" by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below  $+20^{\circ}$  F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of  $60^{\circ}$  F. and the temperature sinks to  $0^{\circ}$ , the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28000000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to  $120^{\circ}$  F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of temperature of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes. Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

**231. Rules for allowing for temperature.** Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them. The thickness of these shims should vary with the temperature. Some roads use such approximate rules as the following: "The proper thickness for coldest weather is  $\frac{5}{16}$  of an inch; during spring and fall use  $\frac{1}{8}$  of an inch, and in the very

hottest weather  $\frac{1}{16}$  of an inch should be allowed." This is on the basis of a 30-foot rail. When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (120° to 150° F.) as a maximum, when the joints should be *tight*; then compute in tabular form the spacing for each temperature, varying by 20°, allowing 0".0468 (almost exactly  $\frac{3}{64}$ " ) for each 20° change. Such a tabular form would be about as follows (rail length 30 feet):

| Temperature . . .  | 150° | 130°             | 110°             | 90°              | 70°              | 50°               | 30°              | 10°               | - 10°           | -30°              |
|--------------------|------|------------------|------------------|------------------|------------------|-------------------|------------------|-------------------|-----------------|-------------------|
| Rail opening . . . | 0    | $\frac{3}{64}$ " | $\frac{3}{32}$ " | $\frac{9}{64}$ " | $\frac{3}{16}$ " | $\frac{15}{64}$ " | $\frac{9}{32}$ " | $\frac{31}{64}$ " | $\frac{3}{8}$ " | $\frac{27}{64}$ " |

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid. A rail lying in the hot sun has a very much higher temperature than the air. The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable. On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.

**232. Chemical composition.** About 98 to 99.5% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts. The manager of a steel-rail mill once declared that their aim was to produce rails having in them—

|                      |                |
|----------------------|----------------|
| Carbon . . . . .     | 0.32 to 0.40%  |
| Silicon . . . . .    | 0.04 to 0.06%  |
| Phosphorus . . . . . | 0.09 to 0.105% |
| Manganese . . . . .  | 1.00 to 1.50%  |

The analysis of 32 specimens of rails on the Chic., Mil. & St. Paul R. R. showed variations as follows:

|                      |                 |
|----------------------|-----------------|
| Carbon . . . . .     | 0.211 to 0.52%  |
| Silicon . . . . .    | 0.013 to 0.256% |
| Phosphorus . . . . . | 0.055 to 0.181% |
| Manganese . . . . .  | 0.35 to 1.63%   |

These quantities have the same general relative proportions as the rail-mill standard given above, the differences lying mainly in the broadening of the limits. Increasing the percentage of carbon by even a few hundredths of one per cent makes the rail harder, but likewise more brittle. If a track is well ballasted and not subject to heaving by frost, a harder and more brittle rail may be used without excessive danger of breakage, and such a rail will wear much longer than a softer tougher rail, although the softer tougher rail may be the better rail for a road having a less perfect roadbed.

A small but objectionable percentage of sulphur is sometimes found in rails, and very delicate analysis will often show the presence, in very minute quantities, of several other chemical elements. The use of a very small quantity of nickel or aluminum has often been suggested as a means of producing a more durable rail. The added cost and the uncertainty of the amount of advantage to be gained has hitherto prevented the practical use or manufacture of such rails.

**233. Testing.** Chemical and mechanical testing are both necessary for a thorough determination of the value of a rail. The chemical testing has for its main object the determination of those minute quantities of chemical elements which have such a marked influence on the rail for good or bad. The mechanical testing consists of the usual tests for elastic limit, ultimate strength, and elongation at rupture, determined from pieces cut out of the rail, besides a "drop test." The drop test consists in dropping a weight of 2000 lbs. from a height of 15 to 20 feet on to the center of a rail which is supported on abutments, placed three or four feet apart. The number of blows required to produce rupture or to produce a permanent set of specified magnitude gives a measure of the strength and toughness of the rail.

**233a. Proposed standard specifications for steel rails.** The following specifications for steel rails are those proposed by a committee of the American Railway Engineering and Maintenance of Way Association in March, 1902:

1. (a) Steel may be made by the Bessemer or open-hearth process.

(b) The entire process of manufacture and testing shall be in accordance with the best standard current practice, and special care shall be taken to conform to the following instructions:

(c) Ingots shall be kept in a vertical position in pit-heating furnaces.

(d) No bled ingots shall be used.

(e) Sufficient material shall be discarded from the top of the ingots to insure sound rails.

#### CHEMICAL PROPERTIES.

2. Rails of the various weights per yard specified below shall conform to the following limits in chemical composition:

|                                     | 50 to 59+<br>lbs.<br>per cent. | 60 to 69+<br>lbs.<br>per cent. | 70 to 79+<br>lbs.<br>per cent. | 80 to 89+<br>lbs.<br>per cent. | 90 to 100<br>lbs.<br>per cent. |
|-------------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| Carbon.....                         | 0.35-0.45                      | 0.38-0.48                      | 0.40-0.50                      | 0.43-0.53                      | 0.45-0.55                      |
| Phosphorus shall not<br>exceed..... | 0.10                           | 0.10                           | 0.10                           | 0.10                           | 0.10                           |
| Silicon shall not ex-<br>ceed.....  | 0.20                           | 0.20                           | 0.20                           | 0.20                           | 0.20                           |
| Manganese.....                      | 0.70-1.00                      | 0.70-1.00                      | 0.75-1.05                      | 0.80-1.10                      | 0.80-1.10                      |

#### PHYSICAL PROPERTIES.

3. One drop test shall be made on a piece of rail not more than 6 feet long, selected from every fifth blow of steel. The test-piece shall be taken from the top of the ingot. The rail shall be placed head upwards on the supports and the various sections shall be subjected to the following impact tests:

| Weight of Rail in Pounds per Yard. |                          | Height of Drop<br>in Feet. |
|------------------------------------|--------------------------|----------------------------|
| More than 45                       | to and including 55..... | 15                         |
| " " 55                             | " " " 65.....            | 16                         |
| " " 65                             | " " " 75.....            | 17                         |
| " " 75                             | " " " 85.....            | 18                         |
| " " 85                             | " " " 100.....           | 19                         |

If any rail break when subjected to the drop test two additional tests will be made of other rails from the same blow of steel, and if either of these latter tests fail, all the rails of the blow which they represent will be rejected; but if both of these additional test-pieces meet the requirements all the rails of the blow which they represent will be accepted. If the rails from the tested blow shall be rejected for failure to meet the requirements of

the drop test, as above specified, two other rails will be subjected to the same tests, one from the blow next preceding and one from the blow next succeeding, the rejected blow. In case the first test taken from the preceding or succeeding blow shall fail two additional tests shall be taken from the same blow of steel, the acceptance or rejection of which shall also be determined as specified above, and if the rails of the preceding or succeeding, blow shall be rejected, similar tests may be taken from the previous or following blows, as the case may be, until the entire group of five blows is tested, if necessary. The acceptance or rejection of all rails from any blow will depend upon the results of the tests thereof.

#### HEAT TREATMENT.

The number of passes and speed of train shall be so regulated that on leaving the rolls at the final pass the temperature of the rail will not exceed that which requires a shrinkage allowance at the hot saws of 6 inches for 85-lb. and  $6\frac{1}{2}$  inches for 100-lb. rails, and no artificial means of cooling the rails shall be used between the finishing pass and the hot saws.

#### TEST-PIECES AND METHODS OF TESTING.

4. The drop-test machine shall have a tup of 2000 lbs. weight, the striking face of which shall have a radius of not more than 5 inches, and the test rail shall be placed head upwards on solid supports 3 feet apart. The anvil-block shall weigh at least 20000 lbs., and the support shall be a part of, or firmly secured to, the anvil.

5. The manufacturer shall furnish the inspector, daily, with carbon determinations of each blow, and a complete chemical analysis every 24 hours, representing the average of the other elements contained in the steel. These analyses shall be made on drillings taken from a small test ingot.

#### FINISH.

6. Unless otherwise specified the section of rail shall be the American standard, recommended by the American Society of Civil Engineers, and shall conform, as accurately as possible, to the templet furnished by the railroad company, consistent with paragraph No. 7, relative to the specified weight. A vari-

ation in height of  $\frac{1}{4}$  inch less and  $\frac{1}{2}$  inch greater than the specified height will be permitted. A perfect fit of the splice-bars, however, shall be maintained at all times.

7. The weight of the rails shall be maintained as nearly as possible, after complying with paragraph No. 6, to that specified in contract. A variation of one-half of one per cent for an entire order will be allowed. Rails shall be accepted and paid for according to actual weights.

8. The standard length of rails shall be 33 feet. Ten per cent of the entire order will be accepted in shorter lengths, varying by even feet down to 27 feet. A variation of  $\frac{1}{4}$  inch in length from that specified will be allowed.

9. Circular holes for splice-bars shall be drilled in accordance with the specifications of the purchaser. The holes shall accurately conform to the drawing and dimensions furnished in every respect, and must be free from burrs.

10. Rails shall be straightened while cold, smooth on head, sawed square at ends, and, prior to shipment, shall have the burr, occasioned by the saw-cutting, removed, and the ends made clean. No. 1 rails shall be free from injurious defects and flaws of all kinds.

#### BRANDING.

11. The name of the maker, the month and year of manufacture shall be rolled in raised letters on the side of the web, and the number of the blow shall be stamped on each rail

#### INSPECTION.

12. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture, prior to shipment.

#### NO. 2 RAILS.

13. Rails that possess any injurious physical defects, or which for any other cause are not suitable for first quality, or No. 1 rails, shall be considered as No. 2 rails, provided, however, that rails which contain any physical defects which seriously impair their strength shall be rejected. The ends of all No. 2 rails shall be painted in order to distinguish them.

**234. Rail wear on tangents.** When the wheel loads on a rail are abnormally heavy, and particularly when the rail has but little carbon and is unusually soft, the concentrated pressure on the rail is frequently greater than the elastic limit, and the metal "flows" so that the head, although greatly abraded, will spread somewhat outside of its original lines, as shown in Fig. 115. The rail wear that occurs on tangents is almost exclusively on top. Statistics show that the rate of rail wear on tangents decreases as the rails are more worn. Tests of a large number of rails on tangents have shown a rail wear averaging nearly one pound per yard per 10 000 000 tons of traffic. There is about 33 pounds of metal in one yard of the head of an 80-lb. rail. As an extreme value this may be worn down one-half, thus giving a tonnage of 165 000 000 tons for the life of the rail. Other estimates bring the tonnage down to 125 000 000 tons. Since the locomotive is considered to be responsible for one-half (and possibly more) of the damage done to the rail, it is found that the rate of wear on roads with shorter trains is more rapid in proportion to the tonnage, and it is therefore thought that the life of a rail should be expressed in terms of the number of trains. This has been estimated at 300 000 to 500 000 trains.



FIG. 115.

**235. Rail wear on curves.** On curves the maximum rail wear occurs on the inner side of the head of the outer rail, giving a worn form somewhat as shown in Fig. 116. The dotted line

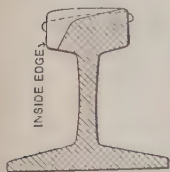


FIG. 116.

shows the nature and progress of the rail wear on the inner rail of a curve. Since the pressure on the outer rail is somewhat lateral rather than vertical, the "flow" does not take place to the same extent, if at all, on the outside, and whatever flow would take place on the inside is immediately worn off by the wheel-flange. Unlike the wear on tangents, the wear on curves is at a greater

rate as the rail becomes more worn.

The inside rail on curves wears chiefly on top, the same as on a tangent, except that the wear is much greater owing to the longitudinal slipping of the wheels on the rail, and the lateral slipping that must occur when a rigid four-wheeled truck is



guided around a curve. The outside rail is subjected to a greater or less proportion of the longitudinal slipping, likewise to the lateral slipping, and, worst of all, to the grinding action of the flange of the wheel, which grinds off the side of the head.

The results of some very elaborate tests, made by Mr. A. M. Wellington, on the Atlantic and Great Western R. R., on the wear of rails, seem to show that the rail wear on curves may be expressed by the formula: "Total wear of rails on a  $d$  degree curve in pounds per yard per 10 000 000 tons duty  $= 1 + 0.03d^2$ ." "It is not pretended that this formula is strictly correct even in theory, but several theoretical considerations indicate that it may be nearly so." According to this formula the average rail wear on a  $6^\circ$  curve will be about twice the rail wear on a tangent. While this is approximately true, the various causes modifying the rate of rail wear (length of trains, age and quality of rails, etc.) will result in numerous and large variations from the above formula, which should only be taken as indicating an approximate law.

**236. Cost of rails.** In 1873 the cost of steel rails was about \$120 per ton, and the cost of iron rails about \$70 per ton. Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they have steadily dropped in price until, during the last few years, steel rails have been manufactured and sold for \$22 per ton. At such prices there is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails.

## CHAPTER X.

### RAIL-FASTENINGS.

#### RAIL-JOINTS

237. **Theoretical requirements for a perfect joint.** A perfect rail-joint is one that has the *same strength and stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail-ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 230), some other contrivance is necessary which will approach this ideal as closely as may be.

238. **Efficiency of the ordinary angle-bar.** Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary “suspended” joint. In order to main-

tain uniform strength and stiffness the angle-bars must supply the deficiency. These theoretical relations are modified to an unknown extent by the unknown and variable yielding of the ties. From some experiments made by the Association of Engineers of Maintenance of Way of the P. R. R.\* the following deductions were made:

1. The capacity of a "suspended" joint is greater than that of a "supported" joint—whether supported on one or three ties. (See § 240)

2. That (with the particular patterns tested) the angle-bars alone can carry only 53 to 56% of a concentrated load placed on a joint.

3. That the capacity of the whole joint (angle-bars and rail) is only 52.4% of the strength of the unbroken rail.

4. That the ineffectiveness of the angle-bar is due chiefly to a deficiency in compressive resistance.

Although it has been universally recognized that the angle-bar is not a perfect form of joint, its simplicity, cheapness, and reliability have caused its almost universal adoption. Within a very few years other forms (to be described later) have been adopted on trial sections and have been more and more extended, until their present use is very large. The present time (1900) is evidently a transition period, and it is quite probable that within a very few years the now common angle-plate will be as unknown in standard practice as the old-fashioned "fish-plate" is at the present time.

**239. Effect of rail gap at joints.** It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint and scarcely at all to the small gap required for expansion. This gap causes a drop equal to the versed sine of the arc having a chord equal to the gap and a radius equal to the radius of the wheel. Taking the extreme case (for a 30-foot rail) of a  $\frac{3}{8}$ " gap and a 33" freight-car wheel, the drop is about  $\frac{1}{16}$ ". In order to test how much the jarring at a joint is due to a gap between the rails, the experiment was tried of cutting shallow notches in the top of an otherwise solid rail and running a locomotive and an inspection car over them. The resulting jarring was practically imperceptible and not comparable to the jar produced at joints. Notwithstanding this fact, many plans have

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\* Roadmasters Association of America—Reports for 1897.

been tried for avoiding this gap. The most of these plans consist essentially of some form of compound rail, the sections breaking joints. (Of course the design of the compound rail has also several other objects in view.) In Fig. 117 are shown a



FIG. 117.—COMPOUND RAIL SECTIONS.

few of the very many designs which have been proposed. These designs have invariably been abandoned after trial. Another plan, which has been extensively tried on the Lehigh Valley R. R., is the use of mitered joints. The advantages gained by their use are as yet doubtful, while the added expense is unquestionable. The "Roadmasters Association of America" in 1895 adopted a resolution recommending mitered joints for double track, but their use does not seem to be growing.

240. "Supported," "suspended," and "bridge" joints. In a supported joint the ends of the rails are on a tie. If the angle-plates are short, the joint is entirely supported on one tie; if very long, it may be possible to place three ties under one angle-bar and thus the joint is virtually supported on three ties rather than one. In a suspended joint the ends of the rails are midway between two ties and the joint is supported by the two. There have always been advocates of both methods, but suspended joints are more generally used than supported joints. The opponents of three-tie joints claim that either the middle tie will be too strongly tamped, thus making it a supported joint, or that, if the middle tie is weakest, the joint becomes a very long (and therefore weak) suspended joint between the outer joint-ties, or that possibly one of the outer joint-ties gives way, thus breaking the angle-plate at the joint. Another objection which is urged is that unless the bars are very long (say 44 inches, as used on the Mich. Cent. R. R.) the ties are too close for proper tamping. The best answer to these objections is the successful use of these joints on several heavy-traffic roads.

"Bridge"-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge joint supports the rail from *underneath* and

there is no transverse stress in the rail, whereas the suspended joint requires the combined transverse strength of both angle-bars and rail. A serious objection to bridge-joints lies in the fact of their considerable thickness between the rail base and the tie. When joints are placed "staggered" rather than "opposite" (as is now the invariable standard practice), the ties supporting a bridge-joint must either be notched down, thus weakening the tie and promoting decay at the cut, or else the tie must be laid on a slope and the joint and the opposite rail do not get a fair bearing.

**241. Failures of rail-joints.** It has been observed on double-track roads that the maximum rail wear occurs a few inches beyond the rail gap at the joint in the direction of the traffic. On single-track roads the maximum rail wear is found a few inches *each* side of the joint rather than at the extreme ends of the rail, thus showing that the rail end deflects down under the wheel until (with fast trains especially) the wheel actually jumps the space and strikes the rail a few inches beyond the joint, the impact producing excessive wear. This action, which is called the "drop," is apt to cause the first tie beyond the joint to become depressed, and unless this tie is carefully watched and maintained at its proper level, the stresses in the angle-bar may actually become reversed and the bar may break at the top. The angle-bars of a suspended joint are normally in compression at the top. The mere reversal of the stresses would cause the bars

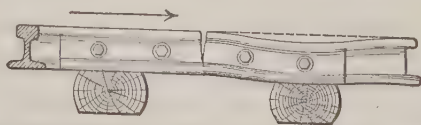


FIG. 118.—EFFECT OF "WHEEL DROP" (EXAGGERATED).

to give way with a less stress than if the stress were always the same in kind. A supported joint, and especially a three-tie joint (see § 240), is apt to be broken in the same manner.

**242. Standard angle-bars.** An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in nearly as great variety in the detailed dimensions of the angle-bars. The sections here illustrated must be considered only as types of the variable forms necessary for each different shape of rail. The

absolutely essential features required for a fit are (1) the angles of the upper and lower surfaces of the bar where they fit against the rail, and (2) the height of the bar. The bolt-holes in the bar and rail must also correspond. The holes in the angle-plates are elongated or made oval, so that the track-bolts, which are

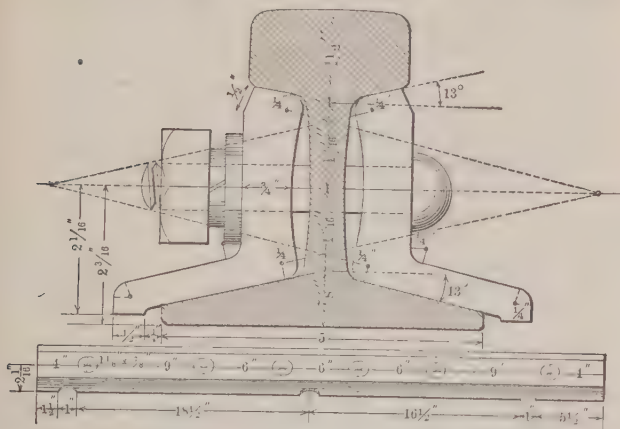


FIG. 119.—STANDARD ANGLE-BAR—80-LB. RAIL. M. C. R.R.

made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails are made of larger diameter (by about  $\frac{1}{8}$ " ) than the bolts, so as to allow the rail to expand with temperature.

243. Later designs of rail-joints. In Plate VII are shown various designs which are competing for adoption. The most prominent of these (judging from the discussion in the convention of the Roadmasters Association of America in 1897) are the "Continuous" and the "Weber." Each of them has been very extensively adopted, and where used are universally preferred to angle-plates. Nearly all the later designs embody more or less directly the principle of the bridge-joint, i.e., support the rail from underneath. An experience of several years will be required to demonstrate which form of joint best satisfies the somewhat opposed requirements of minimum cost (both initial and for maintenance) and minimum wear of rails and rolling stock.

243a. Proposed specifications for steel splice-bars. The following specifications for steel splice-bars were proposed in 1900 by Committee No. 1, American Section, International Association for Testing Materials.

1. Steel for splice-bars may be made by the Bessemer or open-hearth process.

2. Steel for splice-bars shall conform to the following limits in chemical composition:

|                                   | Per cent.    |
|-----------------------------------|--------------|
| Carbon shall not exceed. ....     | 0.15         |
| Phosphorus shall not exceed. .... | 0.10         |
| Manganese. ....                   | 0.30 to 0.60 |

3. Splice-bar steel shall conform to the following physical qualities:

|                                                                   |                |
|-------------------------------------------------------------------|----------------|
| Tensile strength, pounds per square inch. ....                    | 54000 to 64000 |
| Yield point, pounds per square inch. ....                         | 32000          |
| Elongation, per cent in eight inches shall not be less than. .... | 25             |

4. (a) A test specimen cut from the head of the splice-bar shall bend 180° flat on itself without fracture on the outside of the bent portion.

(b) If preferred the bending test may be made on an unpunched splice-bar, which, if necessary, shall be first flattened and shall then be bent 180° flat on itself without fracture on the outside of the bent portion.

5. A test specimen of 8-inch gauged length, cut from the head of the splice-bar, shall be used to determine the physical properties specified in paragraph No. 3.

6. One tensile specimen shall be taken from the rolled splice-bars of each blow or melt, but in case this develops flaws, or breaks outside of the middle third of its gauged length, it may be discarded and another test specimen submitted therefor.

7. One test specimen cut from the head of the splice-bar shall be taken from a rolled bar of each blow or melt, or if preferred the bending test may be made on an unpunched splice-bar which, if necessary, shall be flattened before testing. The bending test may be made by pressure or by blows.

8. For the purposes of this specification, the yield point shall



be determined by the careful observation of the drop of the beam or halt in the gauge of the testing machine.

9. In order to determine if the material conforms to the chemical limitations prescribed in paragraph No. 2 herein, analysis shall be made of drillings taken from a small test ingot.

10. All splice-bars shall be smoothly rolled and true to templet. The bars shall be sheared accurately to length and free from fins or cracks, and shall perfectly fit the rails for which they are intended. The punching and notching shall accurately conform in every respect to the drawing and dimensions furnished.

11. The name of the maker and the year of manufacture shall be rolled in raised letters on the side of the splice-bar.

12. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer, to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture, prior to shipment.

#### TIE-PLATES.

**244. Advantages.** (a) As already indicated in § 204, the life of a soft-wood tie is very much reduced by "rail-cutting" and "spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the "rail-cutting" is not due to the mere pressure of the rail on the tie, even with a maximum load on the rail, but is due to the impact resulting from vibration and to the longitudinal working of the rail. It has been proved that this rail-cutting is practically prevented by the use of tie-plates. (b) On curves there is a tendency to overturn the outer rail due to the lateral pressure on the side of the head. This produces a concentrated pressure of the outer edge of the base on the tie which produces rail-cutting and also draws the inner spikes. Formerly the only method of guarding against this was by the use of "rail-braces," one pattern of which is shown in Fig. 120. But it has been found that tie-plates serve the purpose even better, and rail-braces have been abandoned where tie-plates are used. (c) Driving spikes through holes in the plate enables the spikes on *each* side of the rail to mutually support each other, no matter in which (lateral) direction the rail may tend to move, and this probably accounts in large

measure for the added stability obtained by the use of tie-plates. (d) The wear in spikes, called "necking," caused by the vertical vibration of the rail against them, is very greatly reduced. (e) The cost is very small compared with the value of the added life of the tie, the large reduction in the work of track main-

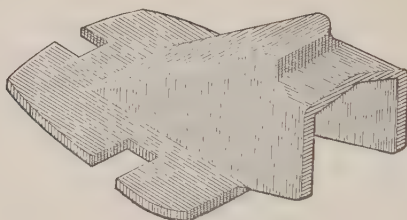


FIG. 120.

tenance, and the smoother running on the better track which is obtained. It has been estimated that by the use of tie-plates the life of hard-wood ties is increased from one to three years, and the life of soft-wood ties is increased from three to six years. From the very nature of the case, the value of tie-plates is greater when they are used to protect soft ties.

**245. Elements of the design.** The earliest forms of tie-plates were flat on the bottom, but it was soon found that they would work loose, allow sand and dirt to get between the rail and the plate and also between the plate and the tie, which would cause excessive wear. Such plates are also apt to produce an objectionable rattle. Another fault of the earlier designs was the use of plates so thin that they would buckle. The latest designs have flanges or "teeth" formed on the lower surface which penetrate the tie about  $\frac{3}{4}$ " to  $1\frac{3}{4}$ ". Opinion is still divided on the question of whether these teeth should run with the grain or across the grain. If the flanges run with the grain, they generally extend the whole length of the tie-plate—as in the Wolhaupter design. If the grain is to be cut crosswise, several teeth about 1" wide will be used—as in the Goldie design.

It is a very important feature that the spike-holes should be so punched that the spikes will fit closely to the base of the rail. Otherwise a lateral motion of the rail will be permitted which will defeat one of the main objects of the use of the plate.

Another unsettled detail is the use of "shoulders" on the upper surface. On the one hand it is claimed that the use of shoulders relieves the spikes of side pressure from the rail and prevents "necking." On the other hand it is claimed that if the

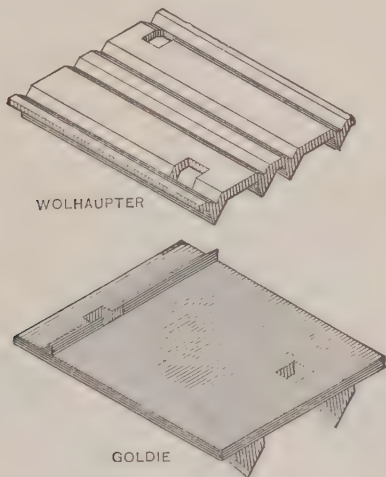


FIG. 121.—TIE-PLATES.

the plain plate is once properly set with new spikes (at least with spikes not already necked) the spikes will not neck appreciably, and that, as the shouldered plates cost more, the additional expenditure is unnecessary.

The above designs should be studied with reference to the manner in which they fulfill the requirements which have been already stated. As in the case of rail-joints, the best forms of tie-plates are of comparatively recent design, and experience with them is still insufficient to determine beyond all question which designs are the best.

**246. Method of setting.** A very important detail in the process of setting the tie-plates on the ties is that the flanges or teeth should penetrate the tie as far as desired when the plates are first put in position. It requires considerable force to press the teeth into a tie. In a few cases trackmen have depended on the easy process of waiting for passing trains to force the teeth

down. Until the teeth are down the spikes cannot be driven home, and this apparently cheap and easy process results in loose spikes and rails. If the trackmen neglect even temporarily to tighten these spikes, it will become impossible to make them tight ultimately. The plates are generally pounded into place with a 10- to 16-pound sledge-hammer. A very good method was adopted once during the construction of a bridge when a pile-driver was at hand. The bridge-ties were placed under the pile-hammer. The plates, accurately set to gauge, were then forced in by a blow from the 3000-lb. hammer falling 2 or 3 feet.

## SPIKES.

**247. Requirements.** The rails must be held to the ties by a fastening which will not only give sufficient resistance, but which will retain its capacity for resistance. It must also be cheap and easily applied. The ordinary track-spike fulfills the last requirements, but has comparatively small resisting power, compared with screws or bolts. Worse than all, the tendency to

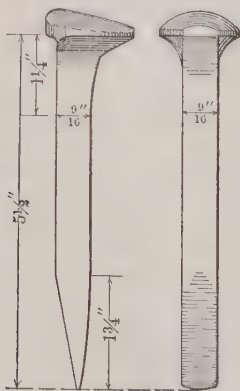


FIG. 122.



FIG. 123.

vertical vibration in the rail produces a series of upward pulls on the spike that soon loosens it. When motion has once begun the capacity for resistance is greatly reduced, and but little more vibration is required to pull the spike out so much that re-driving is necessary. Driving the spike to place again in the same hole

is of small value except as a very temporary expedient, as its holding power is then very small. Redriving the spikes in new holes very soon "spike-kills" the tie. Many plans have been devised to increase the holding power of spikes, such as making them jagged, twisting the spike, swelling the spike at about the center of its length, etc. But it has been easily demonstrated that the fibers of the wood are generally so crushed and torn by driving such spikes that their holding power is less than that of the plain spike.

The ordinary spike (see Fig. 122) is made with a square cross-section which is uniform through the middle of its length, the lower  $1\frac{3}{4}$ " tapering down to a chisel edge, the upper part swelling out to the head. The Goldie spike (see Fig. 123) aims to improve this form by reducing to a minimum the destruction of the fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will therefore cause the fibers to press still harder on the spike and thus increase the resistance.

**248. Driving.** The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered. Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in the two pairs of spikes in any one tie (see Fig. 124). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.

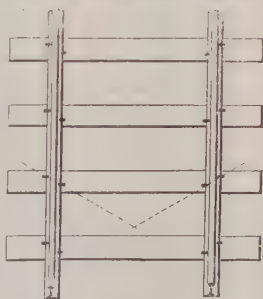


FIG. 124.—SPIKE-DRIVING.

**249. Screws and bolts.** The use of these abroad is very extensive, but their use in this country has not passed the experimental stage. The screws are "wood"-screws (see Fig. 125),

having large square heads, which are screwed down with a track-wrench. Holes, having the same diameter as the base of the screw-heads, should be first bored into the tie, at exactly the right position and at the proper angle with the vertical. A light wooden frame is sometimes used to guide the auger at the proper angle. Sometimes the large head of the screw bears directly against the base of the rail, as with the ordinary spike. Other designs employ a plate, made to fit the rail on one side, bearing on the tie on the other side, and through which the screw passes. These screws cost much more than the spikes and require more work to put in place, but their holding power is much greater and the work of track maintenance is very much less. Screw-bolts, passing entirely through the tie, having the head at the bottom of the tie and the nut on the upper side, are also used abroad. These are quite difficult to replace, requiring that the ballast be dug out beneath the tie, but on the other hand the



FIG. 125.

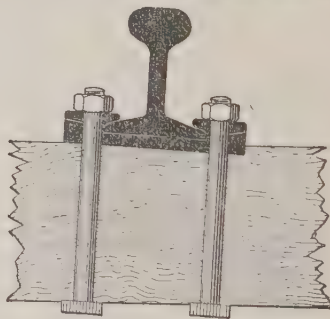


FIG. 126.

occasions for replacing such a bolt are comparatively rare, as their durability is very great. The use of screws or bolts increases the life of the tie by the avoidance of "spike-killing." It is capable of demonstration that the reduced cost of maintenance and the resulting improvement in track would much more than repay the added cost of screws and bolts, but it seems impossible to induce railroad directors to authorize a large and immediate additional expenditure to make an annual saving whose value, although unquestionably considerable, cannot be exactly computed.

250. "Wooden spikes." Among the regulations for track-laying given in § 208, mention was made of wooden "spikes,"



or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are irregular in size and therefore more or less inefficient. It is also quite probable that if the trackmen are required to make their own plugs, they would spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross-section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 127) has been designed to fill these requirements. Being machine-made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to attempt to cut them by hand.



FIG. 127.

#### TRACK-BOLTS AND NUT-LOCKS.

**251. Essential requirements.** The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent slipping during the contraction due to a considerable fall of temperature on a straight track, but when the track is curved, or when expansion takes place, it is conceivable that the resistance of the ties in the ballast to lateral motion may be less than the resistance at the joint. A test to determine this resistance was made by Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., using 80-lb. rails and ordinary angle-bars, the bolts being screwed up as usual. If required a force of about 31000 to



35000 lbs. to start the joint, which would be equivalent to the stress induced by a change of temperature of about  $22^{\circ}$ . But if the central angle of any given curve is small, a comparatively small lateral component will be sufficient to resist a compression of even 35000 lbs. in the rails. Therefore there will ordinarily be no trouble about having the joints screwed too tight. The vibration caused by the passage of a train reduces the resistance to slipping. This vibration also facilitates an objectionable feature, viz., loosening of the nuts of the track-bolts. The bolt is readily prevented from turning by giving it a form which is *not* circular immediately under the head and making corresponding holes in the angle-plate. Square holes would answer the purpose, except that the square corners in the holes in the angle-plates would increase the danger of fracture of the plates. Therefore the holes (and also the bolts, under the head) are made of an oval form, or perhaps a square form with rounded corners, avoiding angles in the outline.

The nut-locks should be simple and cheap, should have a life at least as long as the bolt, should be effective, and should not lose their effectiveness with age. Many of the designs that have been tried have been failures in one or more of these particulars as will be described in detail below.

**252. Design of track-bolts.** In Fig. 128 is shown a common design of track-bolt. In its general form this represents the bolt used on nearly all roads, being used not only with the common angle-plates, but also with many of the improved designs of rail-joints. The variations are chiefly a general increase in size to correspond with the increased weight of rails, besides variations in detail dimensions which are frequently unimportant. The diameter is usually  $\frac{3}{4}$ " to  $\frac{7}{8}$ "; 1" bolts are sometimes used for the heaviest sections of rails. As to length, the bolt should not extend more than  $\frac{1}{2}$ " outside of the nut when it is screwed up. If it extends farther than this it is liable to be

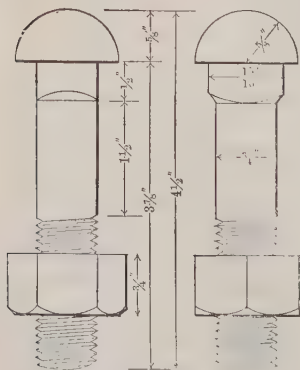


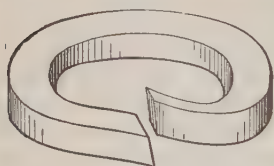
FIG. 128.—TRACK-BOLT.

broken off by a possible derailment at that point. The lengths used vary from  $3\frac{1}{4}$ ", which may be used with 60-lb. rails, to 5", which is required with 100-lb. rails. The length required depends somewhat on the type of nut-lock used.

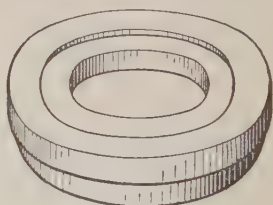
**253. Design of nut-locks.** The designs for nut-locks may be divided into three classes: (a) those depending entirely on an elastic washer which absorbs the vibration which might otherwise induce turning; (b) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (c) the "positive" nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The "vulcanized fiber" nut-lock is an example of the first class. It consists essentially of a rubber washer which is protected by an iron ring. When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless. Another illustration of class (a) is the use of wooden blocks, generally 1" to 2" oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint. This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition. The "Verona" nut-lock is another illustration of class (a) which also combines some of the positive elements of class (c). It is made of tempered steel and, as shown in Fig. 129, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The "National" nut-lock is a type of the second class (b), in which, like the "Harvey" nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The "National" nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed up it is forced slightly open and therefore presses on the threads of the bolt with such force that vibration cannot jar it loose. Unlike the "National" nut, the "Harvey "

nut is solid, but the form of the thread is progressively varied so that the thread pinches the thread of the bolt and the frictional resistance to turning is too great to be affected by vibration.

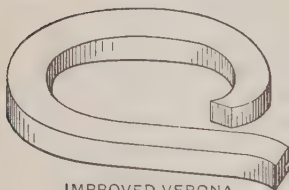
The "Jones" nut-lock, belonging to class (c), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of



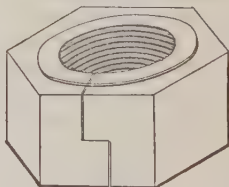
VERONA



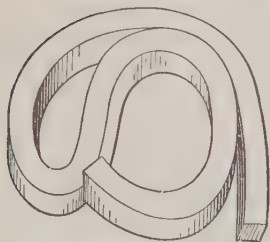
VULCANIZED FIBRE



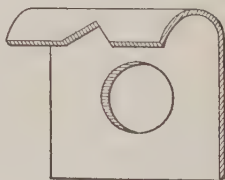
IMPROVED VERONA



NATIONAL



EXCELSIOR



JONES

FIG. 129.—TYPES OF NUT-LOCKS.

which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (c) are not in common use.

## CHAPTER XI.

### SWITCHES AND CROSSINGS.

#### SWITCH CONSTRUCTION.

**254. Essential elements of a switch.** Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a necessity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or "flange spaces" are formed *through* the rails. An ordinary stub-switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side. This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a "frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means of a short *movable* rail which is on occasion placed across the main rail, but such designs have not come into general use.

**255. Frogs.** Frogs are provided with two channel-ways or "flange spaces" through which the flanges of the wheels move.

Each channel cuts out a parallelogram from the tread area. Since the wheel-tread is always wider than the rail, the wing rails will support the wheel not only across the space cut out by the channel, but also until the tread has passed the point of the frog and can obtain a broad area of contact on the tongue of the frog. This is the theoretical idea, but it is very imperfectly

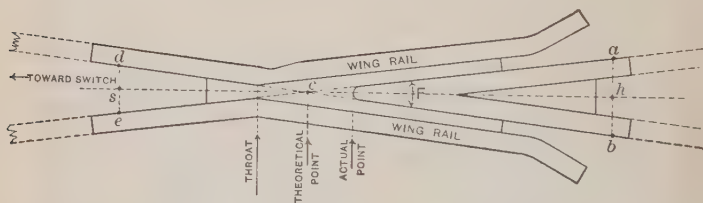


FIG. 130.—DIAGRAMMATIC DESIGN OF FROG.

realized. The wing rails are sometimes subjected to excessive wear owing to "hollow treads" on the wheels—owing also to the frog being so flexible that the point "ducks" when the wheel approaches it. On the other hand the sharp point of the frog will sometimes cause destructive wear on the tread of the wheel. Therefore the tongue of the frog is not carried out to the sharp theoretical point, but is purposely somewhat blunted. But the break which these channels make in the continuity of the tread area becomes extremely objectionable at high speeds, being mutually destructive to the rolling stock and to the frog. The jarring has been materially reduced by the device of "spring frogs"—to be described later. Frogs were originally made of cast iron—then of cast iron with wearing parts of cast steel, which were fitted into suitable notches in the cast iron. This form proved extremely heavy and devoid of that elasticity of track which is necessary for the safety of rolling stock and track at high speeds. The present universal practice is to build the frog up of pieces of rails which are cut or bent as required. These pieces of rails (at least four) are sometimes assembled by riveting them to a flat plate, but this method is now but little used, except for very light work. The usual practice is now chiefly divided between "bolted" and "keyed" frogs. In each case the space between the rails, except a sufficient flange-way, is filled with a cast-iron filler and the whole assemblage of parts

is suitably bolted or clamped together, as is illustrated in Plate VIII. The operation of a spring-rail frog is evident from the figure. Since a siding is usually operated at slow speed, while the main track may be operated at fast speed, a spring-rail frog will be so set that the tread is continuous for the main track and broken for the siding. This also means that the spring-rail will only be moved by trains moving at a (presumably) slow speed on to the siding. For the fast trains on the main line such a frog is substantially a "fixed" frog and has a tread which is practically continuous.

**256. To find the frog number.** The frog number ( $n$ ) equals the ratio of the distance of any part on the tongue of the frog from the theoretical point of the frog divided by the width of the tongue at that point, i.e.  $=hc \div ab$  (Fig. 130). This value may be directly measured by applying any convenient unit of measure (even a knife, a short pencil, etc.) to some point of the tongue where the width just equals the unit of measure, and then noting how many times the unit of measure is contained in the distance from that place to the theoretical point. But since  $c$ , the theoretical point, is not so readily determinable with exactitude, it being the imaginary intersection of the gauge lines, it may be more accurate to measure  $de$ ,  $ab$ , and  $hs$ ; then  $n$ , the frog number,  $=hs \div (ab + de)$ . If the frog angle be called  $F$ , then

$$n = hc \div ab = hs \div (ab + de) = \frac{1}{2} \cot \frac{1}{2}F;$$

i.e.,

$$\cot \frac{1}{2}F = 2n.$$

**257. Stub switches.** The use of these, although once nearly universal, has been practically abandoned as turnouts from *main track* except for the poorest and cheapest roads. In some States their use on main track is prohibited by law. They have the sole merit of cheapness with adaptability to the circumstances of very light traffic operated at slow speed when a considerable element of danger may be tolerated for the sake of economy. The rails from  $A$  to  $B$  (see Fig. 131\*) are not fastened

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\*The student should at once appreciate that in Fig. 131, as well as in nearly all the remaining figures in this chapter, it becomes necessary to use excessively large frog angles, short radii, and a very wide gauge in order to illustrate the desired principles with figures which are sufficiently small for the page. In fact, the proportions used in the figures are such that serious mechanical difficulties would be encountered if they were used. These difficulties are here ignored because they can be neglected in the proportions used in practice.

to the ties; they are fastened to each other by tie-rods which keep them at the proper gauge; at and back of *B* they are

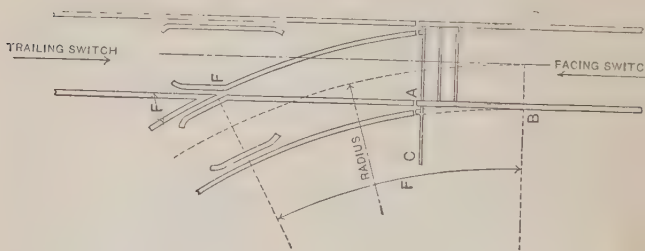


FIG. 131.—STUB SWITCH.

securely spiked to the ties, and at *A* they are kept in place by the connecting bar (*C*) fastened to the switch-stand. One great objection to the switch is that, in its usual form, when operated as a trailing switch, a derailment is inevitable if the switch is misplaced. The very least damage resulting from such a derailment must include the bending or breaking of the tie-rods of the switch-rail. Several devices have been invented to obviate this objection, some of which succeed very well mechanically, although their added cost precludes any economy in the total cost of the switch. Another objection to the switch is the looseness of construction which makes the switches objectionable at high speeds. The gap of the rails at the head-block is always considerable, and is sometimes as much as two inches. A driving-

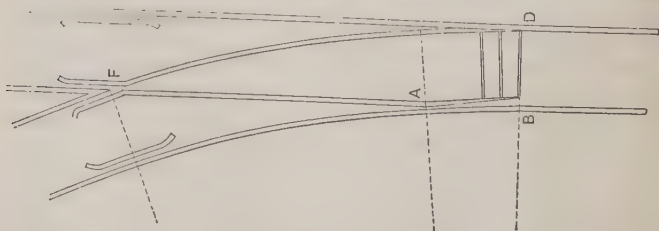


FIG. 132.—POINT SWITCH.

wheel with a load of 12000 to 20000 pounds, jumping this gap with any considerable velocity, will do immense damage to the



farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

**258. Point switches.** The essential principle of a point switch is illustrated in Fig. 132. As is shown, one main rail and also one of the switch-rails is unbroken and immovable. The other main rail (from *A* to *F*) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (*AB*) and an equal length of the opposite lead rail (usually 15 to 24 feet long) are fastened together by tie-rods. The end at *A* is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at *B* includes the web of the rail. In order to retain in it

as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being entirely cut away. As may be seen in Fig. 133, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web and about one-half that of the base—a very fair angle-iron. The planing runs back in *straight* lines, until at about six or seven feet back

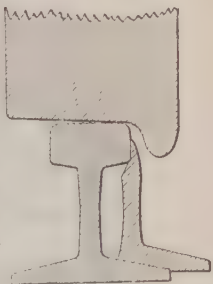


FIG. 133.

from the point the full width of the head is obtained. The full width of the base will only be obtained at about 13 feet from the point. An 80-lb. rail is 5 inches wide at the base. Allowing  $\frac{3}{4}$ " more for a spike between the rails, this gives  $5\frac{3}{4}$ " as the minimum width between rail centers at the joint. The minimum angle of the switch-point (using a 15-foot

point-rail) is therefore the angle whose tangent is  $\frac{5.75}{15 \times 12} = .03914$ , which is the tangent of  $1^\circ 50'$ . Switch-rails are sometimes used with a length of 24 feet, which reduces the angle of the switch-point to  $1^\circ 09'$ .

**259. Switch-stands.** The simplest and cheapest form is the "ground lever," which has no target. The radius of the circle described by the connecting-rod pin is precisely one-half the throw. From the nature of the motion the device is practically

self-locking in either position, padlocks being only used to prevent malicious tampering. The numerous designs of upright stands are always combined with targets, one design of which

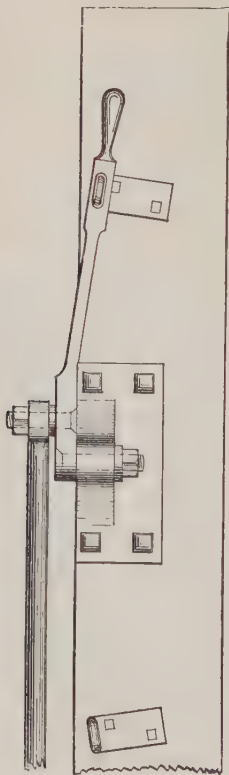


FIG. 134.—GROUND LEVER FOR THROWING A SWITCH.

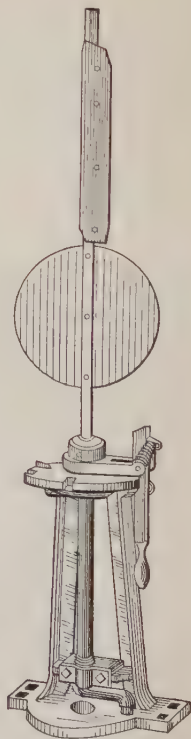


FIG. 135.

illustrated in Fig. 135. When the road is equipped with interlocking signals, the switch-throw mechanism forms a part of the design

260. **Tie-rods.** These are fastened to the webs of the rails by means of lugs which are bolted on, there being usually a hinge-joint between the rod and the lug. Four such tie-rods are

generally necessary. The first rod is sometimes made without hinges, which gives additional stiffness to the comparatively weak rail-points. The old-fashioned tie-rod, having jaws fitting the base of the rail, was almost universally used in the days of stub switches. One great inconvenience in their use lies in the fact that they must be slipped on, one by one, over the *free* ends of the switch-rails. Sometimes the lugs are fastened to the rail-webs by rivets instead of bolts.

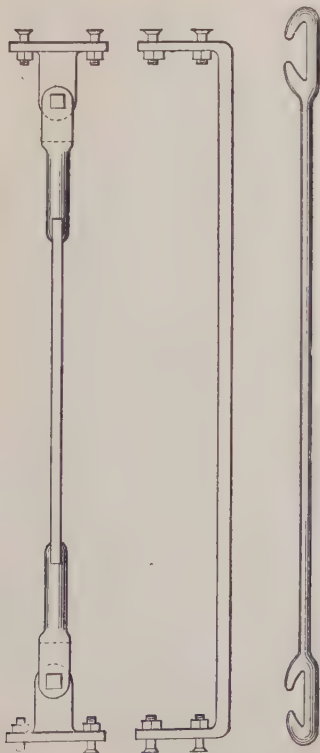


FIG. 136.—FORMS OF TIE-RODS.

**261. Guard-rails.** As shown in Figs. 131 and 132, guard-rails are used on both the main and switch tracks opposite the frog-point. Their function is not only to prevent the possibility of

the wheel-flanges passing on the wrong side of the frog-point, but also to save the side of the frog-tongue from excessive wear. The necessity for their use may be realized by noting the apparent wear usually found on the side of the head of the guard-rail. The flange-way space between the heads of the guard-rail and wheel-rail therefore becomes a definite quantity and should equal about two inches. Since this is less than the space between the heads of ordinary (say 80-pound) rails when placed base to base, to say nothing of the  $\frac{3}{4}$ " necessary for spikes, it becomes necessary to cut the flange of the guard-rail. The length of the rail is made from 10 to 15 feet, the ends being bent as shown in Fig. 132, so as to prevent the possibility of the end of the rail being struck by a wheel-flange.

### MATHEMATICAL DESIGN OF SWITCHES.

In all of the following demonstrations regarding switches, turnouts, and crossovers, the lines are assumed to represent the *gauge-lines*—i.e., the lines of the *inside* of the head of the rails.

**262. Design with circular lead-rails.** The simplest method

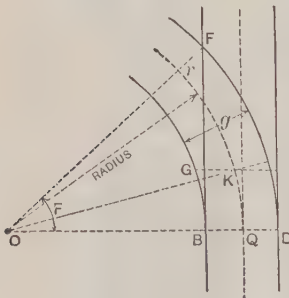


FIG. 137.

is to consider that the lead-rails curve out from the main track-rails by arcs of circles which are tangent to the main rails and which extend to the frog-point *F*. The simple curve from *D* to *F* is of such radius that  $(r + \frac{1}{2}g)$  vers *F* = *g*, in which *F* = the frog angle, *g* = gauge, *L* = the "lead" (*BF*), and *r* = the radius of the center of the switch-rails.

$$\therefore r + \frac{1}{2}g = \frac{g}{\text{vers } F} \quad (74)$$

Also,  $BF \div BD = \cot \frac{1}{2}F$ ;  $BD = g$ ;  $BF = L$ .

$$\therefore L = g \cot \frac{1}{2}F. \quad (75)$$

Also,  $L = (r + \frac{1}{2}g) \sin F$ ;  $(76)$

$$QT = 2r \sin \frac{1}{2}F. \quad (77)$$

These formulæ involve the angle *F*. As shown in Table III, the angles (*F*) are always odd quantities, and their trigonometric functions are somewhat troublesome to obtain closely with

ordinary tables. The formulæ may be simplified by substituting the frog-number  $n$ , from the relation that  $n = \frac{1}{2} \cot \frac{1}{2} F$ . Since

$$r - \frac{1}{2}g = L \cot F \quad \text{and} \quad r + \frac{1}{2}g = L \operatorname{cosec} F,$$

then 
$$\begin{aligned} r &= \frac{1}{2}L (\cot F + \operatorname{cosec} F) \\ &= \frac{1}{2}g \cot \frac{1}{2}F (\cot F + \operatorname{cosec} F) \\ &= \frac{1}{2}g \cot^2 \frac{1}{2}F, \quad \text{since } (\cot a + \operatorname{cosec} a) = \cot \frac{1}{2}a \\ &= 2gn^2. \end{aligned} \quad (78)$$

Also, 
$$L = 2gn, \quad (79)$$

from which 
$$r = n \times L. \quad (80)$$

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (*BA*, Fig. 131) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the next four sections.

The throw ( $t$ ) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at least  $\frac{3}{4}$ " more than that width. The head-block should therefore be placed at such a distance from the heel of the switch ( $B$ ) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii ( $r + \frac{1}{2}g$ ) and ( $r - \frac{1}{2}g$ ), the mean radius  $r$  is used. Then (see Fig. 137)

$$\text{vers } KOQ = t \div r,$$

and the length of the switch-rails is

$$QK = r \sin KOQ. \quad (81)$$

These relations develop another disadvantage in the use of a stub switch. The required value of  $BG$ , using a No. 10 frog and 80-pound rail, is 30.1 feet—slightly more than a full rail length. It would be unsafe to leave so much of the track unspiked from the ties. Whether this is obviated by spiking down a portion of the switch-rails (virtually shortening the lead) or by moving the switch-block nearer the heel of the switch (shortening the switch-rails), but still maintaining the required throw, the theoretical accuracy of the curve is hopelessly lost.

**263. Effect of straight frog-rails.** A portion of the ends of

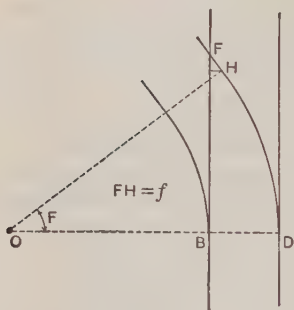


FIG. 138.

the rails of a frog are free and *may* be bent to conform to the switch-rail curve, but there is a considerable portion which is fitted to the cast-iron filler, and this portion is always straight. Call the length of this straight portion back from the frog-point  $f$  ( $=FH$ , Fig. 138). Then we have

$$\begin{aligned} r + \frac{1}{2}g &= (g - f \sin F) \div \text{vers } F \\ &= \frac{g}{\text{vers } F} - f \cot \frac{1}{2}F \\ &= \frac{g}{\text{vers } F} - 2fn. \quad (82) \end{aligned}$$

$$\begin{aligned} BF = L &= (g - f \sin F) \cot \frac{1}{2}F + f \cos F \\ &= 2gn - f \sin F \cot \frac{1}{2}F + f \cos F \\ &= 2gn - f(1 + \cos F) + f \cos F \\ &= 2gn - f. \quad (83) \end{aligned}$$

Since  $r - \frac{1}{2}g = (L - f \sec F) \cot F$ , and

$$r + \frac{1}{2}g = (L - f \cos F) \text{cosec } F,$$

$$r = \frac{1}{2}L(\cot F + \text{cosec } F) - \frac{1}{2}f \sec F \cot F - \frac{1}{2}f \cos F \text{cosec } F$$

$$= Ln - \frac{1}{2}f \left( \frac{1 + \cos f}{\sin f} \right).$$

$$r = Ln - \frac{1}{2}f \cot \frac{1}{2}F$$

$$= Ln - fn. \quad \text{Then from (83)}$$

$$r = 2gn^2 - 2fn. \quad (84)$$

**264. Effect of straight point-rails.** The "point switches," now so generally used, have *straight* switch-rails. This requires







No. 9 frog, standard gauge  $4' 8\frac{1}{2}''$ ,  $f=3'.37$ ,  $k=5\frac{3}{4}''=0'.479$ ,  $DN=15' 0''$ , and  $a=1^\circ 50'$ , we may tabulate the comparative results:

|               | § 262.<br>Simple circle.<br>Curved frog-<br>rail. Curved<br>switch-rail. | § 263.<br>Straight<br>frog-rail.<br>Curved<br>switch-rail. | § 264.<br>Curved frog-<br>rail. Straight<br>switch-rail. | § 265.<br>Straight<br>frog-rail.<br>Straight<br>switch-rail. |
|---------------|--------------------------------------------------------------------------|------------------------------------------------------------|----------------------------------------------------------|--------------------------------------------------------------|
| $r$           | 762.75                                                                   | 702.00                                                     | 747.48                                                   | 681.16                                                       |
| Deg. of curve | $7^\circ 31'$                                                            | $8^\circ 10'$                                              | $7^\circ 40'$                                            | $8^\circ 25'$                                                |
| $L$           | 84.75                                                                    | 81.37                                                      | 74.00                                                    | 72.13                                                        |

This shows that the effect of using straight frog-rails and straight switch-rails is to sharpen the curve and shorten the lead in each case separately, and that the combined effect is still greater. The effect of the straight switch-rails is especially marked in reducing the length of lead, and therefore Eq. 78 to 80, although having the advantage of extreme simplicity, cannot be used for point-switches without material error. The effect of the straight frog-rail is less, and since it can be materially reduced by bending the free end of the frog-rails, the influence of this feature is frequently ignored, the frog-rails are assumed to be curved, and Eq. 85 and 86 are used. (See § 276 for a further discussion of this point.)

#### 267. Dimensions for a turnout from the OUTER side of a curved

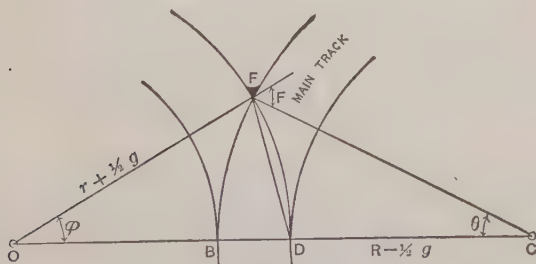


FIG. 141.

track. In this demonstration the switch-rails will be considered as uniformly circular from the switch-points to the frog-point.

In the triangle  $FCD$  (Fig. 141) we have

$$(FC + CD) : (FC - CD) :: \tan \frac{1}{2}(FDC + DFC) : \tan \frac{1}{2}(FDC - DFC);$$

$$\text{but} \quad \frac{1}{2}(FDC + DFC) = 90^\circ - \frac{1}{2}\theta$$

$$\text{and} \quad \frac{1}{2}(FDC - DFC) = \frac{1}{2}F.$$

$$\text{Also,} \quad FC + CD = 2R \quad \text{and} \quad FC - CD = g;$$

$$\therefore 2R : g :: \cot \frac{1}{2}\theta : \tan \frac{1}{2}F$$

$$:: \cot \frac{1}{2}F : \tan \frac{1}{2}\theta;$$

$$\therefore \tan \frac{1}{2}\theta = \frac{gn}{R} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (91)$$

$$\text{Also,} \quad OF : FC :: \sin \theta : \sin \phi; \quad \text{but} \quad \phi = (F - \theta);$$

$$\text{then} \quad r + \frac{1}{2}g = (R + \frac{1}{2}g) \frac{\sin \theta}{\sin (F - \theta)} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (92)$$

$$BF = L = 2(R + \frac{1}{2}g) \sin \frac{1}{2}\theta \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (93)$$

If the curvature of the main track is very sharp or the frog angle unusually small,  $F$  may be less than  $\theta$ ; in which case the center  $O$  will be on the same side of the main track as  $C$ . Eq. 92 will become (by calling  $r = -r$  and changing the signs)

$$(r - \frac{1}{2}g) = (R + \frac{1}{2}g) \frac{\sin \theta}{\sin (\theta - F)} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot (94)$$

If we call  $d$  the degree of curve corresponding to the radius  $r$ , and  $D$  the degree of curve corresponding to the radius  $R$ , also  $d'$  the degree of curve of a turnout from a straight track (the frog angle  $F$  being the same), it may be shown that  $d = d' - D$  (very nearly). To illustrate we will take three cases, a number 6 frog (very blunt), a number 9 frog (very commonly used), and a number 12 frog (unusually sharp). Suppose  $D = 4^\circ 0'$ ; also  $D = 10^\circ 0'$ ;  $g = 4' 8\frac{1}{2}'' = 4.708$ .

A brief study of the tabular form on p. 279 will show that the error involved in the use of the approximate rule for ordinary curves ( $4^\circ$  or less) and for the usual frogs (about No. 9) is really insignificant, and that, even for sharper curves ( $10^\circ$  or more), or for very blunt frogs, the error would never cause damage, considering the lower probable speed. In the most unfavorable case noted above the change in radius is about 1%. On account of the closeness of the approximation the method is frequently used. The remarkable agreement of the computed values of  $L$

| Frog<br>num-<br>ber. | $D=4^{\circ}$ . |             |            |        | "L" for<br>straight<br>track. |
|----------------------|-----------------|-------------|------------|--------|-------------------------------|
|                      | $d$             | $d' - D$    | Error.     | $L$    |                               |
| 6                    | 12° 54' 20"     | 12° 57' 52" | 0° 03' 32" | 56.57  | 56.50                         |
| 9                    | 3 30 27         | 3 31 04     | 0 0 37     | 84.85  | 84.75                         |
| 12                   | 0 13 33         | 0 13 36     | 0 0 03     | 112.72 | 113.00                        |

| Frog<br>num-<br>ber. | $D=10^{\circ}$ . |            |            |        | "L" for<br>straight<br>track. |
|----------------------|------------------|------------|------------|--------|-------------------------------|
|                      | $d$              | $d' - D$   | Error.     | $L$    |                               |
| 6                    | 6° 53' 24"       | 6° 57' 52" | 0° 04' 28" | 56.66  | 56.50                         |
| 9                    | 2 27 54          | 2 28 56    | 0 01 02    | 84.86  | 84.75                         |
| 12                   | 5 44 26          | 5 46 24    | 0 01 58    | 112.91 | 113.00                        |

with the corresponding values for a straight main track (the lead rails circular throughout) shows that the error is insignificant in using the more easily computed values.

268. Dimensions for a turnout from the INNER side of a curved track. (Lead rails circular throughout.) From Fig. 142 we have, from the triangle  $DFC$ ,

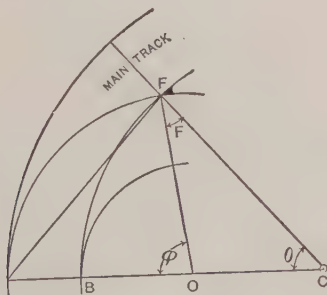


FIG. 142.

$$\begin{aligned}
 DF + FC : DF - FC &:: \tan \frac{1}{2}(DFC + FDC) : \tan \frac{1}{2}(DFC - FDC); \\
 \text{but} \quad \frac{1}{2}(DFC + FDC) &= 90^{\circ} - \frac{1}{2}\theta \\
 \text{and} \quad \frac{1}{2}(DFC - FDC) &= \frac{1}{2}F; \\
 \therefore 2R : g &:: \cot \frac{1}{2}\theta : \tan \frac{1}{2}F \\
 &:: \cot \frac{1}{2}F \tan \frac{1}{2}\theta; \\
 \therefore \tan \frac{1}{2}\theta &= \frac{gn}{R} \quad \dots \dots \dots (95)
 \end{aligned}$$

From  $OFC$ ,

$$OF:FC :: \sin \theta : (F + \theta).$$

$$(r + \frac{1}{2}g) = (R - \frac{1}{2}g) \frac{\sin \theta}{\sin (F + \theta)}. \quad . \quad . \quad . \quad (96)$$

$$L = BF = 2(R - \frac{1}{2}g) \sin \frac{1}{2}\theta. \quad . \quad . \quad . \quad (97)$$

As in § 267, it may be readily shown that the degree of the turnout ( $d$ ) is *nearly* the sum of the degree of the main track ( $D$ ) and the degree ( $d'$ ) of a turnout from a straight track when the frog angle is the same. The discrepancy in this case is somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

269. Double turnout from a straight track. In Fig. 143 the frogs  $F_l$  and  $F_r$  are generally made equal. Then, if there are

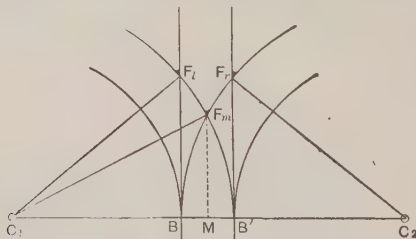


FIG. 143.

uniform curves from  $B'$  to  $F_l$  and from  $B$  to  $F_r$ , the required value of  $F_m$  is obtained from

$$\text{vers } \frac{1}{2}F_m = \frac{g}{2(r + \frac{1}{2}g)}, \quad . \quad . \quad . \quad . \quad (98)$$

$r$  being found from Eq. 78, in which  $n$  is the frog number of  $F_l$  or  $F_r$ .

$$MF_m = r \tan \frac{1}{2}F_m;$$

but since  $n_m = \frac{1}{2} \cot \frac{1}{2}F_m$ ,

$$MF_m = \frac{r}{2n_m}. \quad . \quad . \quad . \quad . \quad (99)$$

Since vers  $F_l = \frac{g}{(r + \frac{1}{2}g)}$ ,

$$\text{vers } \frac{1}{2}F_m = \frac{1}{2} \text{vers } F_l. \quad . \quad . \quad . \quad . \quad (100)$$

Also, since  $(C_1 F_m)^2 = (M F_m)^2 + (C_1 M)^2$ , we have

$$(r + \frac{1}{2}g)^2 = \left(\frac{r}{2n_m}\right)^2 + r^2;$$

$$r^2 + rg + \frac{1}{4}g^2 = \frac{r^2}{4n_m^2} + r^2.$$

Simplifying and substituting,  $r = 2gn^2$ , we have

$$2g^2n^2 + \frac{1}{4}g^2 = \frac{4g^2n^4}{4n_m^2};$$

$$n_m^2 = \frac{n^4}{2n^2 + \frac{1}{4}}.$$

Dropping the  $\frac{1}{4}$ , which is always insignificant in comparison with  $2n^2$ , we have

$$n_m = \frac{n}{\sqrt{2}} = n \times .707 (\text{approx.}). \quad . \quad . \quad . \quad . \quad (101)$$

Frogs are usually made with angles corresponding to integral values of  $n$ , or sometimes in "half" sizes, e.g. 6,  $6\frac{1}{2}$ , 7,  $7\frac{1}{2}$ , etc. If No.  $8\frac{1}{2}$  frogs are used for  $F_l$  and  $F_r$ , the exact frog number for  $F_m$  is 6.01. This is so nearly 6 that a No. 6 frog may be used without sensible inaccuracy. Numbers 7 and 10 are a less perfect combination. If sharp frogs must be used,  $8\frac{1}{2}$  and 12 form a very good combination.

If it becomes necessary to use other frogs because the right combination is unobtainable, it may be done by compounding the curve at the middle frog.  $F_l$  and  $F_r$  should be greater than  $\frac{1}{2}F_m$ . If equal to  $\frac{1}{2}F_m$ , the rails would be straight from the middle frog to the outer frogs. In Fig. 144,  $\theta_1 = F_l - \frac{1}{2}F_m$ .

Drawing the chord  $\overline{F_l F_m}$ ,

$$K \overline{F_l F_m} = F_l - \frac{1}{2}\theta_1 = F_l - \frac{1}{2}F_l + \frac{1}{4}F_m = \frac{1}{2}(F_l + \frac{1}{2}F_m);$$

$$\frac{\overline{F_l F_m}}{\sin \overline{K F_l F_m}} = \frac{\overline{K F_m}}{2 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m)}; \quad . \quad . \quad (102)$$

$$\overline{K F_l} = \overline{K F_m} \cot \overline{K F_l F_m} = \frac{1}{2}g \cot \frac{1}{2}(F_l + \frac{1}{2}F_m); \quad . \quad (103)$$

$$(r_1 + \frac{1}{2}g) = \frac{\overline{F_l F_m}}{2 \sin \frac{1}{2}\theta} = \frac{g}{4 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m) \sin \frac{1}{2}(F_l - \frac{1}{2}F_m)}$$

$$= \frac{\frac{1}{2}g}{\cos \frac{1}{2}F_m - \cos F_l} \quad . \quad . \quad . \quad . \quad (104)$$

If three frogs, all different, *must* be used, the largest may be selected as  $F_m$ ; the radius of the lead rails may be found by an inversion of Eq. 98;  $F_m$  may be located in the center of the tracks by Eq. 99; then each of the smaller frogs may be located

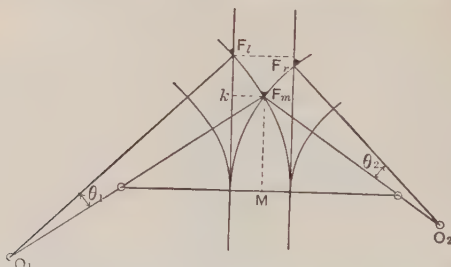


FIG. 144.

by separate applications of Eq. 102 or 103, the radius being determined by Eq. 104.

270. **Two turnouts on the same side.** In Fig. 145, let  $O_1$  bisect  $O_2D$ . Then  $(r_1 + \frac{1}{2}g) = \frac{1}{2}(r_2 + \frac{1}{2}g)$ ; also,  $O_1O_2 = O_1F_l$  and  $F_r = F_l$ .

$$\text{vers } F_m = \frac{g}{r' + \frac{1}{2}g} = \frac{2g}{r + \frac{1}{2}g}; \quad . . . . (105)$$

$$BF_m = (r' + \frac{1}{2}g) \sin F_m. \quad . . . . (106)$$

It may readily be shown that the relative values of  $F_r$ ,  $F_l$ , and  $F_m$  are almost identical with those given in § 269; as may be

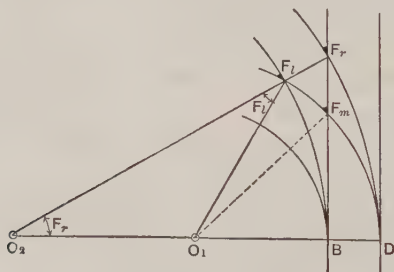


FIG. 145.

apparent when it is considered that the middle switch may be regarded simply as a curved main track, and that, as developed



in § 267, the dimensions of turnouts are nearly the same whether the main track is straight or slightly curved.

**271. Connecting curve from a straight track.** The "connecting curve" is the track lying between the frog and the side track where it becomes parallel to the main track ( $FS$  in Fig. 146 or 147). Call  $d$  the distance between track centers. The angle  $FO_1R = F$  (see Fig. 146). Call  $r'$  the radius of the connecting curve. Then

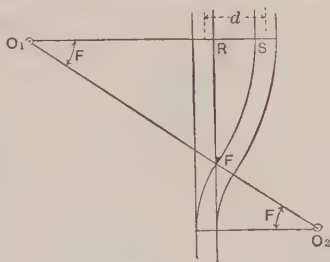


FIG. 146.

$$(r' - \frac{1}{2}g) = \frac{d - g}{\text{vers } F}; \quad (107)$$

$$FR = (r' - \frac{1}{2}g) \sin F. \quad (108)$$

If it is considered that the distance  $FR$  consumes too much track room it may be shortened by the method indicated in Fig. 151.

**272. Connecting curve from a curved track to the OUTSIDE.** When the main track is curved, the required quantities are the

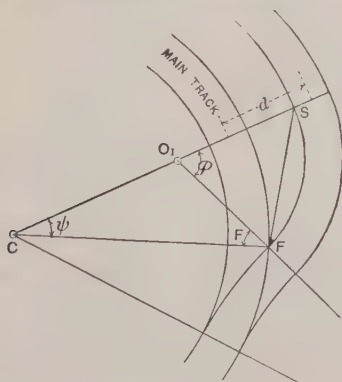


FIG. 147.

radius  $r$  of the connecting curve from  $F$  to  $S$ , Fig. 147, and its length or central angle. In the triangle  $CSF$

$$CS + CF : CS - CF :: \tan \frac{1}{2}(CFS + CSF) : \tan \frac{1}{2}(CFS - CSF);$$

but  ${}_2(CFS + CSF) = 90 - \frac{1}{2}\phi$ ; and, since the triangle  $O_1SF$  is isosceles,  $\frac{1}{2}(CFS - CSF) = \frac{1}{2}F$ ;

$$\begin{aligned} \therefore 2R + d : d - g &:: \cot \frac{1}{2}\phi : \tan \frac{1}{2}F \\ &:: \cot \frac{1}{2}F : \tan \frac{1}{2}\phi; \end{aligned}$$

$$\therefore \tan \frac{1}{2}\phi = \frac{2n(d-g)}{2R+d}. \quad \dots \dots \dots (109)$$

From the triangle  $CO_1F$  we may derive

$$r - \frac{1}{2}g : R + \frac{1}{2}g :: \sin \phi : \sin (F + \phi);$$

$$r - \frac{1}{2}g = (R + \frac{1}{2}g) \frac{\sin \phi}{\sin (F + \phi)}. \quad \dots \dots \dots (110)$$

Also

$$FS = 2(r - \frac{1}{2}g) \sin \frac{1}{2}(F + \phi). \quad \dots \dots \dots (111)$$

273. Connecting curve from a curved track to the INSIDE.

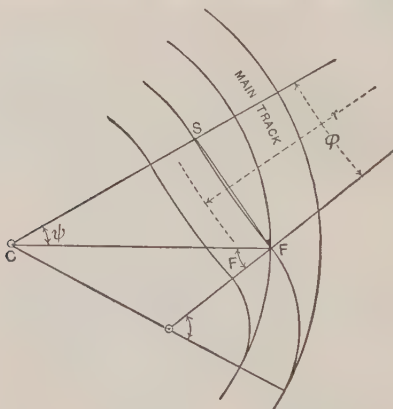


FIG. 148.

As above, it may readily be deduced from the triangle  $CFS$  (see Fig. 148) that

$$(2R - d) : (d - g) :: \cot \frac{1}{2}\phi : \tan \frac{1}{2}F,$$

and finally that

$$\tan \frac{1}{2}\phi = \frac{2n(d-g)}{2R-d}. \quad \dots \dots \dots (112)$$

Similarly we may derive (as in Eq. 110)

$$(r - \frac{1}{2}g) = (R - \frac{1}{2}g) \frac{\sin \phi}{\sin (F - \phi)}. \quad \dots \dots \dots (113)$$

Also  $FS = 2(r - \frac{1}{2}g) \sin \frac{1}{2}(F - \phi) \dots \dots (114)$

Two other cases are possible. (a)  $r$  may increase until it becomes infinite (see Fig. 149), then  $F = \phi$ . In such a case we may write, by substituting in Eq. 112,

$$2R - d = 4n^2(d - g) \dots (115)$$

This equation shows the value of  $R$ , which renders this case possible with the given values of  $n$ ,  $d$ , and  $g$ . (b)  $\phi$  may be greater than  $F$ . As before (see Fig. 150)

$$2R - d : d - g :: \cot \frac{1}{2}\psi : \tan \frac{1}{2}F;$$

$$\tan \frac{1}{2}\phi = \frac{2n(d - g)}{2R - d},$$

the same as Eq. 112, but

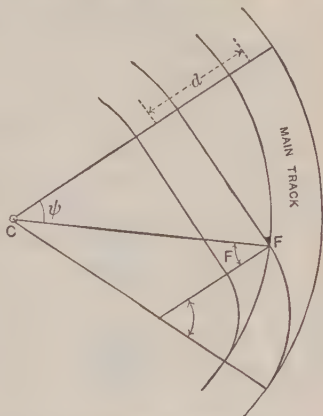


FIG. 149.

$$r + \frac{1}{2}g = (R - \frac{1}{2}g) \frac{\sin \phi}{\sin (\phi - F)} \dots \dots (116)$$

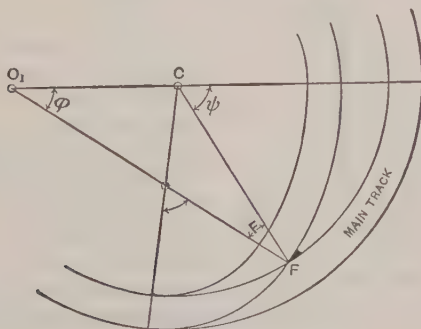


FIG. 150.

*Problem.* To find the dimensions of a connecting curve running to the INSIDE of a curved main track; number 9 frog,  $4^\circ 30'$  curve,  $d = 13'$ ,  $g = 4' 8\frac{1}{2}''$ .



The total distance along the track may be derived as follows:

$$DV = 2DF_1 + F_2Y = 2DF_1 + XY - XF_2;$$

$$XY = (d - g) \cot F_1; \quad XF_2 = g \div \sin F_2;$$

$$\therefore DV = 2DF_1 + (d - g) \cot F_1 - \frac{g}{\sin F_2} \dots \dots (118)$$

If a reversed curve with equal frogs is used, we have

$$\text{vers } \theta = \frac{d}{2r}; \dots \dots \dots (119)$$

also  $DQ = 2r \sin \theta \dots \dots \dots (120)$

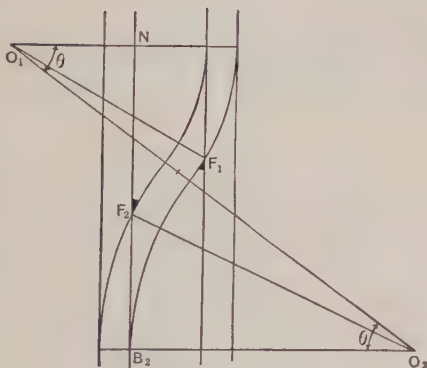


FIG. 152.

If the frogs are unequal, we will have (see Fig. 152)

$$r_2 \text{ vers } \theta + r_1 \text{ vers } \theta = d;$$

$$\therefore \text{vers } \theta = \frac{d}{r_1 + r_2}; \dots \dots \dots (121)$$

also the distance along the track

$$B_2N = (r_1 + r_2) \sin \theta. \dots \dots \dots (122)$$

*Problem.* A crossover is to be placed between two parallel straight tracks, 12' 2" between centers, using a No. 8 and a No. 9

frog, and with a reversed curve between the frogs. Required the total distance between switch-points (the distance  $B_2N$  in Fig. 152).

*Solution.* If straight point rails and straight frog rails are used, the radii,  $r_1$  and  $r_2$ , taken from the middle section of Table III, are 527.91 and 681.16.

Eq. 122.

$$\text{vers } \theta = \frac{d}{r_1 + r_2}$$

$$\begin{aligned} r_1 &= 527.91 \\ r_2 &= 681.16 \\ r_1 + r_2 &= 1209.07 \end{aligned}$$

$$d = 12' 2'' = 12.16,$$

$$\log = 1.08517$$

$$\log (r_1 + r_2) = 3.08245$$

$$\log \text{vers } \theta = 8.00272$$

$$\theta = 8^\circ 08' 06''$$

Eq. 122.

$$\log (r_1 + r_2) = 3.08245$$

$$\log \sin \theta = 9.15077$$

$$\log 171.09 = 2.23322$$

$$B_2N = 171.09$$

The length of the curve from  $B_2 = 100(\theta \div d) = 100(8^\circ 08' 06'' \div 8^\circ 25') = 96.65$ . The length of the other curve is  $100(8^\circ 08' 06' \div$

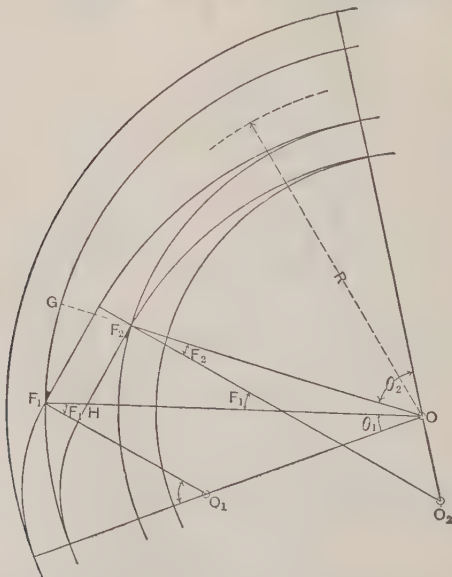


FIG. 153.

$10^\circ 52') = 74.86$ . As a check,  $96.65 + 74.86 = 171.51$ , which is slightly in excess of 171.09, as it should be.

**275. Crossover between two parallel curved tracks.** (a) **Using a straight connecting curve.** This solution has limitations. If one frog ( $F_1$ ) is chosen,  $F_2$  becomes determined, being a function of  $F_1$ . If  $F_1$  is less than some limit, depending on the width ( $d$ ) between the parallel tracks, this solution becomes impossible. In Fig. 153 assume  $F_1$  as known. Then  $F_1H = g \sec F_1$ . In the triangle  $HOF_2$  we have

$$\sin HF_2O : \sin F_2HO :: HO : F_2O;$$

$$\sin F_2HO = \cos F_1; \quad HF_2O = 90^\circ + F_2;$$

$$\therefore \sin HF_2O = \cos F_2.$$

$$HO = R + \frac{1}{2}d - \frac{1}{2}g - g \sec F_1; \quad F_2O = R - \frac{1}{2}d + \frac{1}{2}g;$$

$$\therefore \cos F_2 = \cos F_1 \frac{R + \frac{1}{2}d - \frac{1}{2}g - g \sec F_1}{R - \frac{1}{2}d + \frac{1}{2}g}. \quad \dots \dots (123)$$

Knowing  $F_2$ ,  $\theta_2$  is determinable from Eq. 91. Fig. 153 shows the case where  $\theta_2$  is greater than  $F_2$ . Fig. 154 shows the case where it is less. The demonstration of Eq. 123 is applicable to

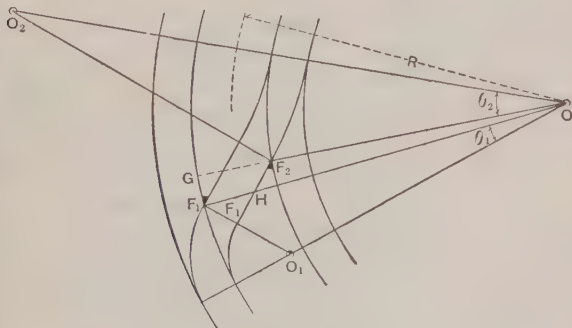


FIG. 154.

**both figures** The relative position of the frogs  $F_1$  and  $F_2$  may be determined as follows, the solution being applicable to both Figs. 153 and 154:

$$HOF_2 = 180^\circ - (90^\circ - F_1) - (90^\circ + F_2) = F_1 - F_2.$$

Then

$$GF_1 = 2(R + \frac{1}{2}d - \frac{1}{2}g) \sin \frac{1}{2}(F_1 - F_2). \quad \dots \dots (124)$$

Since  $F_2$  comes out *any* angle, its value will not be in general that of an even frog number, and it will therefore need to be made to order.



(b) Continuing the switch-rail curves until they meet as a reversed curve. In this case  $F_1$  and  $F_2$  may be chosen at pleasure (within limitations), and they will of course be of regular sizes and equal or unequal as desired.  $F_1$  and  $F_2$  being known,  $\theta_1$  and  $\theta_2$  are computed by Eq. 95 and 91. In the triangle  $OO_1O_2$  (see Fig. 155)

$$\text{vers } \phi = \frac{2(S - OO_2)(S - OO_1)}{(OO_2)(OO_1)},$$

in which

$$S = \frac{1}{2}(OO_1 + OO_2 + O_1O_2);$$

but

$$OO_1 = R + \frac{1}{2}d - r_1,$$

$$OO_2 = R - \frac{1}{2}d - r_2,$$

$$O_1O_2 = r_1 + r_2;$$

$$\therefore S = \frac{1}{2}(2R + 2r_2) = R + r_2;$$

$$S - OO_2 = R + r_2 - R + \frac{1}{2}d - r_2 = \frac{1}{2}d;$$

$$S - OO_1 = R + r_2 - R - \frac{1}{2}d + r_1 = r_1 + r_2 - \frac{1}{2}d;$$

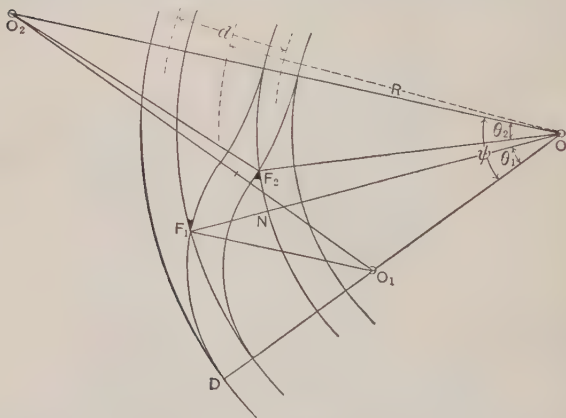


FIG. 155.

$$\text{vers } \phi = \frac{d(r_1 + r_2 - \frac{1}{2}d)}{(R - \frac{1}{2}d + r_2)(R + \frac{1}{2}d - r_1)}; \quad \dots \quad (125)$$

$$\sin OO_2O_1 = \sin \phi \frac{OO_1}{O_1O_2} = \sin \phi \frac{R + \frac{1}{2}d - r_1}{r_1 + r_2}; \quad \dots \quad (126)$$

$$O_2O_1D = \phi + O_1O_2O; \quad \dots \quad (127)$$

$$NF_2 = 2(R - \frac{1}{2}d + \frac{1}{2}g) \sin \frac{1}{2}(\phi - \theta_1 - \theta_2). \quad \dots \quad (128)$$

Although the above method introduces a reversed curve, yet it uses up less track than the first method and permits the use of ordinary frogs rather than those having some special angle which must be made to order.

*Problem.* Required the dimensions of a crossover on a  $4^{\circ} 30'$  curve when the distance between track centers is 13 feet. The frog for the outer main track ( $F_1$  in Fig. 155) is No. 9;  $F_2$  is No. 7. Then  $R=1273.6$ ;  $R_1$ , for the inner main track,  $=1280.1$ ;  $D_1=4^{\circ} 29'$ ;  $R_2=1267.1$ ;  $D_2=4^{\circ} 31'$ ;  $r_1$ =radius for  $(d_1+D_1)^{\circ}$  curve = radius for  $(8^{\circ} 25'+4^{\circ} 29')$  curve  $=445.09$ ;  $r_2$ =radius for  $(d_2-D_2)^{\circ}$  curve = radius for  $(14^{\circ} 27'-4^{\circ} 31')$  curve  $=577.53$ . (See §§ 267-268.)

|                                  |                          |                                           |
|----------------------------------|--------------------------|-------------------------------------------|
| Eq. 125.                         | $d=13$ ;                 | $\log = 1.1139\bar{4}$                    |
| $r_1+r_2-\frac{1}{2}d=1016.12$ ; |                          | $\log = 3.0069\bar{4}$                    |
| $R-\frac{1}{2}d+r_2=1844.63$ ;   | $\log = 3.2658\bar{6}$ ; | $\text{co-log} = 6.7341\bar{4}$           |
| $R+\frac{1}{2}d-r_1=835.01$ ;    | $\log = 2.9216\bar{9}$ ; | $\text{co-log} = 7.0783\bar{1}$           |
| $\Psi = 7^{\circ} 30' 35''$      |                          | $\log \text{ vers } \Psi = 7.9333\bar{4}$ |

|                     |                                |                                           |
|---------------------|--------------------------------|-------------------------------------------|
| Eq. 126.            |                                | $\log \sin \Psi = 9.1162\bar{6}$          |
| $r_1+r_2=1022.62$ ; | $\log = 3.0097\bar{1}$ ;       | $\log (R+\frac{1}{2}d-r_1)=2.9216\bar{9}$ |
|                     |                                | $\text{co-log} = 6.9902\bar{8}$           |
|                     | $OO_1O_2=6^{\circ} 07' 34''$ ; | $\sin OO_1O_2=9.0282\bar{3}$              |

|          |                                                                   |
|----------|-------------------------------------------------------------------|
| Eq. 127. | $OO_1D=7^{\circ} 30' 35''+6^{\circ} 07' 34''=13^{\circ} 38' 09''$ |
|----------|-------------------------------------------------------------------|

Lead from switch point No. 1 up to  $F_1=72.20$ ;  $\theta_1=\frac{72.20}{100}d'$ , where  $d'$  corresponds to the radius  $(R+\frac{1}{2}d-\frac{1}{2}g)$  or 1277.75;  $d'=4^{\circ} 29'$ ;  $\theta_1=3^{\circ} 14'$ .

Lead from switch point No. 2 up to  $F_2=61.65$ ;  $\theta_2=\frac{61.65}{100}d''$ , where  $d''$  corresponds to the radius  $(R-\frac{1}{2}d+\frac{1}{2}g)$  or 1269.45;  $d''=4^{\circ} 31'$ ;  $\theta_2=2^{\circ} 47'$ .

|          |                                                                                        |
|----------|----------------------------------------------------------------------------------------|
| Eq. 128. | $2$ ; $\log = 0.3010\bar{3}$                                                           |
|          | $R-\frac{1}{2}d+\frac{1}{2}g=1269.45$ ; $\log = 3.1036\bar{1}$                         |
|          | $\frac{1}{2}(\phi-\theta_1-\theta_2)=0^{\circ} 44' 48''$ ; $\log \sin = 8.1149\bar{7}$ |
|          | $NF_2=33.08$ . $\log 33.08=1.5196\bar{1}$                                              |

Length of curve with radius  $r_1=100 \frac{13^{\circ} 38' 09''}{12^{\circ} 54'} = 105.70$ ;

“ “ “ “ “  $r_2=100 \frac{6^{\circ} 07' 34''}{9^{\circ} 56'} = 61.67$ ;

Total length of curve between switch points  $= 167.37$ .

As a check, the sum of the two leads and  $NF_2$  equals  $(72.20 +$

$61.65 + 33.08 = 166.93$ , which is a little less than the length of the curve, as it should be.

Note that the point of reversed curve is placed  $.02' (= \frac{1}{4}'')$  beyond the frog point  $F_2$ . If the computations had apparently indicated the point of reversed curve coming between the frog point and the switch point, it would have shown the impracticability of the combination of No. 7 and No. 9 frogs with this particular degree of curve, gauge of track, and distance between track centers. If both frogs were made No. 9, the total length of track between switch points would be increased to over 188 feet and the point of reversed curve would be nearly at the middle point. This shows that the frog numbers should be nearly equal, but also shows that there is some choice "within limitations."

**276. Practical rules for switch-laying.** A consideration of the previous sections will show that the formulæ are comparatively simple when the lead rails are assumed as circular; that they become complicated, even for turnouts from a straight main track, when the effect of straight frog and point rails is allowed for, and that they become hopelessly complicated when allowing for this effect on turnouts from a curved main track. It is also shown (§ 267) that the length of the lead is practically

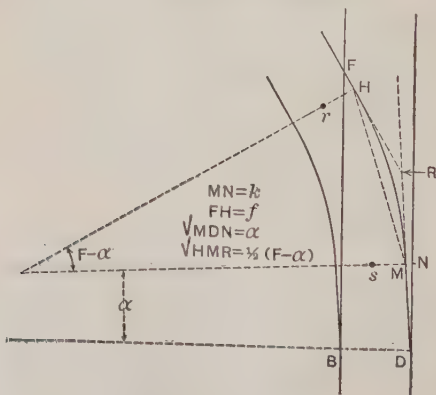


FIG. 140.

the same whether the main track is straight or is curved with such curves as are commonly used, and that the degree of curve of the lead rails from a curved main track may be found with

close approximation by mere addition or subtraction. From this it may be assumed that if the length of lead ( $L$ ) and the radius of the lead rails ( $r$ ) are computed from Eq. 87 and 90 for various frog angles, the same leads may be used for curved main track; also, that the degree of curve of the lead rails may be found by addition or subtraction, as indicated in § 267, and that the approximations involved will not be of practical detriment. In accordance with this plan Table III has been computed from Eq. 87, 88, and 90. The *leads* there given may be used for all main tracks, straight or curved. The table gives the degree of curve of the lead rails for *straight* main track; for a turnout to the *inside*, add the degree of curve of the main track; for a turnout to the *outside*, subtract it.

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 15 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at  $B$ ,  $F$ , and  $D$  (see Fig. 140); measure off the length of the switch-rails  $DN$ ; offset  $\frac{1}{2}g + k$  from  $N$  for the point  $S$ . The point  $H$  may be located (temporarily) by measuring along the rail a distance  $FH (=f)$  and then swinging out a distance of  $f \div n$  ( $n$  being the frog number).  $HT = \frac{1}{2}g$  and is measured at right angles to  $FH$ . Points for track centers between  $S$  and  $T$  may be laid off by a transit or by the use of a string and tape. Substituting in Eq. 31 the value of  $R$  and of chord ( $=ST$ ), we may compute  $x (=db)$ . Locate the middle point  $d$  and the quarter points  $a''$  and  $c''$ . Then  $a''a$  and  $c''c$  each equal three-fourths of  $db$ . Theoretically this gives a parabola rather than a circle, but the difference for all practical cases is too small for measurement.

*Example.* Given a main track on a  $4^\circ$  curve; a turnout to the outside, using a number 9 frog; gauge  $4' 8\frac{1}{2}''$ ;  $f = 3'.37$ ;  $k = 5\frac{3}{4}''$ ;  $DN = 15' 0''$  and  $a = 1^\circ 50'$ . Then for a *straight* track  $r$  would equal 631.16 [ $d = 9^\circ 05'$ ]. For this curved track  $d$  will be nearly  $(9^\circ 05' - 4^\circ) = 5^\circ 05'$ , or  $r$  will be 1131.2.  $L$  for the *straight* track would be 72.20; but since the lead is slightly increased (see § 267) when the turnout is on the outside of a curve,  $L$  may here be called 72.5.  $FH = f = 3'.37$ ;  $f \div n = 3.37 \div 9 = 0'.375 = 4''.5$ .  $H$ ,  $T$ , and  $S$  may be located as de-

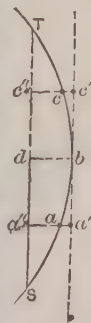


FIG. 146.

scribed above.  $ST$  may be measured on the ground, or it may be computed from Eq. 88, giving the value of 53.80 feet for straight track. Since it is slightly more for a turnout to the outside of a curve, it may be called 54.0. Then  $x=db=$

$$\frac{54.5}{8 \times 1131.2} = 0.322 \text{ foot, and } aa'' \text{ and } cc'' = 0.24 \text{ foot.}$$

#### CROSSINGS.

**277. Two straight tracks.** When two straight tracks cross each other, four frogs are necessary, the angles of two of them

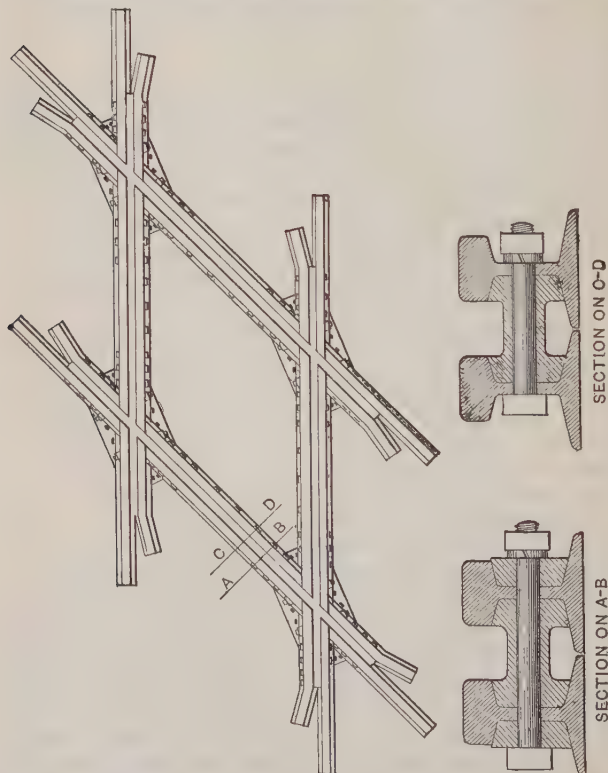


Fig. 157.—Crossing.

being supplementary to the angles of the other. Since such crossings are sometimes operated at high speeds, they should be

very strongly constructed, and the angles should preferably be  $90^\circ$  or as near that as possible. The frogs will not in general be "stock" frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig. 157 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

278. One straight and one curved track. Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 158,  $R$  is known, and the angle  $M$ , made by the center lines of the tracks at their point of intersection, is also known.

$$M = NCM. \quad NC = R \cos M.$$

$$(R - \frac{1}{2}g) \cos F_1 = NC + \frac{1}{2}g;$$

$$\therefore \cos F_1 = \frac{R \cos M + \frac{1}{2}g}{R - \frac{1}{2}g}.$$

Similarly

$$\cos F_2 = \frac{R \cos M + \frac{1}{2}g}{R + \frac{1}{2}g}, \quad (129)$$

$$\cos F_3 = \frac{R \cos M - \frac{1}{2}g}{R + \frac{1}{2}g},$$

$$\cos F_4 = \frac{R \cos M - \frac{1}{2}g}{R - \frac{1}{2}g}.$$

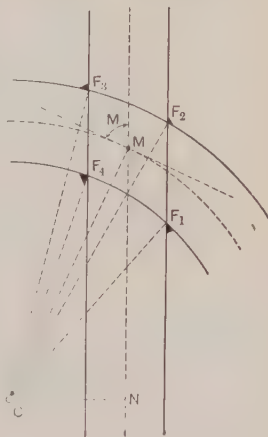


FIG. 158.

$$\begin{aligned} F_3 F_4 &= (R + \frac{1}{2}g) \sin F_3 - (R - \frac{1}{2}g) \sin F_4; \\ H F_4 &= (R - \frac{1}{2}g)(\sin F_4 - \sin F_1). \end{aligned} \quad (130)$$

279. Two curved tracks. The four frogs are unequal, and the angle of each must be computed. The radii  $R_1$  and  $R_2$  are known; also the angle  $M$ .  $r_1, r_2, r_3$ , and  $r_4$  are therefore known by adding or subtracting  $\frac{1}{2}g$ , but the lines are so indicated for brevity. Call the angle  $MC_1C_2 = C_1$ , the angle  $MC_2C_1 = C_2$ , and the line  $C_1C_2 = c$ . Then

$$\frac{1}{2}(C_1 + C_2) = 90^\circ - \frac{1}{2}M$$

and

$$\tan \frac{1}{2}(C_1 - C_2) = \cot \frac{1}{2}M \frac{R_2 - R_1}{R_2 + R_1}. \quad (131)$$

$C_1$  and  $C_2$  then become known and

$$c = C_1 C_2 = R_2 \frac{\sin M}{\sin C_1}. \quad (132)$$

In the triangle  $F_1C_1C_2$ , call  $\frac{1}{2}(c+r_1+r_4)=s_1$ ;  $s_2=\frac{1}{2}(c+r_2+r_4)$ ;

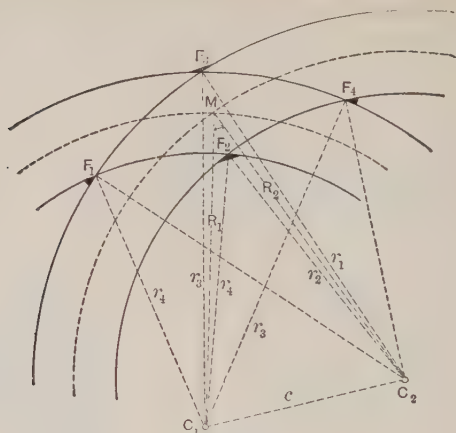


FIG. 159.

$s_3=\frac{1}{2}(c+r_1+r_3)$ ; and  $s_4=\frac{1}{2}(c+r_2+r_3)$ . Then, by formula 29, Table XXX,

$$\left. \begin{aligned} \text{vers } F_1 &= \frac{2(s_1-r_1)(s_1-r_4)}{r_1r_4}, \\ \text{Similarly } \text{vers } F_2 &= \frac{2(s_2-r_2)(s_2-r_4)}{r_2r_4}, \\ \text{vers } F_3 &= \frac{2(s_3-r_1)(s_3-r_2)}{r_1r_3}, \\ \text{vers } F_4 &= \frac{2(s_4-r_2)(s_4-r_3)}{r_2r_3}. \end{aligned} \right\} \dots \dots \dots (133)$$

$$\sin C_1C_2F_4 = \sin F_4 \frac{r_3}{c};$$

$$\sin C_1C_2F_2 = \sin F_2 \frac{r_4}{c};$$

$$\therefore F_2C_2F_4 = C_1C_2F_4 - C_1C_2F_2, \dots \dots \dots (134)$$

$$\sin F_1C_1C_2 = \sin F_1 \frac{r_1}{c};$$

$$\sin F_2C_1C_2 = \sin F_2 \frac{r_2}{c},$$

$$\therefore F_1C_1F_2 = F_1C_1C_2 - F_2C_1C_2; \dots \dots \dots (135)$$

from which the chords  $F_1F_2$  and  $F_2F_4$  are readily computed.



$F_1F_2$  and  $F_2F_4$  are nearly equal. When the tracks are straight and the gauges equal, the quadrilateral is equilateral.

*Problem.* Required the frog angles and dimensions for a crossing of two curves ( $D_1=4^\circ$ ;  $D_2=3^\circ$ ) when the angle of their tangents at the point of intersection  $=62^\circ 28'$  (the angle  $M$  in Fig. 159).

*Solution*

$$R_1=1432.7; R_2=1910.1;$$

$$r_1=R_2+\frac{1}{2}g=1910.1+2.35=1912.45;$$

$$r_2=R_2-\frac{1}{2}g=1910.1-2.35=1907.75;$$

$$r_3=R_1+\frac{1}{2}g=1432.7+2.35=1435.05;$$

$$r_4=R_1-\frac{1}{2}g=1432.7-2.35=1430.35.$$

Eq. 131.

$$\log \cot \frac{1}{2}M=0.21723$$

$$R_2-R_1=477.4; \log =2.67888$$

$$R_2+R_1=3342.8; \log =3.52411; \text{ co-log } \underline{6.47589}$$

$$\frac{1}{2}(C_1-C_2)=13^\circ 15' 07''; \tan 13^\circ 15' 07''=9.37200$$

$$\frac{1}{2}(C_1+C_2)=58^\circ 46' \quad [\frac{1}{2}(C_1+C_2)=90^\circ-\frac{1}{2}M]$$

$$C_1=72^\circ 01' 07''$$

$$C_2=45^\circ 30' 53''$$

Eq. 132.

$$\log R_2=3.28105$$

$$\log \sin M=9.94779$$

$$\log \sin C_1=9.97825; \text{ co-log } \underline{0.02175}$$

$$c=C_1C_2=1780.7;$$

$$\log C_1C_2=3.25059$$

Eq. 133.

|                           |                           |                           |                           |
|---------------------------|---------------------------|---------------------------|---------------------------|
| $c=1780.7$                | $c=1780.7$                | $c=1780.7$                | $c=1780.7$                |
| $r_1=1912.45$             | $r_2=1907.75$             | $r_1=1912.45$             | $r_2=1907.75$             |
| $r_4=1430.35$             | $r_4=1430.35$             | $r_3=1435.05$             | $r_3=1435.05$             |
| $\underline{2}   5123.50$ | $\underline{2}   5118.80$ | $\underline{2}   5128.20$ | $\underline{2}   5123.50$ |
| $s_1=2561.75$             | $s_2=2559.40$             | $s_3=2561.10$             | $s_4=2561.75$             |
| $s_1-r_1=649.30$          | $s_2-r_2=651.65$          | $s_3-r_1=651.65$          | $s_4-r_2=654.00$          |
| $s_1-r_4=1131.40$         | $-r_4=1129.05$            | $s_3-r_3=1129.05$         | $s_1-r_3=1126.70$         |

$$\log 2=0.30103$$

$$(s_1-r_1); \log 649.30=2.81244$$

$$(s_1-r_4); \log 1131.40=3.05361$$

$$\text{co-log}=6.71841$$

$$\text{co-log}=6.84456$$

$$\log \text{vers } 62^\circ 25' 31''=9.73006$$

$$\log 2=0.30103$$

$$(s_2-r_2); \log 651.65=2.81401$$

$$(s_2-r_4); \log 1129.05=3.05271$$

$$\text{co-log}=6.71948$$

$$\text{co-log}=6.84456$$

$$\log \text{vers } 62^\circ 33' 55''=9.73180$$

$$r_1=1912.45; \log =3.28159;$$

$$r_4=1430.35; \log =3.15544;$$

$$\underline{F_1=62^\circ 25' 31''};$$

$$r_2=1907.75; \log =3.28052;$$

$$r_4=1430.35; \log =3.15544;$$

$$\underline{F_2=62^\circ 33' 55''};$$

$$r_1 = 1912.45; \log = 3.28159;$$

$$r_3 = 1435.05; \log = 3.15686;$$

$$\underline{F_3 = 62^\circ 21' 57''};$$

$$r_2 = 1907.75; \log = 3.28052;$$

$$r_3 = 1435.05; \log = 3.15686;$$

$$\underline{F_4 = 62^\circ 30' 14''};$$

$$\log 2 = 0.30103$$

$$(s_3 - r_1); \log 651.65 = 2.81401$$

$$(s_3 - r_3); \log 1129.05 = 3.05271$$

$$\text{co-log} = 6.71841$$

$$\text{co-log} = 6.84313$$

$$\log \text{vers } 62^\circ 21' 57'' = 9.72930$$

$$\log 2 = 0.30103$$

$$(s_4 - r_2); \log 654.00 = 2.81558$$

$$(s_4 - r_3); \log 1126.70 = 3.05181$$

$$\text{co-log} = 6.71948$$

$$\text{co-log} = 6.84313$$

$$\log \text{vers } 62^\circ 30' 14'' = 9.73103$$

As a check, the *mean* of the frog angles  $= 62^\circ 27' 54''$ , which is within  $6''$  of the value of  $M$ .

Eq. 134.

$$\log c = 3.25059;$$

$$C_1 C_2 F_4 = 45^\circ 37' 51'';$$

$$C_1 C_2 F_2 = 45^\circ 28' 17'';$$

$$\underline{F_2 C_2 F_4 = 45^\circ 37' 51'' - 45^\circ 28' 17'' = 0^\circ 09' 34''}.$$

$$\underline{F_2 F_4 = 5.309};$$

Eq. 135.

$$F_1 C_1 C_2 = 72^\circ 10' 22'';$$

$$F_2 C_1 C_2 = 71^\circ 57' 38'';$$

$$\underline{F_1 C_1 F_2 = 72^\circ 10' 22'' - 71^\circ 57' 38'' = 0^\circ 12' 44''}.$$

$$\underline{F_1 F_2 = 5.298};$$

$$\log \sin F_4 = 9.94794$$

$$\log r_3 = 3.15686$$

$$\text{co-log } c = 6.74940$$

$$\sin C_1 C_2 F_4 = 9.85421$$

$$\log \sin F_2 = 9.94818$$

$$\log r_4 = 3.15544$$

$$\text{co-log } c = 6.74940$$

$$\sin C_1 C_2 F_2 = 9.85303$$

$$\log 2 = 0.30103$$

$$\log r_2 = 3.28052$$

$$\frac{1}{2}(0^\circ 09' 34'') = 0^\circ 04' 47''; \log \sin = \begin{pmatrix} 4.68557 \\ 2.45788 \end{pmatrix}$$

$$\log F_2 F_4 = 0.72500$$

$$\sin F_1 = 9.94763$$

$$\log r_1 = 3.28159$$

$$\text{co-log } c = 6.74940$$

$$\sin F_1 C_1 C_2 = 9.97863$$

$$\sin F_2 = 9.94818$$

$$\log r_2 = 3.28052$$

$$\text{co-log } c = 6.74940$$

$$\sin F_2 C_1 C_2 = 9.97811$$

$$\log 2 = 0.30103$$

$$\log r_4 = 3.15544$$

$$\frac{1}{2}(0^\circ 12' 44'') = 0^\circ 06' 22''; \log \sin = \begin{pmatrix} 4.68557 \\ 2.58206 \end{pmatrix}$$

$$\log F_1 F_2 = 0.72411$$

As a check,  $F_2 F_4$  and  $F_1 F_2$  are very nearly equal, as they should be.

## CHAPTER XII.

### MISCELLANEOUS STRUCTURES AND BUILDINGS.

#### WATER-STATIONS AND WATER-SUPPLY.

**280. Location.** The water-tank on the tender of a locomotive has a capacity of from 2500 to 5000 gallons—sometimes less, rarely very much more. The consumption of water is very variable, and will correspond very closely with the work done by the engine. On a long down grade it is very small; on a ruling grade going up it may amount to 150 gallons per mile in exceptional cases, although 60 to 100 gallons would be a more usual figure. Nominally a locomotive could run 40 miles or more on one tankful, but it would be impracticable to separate the water-stations by such an interval. On roads of the smallest traffic, 15 to 20 miles should be the maximum interval between stations; 10 miles is a more common interval on heavy traffic-roads. But these intervals are varied according to circumstances. In the early history of some of the Pacific railroads it was necessary to attach one or more tank-cars to each train in order to maintain the supply for the engine over stretches of 100 miles and over where there was no water. Since then water-stations have been obtained at great expense by boring artesian wells. The individual locations depend largely on the facility with which a sufficient supply of suitable water may be obtained. Streams intersecting the railroad are sometimes utilized, but if such a stream passes through a limestone region the water is apt to be too hard for use in the boilers. More frequently wells are dug or bored. When the local supply at some determined point is unsuitable, and yet it is necessary to locate a water-station there, it may be found justifiable to pipe the water several miles. The construction of municipal water-works at suitable places along the line has led to the frequent utilization of such supplies. In such cases the railroad is generally the largest single consumer and obtains the most favorable rates. When possible, water-stations are located at regular stopping points and at division termini.

**281. Required qualities of water.** Chemically pure water is unknown except as a laboratory product. The water supplied by wells, springs, etc., is always more or less charged with calcium and magnesium carbonates and sulphates, as well as other impurities. The evaporation of water in a boiler precipitates these impurities to the lower surfaces of the boiler, where they sometimes become incrustated and are difficult to remove. The protection of the iron or steel of a boiler from the fierce heat of the fire depends on the presence of water on the other side of the surface, which will absorb the heat and prevent the metal from assuming an excessively high temperature. If the water side of the metal becomes covered or incrustated with a deposit of chemicals, the conduction of heat to the water is much less free, the metal will become more heated and its deterioration or destruction will be much more rapid. An especially common effect is the production of leaks around the joints between tubes and tube-sheets and the joints in the boiler-plates. Such injury can only be prevented by the application of one (or both) of two general methods—(a) the frequent cleaning of the boilers and (b) the chemical purification of the water before its introduction into the boiler. Although “manholes” and “hand-holes” are made in boilers, it is physically impossible to clean out every corner of the inside of a boiler where deposits will form and where they are especially objectionable—on the tube-sheets. Such a cleaning is troublesome and expensive.

Chemical purification is generally accomplished by treating the water before it enters the boiler. The reagents chiefly employed are quicklime and sodium carbonate. Lime precipitates the bicarbonate of lime and magnesia. Sodium carbonate gives, by double decomposition in the presence of sulphate of lime, carbonate of lime, which precipitates, and soluble sulphate of soda, which is non-incrustant. When this is done in a purifying tank, the purified water is drawn off from the top of the tank and supplied pure to the engines. The precipitants are drawn off from the settling-basin at the bottom of the tank. This purification, which makes no pretense of being chemically perfect, may be accomplished for a few cents per 1000 gallons. It is used much more extensively in Europe than in this country, the Southern Pacific being the only railroad which has employed such methods on a large scale. Reliance is frequently placed on the employment of a “non-incrustant” which is introduced

directly into the boiler. When no incrustation takes place the accumulation of precipitant and mud in the bottom of the boiler may be largely removed by mere "blowing off" or by washing out with a hose.

American practice may therefore be summarized as follows: (a) Employing as pure water as possible; (b) cleaning out boilers by "blowing off" or by washing out with a hose or by physical scraping at more or less frequent intervals or when other repairs are being made; (c) the occasional employment of non-incrustants; (d) the occasional chemical treatment of water before it enters the tender-tank.

282. Tanks. Whatever the source, the water must be led or pumped into tanks which are supported on frames so that the bottoms of the tanks are about 12 feet above the rails. Wooden

TABLE XIV.—CAPACITY OF CYLINDRICAL WATER-TANKS IN UNITED STATES STANDARD GALLONS OF 231 CUBIC INCHES.

| Height<br>in<br>feet. | Diameter of tank in feet. |       |       |       |       |       |       |       |
|-----------------------|---------------------------|-------|-------|-------|-------|-------|-------|-------|
|                       | 10                        | 12    | 14    | 16    | 18    | 20    | 22    | 24    |
| 6                     | 3525                      | 5076  | 6909  | 9024  | 11421 | 14101 | 17062 | 20305 |
| 7                     | 4113                      | 5922  | 8061  | 10528 | 13325 | 16451 | 19905 | 23689 |
| 8                     | 4700                      | 6768  | 9212  | 12032 | 15229 | 18801 | 22749 | 27073 |
| 9                     | 5288                      | 7611  | 10364 | 13536 | 17132 | 21151 | 25592 | 30457 |
| 10                    | 5875                      | 8460  | 11515 | 15041 | 19036 | 23501 | 28436 | 33841 |
| 11                    | 6463                      | 9306  | 12667 | 16545 | 20939 | 25851 | 31180 | 37225 |
| 12                    | 7050                      | 10152 | 13819 | 18049 | 22843 | 28201 | 34123 | 40609 |
| 13                    | 7638                      | 10998 | 14970 | 19553 | 24746 | 30551 | 36967 | 43994 |
| 14                    | 8225                      | 11844 | 16122 | 21057 | 26650 | 32901 | 39810 | 47378 |
| 15                    | 8813                      | 12690 | 17273 | 22561 | 28554 | 35251 | 42654 | 50762 |
| 16                    | 9400                      | 13536 | 18425 | 24065 | 30457 | 37601 | 45198 | 54146 |
| 17                    | 9988                      | 14383 | 19576 | 25569 | 32361 | 39951 | 48341 | 57530 |
| 18                    | 10575                     | 15229 | 20728 | 27073 | 34264 | 42301 | 51185 | 60914 |
| 19                    | 11163                     | 16075 | 21879 | 28577 | 36168 | 44652 | 54028 | 63298 |
| 20                    | 11750                     | 16921 | 23031 | 30081 | 38071 | 47002 | 56872 | 67682 |
| 21                    | 12338                     | 17767 | 24182 | 31585 | 39975 | 49352 | 59716 | 71067 |
| 22                    | 12925                     | 18613 | 25334 | 33089 | 41879 | 51702 | 62559 | 74451 |
| 23                    | 13513                     | 19459 | 26485 | 34593 | 43782 | 54052 | 65403 | 77835 |
| 24                    | 14101                     | 20305 | 27637 | 36097 | 45686 | 56402 | 68246 | 81219 |
| 25                    | 14688                     | 21151 | 28789 | 37601 | 47589 | 58752 | 71090 | 84603 |

tanks having a diameter of 24 feet, 16 feet high, and with a capacity of over 50,000 gallons are frequently employed. Iron or steel tanks are also used.

In Table XIV is shown the capacity of cylindrical water-tanks in United States standard gallons of 231 cubic inches. From

this table the dimensions of a tank of any desired capacity may readily be found. Two or more tanks are sometimes used rather than construct one of excessive size. The smaller sizes shown in the table are of course too small for ordinary use but that part of the table was filled out for its possible convenience otherwise. On single-track roads where all engines use one track the tank may be placed 8' 5" from the track center; this gives sufficient clearance and yet permits the use of a single swinging pipe which will reach from the bottom of the tank to the tender manhole. In Fig. 160 is illustrated

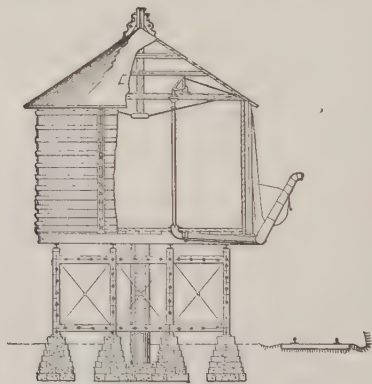


FIG. 160.—WATER-TANK.

one form of wooden tank. They are preferably manufactured by those who make a special business of it and who by the use of special machinery can insure tight joints. When it is inconvenient to place the tank near the track, or when there is a double track, a "stand-pipe" becomes necessary. See § 285. One of the most difficult and troublesome problems is to prevent freezing, particularly in the valves and pipes. Not only are the pipes carefully covered but fires must be maintained during cold weather. When the pumping is accomplished by means of a steam-pump, supplied from a steam-boiler in the pump-house under the tank, coils of steam-pipe may be employed to heat the water or to heat the pipes. Partial protection may be obtained by means of a double roof and double bottom, the spaces being filled with sawdust or some other non-conductor of heat.



**283. Pumping.** The pumping is done most reliably with steam-pumps or gas-engines, although hot-air engines, windmills, and even man-power are occasionally employed. Economy of operation requires that the water-stations shall be so located that each tank shall be used regularly and that each pump shall be regularly operated for maintaining the water-supply. On the other hand, the pump should not be *required* to regularly work at night to maintain the supply and should have an excess capacity of say 25%. When a tank is but little used, it will still require the labor of an attendant, and his time will be largely wasted unless he can be utilized for other labor about the station. In recent years gasoline has been extensively employed as a fuel for the pumping-engines. The chief advantages of its use lies in the extreme simplicity of the mechanism and the very slight attention it requires, which permits their being operated by station-agents and others, who are paid \$10 per month extra, instead of paying a regular pumper \$35 per month. "Screenings," "slack coal," etc., are used as fuel for steam-pumps and may frequently be delivered at the pump-house at a cost not exceeding 30 cents per ton, but even at that price the cost of pumping per thousand gallons, although dependent on the horizontal and vertical distances to the source of supply and to the tank, will generally run at 2 cents to 6 cents per 1000 gallons. In many cases where steam plants have been replaced by gasoline plants, the cost of pumping per 1000 gallons has been reduced to one third or even one fourth of the cost of steam pumping. (Of course the cost, using windmills, is reduced to the mere maintenance of the machinery, but the unreliability of wind as a motive power and the possibility of its failure to supply water when it is imperatively needed has made this form of motive power unpopular. (See report to Ninth Annual Convention of the Association of Railway Superintendents of Bridges and Buildings, Oct. 1899.)

**284. Track tanks.** These are chiefly required as one of the means of avoiding delays during fast-train service. A trough, made of steel plate, is placed between the rails on a stretch of *perfectly level* track. A scoop on the end of a pipe is lowered from under the tender into the tank while the train is in motion. The rapid motion scoops up the water, which then flows into the tender tank. The following brief description of an installation on the Baltimore & Ohio Railroad between Baltimore and



Philadelphia will answer as a general description of the method. The trough is made of  $\frac{3}{16}$ " steel plate, 19" wide, 6" deep, and has a length of 1200 feet. There is riveted on each side a line of  $1\frac{1}{2} \times 2 \times \frac{1}{4}$ " angle bars. These angle bars rest on the ties. Ordinary track spikes hold these angle bars to the ties, but permit expansion as with rails. The tanks are firmly anchored at the center, the ends being free to expand or contract. The plates are 15 feet long and are riveted with  $\frac{7}{16}$ " rivets, 20 rivets per joint. At each end is an inclined plane 13' 8" long. If the fireman should neglect to raise the scoop before the end of the tank is reached, the inclined plane will raise it automatically and a catch will hold it raised. Water is supplied to the tanks by a No. 9 Blake pump having a capacity of 260 gallons per minute. During cold weather, freezing is prevented by injecting into the side of the tanks, at intervals of 45 feet, jets of steam, which come through  $\frac{1}{8}$ " holes. Two boilers of 80 and 95 H.P. are required for pumping and to keep the water from freezing. During warm weather an upright 25 H.P. boiler suffices for the pumping. The cost of installation was about \$10,000 to \$11,000, the cost of maintenance being about \$132.50 per month.

**285. Stand-pipes.** These are usually manufactured by those who make a specialty of such track accessories, and who can ordinarily be trusted to furnish a correctly designed article. In Fig. 161 is shown a form manufactured by the Sheffield Car Co. Attention is called to the position of the valve and to the device for holding the arm parallel to the track when not in use so that it will not be struck by a passing train. When a stand pipe is located between parallel tracks, the strict requirements of clearance demand that the tracks shall be bowed outward slightly. If the tracks were originally straight, they may be shoved over by the trackmen, the shifting gradually running out at about 100 feet each side of the stand-pipe. If the tracks were originally curved, a slight change in radius will suffice to give the necessary extra distance between the tracks.

#### BUILDINGS.

**286. Station platforms.** These are most commonly made of planks at minor stations. Concrete is used in better-class work, also paving brick. An estimate of the cost of a platform of paving brick laid at Topeka, Kan., was \$4.89 per 100 square feet when

laid flat and \$7.24 per 100 square feet when laid on edge. The curbing cost 36 cents per linear foot. Cinders, curbed by timbers

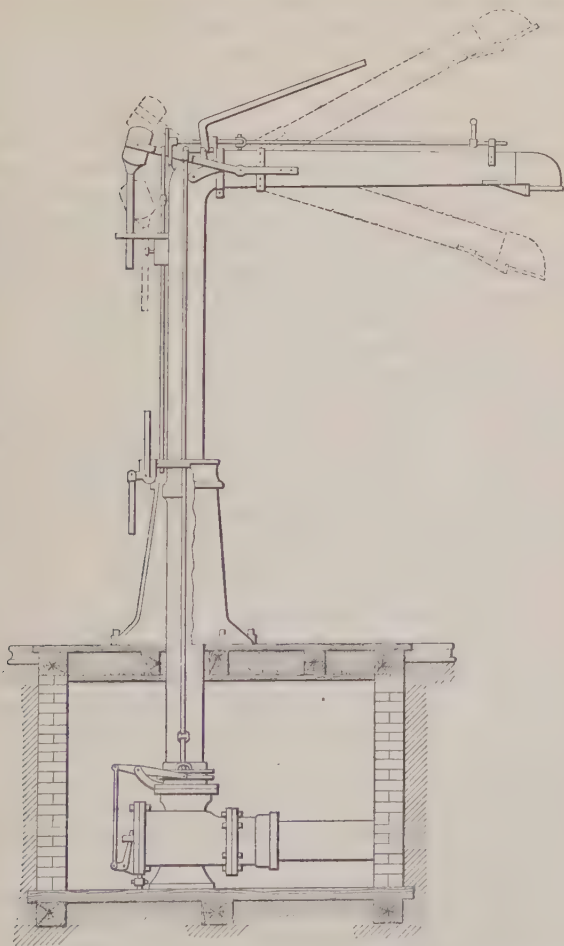


FIG. 161.—STAND-PIPE.

or stone, bound by iron rods, make a cheap and fairly durable platform, but in wet weather the cinders will be tracked into

the stations and cars. Three inches of crushed stone on a cinder foundation is considered to be still better, after it is once thoroughly packed, than a cinder surface.

**Elevation.**—The elevation of the platform with respect to the rail has long been a fruitful source of discussion. Some roads make the platforms on a level with the top of the rail, others 3" above, others still higher. As a matter of convenience to the passengers, the majority find it easier to enter the car from a high platform, but experience proves that accidents are more numerous with the higher platforms, unless steps are discarded altogether and the cars are entered from level platforms, as is done on elevated roads. As a railroad must generally pay damages to the stumbling passenger, they prefer to build the lower platform. Convenience requires that the rise from the platform to the lowest step should not be greater than the rise of the car steps. This rise is variable, but with the figures usually employed the application of the rule will make the platform 5" to 15" above the rail.

**Position with respect to tracks.**—Low platforms are generally built to the ends of the ties, or, if at the level of the top of the rail, are built to the rail head. Car steps usually extend 4' 6" from the track center and are 14" to 24" above the rail. The platform must have plenty of clearance, and when the platform is high its edge is generally required to be 5' 6" from the track center.

**287. Minor stations.** For a complete discussion of the design of stations of all kinds, including the details, the student is referred to "Buildings and Structures of American Railroads," by Walter G. Berg, now Chief Engineer of the Lehigh Valley Railroad. The subject is too large for adequate discussion here, but a few fundamental principles will be referred to.

**Rooms required.** An office and waiting-room is the minimum. A baggage-room, toilet-rooms, and express office are successively added as the business increases. In the Southern States a separate waiting-room for colored people is generally provided. It used to be common to have separate waiting-rooms for men and women. Experience proved that the men's waiting-room became a lounging place and smoking-room for loafers, and now large single waiting-rooms are more common even in the more pretentious designs, smoking being excluded. The office usually has a bay window, so that a more extended view

of the track is obtainable. The women's toilet-room is entered from the waiting-room. The men's toilet-room, although built immediately adjoining the other in order to simplify the plumbing, is entered from outdoors. Old-fashioned designs built the station as a residence for the station-agent; later designs have very generally abandoned this idea. "Combination" stations (passenger and freight) are frequently built for small local stations, but their use seems to be decreasing and there is now a tendency to handle the freight business in a separate building.

**288. Section-houses.** These are houses built along the right-of-way by the railroad company as residences for the trackmen. The liability of a wreck or washout at any time and at any part of the road, as well as the convenience of these houses for ordinary track labor, makes it all but essential that the trackmen should live on the right-of-way of the road, so that they may be easily called on for emergency service at any time of day or night. This is especially true when the road passes through a thinly settled section, where it would be difficult if not impossible to obtain suitable boarding-places. It is in no sense an extravagance for a railroad to build such houses. Even from the direct financial standpoint the expense is compensated by the corresponding reduction in wages, which are thus paid partly in free house rent. And the value of having men on hand for emergencies will often repay the cost in a single night. Where the country is thickly settled the need for such houses is not so great, and railroads will utilize or perhaps build any sort of suitable house, but on Southern or Western roads, where the need for such houses is greater, standard plans have been studied with great care, so as to obtain a maximum of durability, usefulness, comfort, and economy of construction. (See Berg's Buildings, etc., noted above.) On Northwestern roads, protection against cold and rain or snow is the chief characteristic; on Southern roads good ventilation and durability must be chiefly considered. Such houses may be divided into two general classes—(a) those which are intended for trackmen only and which may be built with great simplicity, the only essential requirements being a living-room and a dormitory, and (b) those which are intended for families, the houses being then distinguished as "dwelling-houses for employees."

**289. Engine-houses.** Small engine-houses are usually built rectangular in plan. Their minimum length should be some-

what greater than that of the longest engine on the road. They may be built to accommodate two engines on one track, but then they should be arranged to be entered at either end, so that neither engine must wait for the other. In width there may be as many tracks as desired, but if the demand for stalls is large, it will probably be preferable to build a "roundhouse." Rectangular engine-houses are usually entered by a series of parallel tracks switching off from one or more main tracks, no turn-table being necessary. If a turn-table is placed outside (because one

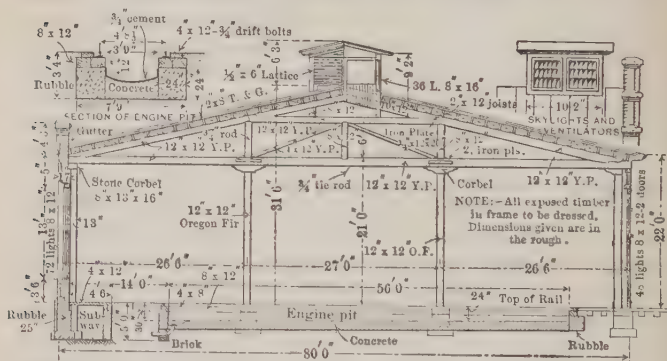


FIG. 162.—ENGINE-HOUSE.

is needed at that part of the road) enough track should be allowed between the house and the turn-table so that engines may be quickly removed from the engine-house in case of fire without depending on the turn-table to get them out of danger.

**Roundhouses.** The plan of these is generally polygonal rather than circular. The straight walls are easier to build; the construction is more simple, and the general purpose is equally well served. They may be built as a part of a circle or a complete circle, a passageway being allowed, so that there are two entrances instead of one. When space is very limited a roundhouse with turn-table will accommodate more engines in proportion to the space required (including the approaches) than a rectangular house. The enlarged space on the outer side of each segment of a roundhouse furnishes the extra space which is needed for the minor repairs which are usually made in a roundhouse. One disadvantage is that supervision is not quite so easy or effec-

tive as in rectangular houses. Of course such houses are used not only for storing and cleaning engines, but also for minor repairs which do not require the engine to be sent to the shops for a general overhauling.

**Construction.** The outer walls are usually of brick. The inner walls consist almost entirely of doors and the piers between them, although there is usually a low wall from the top of the door frames to the roof line, which usually slopes outward so as to turn rain-water away from the central space.

**Roofs.** Many roofs have been built of slate with iron truss framing, with the idea of maximum durability. The slate is good, but experience shows that the iron framing deteriorates very rapidly from the action of the gases of combustion of the engines which must be "fired" in the houses before starting. Roof frames are therefore preferably made of wood.

**Floors.** These are variously constructed of cinders, wood, brick, and concrete. Brick has been found to be the best material. Anything short of brick is a poor economy; concrete is very good if properly done but is somewhat needlessly expensive.

**Ventilation.** This is a troublesome and expensive matter. The general plan is to have "smoke-jacks" which drop down over the stack of each engine as it reaches its precise place in its stall and which will carry away all smoke and gas. Such a movable stack is most easily constructed of thin metal—say galvanized iron—but these will be corroded by the gases of combustion in two or three years. Vitrified pipe, cast iron, expanded metal and cement, and even plain wood painted with "fireproof" paint, have been variously tried, but all methods have their unsatisfactory features. (For an extended discussion of roundhouse floors and ventilation see the Proc. Assoc. of Railway Supts. of Bridges and Buildings for 1898, pp. 112-135.)

#### SNOW STRUCTURES.

**290. Snow-fences.** Snow structures are of two distinct kinds—fences and sheds. A snow-fence implies drifting snow—snow carried by wind—and aims to cause all drifting snow to be deposited away from the track. Some designs actually succeed in making the wind an agent for clearing snow from the track where it has naturally fallen. A snow-fence is placed at right angles to the prevailing direction of the wind and 50 to 100 feet away from the tracks. When the road line is at right angles to



the prevailing wind, the right-of-way fence may be built as a snow-fence—high and with tight boarding. Hedges have sometimes been planted to serve this purpose. When the prevailing wind is oblique, the snow fences must be built in sections where they will serve the best purpose. The fences act as wind breakers, suddenly lowering the velocity of the wind and causing the snow carried by the wind to be deposited along the fence. Portable fences are frequently used, which are placed (by permission of the adjoining property owners) outside of the right-of-way. If a drift forms to the height of the portable fence the fence may be replaced on the top of the drift, where it may act as before, forming a still higher drift. When the prevailing wind runs along the track line, snow-fences built in short sections on the sides will cause snow to deposit around them while it scours its way along the track line, actually clearing it. Such a method is in successful operation at some places on the White Mountain and Concord divisions of the Boston & Maine Railroad. Snow-fences, in connection with a moderate amount of shoveling and plowing, suffice to keep the tracks clear on railroads not troubled with avalanches. In such cases snow-sheds are the only alternative.

**291. Snow-sheds.** These are structures which will actually keep the tracks clear from snow regardless of its depth outside. Fortunately they are only necessary in the comparatively rare situations where the snowfall is excessive and where the snow is liable to slide down steep mountain slopes in avalanches. These avalanches frequently bring down with them rocks, trees, and earth, which would otherwise choke up the road-bed and render it in a moment utterly impassable for weeks to come. The sheds are usually built of 12"×12" timber framed in about the same manner as trestle timbering; the "bents" are sometimes placed as close as 5 feet, and even this has proved insufficient to withstand the force of avalanches. The sheds are therefore so designed that the avalanche will be *deflected* over them instead of spending its force against them. Although these sheds are only used in especially exposed places, yet their length is frequently very great and they are liable to destruction by fire. To confine such a fire to a limited section, "fire-breaks" are made—i.e., the shed is discontinued for a length of perhaps 100 feet. Then, to protect that section of track, a V-shaped deflector will be placed on the uphill side which will deflect all



descending material so that it passes over the sheds. Solid crib work is largely used for these structures. Fortunately suitable timber for such construction is usually plentiful and cheap where these structures are necessary. Sufficient ventilation is obtained by longitudinal openings along one side immediately under the roof. "Summer" tracks are usually built outside the sheds to avoid the discomfort of passing through these semi-tunnels in pleasant weather. The fundamental elements in the design of such structures is shown in Fig. 163, which illustrates some of the sheds used on the Canadian Pacific Railroad.

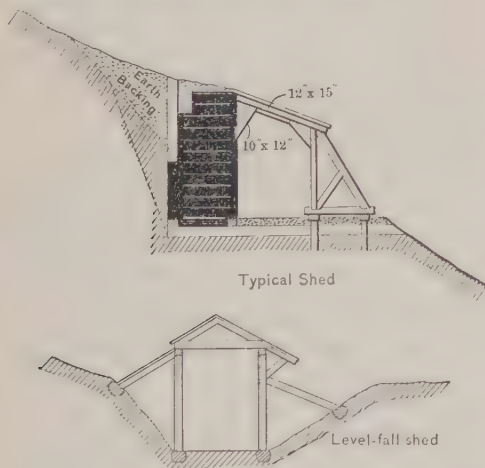


FIG. 163.—SNOW-SHEDS—CANADIAN PACIFIC RAILROAD.

**292. Turn-tables.** The essential feature of a turn-table is a carriage of sufficient size and strength to carry a locomotive, the carriage turning on a pivot of sufficient size to carry such a load. The carriage revolves in a circular pit whose top has the same general level as the surrounding tracks. The carriages were formerly made largely of wood; very many of those still in use are of cast iron. Structural steel is now universally employed for all modern work and since the construction of the carriage and the pivot is a special problem in structures, no further attention will here be paid to the subject, except to that part which the railroad engineer must work out

—laying out the site and preparing the foundation. The *minimum* length of such a carriage (and therefore the diameter of the pit) is evidently the length over all of the longest engine and tender in use on the road. Usually 60-foot turn-tables will suffice for an ordinary road, and for light-traffic roads employing small engines, 50 feet or even less may be sufficient. Many of the heavier freight engines of recent make have a total length of about 65 feet; therefore 70-foot turn-tables are a better standard for heavy-traffic roads. A retaining-wall should be built around the pit. The stability of this wall immediately under the tracks should be especially considered. The most important feature is the stability of the foundation of the pivot, which must sustain a concentrated pressure, more or less eccentric, of perhaps 150 tons. When firm soil or rock may be easily reached, this need give no trouble, but in a soft, treacherous soil a foundation of concrete or piling may be necessary. If the soil is very porous, it may be depended on to carry away all rain-water which may fall into the pit before the foundations are affected, but when the soil is tenacious it may be necessary to drain the subsoil thoroughly and carry off immediately all surface drainage by means of subsoil pipes which have a suitable outfall.

The location of the turn-table in the yard is a part of the general subject of "Yards," and will be considered in the next chapter.

## CHAPTER XIII.

### YARDS AND TERMINALS.

293. **Value of proper design.** A large part of the total cost of handling traffic, particularly freight, is that incurred at terminals and stations. In illustration of this, consider the relative total cost of handling a car-load of coal and a car-load (of equal weight) of mixed merchandise. The coal will be loaded in bulk on the cars at the mines, where land is comparatively cheap, and the cars grouped into a train without regard to order, since they are (usually) uniform in structure, loading, and contents. When the terminal or local station is reached they are run on tracks occupying property which is usually much cheaper than the site of the terminal tracks and freight-houses; they are unloaded by gravity into pockets or machine conveyors and the empty cars are rapidly hauled by the train-load out of the way. On the other hand, the merchandise is loaded by hand on the car from a freight-house occupying a central and valuable location, the car is hauled out into a yard occupying valuable ground, is drilled over the yard tracks for a considerable aggregate mileage before starting for its destination, where the same process is repeated in inverse order. In either case the terminal expenses are evidently a large percentage of the total cost and, once loaded, it makes but little difference just how far the car is hauled to the other terminal. But the very evident increase in terminal charges for general merchandise over those for coal (large as they are) gives a better idea of the magnitude of terminal charges.

Many yards are the result of growth, adding a few tracks at a time, without much evidence of any original plan. In such cases the yard is apt to be very inefficient, requiring a much larger aggregate of drilling to accomplish desired results, requiring much more *time* and hence blocking traffic and finally adding greatly to the cost of terminal service, although the fact of its being a needless addition to cost may be unsuspected or not fully appreciated. An unwillingness or inability to spend money for

the necessary changes, and the difficulty of making the changes while the yard is being used, only prolong the bad state of affairs and an inefficient makeshift is frequently adopted. Assume that an improvement in the design of the yard will permit a saving of the use of one switching engine, or for example, that the work may be accomplished with three switching engines instead of four. Assuming a daily cost of \$25, we have in 313 working days an annual saving of \$7825, which, capitalized at 5%, gives \$156,500, enough to reconstruct any ordinary yard.\*

**294. Divisions of the subject.** The subject naturally divides itself into three heads—(a) Yards for receiving, classifying, and distributing freight cars, called more briefly freight yards; (b) yards and conveniences for the care of engines, such as ash tracks, turn-tables, coal-chutes, sand-houses, water-tanks, or water stand-pipes, etc., and (c) passenger terminals.

#### FREIGHT YARDS.

**295. General principles.** It should be recognized at the start that at many places an ideally perfect yard is impossible, or at least impracticable, generally because ground of the required shape or area is practically unobtainable. But there are some general principles which may and should be followed in every yard and other ideals which should be approached as nearly as possible. Nevertheless every yard is an independent problem. Before taking up the design of freight yards, it is first necessary to consider the general object of such yards and the general principles by which the object is accomplished. These may be briefly stated as follows:

1. A yard is a device, a machine, by which incoming cars are sorted and classified—some sent to warehouses for unloading, some sent to connecting railroads, some made up for local distribution along the road, some sent for repairs, and, in short a device by which all cars are sent *through* and *out* of the yard as quickly as possible.

2. Except when a road's business is decreasing, or when its equipment is greater than its needs and its cars must be stored, efficiency of management is indicated by the rapidity with which the passage of cars *through* the yard is accomplished.

3. When a yard is the terminal of a "division," the freight

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\* Estimate of Mr. H. G. Hetzler, C., B. & Q. Ry.

trains will be pulled into a "receiving track" and the engine and caboose detached. The caboose will be run on to a "caboose track," which should be conveniently near, and the engine is run off to the engine yard. If the train is a "through" train and no change is to be made in its make-up, it will only need to wait for another engine and perhaps another caboose. If the cars are to be distributed, they will be drawn off by a switching engine to the "classification yard."

4. The design of a yard is best studied by first picking out the ladder tracks and the through tracks which lead from one division of the yard to another. These are tracks which must always be kept open for the passage of trains, in contradistinction to the tracks on which cars may be left standing, even though it is only for a few moments, while drilling is being done. Such a set of tracks, which may be called the skeleton of the yard, is shown by heavy lines in Fig. 164. Each line indicates a pair of rails. The tracks of the storage yards are shown by the lighter lines.

5. There is a distinct advantage in having all storage tracks double-ended—except "team tracks." Team tracks are those which have spaces for the accommodation of teams, so that loading or unloading may be done directly between the cars and teams. To avoid the necessity of teams passing over the tracks, these are best placed on the outskirts of the yard and consist of short stubs arranged in pairs. But storage tracks should have an outlet at each end so as to reduce the amount of drilling necessary to reach a car which may be at the extreme end of a long string of cars. This is done usually by means of two "ladder" tracks, parallel to each other, which thus make the storage tracks between them of equal length.

6. The equality of length of these storage tracks is a point insisted on by many, but on the other hand, trains are not always of uniform length even on any one division. Loaded trains and trains of empties will vary greatly in length, and the various styles and weights of freight engines employed necessitate other variations in the weights and lengths of trains hauled. With storage tracks of somewhat variable length a larger percentage of track length may be utilized, there will be less hauling over a useless length of track, and (assuming that the plot of ground available for yard purposes has equally favorable conditions for yard design) more business may be handled in a yard of given area.

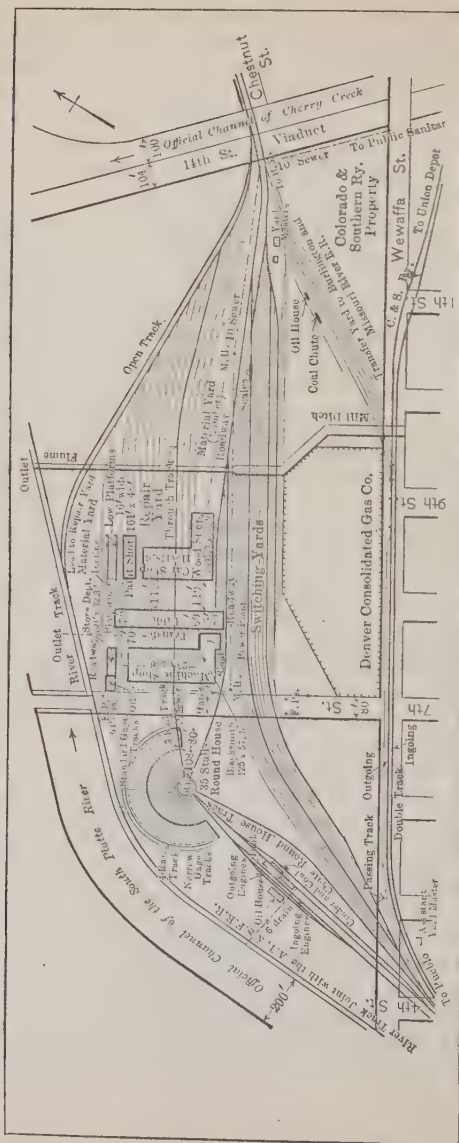


FIG. 164.—PLAN OF THE NEW SHOPS AND YARDS OF THE COLORADO &amp; SOUTHERN RAILWAY AT DENVER.

7 Yards are preferably built so that the tracks have a grade of 0.5%—sometimes a little more than this—in the direction of the traffic through the yard. This grade, which will overcome a tractive resistance of 10 pounds per ton, will permit cars to be started down the ladder tracks by a mere push from the switching engine. They are then switched on to the desired storage track and run down that track by gravity until stopped at the desired place by a brakeman riding on the cars.

8. Although not absolutely necessary, there is an advantage in having all frog numbers and switch dimensions uniform. No. 7 frogs are most commonly used. Sharper-angled frogs make easier riding, less resistance and less chance of derailment, but on the other hand require longer leads and more space. No. 6 and even No. 5 frogs are sometimes used on account of economy of space, but they have the disadvantages of greater tractive resistance, greater wear and tear on track and rolling stock, and greater danger of derailment.

**296. Relation of yard to main tracks.** Safety requires that there should be no connection between the yard tracks and the main tracks except at each end of the yard, where the switches should be amply protected by signals. Sometimes the main tracks run through the yard, making practically two yards—one for the traffic in either direction—but this either requires a double layout of tracks and houses (such as ash tracks, coal-chutes, sand-houses, etc.), or a very objectionable amount of crossing of the main-line tracks. The preferable method is to have the main line tracks entirely on the outside of the yard. A method which is in one respect still better is to spread the main tracks so that they run on each side of the yard. In this case there is never any necessity to cross one main track to pass from the yard to the other main track; a train may pass from the yard to either main track and still leave the other main track free and open. The ideal arrangement is that by which some of the tracks cross *over* or *under* all opposing tracks. By this means all connections between the yard and the main tracks may be by “trailing” switches; that is, trains will run on to the main track in the direction of motion on that main track. Of course all this applies only to *double main track*.

An important element of yard design is to have a few tracks immediately adjoining the main tracks and separate from the yard proper on which outgoing trains may await their orders to take



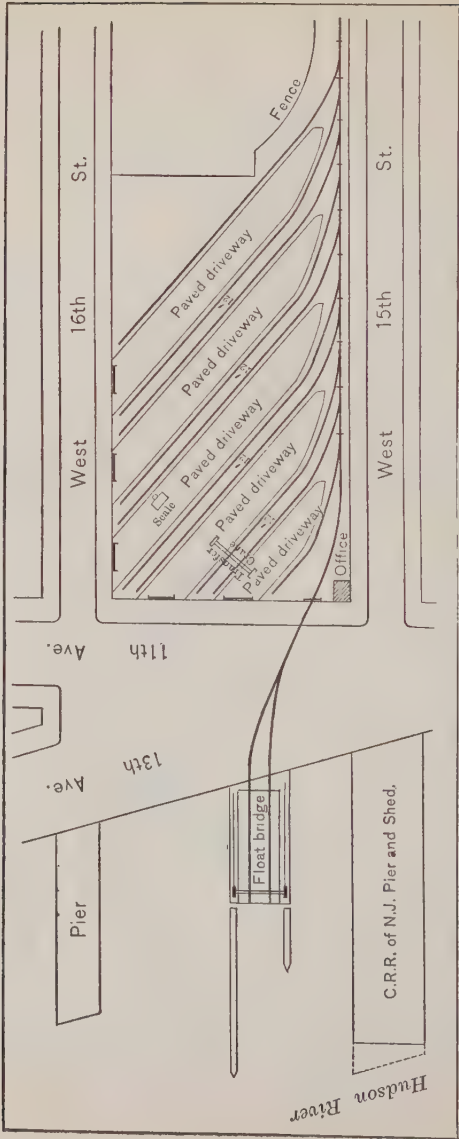


FIG. 165.—MINOR FREIGHT YARD.

the main track. When the orders come, they may start at once without any delay, without interfering with any yard operations, and they are not occupying tracks which may form part of the system needed for switching.

**297. Minor freight yards.** The term here refers to the substations, only found in the largest cities, to which cars will be sent to save in the amount of necessary team hauling and also to relieve a congestion of such loading and unloading at the main freight terminal. The cars are brought to these yards sometimes on floats (as is done so extensively at various points around New York Harbor), or they are run down on a long siding running perhaps through the city streets. But the essential feature of these yards is the maximum utilization of every square foot of yard space, which is always very valuable and which is frequently of such an inconvenient shape that a great ingenuity is required to obtain good results. There is generally a temptation to use excessively sharp curves. When the radii are greater than 150 feet no especial trouble is encountered. Curves with radius as short as 50 feet have been used in some yards. On such curves the long cars now generally used make a sharper angle with each other than that for which the couplers were designed and special coupler-bars become necessary. The two general methods of construction are (a) a series of parallel team tracks (as previously described and as illustrated further in Fig. 165), and (b) the "loop system," as is illustrated in Fig. 166.

**298. Transfer cranes.** These are almost an essential feature for yards doing a large business. The transportation of built-up girders, castings for excessively heavy machinery, etc., which weigh five to thirty tons and even more, creates a necessity for machinery which will easily transfer the loads from the car to the truck and *vice versa*. An ordinary "gin-pole" will serve the purpose for loads which do not much exceed five tons. A fixed framework, covering a span long enough for a car track and a team space, with a trolley traveling along the upper chord, is the next design in the order of cost and convenience. Increasing the span so that it covers two car tracks and two team spaces will very materially increase the capacity. Making the frame movable so that it travels on tracks which are parallel to the car tracks, giving the frame a longitudinal motion equal to two or three car lengths, and finally operating the raising and traveling mechanism by power, the facility for rapidly disposing of



heavy articles of freight is greatly increased. Of course only a very small proportion of freight requires such handling, and the business of a yard must be large or perhaps of a special character to justify and pay for the installation of such a mechanism. Figs. 165 and 166 each indicate a transfer crane, evidently of the fixed type.

**299. Track scales.** The location of these should be on one of the receiving tracks near the entrance to the yard, but not on the main track. It is always best to have a "dead track" over the scales—i.e., a track which has one rail on the solid side wall of the scale pit and the other supported at short intervals by posts which come up through the scale platform and yet do not touch it. These rails and the regular scale rails switch into one track by means of point rails a few feet beyond each end of the scales. The switches should be normally set so that all trains will use the dead track, unless the scales are to be operated. It has been found possible in a gravity yard to weigh a train with very little loss of time by running each car slowly by gravity over the scales and weighing them as they pass over.

#### ENGINE YARDS.

**300. General principles.** Engine yards must contain all the tracks, buildings, structures, and facilities which are necessary for the maintenance, care, and storage of locomotives and for providing them with all needed supplies. The supplies are fuel, water, sand, oil, waste, tallow, etc. Ash-pits are generally necessary for the prompt and economical disposition of ashes; engine-houses are necessary for the storage of engines and as a place where minor repairs can be quickly made. A turn-table is another all but essential requirement. The arrangement of all these facilities in an engine yard should properly depend on the form of the yard. In general they should be grouped together and should be as near as possible to the place where through engines drop the trains just brought in and where they couple on to assembled outgoing trains, so that all unnecessary running light may be avoided. In Figs. 164 and 167 are shown two designs which should be studied with reference to the relative arrangement of the yard facilities.

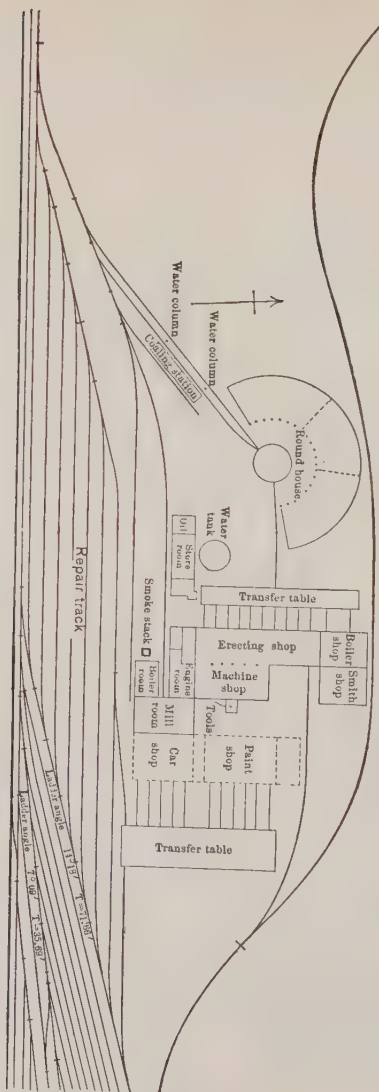


FIG. 167.—ENGINE YARD AND SHOPS, URBANA, ILL.

## PASSENGER TERMINALS.

(Passenger terminals are one of the logical subdivisions of this chapter, but their construction does not concern one engineer in a thousand. The local conditions attending their construction are so varied that each case is a special problem in itself—a problem which demands in many respects the services of the architect rather than the engineer. The student who wishes to pursue this subject is referred to an admirable chapter in “Buildings and Structures of American Railroads,” by Walter G. Berg, Chief Engineer of the Lehigh Valley Railroad.)

## CHAPTER XIV.

### BLOCK SIGNALING.

#### GENERAL PRINCIPLES.

**301. Two fundamental systems.** The growth of systems of block signaling has been enormous within the last few years—both in the amount of it and in the development of greater perfection of detail. The development has been along two general lines: (*a*) the *manual*, in which every change of signal is the result of some definite action on the part of some signalman, but in which every action is so controlled or limited or subject to the inspection of others that a mistake is nearly, if not quite, impossible; (*b*) the *automatic*, in which the signals are operated by mechanism, which cannot set a wrong signal as long as the mechanism is maintained in proper order. The fundamental principles of the two systems will be briefly outlined, after which the chief details of the most common systems will be pointed out.

**302. Manual systems.** Any railroad which has a telegraph line and an operator at all regular stations may (and generally does) operate its trains according to the fundamental principles of the manual block system even though it makes no claim to a block-signal system. The basic idea of such a system is that after a train has passed a given telegraph- or signal-station, no other train will be permitted to follow it into that “block” until word is telegraphed from the next station ahead that the first train has passed out of that block. With a double-track road the operation is very simple; trains may be run at short intervals with long blocks; with an average speed of 30 miles per hour and blocks 5 miles long, trains *could* be run on a ten minute interval (nearly). A road with any such traffic would, of course, have much shorter blocks, and, practically, they would need to be considerably shorter.

With a single-track road the operation is much more complex, since the operator must keep himself informed of the move-



ments of the trains in *both* directions. The ratio of length of block to train interval would be only one half (and practically much less than half) what it could be with a double-track road. When such a system is adhered to rigidly, it is called an *absolute block* system. But when operating on this system, a delay of one train will necessarily delay every other train that follows closely after. A portion, if not all, of the delay to subsequent trains may be avoided, although at some loss of safety, by a system of permissive blocking. By this system an operator may give to a succeeding train a "clearance card" which permits it to pass into the next block, but at a reduced speed and with the train under such control that it may be stopped on very short notice, especially near curves. One element of the danger of this system is the *discretionary* power with which it invests the signalmen, a discretion which may be wrongfully exercised. A modification (which is a fruitful source of collisions on single-track roads) is to order two trains to enter a block approaching each other, and with instructions to pass each other at a passing siding at which there is no telegraph-station. When the instructions are properly made out and literally obeyed, there is no trouble, but every thousandth or ten thousandth time there is a mistake in the orders, or a misunderstanding or disobedience, and a collision is the result. The telegraph line, a code of rules, a corps of operators, and signals under the immediate control of the operators, are all that is absolutely needed for the simple manual system.

**303. Development of the manual system.** One great difficulty with the simple system just described is that each operator is practically independent of others except as he may receive general or specific orders from a train-dispatcher at the division headquarters. Such difficulties are somewhat overcome by a very rigid system of rules requiring the signalmen at each station to keep the adjacent signalmen or the train-dispatcher informed of the movements of all trains past their own stations. When these rules (which are too extensive for quotation here) are strictly observed, there is but little danger of accident, and a neglect by any one to observe any rule will generally be apparent to at least one other man. Nevertheless the safety of trains depends on *each* signalman doing his duty, and a little carelessness or forgetfulness on the part of any one man may cause an accident. The signaling between stations *may* be done by

ordinary telegraphic messages or by telephone, but is frequently done by electric bells, according to a code of signals, since these may be readily learned by men who would have more difficulty in learning the Morse code.

In order to have the signalmen mutually control each other, the "controlled manual" system has been devised. The first successful system of this kind which was brought into extensive use is the "Sykes" system, of which a brief description is as follows: Each signal is worked by a lever; the lever is locked by a latch, operated by an electro-magnet, which, with other necessary apparatus, is inclosed in a box. When a signal is set at danger, the latch falls and locks the lever, which cannot be again set free until the electro-magnet raises the latch. The magnet is energized only by a current, the circuit of which is closed by a "plunger" at the *next* station ahead; just above the plunger is an "indicator," also operated by the current, which displays the words *clear* or *blocked*. (There are variations on this detail.) When a train arrives at a block station (*A*), the signalman should have previously signaled to the station *ahead* (*B*) for permission to free the signal. The man ahead (*B*) pushes in the "plunger" on his instrument (assuming that the previous train has already passed him), which electrically opens the lock on the lever at the previous station (*A*). The signal at *A* can then be set at "safety." As soon as the train has passed *A* the signal at *A* must be set at "danger." A further development is a device by which the mere passage of the train over the track for a few feet beyond the signal will automatically throw the signal to "danger." After the signal once goes to danger, it is automatically locked and cannot be released except by the man in advance (*B*), who will not do so until the train has passed him. The "indicator" on *B*'s instrument shows "blocked" when *A*'s signal goes to danger after the train has passed *A*, and *B*'s plunger is then locked, so that he cannot release *A*'s signal while a train is in the block. As soon as the train has passed *A*, *B* should prepare to get his signals ready by signaling ahead to *C*, so that if the block between *B* and *C* is not obstructed, *B* may have his signals at "safety" so that the train may pass *B* without pausing. The student should note the great advance in safety made by the Sykes system; a signal cannot be set free except by the combined action of two men, one the man who actually operates the signal and

the other the man at the station ahead, who frees the signal electrically and who by his action certifies that the block immediately ahead of the train is clear.

A still further development makes the system still more "automatic" (as described later), and causes the signal to fall to danger or to be kept locked at danger, if even a single pair of wheels comes on the rails of a block, or if a switch leading from a main track is opened.

**304. Permissive blocking.** "Absolute" blocking renders accidents due to collisions almost impossible unless an engineer runs by an adverse signal. The signal mechanism is usually so designed that, if it gets out of order, it will inevitably fall to "danger," i.e., as described later, the signal-board is counterbalanced by a weight which is much heavier. If the wire breaks, the counterweight will fall and the board will assume the horizontal position, which always indicates "danger." But it sometimes happens that when a train arrives at a signal-station, the signalman is unable to set the signal at safety. This may be because the previous train has broken down somewhere in the next block, or because a switch has been left open, or a rail has become broken, or there is a defect of some kind in the electrical connections. In such cases, in order to avoid an indefinite blocking of the whole traffic of the road, the signalman may give the engineer a "caution-card" or a "clearance card," which authorizes him to proceed slowly and with his train under complete control into the block and through it if possible. If he arrives at the next station without meeting any obstruction it merely indicates a defective condition of the mechanism, which will, of course, be promptly remedied. Usually the next section will be found clear, and the train may proceed as usual. On roads where the "controlled manual" system has received its highest development, the rules for permissive blocking are so rigid that there is but little danger in the practice, unless there is an absolute disobedience of orders.

**305. Automatic systems.** By the very nature of the case, such systems can only be used to indicate to the engineers of trains something with reference to the passage of previous trains. The complicated shifting of switches and signals which is required in the operation of yards and terminals can only be accomplished by "manual" methods, and the only automatic features of these methods consist in the mechanical checks

(electric and otherwise), which will prevent wrong combinations of signals. But for long stretches of the road, where it is only required to separate trains by at least one block length, an automatic system is generally considered to be more reliable. As expressed forcibly by a railroad manager, "an automatic system does not go to sleep, get drunk, become insane, or tell lies when there is any trouble." The same cannot always be said of the employés of the manual system.

The basic idea of all such systems is that when a train passes a signal-station (*A*), the signal automatically assumes the "danger" position. This may be accomplished electrically, pneumatically, or even by a direct mechanism. When the train reaches the end of the block at *B* and passes into the next one, the signal at *B* will be set at danger and the signal at *A* will be set at safety. The lengths of the blocks are usually so great that the only practicable method of controlling from *B* a mechanism at *A* is by electricity, although the actual motive power at *A* may be pneumatic or mechanical. At one time the current from *A* to *B* was carried on ordinary wires. This method has the very positive advantage of reliability, definite resistance to the current, and small probability of short-circuiting or other derangement. But now all such systems use the rails for a track circuit and this makes it possible to detect the presence of a single pair of wheels on the track anywhere in the block, or an open switch, or a broken rail. Any such circumstances, as well as a defect in the mechanism, will break or short-circuit the current and will cause the signal to be set at danger. To prevent an indefinite blocking of traffic owing to a signal persistently indicating danger, most roads employing such a system have a rule substantially as follows: When a train finds a signal at danger, after waiting one minute (or more, depending on the rules), it may proceed slowly, expecting to find an obstruction of some sort; if it reaches the next block without finding any obstruction and finds the next signal clear, it may proceed as usual, but must promptly report the case to the superintendent. Further details regarding these methods will be given later. See § 310.

306. "Distant" signals. The close running of trains that is required on heavy-traffic roads, especially where several branches combine to enter a common terminal, necessitates the use of very short blocks. A heavy train running at high speed

can hardly make a "service" stop in less than 2000 feet, while the curves of a road (or other obstructions) frequently make it difficult to locate a signal so that it can be seen more than a few hundred feet away. It would therefore be impracticable to maintain the speed now used with heavy trains if the engineer had no foreknowledge of the condition in which he will find a signal until he arrives within a short distance of it. To overcome this difficulty the "distant" signal was devised. This is placed about 1800 or 2000 feet from the "home" signal, and is interlocked with it so that it gives the *same* signal. The distant signal is frequently placed on the same pole as the home signal of the previous block. When the engineer finds the distant signal "clear," it indicates that the succeeding home signal is also clear, and that he may proceed at full speed and not expect to be stopped at the next signal; for the distant signal cannot be cleared until the succeeding home signal is cleared, which cannot be done until the block succeeding that is clear. A clear distant signal therefore indicates a clear track for two succeeding blocks. When the engineer finds the distant signal blocked, he need not stop (providing the home signal is clear). It simply indicates that he must be prepared to stop at the next home signal and must reduce speed if necessary. It may happen that by the time he reaches the succeeding home signal it has already been cleared, and he may proceed without stopping. This device facilitates the rapid running of trains, with no loss of safety, and yet with but a moderate addition to the signaling plant.

307. "Advance" signals. It sometimes becomes necessary to locate a signal a few hundred feet short of a regular passenger-station. A train might be halted at such a signal because it was not cleared from the signal-station ahead—perhaps a mile or two ahead. For convenience, an "advance" signal may be erected immediately beyond the passenger-station. The train will then be permitted to enter the block as far as the advance signal and may deliver its passengers at the station. The advance signal is interlocked with the home signal back of it, and cannot be cleared until the home signal is cleared and the entire block ahead is clear. In one sense it adds another block, but the signal is entirely controlled from the signal station back of it.



## MECHANICAL DETAILS.

**308. Signals.** The primitive signal is a mere cloth flag. A better signal is obtained when the flag is suspended in a suitable place from a fixed horizontal support, the flag weighted at the bottom, and so arranged that it may be drawn up and out of sight by a cord which is run back to the operator's office. The next step is the substitution of painted wood or sheet metal for the cloth flag, and from this it is but a step to the standard semaphore on a pole, as is illustrated in Fig. 168. The simple flag, operated for convenience with a cord, is the signal employed on thousands of miles of road, where they perhaps make no claim to a block-signal system, and yet where the trains are run according to the fundamental rules of the simple manual block method.

**Semaphore boards.** These are about 5 feet long, 8 inches wide at one end, and tapered to about 6 inches wide at the hinge end. The boards are fastened to a casting which has a ring to hold a red glass which may be swung over the face of a lantern, so as to indicate a red signal. "Distant" signal-boards usually have their ends notched or pointed; the "home" signal-boards are square ended. The boards are always to the *right* of the hinge when a train is approaching them. The "home" signals are generally painted red and the "distant" signals green, although these colors are not invariable. The backs of the boards are painted white. Therefore any signal-board which appears on the *left* side of its hinge will also appear *white*, and is a signal for traffic in the opposite direction, and is therefore of no concern to an engine-man.

**Poles and bridges.** When the signals are set on poles, they are generally placed on the right-hand side of the track. When there are several tracks, four or more, a bridge is frequently built and then each signal is over its own track. When switches run off from a main track, there may be several signal-boards over one track. The upper one is the signal for the main track and the lower ones for the several switches. In Fig. 169 is shown a "bridge" with its various signal-boards controlling the several tracks and the switches running off from them.

**"Banjo" signals.** This name is given to a form of signal, illustrated in Fig. 170, in which the indication is taken from the

*(To face page 330.)*



FIG. 168.—SEMAPHORES.





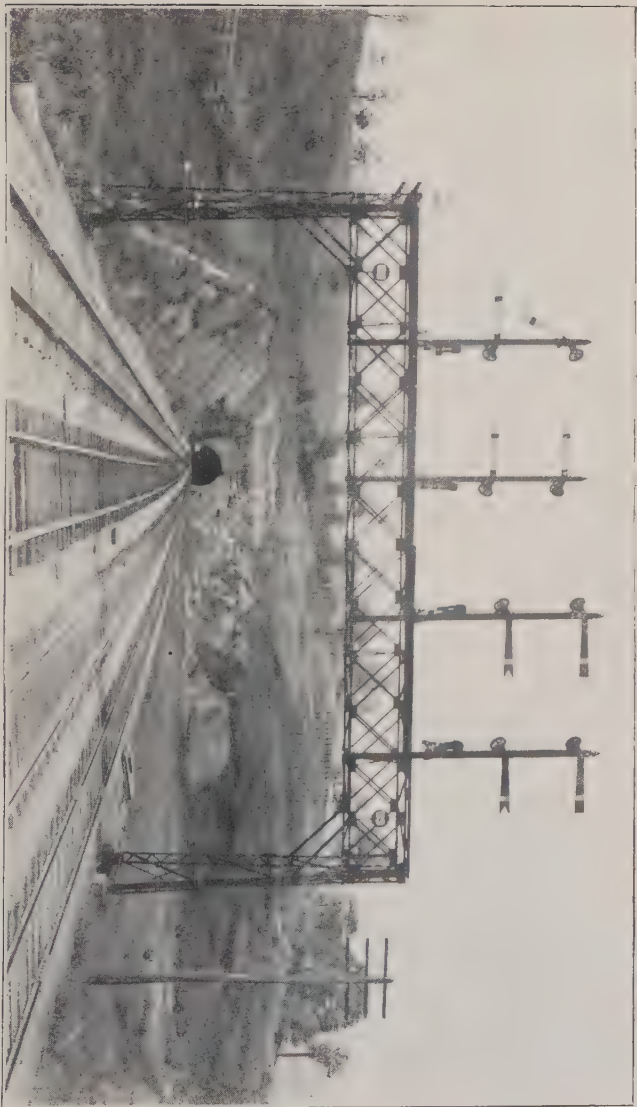


FIG. 169.—SIGNAL BRIDGE.



*(To face page 330.)*

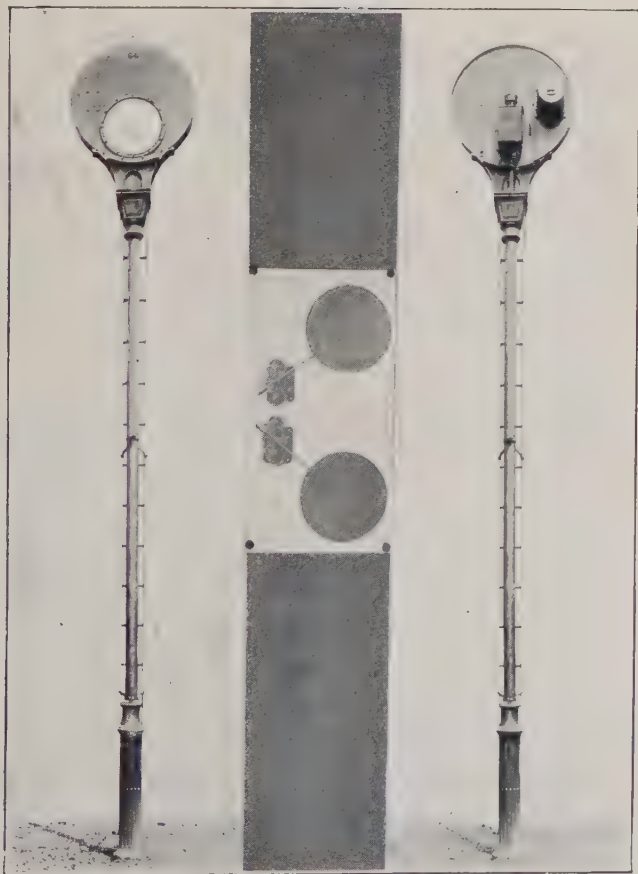


FIG. 170.—“BANJO” SIGNALS.



*color* of a round disk inclosed with glass. This is the distinctive signal of the Hall Signal Company, and is also used by the Union Switch and Signal Company. The great argument in their favor is that they may be worked by an electric current of low voltage, which is therefore easily controlled; that the mechanism is entirely inside of a case, is therefore very light, and is not exposed to the weather. The argument urged against them is that it is a signal of *color* rather than *form* or *position*, and that in foggy weather the signal cannot be seen so easily; also that unsuspected color-blindness on the part of the engineman may lead to an accident. Notwithstanding these objections, this form of signal is used on thousands of miles of line in this country.

**309. Wires and pipes.** Signals are usually operated by levers in a signal-cabin, the levers being very similar to the reversing-lever of a locomotive. The distance from the levers to the signals is, of course, very variable, but it is sometimes 2000 feet. The connecting-link for the most distant signals is usually No. 9 wire; for nearer signals and for all switches operated from the cabin it may be 1-inch pipe. When not too long, one pipe will serve for both motions, forward and back. When wires are used, it is sometimes so designed (in the cheaper systems) that one wire serves for one motion, gravity being depended on for the other, but now all good systems require two wires for each signal.

**Compensators.** Variations of temperature of a material with as high a coefficient as iron will cause very appreciable difference of length in a distance of several hundred feet, and a dangerous lack of adjustment is the result. To illustrate: A fall of 60° F. will change the length of 1000 feet of wire by

$$1000 \times 60 \times .0000065 = 0.39 \text{ foot} = 4.68 \text{ inches.}$$

A much less change than this will necessitate a readjustment of length, unless automatic compensators are used. A compensator for pipes is very readily made on the principle illustrated in Fig. 171. The problem is to preserve the distance between *a* and *d* constant regardless of the temperature. Place the compensator half-way between *a* and *d*, or so that  $ab = cd$ . A fall of temperature contracts *ab* to *ab'*. Moving *b* to *b'* will cause *c* to move to *c'*, in which  $bb' = cc'$ . But *cd* has also shortened to *c'd*; therefore *d* remains fixed in position. To avoid

too great angular motion, one such compensator should be used for each 500 feet. If a line 1000 feet long is to be provided for, two compensators would be used, 250 feet from each end. Note that in operating through a compensator the *direction* of motion changes: i.e., if *a* moves to the right, *d* moves to the left, or if there is compression in *ab* there is tension in *cd*, and

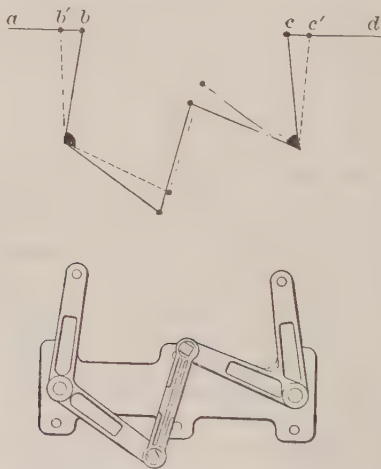


FIG. 171.—STANDARD PIPE COMPENSATOR.

*vice versa*. Therefore this form of compensator can only be used with pipes which will withstand compression. It has seemed impracticable to design an equally satisfactory compensator for wires, although there are several designs on the market.

**Guides around curves and angles.** When wires are required to pass around curves of large angle, pulleys are used, and a length of chain is substituted for the wire. For pipes, when the curve is easy the pipes are slightly bent and are guided through pulleys. When the angle is sharper, "angles" are used. The operation of these details is self-evident from an inspection of Fig. 172.

**310. Track circuit for automatic signaling.** The several systems of automatic signaling differ in the minor details, but



nearly all of them agree in the following particulars. A current of low potential is run from a battery at one end of a section through one line of rails to the other end of the section, then through a relay, and then back to the battery through the other

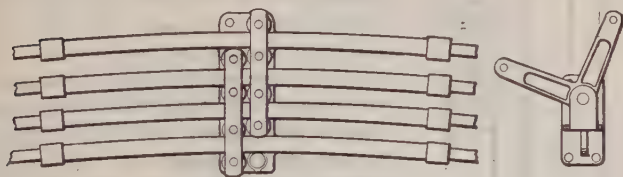


FIG. 172.—DEFLECTING-RODS.

line of rails. To avoid the excessive resistance which would occur at rail joints which may become badly rusted, a wire suitably attached to the rails is run around each joint. In order to insulate the rails of one section from the rails at either end and yet maintain the rails structurally continuous, the ends of the rails at these dividing points are separated by an insulator and the joint pieces are either made of wood or have some insulating material placed between the rails and the ordinary metal joint. The bolts must also be insulated. When the relay is energized by a current, it closes a local circuit at the signal-station, which will set the signal there at "safety." The resistance of the relay is such that it requires nearly the whole current to work it and to keep the local circuit closed. Therefore, when there is any considerable loss of current from one rail to the other, the relay will not be sufficiently energized, the local circuit will be broken, and the signal will automatically fall to danger. This diversion of current from one rail to the other before the current reaches the relay may be caused in several ways: the presence of a pair of wheels on the rails anywhere in the section will do it; also the breakage of a rail; also the opening of a switch anywhere in the section; also the presence of a pair of wheels on a siding between the "fouling point" and the switch. (The "fouling point" of a siding is that point where the rails first commence to approach the main track.) In Fig. 173 is shown all of the above details, as well as some others. At *A*, *B*, and the "fouling point" are shown the in-

ulated joints. The batteries and signals are arranged for train motion to the *right*. When a train has passed the points near *A*, where the wires leave the rails for the relay, the current from the "track battery" at *B* will pass through the wheels and axles, and although no electrical connection is broken, so much current will be shunted through the wheels and axles that the weak current still passing through the relay is not strong enough to energize it against its spring and the "signal-magnet" circuit is broken, and the signal *A* goes to "danger." At the turnout the rails between the fouling point and the switch are so connected (and insulated) that a pair of wheels on these rails will produce the same effect as a pair on the main track. This is to guard against the effect of a car standing too near the switch, even though it is not on the main track. When the train passes *B*, if there is no other interruption of the current, the track battery at *B* again energizes the relay at *A*, the signal-magnet circuit at *A* is closed, and the signal is drawn to "safety."

(The present edition has omitted several subdivisions of this general subject, notably the "staff system," used chiefly in England, and all discussions of "interlocking" which is an essential feature of the operation of large terminal yards. A future edition may supply these deficiencies, although an exhaustive treatment of the subject of Signaling would require a separate volume.)

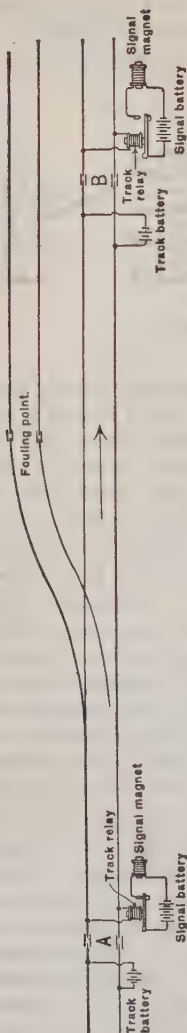


FIG. 173.

## CHAPTER XV.

### ROLLING-STOCK.

(It is perhaps needless to say that the following chapter is in no sense a course in the design of locomotives and cars. Its chief idea is to give the student the elements of the construction of those vehicles which are to use the track which he may design—to point out the mutual actions and reactions of vehicle against track and to show the effect on track wear of variations in the design of rolling-stock. The most of the matter given has a direct practical bearing on track-work, and it is considered that all of it is so closely related to his work that the civil engineer may study it with profit.)

#### WHEELS AND RAILS.

**311. Effect of rigidly attaching wheels to their axles.** The wheels of railroad rolling-stock are invariably secured rigidly to the axles, which therefore revolve with the wheels. The chief reason for this is to avoid excessive wear between the axles and the wheels.

Any axle must always be somewhat loose in its journals. A sidewise force  $P$  (see Fig. 174) acting against the circumference of the wheel will produce a much greater pressure on the axle at  $S$  and  $S'$ , and if the wheel moves on the axle, the wear at  $S$  and  $S'$  will be excessive. But when the axle is fitted to the wheel with a "forced fit" and does not revolve, the mere pressure produced at  $S$  is harmless. When two wheels are fitted tight to an axle, as in Fig. 175, and the axle revolves in the journals  $aa$ , a sidewise pressure of the rail against the wheel flange will only produce a slight and harmless increase of the journal pressure  $Q$ , although at  $Q$  there is sliding contact. Twist-

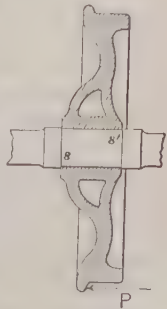


FIG. 174.

ing action in the journals is thus practically avoided, since a small pressure at the journal-boxes at each end of the axle suffices to keep the axle truly in line.

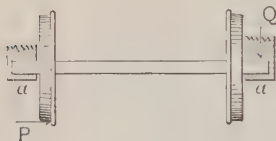


FIG. 175.

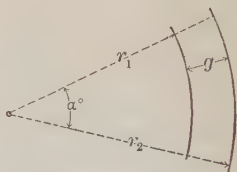


FIG. 176.

On the other hand, when the wheels are rigidly attached to their axles, both wheels must turn together, and when rounding curves, the inner rail being shorter than the outer rail, one wheel must slip by an amount equal to that difference of length. The amount of this slip is readily computable:

$$\text{Longitudinal slip} = \frac{2\pi\alpha^\circ}{360^\circ}(r_2 - r_1) - \frac{2\pi g}{360^\circ}\alpha^\circ - C\alpha^\circ, \quad (136)$$

in which  $C$  is a constant for any one gauge, and  $g$  = the track gauge =  $(r_2 - r_1)$ . For standard gauge (4.708) the slip is .08218 foot per degree of *central angle*. This shows that the longitudinal slipping around any curve of any given central angle will be *independent of the degree of the curve*. The constant (.08218) here given is really somewhat too small, since the true gauge that should be considered is the distance between the lines of tread on the rails. This distance is a somewhat indeterminate and variable quantity, and probably averages 4.90 feet, which would increase the constant to .086. The slipping may occur by the inner wheel slipping ahead or the outer wheel slipping back, or by both wheels slipping. The total slipping will be constant in any case. The slipping not only consumes power, but wears both the wheels and the rail. But even these disadvantages are not sufficient to offset the advantages resulting from rigid wheels and axles.

**312. Effect of parallel axles.** Trucks are made with two or three parallel axles (except as noted later), in order that the axles shall mutually guide each other and be kept approximately

perpendicular to the rails. If the curvature is very sharp and the wheel-base comparatively long (as is notably the case on street railways at street corners), the front and rear wheels

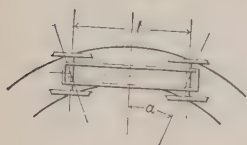


FIG. 177.

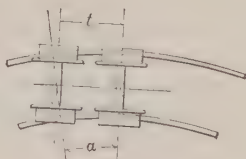


FIG. 178.

will stand at the same angle ( $a$ ) with the track, as shown in Fig. 177. But it has been noticed that for ordinary degrees of curvature, the rear wheels stand radial to the curve (see Fig. 178), and for steam railroad work this is the normal case. When the two parallel axes are on a curve (as shown), the wheels tend to run in a straight line. In order that they shall run on a curve they must slip laterally. The principle is illustrated in an exaggerated form in Fig. 179. The wheel tends to roll from  $a$  toward  $b$ . Therefore in passing along the track from  $a$  to  $c$  it must actually slip laterally an amount  $bc$  which equals  $ac \sin a$ .



FIG. 179.

Let  $l$  = length of the wheel-base (Figs. 177 and 178);  $r$  = radius of curve; then for the first case (Fig. 177),  $\sin a = l \div 2r$ ; for the second and usual case (Fig. 178),  $\sin a = l \div r$ ; for  $l = 5$  feet and  $r$  = radius of a  $1^\circ$  curve,  $a = 0^\circ 03'$  for the second case.  $a$  varies (practically) as the degree of curve. The lateral slipping *per unit* of distance traveled therefore equals  $\sin a$ . As an illustration, given a 5-foot wheel-base on a  $5^\circ$  curve,  $a = 0^\circ 15'$ ,  $\sin a = .00436$ , and for each 100 feet traveled along the curve the lateral slip of the front wheels would be 0.436 foot. There would be no lateral slipping of the rear wheels, assuming that the rear axle maintained itself radial.

From the above it might be inferred that the flanges of the forward wheels will have much greater wear than those of the rear wheels. Since cars are drawn in both directions about equally, no difference in flange wear due to this cause will occur, but locomotives (except switching-engines) run forward almost

exclusively, and the excess wear of the front wheels of the pilot- and tender-trucks is plainly observable.

For a given curve the angle  $\alpha$  (and the accompanying resistance) is evidently greater the greater the distance between the axles. On the other hand, if the two axles are very close together, there will be a tendency for the truck to twist and the wheels to become jammed, especially if there is considerable play in the gauge. The flange friction would be greater and would perhaps exceed the saving in lateral slipping. A general rule is that the axles should never be closer together than the gauge.

Although the slipping per unit of length along the curve varies directly as the degree of curvature, the length of curve necessary to pass between two tangents is inversely as the degree of curve, and the total slipping between the two tangents is independent of the degree of curve. Therefore when a train passes between

two tangents, the total slipping of the wheels on the rails, longitudinal and lateral, is a quantity which depends **only** on the **central angle** and is **independent** of the **radius** or **degree of curve**.

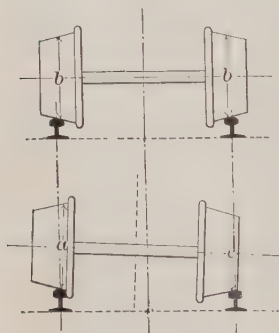


FIG. 180.

**313. Effect of coning wheels.** The wheels are always set on the axle so that there is some "play" or chance for lateral motion between the wheel-flanges and the rail. The treads of the wheel are also "coned." This coning and play of gauge are shown in an exaggerated form in Fig. 180. When the

wheels are on a tangent, although there will be occasional oscillations from side to side, the normal position will be the symmetrical position in which the circles of tread  $bb$  are equal. When centrifugal force throws the wheel-flange against the rail, the circle of tread  $a$  is larger than  $b$ , and much larger than  $c$ ; therefore the wheels will tend to roll in a circle whose radius equals the slant height of a cone whose elements would pass through the unequal circles  $a$  and  $c$ . If this radius equaled the radius of the track, and if the axle were free to assume a radial position, the wheels would roll freely on the rails without any



slipping or flange pressure. Under such ideal conditions, coning would be a valuable device, but it is impracticable to have all axles radial, and the radius of curvature of the track is an extremely variable quantity. It has been demonstrated that with parallel axles the influence of coning diminishes as the distance between the axle increases, and that the effect is practically inappreciable when the axles are spaced as they are on locomotives and car-trucks. The coning actually used is very slight (see Chapter XV, § 332) and has a different object. It is so slight that even if the axles were radial it would only prevent the slipping on a very light curve—say a  $1^\circ$  curve.

**314. Effect of flanging locomotive driving-wheels.** If all the wheels of all locomotives were flanged it would be practically impossible to run some of the longer types around sharp curves. The track-gauge is always widened on curves, and especially on sharp curves, but the widening would need to be excessive to permit a consolidation locomotive to pass around an  $8^\circ$  or  $10^\circ$  curve if all the drivers were flanged. The action of the wheels on a curve is illustrated in Figs. 181, 182, and 184. All small truck-wheels are flanged. The rear drivers are always flanged and four-driver engines usually have all the drivers flanged. Consolidation engines have only the front and rear drivers flanged. Mogul and ten-wheel engines have one pair of drivers blank. On Mogul engines it is always the middle pair. On ten-wheel engines, when used on a road having sharp curves, it is preferable to flange the front and rear driving-wheels and use a "swing bolster" (see § 315); when the curvature is easy, the middle and rear drivers may be flanged and the truck made with a rigid center. The blank drivers have the same total width as the other drivers and of course a much wider tread, which enables these drivers to remain on the rail, even though the curvature is so sharp that the tread overhangs the rail considerably.

**315. Action of a locomotive pilot-truck.** The purpose of the pilot-truck is to guide the front end of a locomotive around a curve and to relieve the otherwise excessive flange pressure that would be exerted against the driver-flanges. There are two classes of pilot-trucks—(a) those having fixed centers and (b) those having shifting centers. This second class is again subdivided into two classes, which are radically different in their action—(b<sub>1</sub>) four-wheeled trucks having two parallel axles



and ( $b_2$ ) two-wheeled trucks which are guided by a "radius-bar." The action of the four-wheeled fixed-centered truck ( $a$ ) is shown in Fig. 181. Since the center of the truck is forced

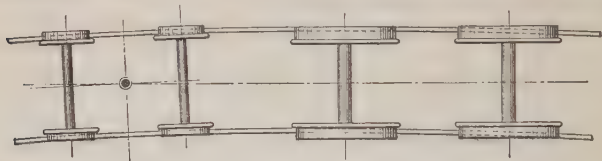


FIG. 181.—FIXED CENTER PILOT-TRUCK.

to be in the center of the track, the front drivers are drawn away from the outer rail. The rear outer driver tends to roll away from the outer rail rather than toward it, and so the effect

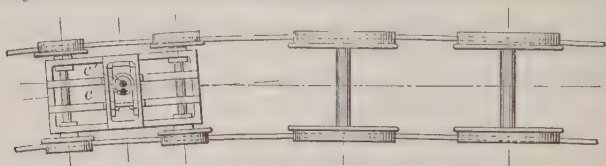


FIG. 182.—FOUR-WHEELED TRUCK—SHIFTING CENTER.

of the truck is to relieve the driver-flanges of any excessive pressure due to curvature. The only exception to this is the case where the curvature is sharp. Then the front inner driver may be pressed against the *inner* rail, as indicated in Fig. 181.

This limits the use of this type of wheel-base on the sharper curves.

The next type—( $b_1$ ) four-wheeled trucks with shifting centers—is much more flexible on sharp curvature; it likewise draws the front drivers away from the outer rail. The relative position of the wheels is shown in Fig. 182, in which  $c'$  represents the position of center-pin and  $c$  the displaced truck center. The structure and action of the truck is shown in Fig. 183. The "center-pin" (1) is

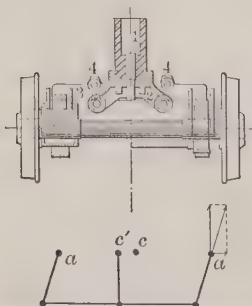


FIG. 183.—ACTION OF SHIFTING CENTER.

supported on the "truck-bolster" (2), which is hung by the "links" (4) from the "cross-ties" (3). The links are therefore

in tension and when the wheels are forced to one side by the rails the *links* are inclined and the front of the engine is drawn inward by a force equal to the weight on the bolster times the tangent of the angle of inclination of the links. This assumes that all links are vertical when the truck is in the center. Frequently the opposite links are normally inclined to each other, which somewhat complicates the above simple relation of the forces, although the general principle remains identical.

The two-wheeled pilot-truck with shifting center is illustrated in Fig. 184. The figure shows the facility with which

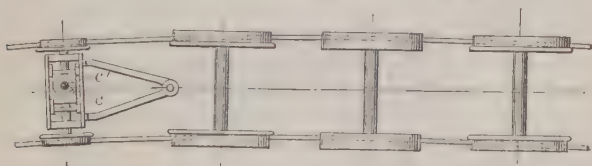


FIG. 184.—TWO-WHEELED TRUCK—SHIFTING CENTER.

an engine with long wheel-base may be made to pass around a comparatively sharp curve by omitting the flanges from the middle drivers and using this form of pilot-truck. As in the previous case, the eccentricity of the center of the truck relative to the center-pin induces a centripetal force which draws the front of the engine inward. But the swing-truck is not the only source of such a force. If the "radius-bar pin" were placed at  $O'$  (see Fig. 185), the truck-axle would be radial. But the radius-bar is always made somewhat shorter than this, and the pin is placed at  $O$ , a considerable distance ahead of  $O'$ , thus creating a tendency for the truck to run toward the inner rail and draw the front of the locomotive in that direction. This tendency will be objectionably great if the radius-bar is made too short, as has been practically demonstrated in cases when the radius-bar has been subsequently lengthened with a resulting improvement in the running of the engine.

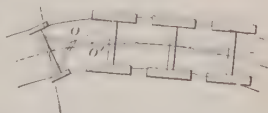


FIG. 185. ACTION OF TWO-WHEELED TRUCK.

## LOCOMOTIVES.

## GENERAL STRUCTURE.

**316. Frame.** The frame or skeleton of a locomotive consists chiefly of a collection of forged wrought-iron bars, as shown in Figs. 186 and 187. These bars are connected at the



FIG. 186.—ENGINE-FRAME.

front end by the "bumper" (c), which is usually made of wood. A little further back they are rigidly connected at *bb* by the cylinders and boiler-saddle. The boilers rest on the frames at *aaaa* by means of "pads," which are bolted to the fire-box, but which permit a free expansion of the boiler along the frame. This expansion is sometimes as much as  $\frac{5}{16}$ ". On a "consolidation" engine (frame shown in Fig. 187) it is frequently



FIG. 187.—ENGINE-FRAME—CONSOLIDATION TYPE.

necessary to use vertical swing-levers about 12" long instead of "pads." The swinging of the levers permit all necessary expansion. At the back the frames are rigidly connected by the iron "foot-plate." The driving-axes pass through the "jaws" *dddd*, which hold the axle-boxes. The frame-bars have a width (in plan) of 3" to 4". The depth (at *a*) is about the same. Fig. 186 shows a frame for an "American" type of locomotive; Fig. 187 shows a frame for a "Consolidation" type (see § 323).

**317. Boiler.** A boiler is a mechanism for transferring the latent heat of fuel to water, so that the water is transformed from cold water into high-pressure steam, which by its expansion will perform work. The efficiency of the boiler depends largely on its ability to do its work rapidly and to reduce to a minimum the waste of heat through radiation. The boiler contains a fire-box (see Fig. 188), in which the fuel is burned. The gases of consumption pass from the fire-box through the numerous boiler-tubes into the "smoke-box" *S* and out through the smoke-stack. The fire-box consists of an inner and outer

shell separated by a layer of water about 3" thick. The exposure of water-surface to the influence of the fire is thus very complete. The efficiency of this transferal of heat is somewhat indicated by the fact that, although the temperature of the gases in the fire-box is probably from 3000° to 4000° F., the temperature in the smoke-box is generally reduced to 500° to

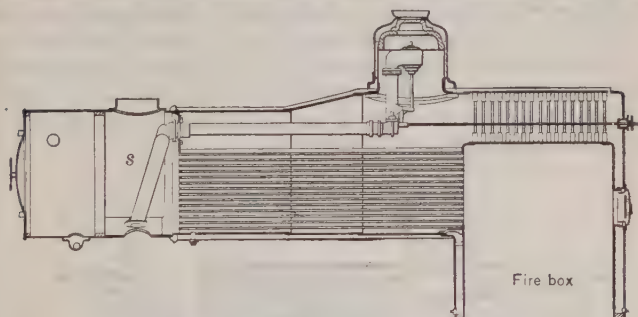


FIG. 188.—LOCOMOTIVE-BOILER.

600° F. If the steam pressure is 180 lbs., the temperature of the water is about 380° F., and, considering that heat will not pass from the gas to the water unless the gas is hotter than the water, the water evidently absorbs a large part of the theoretical maximum. Nevertheless gases at a temperature of 600° F. pass out of the smoke-stack and such heat is utterly wasted.

The tubes vary from 1 $\frac{3}{4}$ " to 2", inside diameter, with a thickness of about 0".10 to 0".12. The aggregate cross-sectional area of the tubes should be about one eighth of the grate area. The number will vary from 140 to 250. They are made as long as possible, but the length is virtually determined by the type and length of engine.

**318. Fire-box.** The fire-box is surrounded by water on the four sides and the top, but since the water is subjected to the boiler pressure, the plates, which are about  $\frac{5}{16}$ " thick, must be stayed to prevent the fire-box from collapsing. This is easily accomplished over the larger part of the fire-box surface by having the outside boiler-plates parallel to the fire-box plates and separated from them by a space of about 3". The plates are then mutually held by "stay-bolts." See Fig. 189. These are about  $\frac{7}{8}$ " in diameter and spaced 4" to 4 $\frac{1}{2}$ ". The  $\frac{5}{16}$ " hole,

drilled  $1\frac{1}{4}$ " deep, indicated in the figure, will allow the escape of steam if the bolt breaks just behind the plate, and thus calls attention to the break. The stay-bolts are turned down to a diameter equal to that at the root of the screw-threads. This method of supporting the fire-box sheets is used for the two sides, the entire rear, and for the front of the fire-box up to the boiler-barrel. The "furnace tube-sheet"—the upper part of the front of the fire-box—is stayed by the tubes. But the top of the fire-box is troublesome. It must always be covered with water so that it will not be "burned" by the intense heat. It must therefore be nearly, if not quite, flat. There are three general methods of accomplishing this.

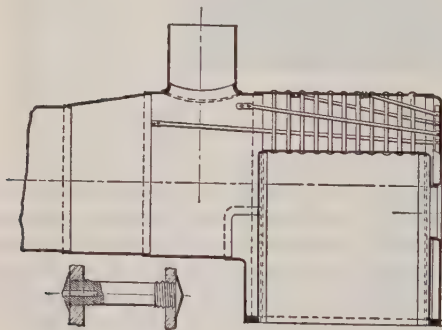


FIG. 189.

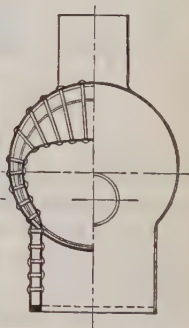


FIG. 190.

(a) **Radial stays.** This construction is indicated in Fig. 190. Incidentally there is also shown the diagonal braces for resisting the pressure on the back end of the boiler above the fire-box. It may be seen that the stays are not perpendicular to either the crown-sheet or the boiler-plate. This is objectionable and is obviated by the other methods.

(b) **Crown-bars.** These bars are in pairs, rest on the side furnace-plates, and are further supported by stays. See Fig. 191.

(c) **Belpaire fire-box.** The boiler above the fire-box is rectangular, with rounded corners. The stays therefore are perpendicular to the plates. See Fig. 192.

**Fire-brick arches.** These are used, as shown in Fig. 193, to force all the gases to circulate through the upper part of the fire-

box. Perfect combustion requires that all the carbon shall be turned into carbon dioxide, and this is facilitated by the forced circulation.

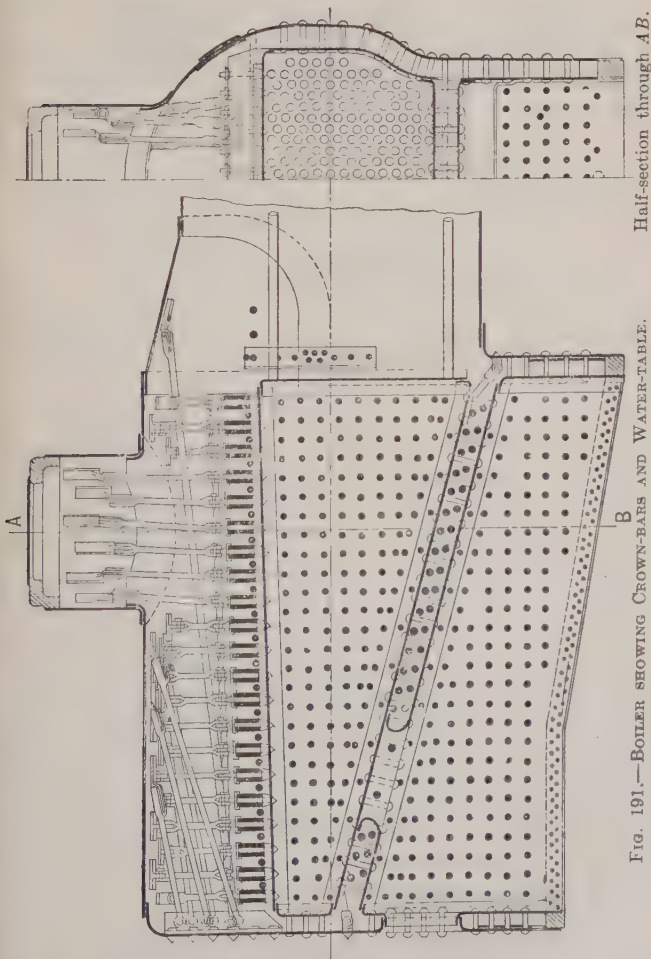


FIG. 191.—BOILER SHOWING CROWN-BARS AND WATER-TABLE.

**Water-tables.** The same object is attained by using a water-table instead of a brick arch—as shown in Fig. 191. But it has



the further advantages of giving additional heating-surface and avoiding the continual expense of maintaining the bricks. One

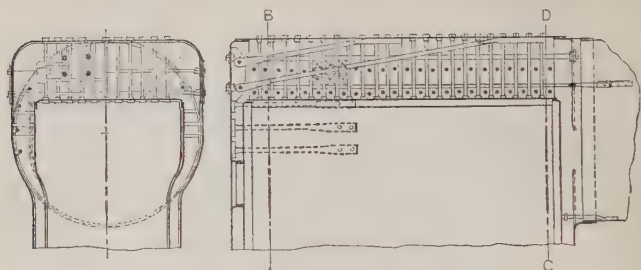


FIG. 192.—"BÉLPAIRE" FIRE-BOX.

Half-section through AB.

Half-section through CD.

feature of the design is the use of a number of steam-jets which force air into the fire-box and assist the combustion.

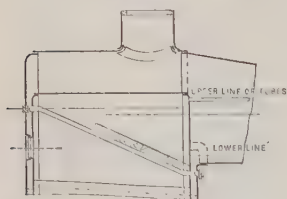


FIG. 193. — FIRE-BRICK ARCH.

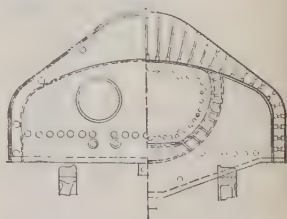


FIG. 194.—WOOTTEN FIRE-BOX.

**Area.** Fire-boxes are usually limited in width to the practicable width between the wheels—thus giving a net inside width of about 3 feet and a maximum length of 10 to 11 feet—this being about the maximum distance over which the firemen can properly control the fire. About 37 square feet is the maximum area obtainable except when the "Wootten" fire-box is used—illustrated in Fig. 194. Here the grate is raised above the driving-wheels and has (in the case shown) a width of 8' 0 $\frac{1}{2}$ ". The fire-box area is over 76 square feet. Note that *two* furnace-doors are used.

**9. Coal consumption.** No form of steam-boiler (except a boiler for a steam fire-engine) requires as rapid production of steam, considering the size of the boiler and fire-box, as a



locomotive. The combustion of coal per square foot of grate per hour for stationary boilers averages about 15 to 25 lbs. and seldom exceeds that amount. An ordinary maximum for a locomotive is 125 lbs. of coal per square foot of grate-area per hour, and in some recent practice 220 lbs. have been used. Of course such excessive amounts are wasteful of coal, because a considerable percentage of the coal will be blown out of the smoke-stack unconsumed, the draft necessary for such rapid consumption being very great. The only justification of such rapid and wasteful coal consumption is the necessity for rapid production of steam. The best quality of coal is capable of evaporating about 14 lbs. of water per pound of coal, i.e., change it from water at  $212^{\circ}$  to steam at  $212^{\circ}$ ; the heat required to change water at ordinary temperatures to steam at ordinary working pressure is (roughly) about 20% more. From 6 to 9 lbs. of water per pound of coal is the average performance of ordinary locomotives, the efficiency being less with the higher rates of combustion. Some careful tests of locomotive coal consumption gave the following figures: when the consumption of coal was 50 lbs. per square foot of grate-area per hour, the rate of evaporation was 8 lbs. of water per pound of coal. When the rate of coal consumption was raised to 180, the evaporation dropped to 5 lbs. of water per pound of coal. It has been demonstrated that the efficiency of the boiler is largely increased by an increased length of boiler-tubes. The actual consumption of coal per mile is of course an exceedingly variable quantity, depending on the size and type of the engine and also on the work it is doing—whether climbing a heavy grade with its maximum train-load or running easily over a level or down grade. A test of a 50-ton engine, running without any train at about 20 to 25 miles per hour, showed an average consumption of 21 lbs. of coal per mile. Statistics of the Pennsylvania Railroad show a large increase (as might be expected, considering the growth in size of engines and weight of trains) in the average number of pounds of coal burned per *train-mile*—some of the figures being 55 lbs. in 1863, 72 lbs. in 1872, and nearly 84 lbs. in 1883. Figures are published showing an average consumption of about 10 lbs. of coal per passenger-car mile, and 4 to 5 lbs. per freight-car mile. But these figures are always obtained by dividing the total consumption per train-mile by the number of cars, the coal due to the weight of the engine

being thrown in Wellington developed a rule, based on the actual performance of a very large number of passenger-trains, that the number of pounds of coal per mile =  $21.1 + 6.74$  times the number of passenger-cars. The amount of coal assigned to the engine agrees remarkably with the test noted above. For freight-trains the amount assigned to the engine should be much greater (since the engine is much heavier), and that assigned to the individual cars much less, although the great increase in freight-car weights in recent years has caused an increase in the coal required per car.

**320. Heating-surface.** The rapid production of steam requires that the hot gases shall have a large heating-surface to which they can impart their heat. From 50 to 75 square feet of heating-surface is usually designed for each square foot of grate-area. A more recently used rule is that there should be from 60 to 70 square feet of *tube* heating-surface per square foot of grate-area for bituminous coal. 40 or 50 to 1 is more desirable for anthracite coal. Almost the whole surface of the fire-box has water behind it, and hence constitutes heating-surface. Although this surface forms but a small part of the total (nominally), it is really the most effective portion, since the difference of temperature of the gases of combustion and the water is here a maximum, and the flow of heat is therefore the most rapid. The heating-surface of the tubes varies from 85 to 93% of the total, or about 7 to 15 times the heating-surface in the fire-box. Sometimes the heating-surface is as much as 2300 square feet, but usually it is less than 2000, even for engines which must produce steam rapidly.

Some of the most recent locomotives have greatly exceeded these figures. One just constructed for the New York Central and Hudson River Railroad has the following figures: heating-surface, 3500 sq. ft; grate-area, 50 sq ft; cylinders,  $21'' \times 26''$ ; total weight, 176000 lbs; weight on drivers, 95000 lbs.; drivers, 79" diameter; with 85% of the boiler pressure, it developed an adhesion of 24700 lbs., which represented a factor of adhesion of  $\frac{1}{3.85}$ .

Another rule used by designers is that the engine should have 1 sq ft of heating-surface for each 50 or 60 lbs of weight, efficiency being indicated by a low weight. For the above engine the ratio is 53

321. **Loss of efficiency in steam pressure.** The effective work done by the piston is never equal to the theoretical energy contained in the steam withdrawn from the boiler. This is due chiefly to the following causes:

(a) The steam is "wire-drawn," i.e., the pressure in the cylinder is seldom more than 85 to 90% of the boiler pressure. This is due largely to the fact that the steam-ports are so small that the steam cannot get into the cylinder fast enough to exert its full pressure. It is often purposely wire-drawn by partially closing the throttle, so that the steam may be used less rapidly.

(b) **Entrained water.** Steam is always drawn from a dome placed over the boiler so that the steam shall be as far above the water-surface as possible, and shall be as dry as possible. In spite of this the steam is not perfectly dry and carries with it water at a temperature of, say,  $361^{\circ}$ , and pressure of 140 lbs per square inch. When the pressure falls during the expansion and exhaust, this hot water turns into steam and absorbs the necessary heat from the hot cylinder-walls. This heat is then carried out by the exhaust and wasted.

(c) The back pressure of the exhaust-steam, which depends on the form of the exhaust-passages, etc. This amounts to from 2 to 20% of the power developed.

(d) **Clearance-spaces.** When cutting off at full stroke this waste is considerable (7 to 9%), but when the steam is used expansively the steam in these clearance-spaces expands and so its power is not wholly lost.

(e) **Radiation.** In spite of all possible care in jacketing the cylinders, some heat is lost by radiation.

(f) **Radiation into the exhaust-steam.** This is somewhat analogous to (b). Steam enters the cylinder at a temperature of, say,  $361^{\circ}$ ; the walls of the cylinder are much cooler, say  $250^{\circ}$ ; some heat is used in raising the temperature of the cylinder-walls; some steam is vaporized in so doing; when the exhaust is opened the temperature and pressure fall; the heat temporarily absorbed by the cylinder-walls is reabsorbed by the exhaust-steam, re-evaporating the vapor previously formed, and thus a certain portion of heat-energy goes through the cylinder without doing any useful work. With an early cut-off the loss due to this cause is very great.

The sum of all these losses is exceedingly variable. They are usually less at lower speeds. The loss in *initial pressure*

(the difference between boiler pressure and the cylinder pressure at the beginning of the stroke) is frequently over 20%, but this is not all a net loss. With an early cut-off the average cylinder pressure for the whole stroke is but a small part of the boiler pressure, yet the horse-power developed may be as great as, or greater than that developed at a lower speed, later cut-off, and higher average pressure.

**322. Tractive power** The work done by the two cylinders during a complete revolution of the drivers evidently = area of pistons  $\times$  average steam pressure  $\times$  stroke  $\times 2 \times 2$ . The resistance overcome evidently = tractive force at circumference of drivers times distance traveled by drivers (which is the circumference of the drivers). Therefore

$$\text{Tractive force} = \frac{\left\{ \begin{array}{l} \text{area pistons} \times \text{average steam pressure} \\ \times \text{stroke} \times 2 \times 2. \end{array} \right.}{\text{circumference of drivers}}.$$

Dividing numerator and denominator by  $\pi$  (3.1415), we have

$$\text{Tractive force} = \left\{ \frac{(\text{diam piston})^2 \times \text{average steam pressure} \times \text{stroke}}{\text{diameter of driver}} \right\}, \quad (137)$$

which is the usual rule. Although the rule is generally stated in this form, there are several deductions. In the first place the net effective area of the piston is less than the nominal on account of the area of the piston-rod. The ratio of the areas of the piston-rod and piston varies, but the effect of this reduction is usually from 1.3 to 1.7%. No allowance has been made for friction—of the piston, piston-rod, cross-head, and the various bearings. This would make a still further reduction of several per cent. Nevertheless the above simple rule is used, because, as will be shown, no great accuracy can be utilized.

The tractive force is limited by the adhesion between the drivers and the rails, and this is a function of the weight on the drivers. Under the most favorable conditions this has been tested to amount to one-third the weight on the drivers, but such a ratio cannot be depended on. Wellington used the ratio one-fourth. The Baldwin Locomotive Works in their "Locomotive Data" give tables and diagrams based on  $\frac{1}{4}$ ,  $\frac{2}{5}$ ,

and  $\frac{1}{3}$  adhesion. As low a value as  $\frac{1}{6}$  or even  $\frac{1}{7}$  is occasionally used, but such a low rate of adhesion would only be found when the rails were abnormally slippery. In a well-designed locomotive the tractive force, as computed above, and the tractive adhesion should be made about equal. The uncertainty in the coefficient of adhesion shows the futility of any refinement in the computation of the tractive force.

It is only at very slow speeds that an engine can utilize all of its tractive force. When running at a high speed, the utmost horse-power that the engine can develop will only produce a draw-bar pull, which is but a small part of the possible tractive force. Power is the product of force times velocity. If the power is constant and the velocity increases, the force must decrease. This fact is well shown in the figures of some tests of a locomotive. The dimensions were as follows: cylinders, 18"×24"; drivers, 68"; weight on drivers, 60000 lbs.; heating-surface, 1458 sq. ft.; grate-area, 17 sq. ft. During one test the average cylinder pressure was 83.3 lbs. (boiler pressure, 145); 14-inch cut-off and throttle  $\frac{3}{4}$  open). By the above formula (137),

$$\text{Tractive force} = \frac{18^2 \times 83.3 \times 24}{68} = 9525 \text{ lbs.}$$

At  $\frac{1}{4}$  adhesion the tractive force was 15000 lbs; even at  $\frac{1}{3}$  adhesion, it would be 12000 lbs. This shows that at the speed of this test (26.3 in. per hour) scarcely more than  $\frac{2}{3}$  of the tractive power was utilized. A still more marked case, shown by another test with the same engine, taken when the speed was 53.4 miles per hour, indicated an average cylinder pressure of 37.2 lbs., the throttle being  $\frac{1}{2}$  open and the valves cutting off at 8". In this case the tractive power, computed as before, equals 4254 lbs., about  $\frac{1}{4}$  of the weight on the drivers and about  $\frac{1}{3}$  of the tractive force which is possible at slow speeds. In the first case, the tractive power (9525) times the speed in feet per second (38.57) divided by 550 gives the indicated horse-power, 668. In the second case, although the tractive forces developed was so much less, the speed was much greater and the horse-power was about the same, 606.

The above figures illustrate some of the foregoing statements regarding loss of efficiency. In both cases the steam was wire-drawn. The boiler pressure was 145 lbs., but when the throttle

was only  $\frac{3}{4}$  open and the steam was cut-off at 14" (24" stroke) the average steam pressure in the cylinder was reduced to 83.3 lbs. With the throttle but  $\frac{1}{2}$  open and the valves cutting off at 8" ( $\frac{1}{3}$  of the stroke), the average pressure was cut down to 37.2 lbs.—about  $\frac{1}{4}$  of the boiler pressure. Note that the heating-surface per square foot of grate-area ( $1458 \div 17 = 86$ ) is very large (see § 320). Note also that the horse-power developed divided by the grate-area (17) gives 39 and 36 H.P. per square foot of grate-area. This is exceptionally large—25 or 30 being a more common figure.

The maximum tractive power is required when a train is starting, and fortunately it is at low velocities that the maximum tractive force can be developed. The motion of the piston is so slow that there is but little reduction of steam pressure, and the valves are generally placed to cut off at full stroke. For the above engine, with 145 lbs. boiler pressure, the absolute maximum of tractive force is  $\frac{18^2 \times 145 \times 24}{68} =$

16581 lbs. Of course, this maximum would never be reached unless the boiler pressure were increased. A common rule is to consider that the average effective cylinder pressure for slow speed and full stroke will be 80% of the boiler pressure. This would reduce the tractive force to the (nominal) value of 13265 lbs., and the corresponding cylinder pressure would be 116 lbs. per square inch. With an effective cylinder pressure of about 131 lbs. the tractive power is 15000 lbs., which is  $\frac{1}{4}$  of the total weight on the drivers. This illustrates the general rule, stated above, that the cylinders, drivers, and boiler pressure should be so proportioned that the maximum tractive force should about equal the maximum adhesion which could be obtained.

As another numerical example, the dimensions of a recently constructed heavy consolidation engine are quoted. The cylinders are 24"  $\times$  32"; diameter of drivers, 54"; total weight of engine and tender, 391400 lbs.; weight of engine, 250300 lbs.; weight on drivers, 225200 lbs.; capacity of tender, 7500 gallons; the boiler has 406 tubes, 21" in diameter and 15' long; fire-box, 132"  $\times$  40 $\frac{1}{2}$ "; heating-surface of tubes, 3564 sq. ft.; of fire-box, 241 sq. ft.—total, 3805 sq. ft.; boiler pressure, 220 lbs. per square inch. Applying Eq. 132, we may compute 75093 lbs. as the absolute maximum of tractive power. In fact this is an unattainable limit, for reasons before stated. The trac-



tive force is given as 63000, which corresponds to an effective cylinder pressure of about 185 lbs., about 84% of the boiler pressure. This tractive force is 28% of the weight on the drivers, a tractive ratio of 1:3.6.

#### RUNNING GEAR.

323. **Types of running gear.** (a) "**American.**" This was once the almost universal type for both passenger and freight service. It is still very commonly used for passenger service, but it is not the best form for heavy freight work.

(b) "**Columbia.**" Four drivers, one pair of pilot-truck wheels and one pair of trailing wheels behind the drivers. The low trailing wheels permit a desirable enlargement of the fire-box. This is a recent type, used exclusively for passenger service.

(c) "**Atlantic.**" Similar to b except that the pilot-truck has four wheels instead of two.

(d) "**Mogul.**" These are used for both passenger and freight service, but are not well adapted for either high speed or great tractive power.

(e) "**Ten-wheel.**" Similar to d except that the pilot-truck has four wheels instead of two. The use is similar to that of d.

(f) "**Consolidation.**" The present standard for freight service. It permits great tractive power without excessive concentrated loads on the track.

(g) **Switching-engines.** These have four or six (and exceptionally even eight or ten) drivers and no truck-wheels. They are only adapted for slow speed when a maximum of tractive power is needed. Sometimes the water-tank and even a small fuel-box is loaded on. Since fuel is always near at hand for a yard-engine, the fuel-box need not be large.

(h) "**Double-enders.**" As explained in § 315, truck-wheels are needed in front of the drivers to guide them around curves. If an ordinary engine is run backward, the flanges of the rear



drivers will become badly worn, and if the speed is high, the danger of derailment is considerable. In suburban service, when the runs are short, it is preferable to run the engines forward and backward, rather than turn them at each end of the route. Therefore a pilot-truck is placed at each end.

(i) "**Miscellaneous types.**" Almost every conceivable combination of drivers and truck-wheels has been used. The "Mastodon" is similar to the "Consolidation" except that the pilot-truck has four wheels instead of two. The "Decapod" has ten driving-wheels. The "Forney" (named after the inventor) has been very extensively used on elevated roads. The weight of the boiler and machinery is carried on four driving-wheels; the engine-frame is extended so as to include a small tank and fuel-box, the weight of which is chiefly supported by a truck of two or four wheels. They run best when running "backward," i.e., tender first.

**324. Equalizing-levers.** The ideal condition of track, from the standpoint of smooth running of the rolling stock, is that the rails should always lie in a plane surface. While this condition is theoretically possible on tangents, it is unobtainable on curves, and especially on the approaches to curves when the outer rail is being raised. Even on tangents it is impossible to *maintain* a perfect surface, no matter how perfectly the track may have been laid. In consequence of this, the points of contact of the wheels of a locomotive, or even of a four-wheeled truck, will not ordinarily lie in one plane. The rougher and more defective the track, the worse the condition in this respect. Since the frame of a locomotive is practically rigid, if the frame rests on the driver-axes through the medium of springs at each axle-bearing, the compression of the springs (and hence the pressure of the drivers on the rail) will be variable if the bearing-points of the drivers are not in one plane. This variable pressure affects the tractive power and severely strains the frame. Applying the principle that a tripod will stand on an uneven surface, a mechanism is employed which *virtually* supports the locomotive on three points, of which one is usually the center-bearing of the forward truck. On each side the pressure is so distributed among the drivers that even if a driver rises or falls with reference to the others, the load carried by each driver is unaltered, and that side of the engine

risers or falls by one  $n$ th of the rise or fall of the single driver, where  $n$  represents the number of wheels. The principle involved is shown in an exaggerated form in Fig. 195. In the diagram,  $MN$  represents the normal position of the frame when the wheels are on line. The frame is supported by the hangers at  $a, c, f$ , and  $h$ .  $ab, de$ , and  $gh$  are horizontal levers vibrating about the points  $H, K$ , and  $L$ , which are supported by the axles. While it is possible with such a system of levers to make  $MN$  assume a position not parallel with its natural position, yet, by an extension of the principle that a beam balance loaded with equal weights will always be horizontal, the effect of raising or lowering a wheel will be to move  $MN$  parallel to itself.

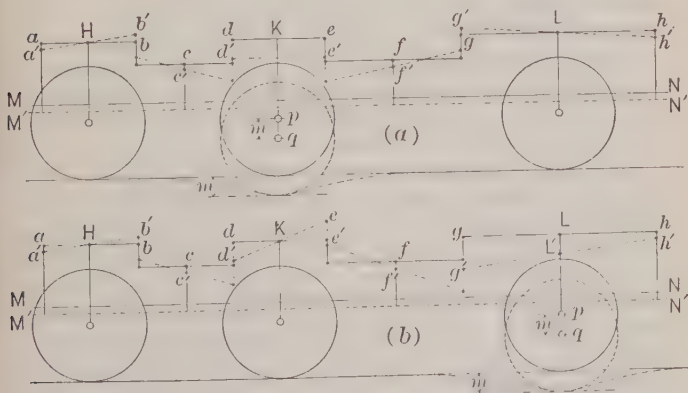


FIG. 195.—ACTION OF EQUALIZING-LEVERS.

It only remains to determine *how much* is the motion of  $MN$  relative to the rise or drop of the wheel.

The dotted lines represent the positions of the wheels and levers when one wheel drops into a depression. The wheel center drops from  $p$  to  $q$ , a distance  $m$ .  $L$  drops to  $L'$ , a distance  $m$  (see Fig. 195, *b*);  $M$  drops to  $M'$ , an unknown distance  $x$ ; therefore  $aa' = x$ ;  $bb' = x$ ;  $cc' = x$ ;  $dd' = 3x = ee'$ ;  $ff' = x$ ;  $\therefore gg' = 5x$ ;  $hh' = x$ ;  $LL' = \frac{1}{2}(gg' + hh') = \frac{1}{2}(6x) = m$ ;  $\therefore x = \frac{1}{6}m$ ; i.e.,  $MN$  drops, parallel to itself,  $1/n$  as much as the wheel drops, where  $n$  is the number of wheels. The resultant effect caused by the simultaneous motion of two wheels with refer-

ence to the third is evidently the algebraic sum of the effects of each wheel taken separately.

The practical benefits of this device are therefore as follows:

(a) When any driver reaches a rough place in the track, a high place or a low place, the stress in all the various hangers and levers is unchanged.

(b) The motion of the frame (represented by the bar *MN* in Fig. 195) is but  $1/n$  of the motion of the wheel, and the jar and vibration caused by a roughness in the track is correspondingly reduced.

The details of applying these principles are varied, but in general it is done as follows;

(a) **American and ten-wheeled types.** Drivers on each side form a system. The center-bearing pilot-truck is the third point of support. The method is illustrated in Fig. 196.

(b) **Mogul and consolidation types.** The front pair of drivers is connected with the two-wheeled pilot-truck (as illustrated in Fig. 197) to form one system. The remaining drivers on each side are each formed into a system.

The device of equalizers is an American invention. Until recently it has not been used on foreign locomotives. The necessity for its use becomes less as the track is maintained with greater perfection and is more free from sharp curves. A locomotive not equipped with this device would deteriorate very rapidly on the comparatively rough tracks which are usually found on light-traffic roads. It is still an open question to what extent the neglect of this device is responsible for the statistical fact that average freight-train loads on foreign trains are less in proportion to the weight on the drivers than is the case with American practice. The recent increasing use of this device on foreign heavy freight locomotives is perhaps an acknowledgment of this principle.

**325. Counterbalancing.** At very high velocities the centrifugal force developed by the weight of the rotating parts becomes a quantity which cannot be safely neglected. These rotating parts include the crank-pin, the crank-pin boss, the side rod, and that part of the weight of the connecting-rod which may be considered as rotating about the center of the crank-driver. As a numerical illustration, a driving-wheel 62" in diameter, running 60 miles per hour, will revolve 325 times per minute. The weights are:

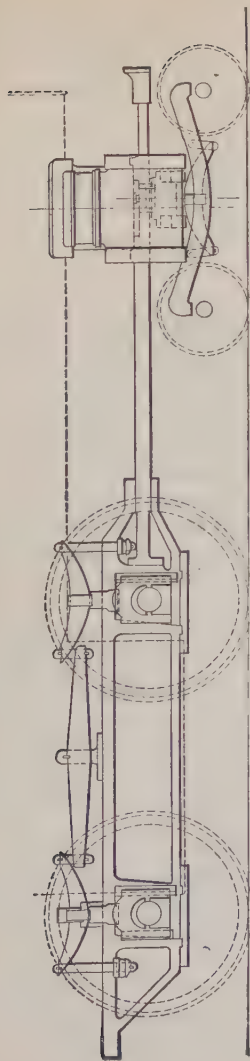


FIG. 196.—EQUALIZING-LEVERS FOR "AMERICAN" TYPE.

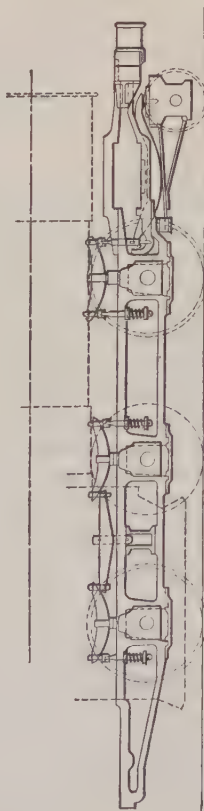


FIG. 197.—EQUALIZING-LEVERS FOR "MOGUL" TYPE.

|                                 |          |
|---------------------------------|----------|
| Crank-pin.....                  | 110 lbs. |
| "    boss.....                  | 150 "    |
| One-half side rod.....          | 240 "    |
| Back end of connecting-rod..... | 190 "    |
| Total.....                      | 690 lbs. |

If the stroke is 24", the radius of rotation is 12", or 1 foot. Then

$$\frac{Gv^2}{gr} = \frac{690 \times 4\pi^2 \times 1^2 \times 325^2}{32.2 \times 1 \times 60^2} = 24821 \text{ lbs.,}$$

which is half as much again as the weight on a driver, 16000 lbs. Therefore if *no* counterbalancing were used, the pressure between the drivers and the rail would always be less (at any velocity) when the crank-pin was at its highest point. At a velocity of about 48 miles per hour the pressure would become zero, and at higher velocities the wheel would actually be thrown from the rail. As an additional objection, when the crank-pin was at the lowest point, the rail pressure would be increased (velocity 60 miles per hour) from 16000 lbs. to nearly 41000 lbs., an objectionably high pressure. These injurious effects are neutralized by "counterbalancing." Since all of the above-mentioned weights can be considered as concentrated at the center of the crank-pin, if a sufficient weight is so placed in the drivers that the center of gravity of the eccentric weight is diametrically opposite to the crank-pin, this centrifugal force can be wholly balanced. This is done by filling up a portion of the space between the spokes. If the center of gravity of the counterbalancing weight is 20" from the center, then, since the crank-pin radius is 12", the required weight would be  $690 \times \frac{12}{20} = 414$  lbs.

In addition to the effect of these revolving parts there is the effect of the sudden acceleration and retardation of the reciprocating parts. In the engine above considered the weights of these reciprocating parts will be:

|                                  |          |
|----------------------------------|----------|
| Front end of connecting-rod..... | 150 lbs. |
| Cross-head.....                  | 174 "    |
| Piston and piston-rod.....       | 300 "    |
| Total.....                       | 624 lbs. |

Assume as before that the reciprocating parts may be considered as concentrated at one point, the point  $P$  of the diagram in Fig. 198. Since the motion of  $P$  is horizontal only,

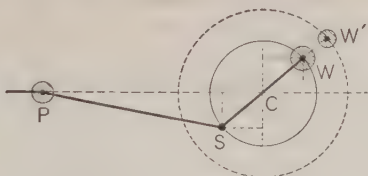


FIG. 198.—ACTION OF COUNTERBALANCE.

the force required to overcome its inertia at any point will exactly equal the *horizontal component* of the force required to overcome the inertia of an *equal* weight at  $S$ , revolving in a circular path. Then evidently the horizontal component of the force required to keep  $W$  in the circular path will exactly balance the force required to overcome the inertia of  $P$ . Of course  $W = P$ . But a smaller weight  $W'$ , whose weight is inversely proportional to its radius of rotation, will evidently accomplish the same result. In the above numerical case, if the center of gravity of the counterweights is 20" from the center, the required weight to completely counterbalance the reciprocating parts would be  $624 \div \frac{1}{20} = 374.4$  lbs. This counterweight need not be all placed on the driver carrying the main crank-pin, but can be (and is) distributed among all the drivers. Suppose it were divided between the two drivers in the above case. At 60 miles per hour such a counterweight would produce an additional pressure of 11211 lbs. when the counterweight was down, or a lifting force of the same amount when the counterweight was up. Although this is not sufficient to lift the driver from the rail, it would produce an objectionably high pressure on the rail (over 27000 lbs.), thus inducing just what it was desired to avoid on account of the eccentric rotating parts. Therefore a compromise must be made. Only a portion (one half to three fourths) of the weight of the reciprocating parts is balanced. Since the effect of the rotating weights is to cause variable pressure on the rail, while the effect of the reciprocating parts is to cause a horizontal wobbling or "nosing" of the locomotive, it is impossible to balance both. Enough counterweight is introduced to partially neutralize the

effect of the reciprocating parts, still leaving some tendency to horizontal wobbling, while the counterweights which were introduced to reduce the wobbling cause some variation of pressure. By using hollow piston-rods of steel, ribbed cross-heads, and connecting- and side-rods with an I section, the weight of the reciprocating parts may be greatly lessened without reducing their strength, and with a decrease in weight the effect of the unbalanced reciprocating parts and of the "excess balance" (that used to balance the reciprocating parts) is largely reduced.

Current practice is somewhat variable on three features:

(a) The proportion of the weight of the connecting-rod which should be considered as revolving weight.

(b) The proportion of the total reciprocating weight that should be balanced.

(c) The distribution among the drivers of the counterweight to balance the reciprocating parts.

An exact theoretical analysis of (a) shows that it is a function of the weights and dimensions of the reciprocating parts. The weight which may be considered as revolving equals \*

$$W_1 \left( \frac{r^2 + k^2 - rd \left( 1 + \frac{r}{l} \right)}{l^2 - r^2} \right) + W_2 \frac{r^2}{l^2 - r^2},$$

in which  $r$ =radius of the crank,  $l$ =length of connecting-rod,  $k$ =distance of center of gyration from wrist-pin,  $d$ =distance of center of gravity from wrist-pin,  $W_1$ =weight of connecting-rod in pounds, and  $W_2$ =weight of piston, piston-rod, and cross-head in pounds; all dimensions in feet. An application of this formula will show that for the dimensions of usual practice, from 51 to 57% of the weight of the connecting-rod should be considered as revolving weight.

The principal rules which have been formulated for counterbalancing may be stated as follows:

1. Each wheel should be balanced correctly for the revolving parts connected with it.

2. *In addition*, introduce counterbalance sufficient for 50% of the weight of the reciprocating parts for ordinary engines, increasing this to 75% when the reciprocating parts are excessively heavy (as in compound locomotives) or when the engine

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\* R. A. Parke, in R. R. Gazette, Feb. 23, 1894.



is light and unable to withstand much lateral strain or when the wheel-base is short.

3. Consider the weight of the connecting-rod as  $\frac{1}{2}$  revolving and  $\frac{1}{2}$  reciprocating when it is over 8 feet long; when shorter than 8 feet, consider  $\frac{6}{10}$  of the weight as revolving and  $\frac{4}{10}$  as reciprocating.

4. The part of the weight of the connecting-rod considered as revolving should be entirely balanced in the crank-driver wheel.

5. The "excess balance" should be divided equally among the drivers.

6. Place the counterbalance as near the rim of the wheel as possible and also as near the outside of the wheel as possible in order that the center of gravity shall be as near as possible opposite the center of gravity of the rods, etc., which are all outside of even the plane of the face of the wheel.

In Fig. 199 is shown a section of a locomotive driver with the cavities in the casting for the accommodation of the lead which is used for the counterbalance weight. Incidentally several other features and dimensions are shown in the illustration.

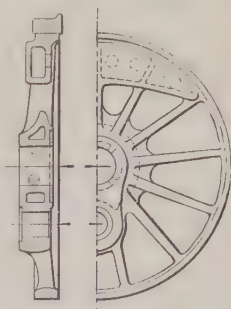


FIG. 199.—SECTION OF LOCOMOTIVE-DRIVER.

326. Mutual relations of the boiler power, tractive power, and cylinder power for various types. The design of a locomotive includes three *distinct* features which are varied in their mutual relations according to the work which the engine is expected to do.

(a) **The boiler power.** This is limited by the rate at which steam may be generated in a boiler of admissible size and weight. Engines which are designed to haul very fast trains which are comparatively light must be equipped with very large grates and heating surfaces so that steam may be developed with great rapidity in order to keep up with the very rapid consumption. Engines for very heavy freight work are run at very much lower velocity and at a lower piston speed in spite of the fact that more strokes are required to cover a given distance and the demand on the boiler for *rapid* steam production is not

as great as with high-speed passenger-engines. The capacity of a boiler to produce steam is therefore limited by the limiting weight of the general type of engine required. Although improvements may be and have been made in the design of fire-boxes so as to increase the steam-producing capacity without adding proportionately to the weight, yet there is a more or less definite limit to the boiler power of an engine of given weight.

(b) **The tractive power.** This is a function of the weight on the drivers. The absolute limit of tractive adhesion between a steel-tired wheel and a steel rail is about one third of the pressure, but not more than one fourth of the weight on the drivers can be depended on for adhesion and wet rails will often reduce this to one fifth and even less. The tractive power is therefore absolutely limited by the practicable weight of the engine. In some designs, when the maximum tractive power is desired, not only is the entire weight of the boiler and running gear thrown on the drivers, but even the tank and fuel-box are loaded on. Such designs are generally employed in switching-engines (or on engines designed for use on abnormally heavy mountain grades) in which the maximum tractive power is required, but in which there is no great tax on the boiler for *rapid* steam production (the speed being always very low), and the boiler and fire-box, which furnish the great bulk of the weight of an engine, are therefore comparatively light, and the requisite weight for traction must, therefore, be obtained by loading the drivers as much as possible. On the other hand, engines of the highest speed cannot possibly produce steam fast enough to maintain the required speed unless the load be cut down to a comparatively small amount. The tractive power required for this comparatively small load will be but a small part of the weight of the engine, and therefore engines of this class have but a small proportion of their weight on the drivers; generally have but two driving-axes and sometimes but one.

(c) **Cylinder power.** The running gear forms a mechanism which is simply a means of transforming the energy of the boiler into tractive force and its power is unlimited, within the practical conditions of the problem. The power of the running gear depends on the steam pressure, on the area of the piston, on the diameter of the drivers, and on the ratio of crank-pin radius to wheel radius, or of stroke to driver diameter. It

is always possible to increase one or more of these elements by a relatively small increase of expenditure until the cylinders are able to make the drivers slip, assuming a sufficiently great resistance. Since the power of the engine is limited by the power of its weakest feature, and since the running gear is the most easily controlled feature, the power of the running gear (or the "cylinder power") is always made somewhat excessive on all well-designed engines. It indicates a badly designed engine if it is stalled and unable to move its drivers, the steam pressure being normal. If it is attempted to use a freight-engine on *fast* passenger service, it will probably fail to attain the desired speed on account of the steam pressure falling. The tractive power and cylinder power are superabundant, but the boiler cannot make steam as fast as it is needed for high speed, especially when the drivers are small. The practical result would be a comparatively low speed kept up with a forced fire. If it is attempted to use a high-speed passenger-engine on heavy freight service, the logical result is a slipping of the drivers until the load is reduced. The boiler power and cylinder power are ample, but the weight on the drivers is so small that the tractive power is only sufficient to draw a comparatively small load.

These relations between boiler, cylinder, and tractive power are illustrated in the following comparative figures referring to a fast passenger-engine, a heavy freight-engine, and a switching-engine. The weights of the passenger- and freight-engines are about the same, but the passenger-engine has only 72% of

| Kind.             | Cylinders. | Total W ght. | Wt. on Driv'rs | Heat- ing Sur- face, sq. ft. | Grate area sq. ft. | Steam Pres- sure in Boiler. | Stroke. Diam. Driver. |
|-------------------|------------|--------------|----------------|------------------------------|--------------------|-----------------------------|-----------------------|
| Fast passenger.   | 19" × 24"  | 126700       | 81500          | 1831.8                       | 26.2               | 180                         | $\frac{24}{78} = .31$ |
| Heavy freight.    | 20" × 24"  | 128700       | 112600         | 1498.3                       | 31.5               | 140                         | $\frac{24}{50} = .48$ |
| Switcher. . . . . | 19" × 24"  | 109000       | 109000         | 1498.0                       | 22.8               | 160                         | $\frac{24}{50} = .48$ |

the tractive power of the freight. But the passenger-engine has 22% more heating-surface and can generate steam much faster; it makes less than two thirds as many strokes in covering a given distance, but it runs at perhaps twice the speed

and probably consumes steam much faster. The switch-engine is lighter in total weight, but the tractive power is nearly as great as the freight and much greater than the passenger-engine. While the heating-surfaces of the freight- and switching-engines are practically identical, the grate area of the switcher is much less; its speed is always low and there is but little necessity for rapid steam development.

While these figures show the general tendency for the relative proportions, and in this respect may be considered as typical, there are large variations. The recent enormous increase in the dead weight of passenger-trains has necessitated greater tractive power. This has been provided sometimes by using "Mogul" and "ten-wheel" engines, which were originally designed for freight work. On the other hand, the demand for fast freight service, and the possibility of safely operating such trains by the use of air-brakes, has required that heavy freight-engines shall be run at comparatively high speeds, and that requires the rapid production of steam, large grate areas, and heating surfaces. But in spite of these variations, the normal standard for passenger service is a four-driver engine carrying about two thirds of the weight of the engine on the drivers, which are very large; the normal standard for freight work is the "consolidation," with perhaps 90% of the weight on the drivers, which are small, but which must have the pony truck for such speed as it uses; and finally the normal standard for switching service has all the weight on the drivers and has comparatively low steam-producing capacity.

**327. Life of locomotives.** The life of locomotives (as a whole) may be taken as about 800000 miles or about 22 to 24 years. While its life should be and is considered as the period between its construction and its final consignment to the scrap pile, parts of the locomotive may have been renewed more than once. The boiler and fire-box are especially subject to renewal. The mileage life is much longer than formerly. This is due partly to better design and partly to the custom of drawing the fires less frequently and thereby avoiding some of the destructive strains caused by extreme alterations of heat and cold. Recent statistics give the average annual mileage on twenty-three leading roads to be 41000 miles.

## CARS.

**328. Capacity and size of cars.** The capacity of freight-cars has been enormously increased of late years. About thirty years ago the usual live-load capacity for a box-car was about 20000 lbs. In 1893 the standard box-car, gondola-cars, etc., of the Pennsylvania Railroad on exhibition at the Chicago Exposition, had a live-load capacity of 60000 lbs. and a dead weight of 30000 to 33000 lbs. With a full load, the weight on each wheel is nearly 12000 lbs., which equals or exceeds the load usually placed on the drivers of ordinary locomotives. But now cars with a live-load capacity of 80000 lbs. are almost standard, 100000-lb. cars are very common, and even larger cars are made for special service. (See Fig. 200.)

The limitation of the carrying capacity for some kinds of freight depends somewhat on the amount of live load that can be carried within given dimensions; for the cross-section of a car is limited to the extreme dimensions which may be safely run through the tunnels and through bridges as at present constructed, and the length is somewhat limited by the difficulty of properly supporting an excessively heavy load, distributed over an unusually long span, by a structure which is subjected to excessive jar, concussion, compression, and tension. The cross-sectional limit seems to have been scarcely reached yet, except, perhaps, in the case of furniture and carriage-cars, whose load per cubic foot is not great. The usual width of freight-cars is about 8 to 9 feet, while parlor-cars and sleepers are generally 10 feet wide and sometimes 11 feet. The highest point of a train is usually the smoke-stack of the locomotive which is generally 14 feet above the rails and occasionally over 15 feet. A sleeping-car usually has the highest point of the car about 14 feet above the rails. Box-cars are usually about 8 feet high (above the sills), with a total height of about 11' 3". Refrigerator-cars are usually about 9' high and furniture-cars about 10' above the sills, the truck adding about 3' 3". The usual length is 34 feet, but 35 to 40 feet is not uncommon. Passenger-cars (day coaches) are usually 50 feet long, exclusive of the end platforms and weigh 45000 to 50000 lbs. Sixty passengers at 150 pounds apiece (a high average) will only add 9000 lbs. to the weight. A parlor-car or sleeper is generally about 65 feet long exclusive of the platforms, which add about 6' 6". The weight is anywhere from 60000 to 80000 lbs.



The weight of the 25 or 30 passengers it *may* carry is hardly worth considering in comparison.

329. **Stresses to which car-frames are subjected.** A car is structurally a truss, supported at points at some distance from the ends and subjected to transverse stress. There is therefore, a change of flexure at two points between the truss. Besides this stress the floor is subjected to compression when the cars are suddenly stopped and to tension when in ordinary motion, the tension being greater as the train resistance is greater and as the car is nearer the engine. The tension is sometimes relieved by means of continuous drawbars (see § 331), but this affords no relief against impact during compression, which is really more destructive. The shocks, jolts and sudden strains to which the car-frames are subjected are very much harder on them than the mere static strains due to their maximum loads if the loads were quiescent. Consequently any calculations based on the static loads are practically valueless, except as a very rough guide, and previous experience must be relied on in designing car bodies. As evidence of the increasing demand for strength in car-frames, it has been recently observed that freight-cars, built some years ago and built almost entirely of wood, are requiring repairs of wooden parts which have been *crushed* in service, the wood being perfectly sound as regards decay.

330. **The use of metal.** The use of metal in car construction is very rapidly increasing. The demand for greater strength in car-frames has grown until the wooden framing has become so heavy that it is found possible to make steel frames and trucks at a small additional cost, the steel frames being twice as strong and yet reducing the dead weight of the car about 5000 lbs., a consideration of no small value, especially on roads having heavy grades. Another reason for the increasing use of metal is the great reduction in the price of rolled or pressed steel, while the cost of wood is possibly higher than before. The advocates of the use of steel advise steel floors, sides, etc. For box-cars a wooden floor has advantages. For ore and coal-cars an all-metal construction has advantages. (Fig. 201.) In Germany, where steel frames have been almost exclusively in use for many years, they have not yet been able to determine the normal age limit of such frames; none have yet *worn out*. The life is estimated at 50 to 80 years.



FIG. 200.—100,000-LB. BOX CAR.



FIG. 201.—STEEL COAL CAR.



FIG. 202.—WOODEN BOX CAR; STEEL FRAME.

(To face page 366.)





**Brake-beams** are also best made of metal rather than wood, as was formerly done. Metal brake-beams are generally used on cars having air-brakes, as a wooden beam must be excessively large and heavy in order to have sufficient rigidity.

**Truck-frames** (see Fig. 203), which were formerly made principally of wood, are now largely made of pressed steel. It makes a reduction in weight of about 3000 lbs. per car. The increased durability is still an uncertain quantity.

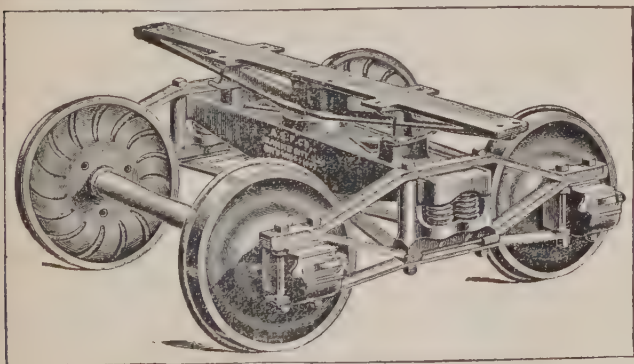


FIG. 203.

**331. Draft-gear.** These are of necessity made with springs for all passenger- and freight-cars. Coal-jimmies are often fastened together by links dropped over hooks, but the larger coal-cars require springs to absorb the shocks. There is a considerable theoretical advantage in "continuous draft-gear," i.e. having a rod (or pair of rods) running continuously from end to end of the car so that there shall be no tensile stress on the car-body itself. But there are several objections in practice. (a) The draft-rod, if there is but one, should be in the center line of the car, i.e. pass through the two truck-centers and the king-pins, which is impracticable. This difficulty is sometimes obviated in an objectionable way by running the draw-bar *above* the truck-center. A better method is to use a pair of rods. (b) The rod is of no value during *compression*, and it is the compression a car receives by minor collisions during switching which produces maximum injury to the car-body and the draft-gear. (c) The rod is much more liable to injury and requires much more expensive repairs when injured.

The older method is to bolt the beams holding the draft-gear to the under side of the car-body. This form is objectionable owing to the fact that the push and pull, being transmitted through the car-body, act eccentrically, tend to loosen the draft-beams from the car-body, and in case of a violent collision have been known to actually buckle the car-body upward (the cars being "flats"). The fastening of the draft-gear to the car-body has been made more secure by using cast-iron keys, then still more so by running the beams back to the "transoms" (the heavy cross-beams which support the car and transfer its weight to the trucks), then by making a *double center sill* extending through the length of the car. Another device is to run the draft-gear *through* the end sill and then the line of push and pull running through the car-frame instead of under it, the car-frame can furnish its maximum resistance.

**332. Gauge of wheels and form of wheel-tread.** In Fig. 204 is shown the standard adopted by the Master Car Builders' Association at their twentieth annual convention. Note the normal position of the gauge-line on the wheel-tread. In Fig. 114, p. 238, the relation of rail to wheel-tread is shown on a smaller scale. It should be noted that there is no definite position where the wheel-flange is absolutely "chock-a-block" against the rail. As the pressure increases the wheel mounts a little higher on the rail until a point is soon reached when the resistance is too great for it to mount still higher. By this means is avoided the shock of unyielding impact when the car sways from side to side. When the gauge between the inner faces of the wheels is greater or less than the limits given in the figure, the interchange rules of the Master Car Builders' Association authorize a road to refuse to accept a car from another road for transportation. At junction points of railroads inspectors are detailed to see that this rule (as well as many others) is complied with in respect to all cars offered for transfer.

#### TRAIN-BRAKES.

**333. Introduction.** Owing to the very general misapprehension that exists regarding the nature and intensity of the action of brakes, a complete analysis of the problem is considered justifiable. This misapprehension is illustrated by the common notion (and even practice) that the effectiveness of



braking a car is proportional to the brake pressure, and therefore a brakeman is frequently seen using a bar to obtain a greater leverage on the brake-wheel and using his utmost strength to obtain the maximum pull on the brake-chain while the car is skidding along with locked wheels.

When a vehicle is moving on a track with a considerable velocity, the mass of the vehicle possesses kinetic energy of translation and the wheels possess kinetic energy of rotation. To stop the vehicle, this energy must be destroyed. The rotary kinetic energy will vary from about 4 to 8% of the kinetic energy of translation, according to the car loading (see § 347). On steam railroads brake action is obtained by pressing brake-shoes against car-wheel treads. As the brake-shoe pressure increases, the brake-shoes retard with increasing force the rotary action of the wheels. As long as the wheels do not slip or "skid" on the rails, the adhesion of the rails forces them to rotate with a circumferential velocity equal to the train velocity. The retarding action of the brake-shoe checks first the rotative kinetic energy (which is small), and the remainder develops a *tendency* for the wheel to slip on the rail. Since the rotative kinetic energy is such a small percentage of the total, it will hereafter be ignored, except as specifically stated, and it will be assumed for simplicity that the only work of the brakes is to overcome the kinetic energy of translation. The possible effect of grade in assisting or preventing retardation, and the effect of all other track resistances, is also ignored. The amount of the developed force which retards the train movement is limited to the possible adhesion or *static* friction between the wheel and the rail. When the friction between the brake-shoe and the wheel exceeds the adhesion between the wheel and the rail, the wheel skids, and then the friction between the wheel and the rail at once drops to a much less quantity. It must therefore be remembered at the outset that the retarding action of brake-shoes on wheels as a means of stopping a train is absolutely limited by the possible static friction between the braked wheels and the rails.

**334. Laws of friction as applied to this problem.** Much of the misapprehension regarding this problem arises from a very common and widespread misstatement of the general laws of friction. It is frequently stated that friction is independent

of the velocity and of the unit of pressure. The first of these so-called laws is not even approximately true. A very exhaustive series of tests were made by Capt. Douglas Galton on the Brighton Railway in England in 1878 and 1879, and by M. George Marié on the Paris and Lyons Railway in 1879, with trains which were specially fitted with train-brakes and with dynagraphs of various kinds to measure the action of the brakes. Experience proved that variations in the condition of the rails (wet or dry), and numerous irregularities incident to measuring the forces acting on a heavy body moving with a high velocity, were such as to give somewhat discordant results, even when the conditions were made as nearly identical as possible. But the tests were carried so far and so persistently that the general laws stated below were demonstrated beyond question, and even the numerical constants were determined as closely as they may be practically utilized. These laws may be briefly stated as follows:

(a) The coefficient of friction between cast-iron brake-blocks and steel tires is about .3 when the wheels are "just moving"; it drops to about .16 when the velocity is about 30 miles per hour, and is less than .10 when the velocity is 60 miles per hour. These figures fluctuate considerably with the condition of the rails, wet or dry.

(b) The coefficient of friction is greatest when the brakes are first applied; it then reduces very rapidly, decreasing nearly one third after the brakes have been applied 10 seconds, and dropping to nearly one half in the course of 20 seconds. Although the general truth of this law was established beyond question, the tests to demonstrate the law of the variation of friction with time of application were too few to determine accurately the numerical constants.

(c) The friction of skidded wheels on rails is always very much less than the adhesion when the wheel is rolling on the rail—sometimes less than one third as much.

(d) An analysis of the tests all pointed to a law that the friction developed does *not* increase as rapidly as the *intensity* of pressure increases, but this may hardly be considered as an established law.

(e) The adhesion between the wheel and the rail appears to be independent of velocity. The adhesion here means the force that must be developed before the wheel will slip on the rail.

The practical effect of these laws is shown by the following observed phenomena:

(a) When the brakes are first applied (the velocity being very high), a brake pressure far in excess of the weight on the wheel (even three or four times as much) may be applied without skidding the wheel. This is partly due to the fact that the wheel has a very high rotative kinetic energy (which varies as the square of the velocity, and which must be overcome first), but it is chiefly due to the fact that the coefficient of friction at the higher velocity is very small (at 60 miles per hour it is about .07), while the adhesion between the wheel and the rail is independent of the velocity.

(b) As the velocity decreases the brake pressure must be decreased or the wheels will skid. Although the friction decreases with the time required to stop and increases with the reduction of speed, and these two effects tend to neutralize each other, yet unless the stop is very slow, the increase in friction due to reduction of speed is much greater than the decrease due to time, and therefore the brake pressure must not be greater than the weight on the wheel, unless momentarily while the speed is still very high.

(c) The adhesion between wheels and rails varies from .20 to .25 and over when the rail is dry. When wet and slippery it may fall to .18 or even .15. The use of sand will always raise it above .20, and on a dry rail, when the sand is not blown away by wind, it may raise it to .35 or even .40.

(d) Experiments were made with an automatic valve by which the brake-shoe pressure against the wheel should be reduced as the friction increased, but since (1) the essential requirement is that the friction produced by the brake-shoes shall not exceed the adhesion between rail and wheel, and since (2) the rail-wheel adhesion is a very variable quantity, depending on whether the rail is wet or dry, it has been found impracticable to use such a valve, and that the best plan is to leave it to the engineer to vary the pressure, if necessary, by the use of the brake-valve.

#### MECHANISM OF BRAKES.

**335. Hand-brakes.** The old style of brakes consists of brake-shoes of some type which are pressed against the wheel-treads



by means of a brake-beam, which is operated by means of a hand-windlass and chain operating a set of levers. It is desirable that brakes shall not be set so tightly that the wheels shall be locked, and then slide over the track, producing flat places on them, which are very destructive to the rolling-stock and track afterward, on account of the impact occasioned at each revolution. With air-brakes the maximum pressure of the brake-shoes can be quite carefully regulated, and they are so designed that the maximum pressure exerted by any pair of brake-shoes on the wheels of any axle shall not exceed a certain per cent. of the weight carried by that axle when the car is *empty*, 90% being the figure usually adopted for passenger-cars and 70% for freight-cars. Consider the case of a freight-car of 100000 lbs. capacity, weighing 33100 lbs., or 8275 lbs. on an axle, and equipped with a hand-brake which operates the levers and brake-beams, which are sketched in Fig. 205. The dead weight on an axle is 8275 lbs.; 70% of

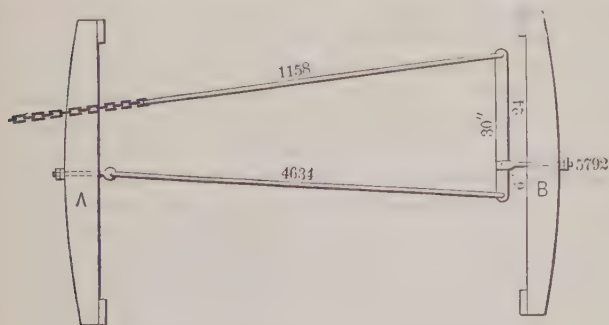


FIG. 205.—SKETCH OF MECHANISM OF HAND-BRAKE.

this is 5792 lbs., which is the maximum allowable pressure per brake-beam, or 2896 lbs. per brake-shoe. With the dimensions shown, such a pressure will be produced by a pull of about 1158 lbs. on the brake-chain. The power gained by the brake-wheel is not equal to the ratio of the brake-wheel diameter to the diameter of the shaft, about which the brake-chain winds, which is about 16 to 1½. The ratio of the circumference of the brake-wheel to the length of chain wound up by one complete turn would be a closer figure. The loss of effi-

iciency in such a clumsy mechanism also reduces the effective ratio. Assuming the *effective* ratio as 6:1 it would require a pull of 193 lbs. at the circumference of the brake-wheel to exert 1158 lbs. pull on the brake-chain, or 5792 lbs. pressure on the wheels at *B*, and even this will not lock the wheels when the car is empty, much less when it is loaded. Note that the pressures at *A* and *B* are unequal. This is somewhat objectionable, but it is unavoidable with this simple form of brake-beam. More complicated forms to avoid this are sometimes used. Hand-brakes are, of course, cheapest in first cost, and even with the best of automatic brakes, additional mechanism to operate the brakes by hand in an emergency is always provided, but their slow operation when a quick stop is desired makes it exceedingly dangerous to attempt to run a train at high speed unless some automatic brake directly under the control of the engineer is at hand. The great increase in the average velocity of trains during recent years has only been rendered possible by the invention of automatic brakes.

**336. "Straight" air-brakes.** The essential constructive features of this form of brake are (1) an air-pump on the engine, operated by steam, which compresses air into a reservoir on the engine; (2) a "brake-pipe" running from the reservoir to the rear of the engine and pipes running under each car, the pipes having flexible connections at the ends of the cars and engine; (3) a cylinder and piston under each car which operates the brakes by a system of levers, the cylinder being connected to the brake-pipe. The reservoir on the engine holds compressed air at about 45 lbs. pressure. To operate the brakes, a valve on the engine is opened which allows the compressed air to flow from the reservoir through the brake-pipe to each cylinder, moving the piston, which thereby moves the levers and applies the brakes. The *defects* of this system are many: (1) With a long train, considerable time is required for the air to flow from the reservoir on the engine to the rear cars, and for an emergency-stop even this delay would often be fatal; (2) if the train breaks in two, the rear portion is not provided with power for operating the brakes, and a dangerous collision would often be the result; (3) if an air-pipe coupling bursts under any car, the whole system becomes absolutely helpless, and as such a thing might happen during some emergency, the accident would then be especially fatal.

This form of brake has almost, if not entirely, passed out of use. It is here briefly described in order to show the logical development of the form which is now in almost universal use, the automatic.

**337. Automatic air-brakes.** The above defects have been overcome by a method which may be briefly stated as follows: A reservoir for compressed air is placed under each car and the tender; whenever the pressure in these reservoirs is reduced for any reason, it is automatically replenished from the main reservoir on the engine; whenever the pressure in the brake-pipe is reduced for **any** cause (opening a valve at any point of its length, parting of the train, or bursting of a pipe or coupler), valves are automatically moved under each car to operate the piston and put on the brakes. *All* the brakes on the train are thus applied almost simultaneously. If the train breaks in two, *both* sections will at once have *all* the brakes applied automatically; if a coupling or pipe bursts, the brakes are at once applied and attention is thereby attracted to the defect; if an emergency should arise, such that the conductor desires to stop the train instantly without even taking time to signal to the engineer, he can do so by opening a valve placed on each car, which admits air to the train-pipe, which will set the brakes on the whole train, and the engineer, being able to discover instantly what had occurred, would shut off steam and do whatever else was necessary to stop the train as quickly as possible. The most important and essential detail of this system is the "automatic triple valve" placed under each car. Quoting from the Westinghouse Air-brake Company's Instruction Book, "A moderate reduction of air pressure in the train-pipe causes the greater pressure remaining stored in the auxiliary reservoir to force the piston of the triple valve and its slide-valve to a position which will allow the air in the auxiliary reservoir to pass directly into the brake-cylinder and apply the brake. A sudden or violent reduction of the air in the train-pipe produces the same effect, and in addition causes supplemental valves in the triple valve to be opened, permitting the pressure from the train-pipe to also enter the brake-cylinder, augmenting the pressure derived from the auxiliary reservoir about 20%, producing practically instantaneous action of the brakes to their highest efficiency throughout the entire train. When the pressure in the brake-pipe is again restored to an

amount in excess of that remaining in the auxiliary reservoir, the piston- and slide-valves are forced in the opposite direction to their normal position, opening communication from the train-pipe to the auxiliary reservoir, and permitting the air in the brake-cylinder to escape to the atmosphere, thus releasing the brakes. If the engineer wishes to apply the brake, he moves the handle of the engineer's brake-valve to the right, which first closes a port, retaining the pressure in the main reservoir, and then permits a portion of the air in the train-pipe to escape. To release the brakes, he moves the handle to the extreme left, which allows the air in the main reservoir to flow freely into the brake-pipe, restoring the pressure therein."

**338. Tests to measure the efficiency of brakes.** Let  $v$  represent the velocity of a train in feet per second;  $W$ , its weight;  $F$ , the retarding force due to the brakes;  $d$ , the distance in feet required to make a stop; and  $g$ , the acceleration of gravity (32.16 feet per square second); then the kinetic energy possessed by the train (disregarding for the present the rotative kinetic energy of the wheels) =  $\frac{Wv^2}{2g}$ . The work done in stopping the train =  $Fd$ .  $\therefore Fd = \frac{Wv^2}{2g}$ . The ratio of the retarding force to the weight,

$$\frac{F}{W} = \frac{v^2}{2gd} = .0155 \frac{v^2}{d}.$$

In order to compare tests made under varying conditions, the ratio  $F \div W$  should be corrected for the effect of grade (+ or -), if any, and also for the proportion of the weight of the train which is on *braked* wheels. For example, a train weighed 146076 lbs., the proportion on braked wheels was 67%, speed 60 feet per second, length of stop 450 feet, track level. Substituting these values in the above formula, we find  $(F \div W) = .124$ . This value is really unduly favorable, since the ordinary track resistance helps to stop the train. This has a value of from 6 to 20 lbs. per ton, averaging say 10 lbs. per ton during the stop, or .005 of the weight. Since the effect of this is small and is nearly constant for all trains, it may be ignored in comparative tests. The grade in this case was level, and therefore grade had no effect. But since only 67% of the weight was on braked wheels, the ratio, on the basis of *all* the

wheels braked, or of the weight reduced to that actually on the braked wheels, is  $0.124 \div 67 = 0.185$ . This was called a "good" stop, although as high a ratio as 0.200 has been obtained.

**339. Brake-shoes.** Brake-shoes were formerly made of wrought iron, but when it was discovered that cast-iron shoes would answer the purpose, the use of wrought-iron shoes was abandoned, since the cast-iron shoes are so much cheaper. A cheap practice is to form the brake-shoe and its head in one piece, which is cheaper in first cost, but when the wearing-surface is too far gone for further use, the whole casting must be renewed. The "Christie" shoe, adopted by the Master Car Builders' Association as standard, has a separate shoe which is fastened to the head by means of a wrought-iron key. The shoe is beveled  $\frac{1}{4}$ " in a width of  $3\frac{3}{8}$ " to fit the coned wheel. This is a greater bevel than the standard coning of a car-wheel. It is perhaps done to allow for some bending of the brake-beam and also so that the maximum pressure (and wear) should come on the outside of the tread, rather than next to the flange, where it might tend to produce sharp flanges. By concentrating the brake-shoe wear on the outer side of the tread, the wear on the tread is more nearly equalized, since the rail wears the wheel-tread chiefly near the flange. This same idea is developed still further in the "flange-shoes," which have a curved form to fit the wheel-flange and which bear on the wheel on the flange and on the outside of the tread. It is claimed that by this means the standard form of the tread is better preserved than when the wear is entirely on the tread. The Congdon brake-shoe is one of a type in which wrought-iron pieces are inserted in the face of a cast-iron shoe. It is claimed that these increase the life of the shoe.

## CHAPTER XVI.

### TRAIN RESISTANCE.

**340. Classification of the various forms.** The various resistances which must be overcome by the power of the locomotive may be classified as follows:

(a) *Resistances internal to the locomotive*, which include friction of the valve-gear, piston- and connecting-rods, journal friction of the drivers; also all the loss due to radiation, condensation, friction of the steam in the passages, etc. In short, these resistances are the sum-total of the losses by which the power at the circumference of the drivers is less than the power developed by the boiler.

(b) *Velocity resistances*, which include the atmospheric resistances on the ends and sides; oscillation and concussion resistances, due to uneven track, etc.

(c) *Wheel resistances*, which include the rolling friction between the wheels and the rails of *all* the wheels (including the drivers); also the journal friction of all the axles, except those of the drivers.

(d) *Grade and curve resistances*, which include those resistances which are due to grade and to curves, and which are not found on a straight and level track.

(e) *Brake resistances*. As shown later, brakes consume power and to the extent of their use increase the energy to be developed by the locomotive.

(f) *Inertia resistances*. The resistance due to inertia is not generally considered as a train resistance because the energy which is stored up in the train as kinetic energy may be utilized in overcoming future resistances. But in a discussion of the demands on the tractive power of the engine, one of the chief items is the energy required to *rapidly* give to a starting train its normal velocity. This is especially true of suburban trains, which must acquire speed very quickly in order that



their general average speed between termini may be even reasonably fast.

**341. Resistances internal to the locomotive.** These are resistances which do not tax the adhesion of the drivers to the rails, and hence are frequently considered as not being a part of the train resistance properly so called. If the engine were considered as lifted from the rails and made to drive a belt placed around the drivers, then all the power that reached the belt would be the power that is ordinarily available for adhesion, while the remainder would be that consumed internally by the engine. The power developed by an engine may be obtained by taking indicator diagrams which show the actual steam pressure in a cylinder at any part of a stroke. From such a diagram the average steam pressure is easily obtained, and this average pressure, multiplied by the length of the stroke and by the net area of the piston, gives the energy developed by one half-stroke of one piston. Four times this product divided by 550 times the time in seconds required for one stroke gives the "indicated horse-power." Even this calculation gives merely the power behind the piston, which is several per cent. greater than the power which reaches the circumference of the drivers, owing to the friction of the piston, piston-rod, cross-head, connecting-rod bearings, and driving-wheel journals. (See § 322, Chapter XV.) By measuring the amount of water used and turned into steam, and by noting the boiler pressure, the energy possessed by the steam used is readily computed. The indicator diagrams will show the amount of steam that has been effective in producing power at the cylinders. The steam accounted for by the diagrams will ordinarily amount to 80 or 85% of the steam developed by the boiler, and the other 15 or 20% represents the loss of energy due to radiation, condensation, etc. From actual tests it has been found that the power consumed by an engine running light is about 11% of that required by the engine when working hard in express freight service. But since the engine resistances (friction, etc.) are increased when it is pulling a load, it was estimated, after allowing for this fact, that about 15 or 16% of the power developed by the pistons was consumed by the engine, leaving about 84 to 85% for the train.

**342. Velocity resistances.** (a) *Atmospheric.* This consists of the head and tail resistances and the side resistance. The head



and tail resistances are nearly constant for all trains of given velocity, varying but slightly with the varying cross-sections of engines and cars. The side resistance varies with the length of the train and the character of the cars, box-cars or flats, etc. Vestibuling cars has a considerable effect in reducing this side resistance by preventing much of the eddying of air-currents between the cars, although this is one of the least of the advantages of vestibuling. Atmospheric resistance is generally assumed to vary as the square of the velocity, and although this may be nearly true, it has been experimentally demonstrated to be at least inaccurate. The head resistance is generally assumed to vary as the area of the cross-section, but this has been definitely demonstrated to be very far from true. A freight-train composed partly of flat-cars and partly of box-cars will encounter considerably more atmospheric resistance than one made exclusively of either kind, other things being equal. The definite information on this subject is very unsatisfactory, but this is possibly due to the fact that it is of little practical importance to know just how much such resistance amounts to.

(b) *Oscillatory and concussive.* These resistances are considered to vary as the square of the velocity. Probably this is nearly, if not quite, correct on the general principle that such resistances are a succession of impacts and the force of impacts varies as the square of the velocity. These impacts are due to the defects of the track, and even though it were possible to make a precise determination of the amount of this resistance in any particular case, the value obtained would only be true for that particular piece of track and for the particular degree of excellence or defect which the track *then* possessed. The general improvement of track maintenance during late years has had a large influence in increasing the possible train-load by decreasing the train resistance. The expenditure of money to improve track will give a road a large advantage over a competing road with a poorer track, by reducing train resistance, and thus reducing the cost of handling traffic.

**343. Wheel resistances.** (a) *Rolling friction of the wheels.* To determine experimentally the rolling friction of wheels, apart from all journal friction, is a very difficult matter and has never been satisfactorily accomplished. Theory as well as practice shows that the higher and the more perfect the

elasticity of the wheel and the surface, the less will be the rolling friction. But the determination, if made, would be of theoretical interest only.

The combined effect of rolling friction and journal friction is determinable with comparative ease. From the nature of the case no great reduction of the rolling friction by any device is possible. It is only a very insignificant part of the total train resistance.

(b) *Journal friction of the axles.* This form of resistance has been studied quite extensively by means of the measurement of the force required to turn an axle in its bearings under various conditions of pressure, speed, extent of lubrication, and temperature. The following laws have been fairly well established: (1) The coefficient of friction increases as the pressure diminishes; (2) it is higher at very slow speeds, gradually diminishing to a minimum at a speed corresponding to a train velocity of about 10 miles per hour, then slowly increasing with the speed; it is very dependent on the perfection of the lubrication, it being reduced to one sixth or one tenth, when the axle is lubricated by a bath of oil rather than by a mere pad or wad of waste on one side of the journal; (3) it is much lower at higher temperature, and *vice versa*. The practical effect of these laws is shown by the observed facts that (1) loaded cars have a less resistance per ton than unloaded cars, the figures being (for speeds of about 10 to 20 miles per hour):

|                                           |                |
|-------------------------------------------|----------------|
| For passenger- and loaded freight-cars... | 4 lbs. per ton |
| “ empty freight-cars. ....                | 6 “ “ “        |
| “ street-cars. ....                       | 10 “ “ “       |
| “ freight-trucks without load. ....       | 14 “ “ “       |

(2) When starting a train, the resistances are about 20 lbs. per ton, notwithstanding the fact that the velocity resistances are practically zero; at about 2 miles per hour it will drop to 10 lbs. per ton and above 10 miles per hour it may drop to 4 lbs. per ton if the cars are in good condition. (3) The resistance could probably be materially lowered if some practicable form of journal-box could be devised which would give a more perfect lubrication. (4) It is observed that freight-train loads must be cut down in winter by about 10 or 15% of the loads that the same engine can haul over the same track in summer. This is due partly to the extra roughness and inelasticity of the

track in winter, and partly to increased radiation from the engine wasting some energy, but this will not account for all of the loss, and the effect, which is probably due largely to the lower temperature of the journal-boxes, is very marked and costly. It has been suggested that a jacketing of the journal-boxes, which would prevent rapid radiation of heat and enable them to retain some of the heat developed by friction, would result in a saving amply repaying the cost of the device.

Roller journals for cars have been frequently suggested, and experiments have been made with them. It is found that they are very effective at low velocities, greatly reducing the starting resistance, which is very high with the ordinary forms of journals. But the advantages disappear as the velocity increases. The advantages also decrease as the load is increased, so that with heavily loaded cars the gain is small. The excess of cost for construction and maintenance has been found to be more than the gain from power saved.

**344. Grade resistance.** The amount of this may be computed with mathematical exactness. Assume that the ball or cylinder (see Fig. 206) is being drawn up the plane. If  $W$

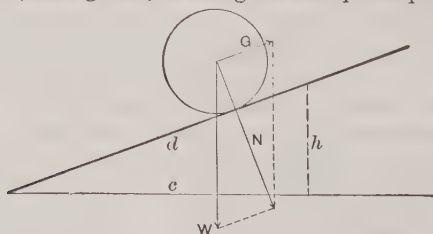


FIG. 206.

is the weight,  $N$  the normal pressure against the rail, and  $G$  the force required to hold it or to draw it up the plane with uniform velocity, the rolling resistances being considered zero or considered as provided for by other forces, then

$$G:W:h:d, \text{ or } G = \frac{Wh}{d};$$

but for all ordinary railroad grades,  $d=c$  to within a tenth of 1%, i.e.,  $G = \frac{Wh}{c} = W \times \text{rate of grade}$ . In order that the student may appreciate the exact amount of this approximation the percentage of slope distance to its horizontal projection is given in the following tabular form;

| Grade in per cent.                                                        | 1       | 2       | 3       | 4       | 5       |
|---------------------------------------------------------------------------|---------|---------|---------|---------|---------|
| $\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100 \dots\dots\dots$ | 100.005 | 100.020 | 100.045 | 100.080 | 100.125 |

| Grade in per cent.                                                        | 6       | 7       | 8       | 9       | 10      |
|---------------------------------------------------------------------------|---------|---------|---------|---------|---------|
| $\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100 \dots\dots\dots$ | 100.180 | 100.245 | 100.319 | 100.404 | 100.499 |

This shows also the error on various grades of measuring with the tape on the ground rather than held horizontally. Since almost all railroad grades are less than 2% (where the error is but .02 of 1%), and anything in excess of 4% is unheard of for normal construction, the error in the approximation is generally too small for practical consideration.

If the rate of grade is 1 : 100,  $G = W \times \frac{1}{100}$ , i.e.,  $G = 20$  lbs. per ton;  $\therefore$  for *any* per cent. of grade,  $G = (20 \times \text{per cent. of grade})$  pounds per ton. When moving up a grade this force  $G$  is to be overcome in addition to all the other resistances. When moving down a grade, the force  $G$  assists the motion and may be more than sufficient to move the train at its highest allowable velocity. The force required to move a train on a level track at ordinary freight-train speeds (say 20 miles per hour) is about 7 lbs. per ton. A down grade of  $\frac{7}{20}$  of 1% will furnish the same power; therefore on a down grade of 0.35%, a freight-train would move indefinitely at about 20 miles per hour. If the grade were higher and the train were allowed to gain speed freely, the speed would increase until the resistance at that speed would equal  $W$  times the rate of grade, when the velocity would become uniform and remain so as long as the conditions were constant. If this speed was higher than a safe permissible speed, brakes must be applied and power wasted. The fact that one terminal of a road is considerably higher than the other does not necessarily imply that the extra power needed to overcome the difference of elevation is a total waste of energy, especially if the maximum grades are so low that brakes will never need to be applied to reduce a dangerously high velocity, for although more power must be

used in ascending the grades, there is a considerable saving of power in descending the grades. The amount of this saving will be discussed more fully in Chapter XXIII.

**345. Curve resistance.** Some of the principal laws will be here given without elaboration. A more detailed discussion will be given in Chapter XXII.

(a) While the total curve resistance increases as the degree of curve increases, the resistance *per degree of curve* is much greater for easy curves than for sharp curves; *e.g.*, the resistance on the excessively sharp curves (radius 90 feet) of the elevated roads of New York City is very much less *per degree of curve* than that on curves of  $1^{\circ}$  to  $5^{\circ}$ . (b) Curve resistance increases with the velocity. (c) The total resistance on a curve depends on the central angle rather than on the radius; *i.e.*, two curves of the same central angle but of different radius would cause about the same total curve resistance. This is partly explained by the fact that the longitudinal slipping will be the same in each case. (See § 311, Chapter XV.) In each case also the trucks must be twisted around and the wheels slipped laterally on the rails by the same amount  $\frac{1}{2}^{\circ}$ . (See § 312, Chapter XV.)

**346. Brake resistances.** If a down grade is excessively steep so that brakes must be applied to prevent the train acquiring a dangerous velocity, the energy consumed is hopelessly lost without any compensation. When trains are required to make frequent stops and yet maintain a high average speed, considerable power is consumed by the application of brakes in stopping. All the energy which is thus turned into heat is hopelessly lost, and in addition a very considerable amount of steam is drawn from the boiler to operate the air-brakes, which consume the power already developed. It can be easily demonstrated that engines drawing trains in suburban service, making frequent stops, and yet developing high speed between stops, will consume a very large proportion of the total power developed by the use of brakes. Note the double loss. The brakes consume power already developed and stored in the train as kinetic or potential energy, while the operation of the brakes requires additional steam power from the engine.

**347. Inertia resistance.** The two forms of train resistance which under some circumstances are the greatest resistances to be overcome by the engine are the grade and inertia resist-

ances, and fortunately both of these resistances may be computed with mathematical precision. The problem may be stated as follows: What constant force  $P$  (in addition to the forces required to overcome the various frictional resistances, etc.) will be required to impart to a body a velocity of  $v$  feet per second in a distance of  $s$  feet? The required number of foot-pounds of energy is evidently  $Ps$ . But this work imparts a kinetic energy which may be expressed by  $\frac{Wv^2}{2g}$ . Equating

these values, we have  $Ps = \frac{Wv^2}{2g}$ , or

$$P = \frac{Wv^2}{2gs} \quad \dots \quad (138)$$

The force required to increase the velocity from  $v_1$  to  $v_2$  may likewise be stated as  $P = \frac{W}{2gs}(v_2^2 - v_1^2)$ . Substituting in the formula the values  $W = 2000$  lbs. (one ton),  $g = 32.16$ , and  $s = 5280$  feet (one mile), we have

$$P = .00588(v_2^2 - v_1^2).$$

Multiplying by  $(5280 \div 3600)^2$  to change the unit of velocity to miles per hour, we have

$$P = .01267(V_2^2 - V_1^2).$$

But this formula must be modified on account of the rotative kinetic energy which must be imparted to the wheels of the cars. The precise additional percentage depends on the particular design of the cars and their loading and also on the design of the locomotive. Consider as an example a box-car, 60000 lbs. capacity, weighing 33000 lbs. The wheels have a diameter of 36" and their radius of gyration is about 13". Each wheel weighs 700 lbs. The rotative kinetic energy of each wheel is 4877 ft.-lbs. when the velocity is 20 miles per hour, and for the eight wheels it is 39016 ft.-lbs. For greater precision (really needless) we may add 192 ft.-lbs. as the rotative kinetic energy of the axles. When the car is fully loaded (weight 93000 lbs.) the kinetic energy of translation is 1,244,340 ft.-lbs.; when empty (weight 33000 lbs.) the energy is 441540 ft.-lbs. The rotative kinetic energy thus adds (for this particular car) 3.15% (when the car is loaded) and 8.9% (when the car is empty) to the kinetic energy of translation. The kinetic



energy which is similarly added, owing to the rotation of the wheels and axles of the locomotive, might be similarly computed. For one type of locomotive it has been figured at about 8%. The variations in design, and particularly the fluctuations of loading, render useless any great precision in these computations. For a train of "empties" the figure would be high, probably 8 to 9%; for a fully loaded train it will not much exceed 3%. Wellington considered that 6% is a good average value to use (actually used 6.11% for "ease of computation"), but considering (*a*) the increasing proportion of live load to dead load in modern car design, (*b*) the greater care now used to make up *full* train-loads, and (*c*) the fact that *full* train-loads are the critical loads, it would appear that 5% is a better average for the conditions of modern practice. Even this figure allows something for the higher percentage for the locomotive and something for a few empties in the train. Therefore, adding 5% to the coefficient in the above equation, we have the true equation

$$P = .0133(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad (139)$$

in which  $V_2$  and  $V_1$  are the higher and lower velocities respectively in *miles per hour*, and  $P$  is the force required *per ton* to impart that difference of velocity in a distance of *one mile*. If more convenient, the formula may be used thus:

$$P_1 = \frac{70.224}{s}(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad (140)$$

in which  $s$  is the distance in feet and  $P_1$  is the corresponding force.

As a numerical illustration, the force required per ton to impart a kinetic energy due to a velocity of 20 miles per hour in a distance of 1000 feet will equal

$$P_1 = \frac{70.224(400 - 0)}{1000} = 28 \text{ lbs.,}$$

which is the equivalent (see § 344) of a 1.4% grade. Since the velocity enters the formula as  $V^2$ , while the distance enters only in the first power, it follows that it will require *four* times the force to produce twice the velocity in the same distance, or that with the *same* force it will require four times the distance to attain twice the velocity.



As another numerical illustration, if a train is to increase its speed from 15 miles per hour to 60 miles per hour in a distance of 2000 feet, the force required (in addition to all the other resistances) will be

$$P_1 = \frac{70.224(3600 - 225)}{2000} = 118.50 \text{ lbs. per ton.}$$

This is equivalent to a 5.9% grade and shows at once that it would be impossible unless there were a very heavy down grade, or that the train was very light and the engine very powerful.

**348. Formulæ for train resistance.** These are generally given in one of the forms

$$\left. \begin{aligned} R &= fV + c \dots\dots (a) \\ R &= fV^2 + c \dots\dots (b) \end{aligned} \right\}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (141)$$

in which  $R$  is the resistance per ton,  $f$  is a coefficient to be determined,  $V$  is the velocity in miles per hour, and  $c$  is a constant, also to be determined. These formulæ disregard grade and curve resistances, inertia resistance, and the active resistance (or assistance) of *wind*, as distinct from mere atmospheric resistance. In short, they are supposed to give the resistance of a train moving at a uniform velocity over a straight and level track, there being no appreciable wind. Both formulæ are empirical, since the resistances do not vary either directly or as the square of the velocity. Some resistances vary nearly as the square and some nearly as the first power.

The quantity  $c$  represents the journal friction and rolling friction, and these are assumed to be constant, although careful tests of journal friction show that its variation with velocity is irregular (see § 343). This shows that such simple formulæ must always be inaccurate, but some formulæ have been suggested, having either of these general forms, which agree very closely with the results of actual tests.

(a) *Searles's formula,*

$$R = 4.82 + .00536V^2 + \frac{.00048V^2 (\text{wt. of eng. and tender})^2}{\text{gross weight of train}} \quad (142)$$

in which  $R$  = total resistance in pounds per ton and  $V$  is the velocity in miles per hour. This formula does not take account of any difference in the form of the train (whether box-cars or

flats), which would have a great influence on the atmospheric resistance; neither does it take into account the relation of length to weight, or whether the cars are loaded or empty. Nevertheless the results agree very closely with the determinations of actual train tests. If the resistance is computed according to this formula for a given class of engine (e.g., a heavy consolidation), and for various lengths of train, it is found that the resistance *per ton of the gross weight of the train* is much less when the train is long, and for a train of ordinary length the resistance hardly increases as fast as the velocity until the velocity is great.

According to this formula, a heavy consolidation engine drawing forty loaded freight-cars would have to overcome a resistance of about 8.2 lbs. per ton of the gross weight of the train at a velocity of 20 miles per hour. At a velocity of 10 miles per hour this resistance drops to about 5.7 lbs. per ton. And so the value of 8 lbs per ton, used by Wellington in his computations of the total power of locomotives on grades, may be considered a safe figure, especially as the velocity at critical places may be assumed to be reduced as much as necessary.

(b) *Wellington's formulæ,*

$$R = \left\{ \begin{array}{l} 3.9 + .0065V^2 + \frac{.57V^2}{W} \dots \text{for loaded flat-cars} \\ 3.9 + .0075V^2 + \frac{.64V^2}{W} \dots \text{for loaded box-cars} \\ 6.0 + .0083V^2 + \frac{.57V^2}{W} \dots \text{for empty flat-cars} \\ 6.0 + .0106V^2 + \frac{.64V^2}{W} \dots \text{for empty box-cars} \end{array} \right\}. \quad (143)$$

Notice in these formulæ the additional journal resistance (indicated by the constant term) for unloaded cars. The last term evidently indicates the atmospheric resistance. The middle term allows for the oscillatory resistances. Assuming the constant term and the coefficients to have been correctly determined, these formulæ should be better than Searles's, since a choice of formulæ can be made depending on the conditions. A train consisting partly of box-cars and partly of flat-cars will have a higher resistance than is shown by any of the above

formula (and *not* a mean value), on account of the increased atmospheric resistance acting on the irregular form of the train.

(c) *Engineering News formula*,

$$R = \frac{V}{4} + 2. \quad . \quad . \quad . \quad . \quad . \quad (144)$$

This formula belongs to class (a), Eq. 141. Its very simplicity makes it valuable for general use, but like the succeeding formula, it does not take account of variations in the form of the train, which have a very material influence on the train resistance.

(d) *D. K. Clark's formula*,

$$\left. \begin{aligned} R &= \frac{V^2}{171} + 8 \dots \dots \dots (\text{for tons of 2240 lbs.}) \\ R &= .00522V^2 + 7.14 \dots \dots (\text{for tons of 2000 lbs.}) \end{aligned} \right\} \quad (145)$$

This is a very old formula, and is mentioned because all of Clark's formulae carry much weight. But in this case the formula is quite defective. The constant term (7.14) representing the journal and rolling friction is too large and thus the formula gives too large a resistance at low velocities; the coefficient of  $V^2$  (.00522) is less than in the other formulae, and so at very high velocities the figures would be less than those given by Searles's or Wellington's formulae, and less than the results of actual tests. For mean velocities the figures accord fairly well with those given by the other formulae and by actual tests.

(e) *Baldwin Locomotive Works formula*. The Baldwin Locomotive Works have adopted a formula of their own as the result of the experience they have been able to accumulate. It is stated

$$R = \frac{V}{6} + 3. \quad . \quad . \quad . \quad . \quad . \quad (146)$$

It is claimed that this formula agrees well with actual tests, and in fact is based on the results of tests, but it evidently cannot allow for known variations in the length or character of the train. As a general formula for locomotives which are to pull *any* kind of a load, the formula is of more value for practical use than Searles's or Wellington's.

In Plate IX is shown graphically the resistance per ton of

four trains according to these five formulæ. For purposes of comparison of the formulæ, the weight of engine and total weight of cars is made the same for the four trains. The resistance would therefore be the same by formulæ (a), (c), (d), and (e). The differences would only appear when applying Wellington's formula. Assume the following as train-loads:

|     |                                             |
|-----|---------------------------------------------|
| (a) | Engine, 64 tons; loaded flat-cars, 648 tons |
| (b) | “ 64 “ “ box- “ 648 “                       |
| (c) | “ 64 “ empty flat- “ 648 “                  |
| (d) | “ 64 “ “ box- “ 648 “                       |

When applying any of these formulæ, due allowance must be made for grade and curve resistances, inertia resistances, and the possible retarding influence of a high wind must be considered if it is a question of the power of a locomotive of given type to draw a given load up a given grade.

**349. Dynamometer tests.** These are made by putting a “dynamometer-car” between the engine and the cars to be tested. Suitable mechanism makes an automatic record of the force which is transmitted through the dynamometer at any instant, and also a record of the velocity at any instant. One of the practical difficulties is the accurate determination of the velocity at any instant when the velocity is fluctuating. When the velocity is decreasing, the kinetic energy of the train is being turned into work and the force transmitted through the dynamometer is less than the amount of the resistance which is actually being overcome. On the other hand, when the velocity is increasing, the dynamometer indicates a larger force than that required to overcome the resistances, but the excess force is being stored up in the train as kinetic energy. Grade has a similar effect, and the force indicated by the dynamometer may be greater or less than that required at the given velocity on a level by the force which is derived from, or is turned into, potential energy. Therefore the resistance indicated by the dynamometer of a train will not be that on a level track at uniform velocity, unless the track is actually level and the velocity really uniform.

Dynamometer tests under other circumstances are therefore of no value unless it is possible to determine the true velocity at any instant and its rate of change, and also to determine the grade. Of course, the grade is easily found. An

allowance for an increase or decrease of kinetic or potential energy must therefore be made before it is possible to know how much force is being spent on the ordinary resistances.

**350. Gravity or "drop" tests.** Dynamometer tests require the use of a dynamometer which is capable of measuring a force of several thousands of pounds, and which therefore cannot determine such values with a close percentage of accuracy, especially if the force is small. A drop test utilizes the force of gravity which may be measured with mathematical accuracy. The general method is to select a stretch of track which has a uniform grade of about  $0.7\%$  and which is preferably straight for two or three miles. On such a grade cars with running gear in good condition may be started by a push. The velocity will gradually increase until at some velocity, depending on the resistances encountered, the cars will move uniformly. The only work requiring extreme care with this method is the determination of the velocity. If the velocity is fluctuating, as it is during the time when it is of the greatest importance to know the velocity, it is not sufficient to determine the time required to run some long measured distance, for the average velocity thus obtained would probably differ

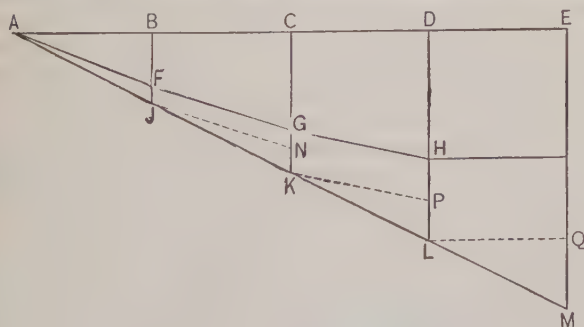


FIG. 207.—LOSS IN VELOCITY-HEAD.

considerably from the velocity at the beginning and end of that space. If the train consists of five cars or more, the velocity may be determined electrically (as described by Wellington in his "Economic Location," etc., p. 793 *et seq.*) from the automatic record made on a chronograph of the passage of the first wheel and the last, the chronograph also recording auto-

matically the ticks of a clock beating seconds. From this the exact time of the passage of the first and last wheels of the train of cars may be determined to the tenth or twentieth of a second.

*Velocity-head.* From theoretical mechanics we know that if a body descends through any path by the action of gravity, and is unaffected by friction, its velocity at any point in the direction of the path of motion is  $V = \sqrt{2gh}$ . If the body is retarded by resistances, its velocity at any point will be less than this. If  $AM$ , Fig. 207, represents any grade (exaggerated of course), then  $BJ$ ,  $CK$ , etc., represent the actual fall at any point. Let  $BF$  represent the fall  $h_1$ , determined from  $h_1 = \frac{v_1^2}{2g}$ , in which  $v_1$  is the actual observed velocity at  $J$ . Then  $JF$  = the velocity-head consumed by the resistances between  $A$  and  $J$ . If the train continues to  $K$ , the corresponding  $h_2$  is  $CG$ ; the remaining fall  $GK$  consists of  $GN$  ( $=JF$ , which is the velocity-head lost back of  $J$ ) and  $NK$ , the velocity-head lost between  $J$  and  $K$ . At some velocity ( $V_n$ ) on any grade, the velocity will not further increase and the line  $AFGHI$  will then be horizontal and at a distance  $(h_n) = EI$  below  $A \dots E$ . The grade  $AM$  is the "grade of repose" for that velocity ( $V_n$ ); i.e., it is the grade that would just permit the train to move indefinitely at the velocity  $V_n$ . The broken line  $AFGHI$  should really be a curve, and the grade of repose at any joint is the angle between  $AM$  and the tangent to that curve at the given point. The "grade of repose" by its definition gives the total resistance of the train at the particular velocity, or multiplying the grade of repose in per cent. by 20 gives the pounds per ton of resistance. Thus being able to determine the total resistance in pounds per ton at any velocity, the variation of total resistance with velocity may be determined, and then by varying the resistances, using different kinds of cars, empty and loaded, box-cars and flats, the resistances of the different kinds at various velocities may be determined.

## CHAPTER XVII

### COST OF RAILROADS.

**351. General considerations.** Although there are many elements in the cost of railroads which are roughly constant per mile of road, yet the published reports of the cost of railroads differ very widely. The variation in the figures is due to several causes. (a) Economy requires that a road shall be operated and placed on an earning basis as soon as possible. Therefore the reported cost of a road during the first few years of its existence is somewhat less than that reported later. This is well illustrated when a long series of consecutive reports from an old-established road is available; nearly every year there will be shown an addition to the previous figures. And this is as it should be. The magnificent road-beds of some old roads cannot be the creation of a single season. It takes many years to produce such settled perfect structures. (b) A large part of the variation is due to a neglect to charge up "permanent improvements" as additions to the cost of the road. For the first few years of the life of a road a great deal of work is done which is in reality a completion of the work of construction, and yet the cost of it is buried under the item "maintenance of way." For example, a long wooden trestle is replaced by an earth embankment and a culvert. Since the original trestle is to be considered a temporary structure, the *excess* of the cost of the permanent structure over that of the temporary structure should evidently be considered as an addition to the cost of the road. But if the filling-in was done slowly, a few train-loads at a time, and the work scattered over many years, the cost of operating the "mud-train" has perhaps been buried under "maintenance" charges. (c) The reports from which many of the following figures were taken have not always analyzed the items of cost with the same detail as has been here attempted, and to that is probably due many of the variations and apparent discrepancies.



The various items of cost will be classified as follows:

1. Preliminary financiering.
2. Surveys and engineering expenses.
3. Land and land damages.
4. Clearing and grubbing.
5. Earthwork.
6. Bridges, trestles, and culverts
7. Trackwork.
8. Buildings and miscellaneous structures.
9. Interest on construction.
10. Telegraph line.

**352. Item 1. PRELIMINARY FINANCIERING.** The cost of this preliminary work is exceedingly variable. The work includes the clerical and legal work of organization, printing, engraving of stocks and bonds, and (sometimes the most expensive of all) the securing of a charter. This sometimes requires special legislative enactments, or may sometimes be secured from a State railroad commission. It has been estimated that about 2% of the railway capital of Great Britain has been spent in Parliamentary expenses over the charters. These expenses are usually but a small percentage of the total cost of the enterprise, but for important lines the gross cost is large, while the amount of money thus spent by organizations which have never succeeded in constructing their roads is sometimes enormous.

**353. Item 2. SURVEYS AND ENGINEERING EXPENSES.** The comparison of a large number of itemized reports on the cost of construction shows that the cost of the "engineering" will average about 2% of the total cost of construction. This includes the cost of surveys and the cost of laying out and superintending the constructive work. The cost of mere surveying up to the time when construction actually commences has been variously quoted at \$60, \$75, and even \$150 per mile. In exceptional cases the surveying for a few miles through some gorge might cost many times this amount, but \$150 per mile may be considered an ordinary maximum for difficult country. On the other hand, much construction has been done over the western prairies after hasty surveys costing not much over \$10 per mile.

**354. Item 3. LAND AND LAND DAMAGES.** The cost of this item varies from the extreme, in which not only the land for

right-of-way but also grants of public land adjoining the road are given to the corporation as a subsidy, to the other extreme, where the right-of-way can only be obtained at exorbitant prices. The width required is variable, depending on the width that may be needed for deep cuts or high fills, or the extra land required for yards, stations, etc. A strip of land 1 mile long and 8.25 feet wide contains precisely 1 acre. An average width of 4 rods (66 feet), therefore, requires 8 acres per mile. On the Boston & Albany Railroad the expenditure assigned to "land and land damages" averages over \$25000 per mile. Of course this includes some especially expensive land for terminals and stations in large cities. Less than \$300 per mile was assigned to this item by an unimportant 18-mile road.

**355. Item 4. CLEARING AND GRUBBING.** The cost of this may vary from zero to 100% for miles at a time, but as an average figure it may be taken as about 3 acres per mile at a cost of say \$50 per acre. The possibility of obtaining valuable timber, which may be utilized for trestles, ties, or otherwise, and the value of which may not only repay the cost of clearing and grubbing, but also some of the cost of the land, should not be forgotten.

**356. Item 5. EARTHWORK.** This item also includes rockwork. The methods of estimating the cost of earthwork and rockwork have been discussed in Chapter III. The percentage of this item to the total cost is very variable. On a western prairie it might not be more than 5 to 10%. On a road through the mountains it will run up to 20 or 25%, and even more. The item also includes tunneling, which on some roads is a heavy item.

**357. Item 6. BRIDGES, TRESTLES, AND CULVERTS.** This item will usually amount to 5 or 6% of the total cost of the road. In special cases, where extensive trestling is necessary, or several large bridges are required, the percentage will be much higher. On the other hand, a road whose route avoids the watercourses may have very little except minor culverts. On the Boston & Albany the cost is given as \$5860 per mile; on the Adirondack Railroad, \$2845 per mile. Considering their relative character (double and single track), these figures are relatively what we might expect,

**358. Item 7. TRACKWORK.** This item will be considered as including everything above subgrade, except as otherwise itemized.

(a) **Ballast.** With an average width, for single track, of 10 feet and an average of 15 inches, 2444 cubic yards of ballast will be required. The Pennsylvania Railroad estimate is 2500 yards of gravel per mile of single track. At an estimate of 60 c. per yard, this costs \$1500 per mile. Broken-stone ballast must be filled out over the ends of the ties and therefore more is required; 2800 cubic yards of broken stone at \$1.25 per yard in place will cost \$3500 per mile.

(b) **Ties.** Ties cost anywhere from 80 c. down to 35 c. and even 25 c. At an average figure of 50 c., 2640 ties per mile will cost \$1320 per mile of single track. The cheaper ties are usually smaller and more must be used per mile, and this tends to compensate the difference in cost.

The following tabular form is convenient for reference:

TABLE XV.—NUMBER OF CROSS TIES PER MILE.

| Spacing<br>center to<br>center. | Number per<br>30' rail. | Number per<br>mile. |
|---------------------------------|-------------------------|---------------------|
| 18 inches                       | 20                      | 3520                |
| 20 "                            | 18                      | 3168                |
| 21 "                            | 17 $\frac{1}{2}$        | 3017                |
| 22.5 "                          | 16                      | 2816                |
| 24 "                            | 15                      | 2640                |
| 25.71 "                         | 14                      | 2464                |
| 27 "                            | 13 $\frac{1}{2}$        | 2347                |
| 30 "                            | 12                      | 2112                |

(c) **Rails.** The total weight of the rails used per mile may best be seen by the tabular form.

A convenient and useful rule to remember is that the number of long tons (2240 lbs.) per mile of single track equals the weight of the rail per yard times  $\frac{1}{7}$ . The rule is exact. For example, there are 3520 yards of rail in a mile of single track; at 70 lbs. per yard this equals 246400 lbs., or 110 long tons (exactly); but  $70 \times \frac{1}{7} = 110$ .

Any calculation of the required weight of rail for a given weight of rolling-stock necessarily depends on the assumptions which are made regarding the support which the rails receive from the ties. This depends not only on the width and spacing

TABLE XVI.—TONS PER MILE (WITH COST) OF RAILS OF VARIOUS WEIGHTS.

| Weight in lbs. per yd. | Tons (2240 lb.) per mile of single track. | Cost at \$26 per ton. | Cost at \$30 per ton. | Weight in lbs. per yd. | Tons (2240 lb.) per mile of single track. | Cost at \$26 per ton. | Cost at \$30 per ton. |
|------------------------|-------------------------------------------|-----------------------|-----------------------|------------------------|-------------------------------------------|-----------------------|-----------------------|
| 8                      | 12.571                                    | \$326.86              | \$377.14              | 65                     | 102.143                                   | \$2655.71             | \$3064.29             |
| 10                     | 15.714                                    | 408.57                | 471.43                | 66                     | 103.714                                   | 2696.57               | 3111.43               |
| 12                     | 18.857                                    | 490.29                | 565.71                | 67                     | 105.286                                   | 2737.43               | 3158.59               |
| 14                     | 22.000                                    | 572.00                | 660.00                | 68                     | 106.857                                   | 2778.29               | 3205.79               |
| 16                     | 25.143                                    | 653.71                | 754.20                | 70                     | 110.000                                   | 2860.00               | 3300.00               |
| 20                     | 31.429                                    | 817.14                | 942.86                | 71                     | 111.571                                   | 2900.86               | 3347.14               |
| 25                     | 39.286                                    | 1021.43               | 1178.57               | 72                     | 113.143                                   | 2941.71               | 3394.29               |
| 30                     | 47.143                                    | 1225.71               | 1414.29               | 73                     | 114.714                                   | 2982.57               | 3441.43               |
| 35                     | 55.000                                    | 1430.00               | 1650.00               | 75                     | 117.857                                   | 3064.29               | 3535.71               |
| 40                     | 62.857                                    | 1634.29               | 1885.71               | 78                     | 122.571                                   | 3186.86               | 3677.14               |
| 45                     | 70.714                                    | 1838.57               | 2121.43               | 80                     | 125.714                                   | 3268.57               | 3771.43               |
| 48                     | 75.429                                    | 1961.14               | 2262.86               | 82                     | 128.857                                   | 3350.29               | 3865.71               |
| 50                     | 78.571                                    | 2042.86               | 2357.14               | 85                     | 133.571                                   | 3472.86               | 4007.14               |
| 52                     | 81.714                                    | 2124.57               | 2451.43               | 88                     | 138.286                                   | 3595.43               | 4148.57               |
| 56                     | 88.000                                    | 2288.00               | 2640.00               | 90                     | 141.429                                   | 3677.14               | 4242.86               |
| 57                     | 89.571                                    | 2328.86               | 2687.14               | 92                     | 144.571                                   | 3758.86               | 4337.14               |
| 60                     | 94.286                                    | 2451.43               | 2828.57               | 95                     | 149.286                                   | 3881.43               | 4478.57               |
| 61                     | 95.857                                    | 2492.29               | 2875.71               | 98                     | 154.000                                   | 4004.00               | 4620.00               |
| 63                     | 99.000                                    | 2574.00               | 2970.00               | 100                    | 157.143                                   | 4085.71               | 4714.29               |

About two per cent. (2%) extra should be allowed for waste in cutting.

of the ties (which are determinable), but also on the support which the ties receive from the ballast, which is not only very uncertain but variable. No general rule can therefore claim any degree of precision, but the following is given by the Baldwin Locomotive Works: "Each ten pounds weight per yard of ordinary steel rail, properly supported by cross-ties (not less than 14 per 30-foot rail), is capable of sustaining a safe load per wheel of 2240 pounds." For example, a consolidation locomotive with 112600 lbs. on 8 drivers has a load of 14075 lbs. per wheel. This divided by 2240 gives 6.28. According to the rule, the rails for such a locomotive should weigh at least 62.8 lbs. per yard.

(d) **Splice-bars, track-bolts, and spikes.** These are usually sold by the pound, except the patented forms of rail-joints, which are sold by the pair. In any case they are subject to market fluctuations in price. As an approximate value the following prices are quoted: Splice-bars, 1.35 c. per pound; track-bolts, 2.4 c.; spikes, 1.75 c. The weight of the splice-bars will depend on the precise pattern adopted—its cross-section and length. For a 45-lb. rail an angle-bar whose original weight in the rolled section is 6.3 lbs. per foot might

be used. A pair 21 inches long would weigh 21.5 lbs. For a 70-lb. rail an angle-bar section weighing 9 to 12 lbs. per yard would be used. A pair of the 10-lb. section, with the long 44-inch 6-hole bar, used by the Michigan Central Railroad, would weigh about 70 lbs. Angle-bars suitable for a 100-lb. rail will weigh about 14 to 16 lbs. per foot. The following tables will be useful for reference.

TABLE XVII. SPLICE-BARS AND BOLTS PER MILE OF TRACK.

| Length of rail. | Number of pairs of splice-bars. | Number of bolts required. |                |
|-----------------|---------------------------------|---------------------------|----------------|
|                 |                                 | 4-hole splice.            | 6-hole splice. |
| 24 feet         | 440                             | 1760                      | 2640           |
| 25 "            | 422                             | 1688                      | 2532           |
| 26 "            | 406                             | 1624                      | 2436           |
| 27 "            | 391                             | 1564                      | 2346           |
| 28 "            | 377                             | 1508                      | 2262           |
| 30 "            | 352                             | 1408                      | 2112           |
| 33 "            | 320                             | 1880                      | 1920           |

TABLE XVIII.—RAILROAD SPIKES.

| Size measured under head. | Average number per keg of 200 pounds | Ties 24" between centers, 4 spikes per tie, number per Mile. |       | Suitable weight of rail. |
|---------------------------|--------------------------------------|--------------------------------------------------------------|-------|--------------------------|
|                           |                                      | Pounds.                                                      | Kegs. |                          |
| 5½" × ⅞"                  | 375                                  | 5632                                                         | 28.16 | 45 to 100                |
| 5" × ⅞"                   | 400                                  | 5280                                                         | 26.40 | 40 " 56                  |
| 5" × ½"                   | 450                                  | 4692                                                         | 23.46 | 40                       |

TABLE XIX. TRACK-BOLTS.

Average number in a keg of 200 pounds.

| Size of bolt. | Square nut. | Hexagonal nut. | Suitable rail.        |
|---------------|-------------|----------------|-----------------------|
| 3" × ¾"       | 250         | 270            | 50 pounds and upward. |
| 3¼" × ¾"      | 243         | 261            |                       |
| 3½" × ¾"      | 236         | 253            |                       |
| 3¾" × ¾"      | 229         | 244            |                       |
| 4 " × ¾"      | 222         | 236            |                       |
| 3½" × ½"      | 170         | 180            |                       |
| 3¼" × ½"      | 165         | 175            |                       |
| 4 " × ½"      | 161         | 170            |                       |
| 4½" × ½"      | 157         | 165            |                       |
| 4½" × ⅜"      | 153         | 160            |                       |

(e) **Track-laying.** Much depends on the force of men employed and the use of systematic methods; \$528 per mile is the estimate employed by the Pennsylvania Railroad. \$500 per mile is the estimate given in § 362.

**359. Item 8. BUILDINGS AND MISCELLANEOUS STRUCTURES.** Except for rough and preliminary estimates, these items must be individually estimated according to the circumstances. The subitems include depots, engine-houses, repair-shops, water-stations, section- and tool-houses, besides a large variety of smaller buildings. The structures include turn-tables, cattle-guards, fencing, road-crossings, overhead bridges, etc. The detailed estimate, given in § 362, illustrates the cost of these smaller items.

**360. Item 9. INTEREST ON CONSTRUCTION.** The amount of capital that must be spent on a railroad before it has begun to earn anything is so very large that the interest on the cost during the period of construction is a very considerable item. The amount that must be charged to this head depends on the current rate of money on the time required for construction and on the ability of the capitalists to retain their capital where it will be earning something until it is actually needed to pay the company's obligations. Of course, it is not necessary to have the entire capital needed for construction on hand when construction commences. Assuming money to be worth 6%, that the work of construction will require one year, that the money may be retained where it will earn something for an average period of six months after construction commences, or, in other words, it will be out of circulation six months before the road is opened for traffic and begins to earn its way, then we may charge 3% on the total cost of construction.

**361. Item 10. TELEGRAPH LINES.** This evidently depends on the scale of the road and the magnitude of the business to be operated. In the following estimate it is given as \$200 per mile, which evidently is intended to apply to the business of a small road.

**362. Detailed estimate of the cost of a line of road.** The following estimate was given in the *Engineering News* of Dec. 27, 1900, of the cost of the Duluth, St. Cloud, Glencoe & Mankato Railroad, 157.2 miles long.

The estimate is exactly as copied from the *Engineering News*. There are some numerical discrepancies. Item 26 should evi-



dently be based on the sum of the first 25 items, and item 27 on the sum of the first 26. The figures in parentheses ( ) are deduced from the figures given.

|                                                                                          |                  |
|------------------------------------------------------------------------------------------|------------------|
| 1. Right-of-way: 1905.3 acres (12.12 acres per mile) @ \$100 per acre.....               | \$190530         |
| 2. Clearing and grubbing. 144 acres (0.916 acre per mile) @ \$50 per acre.....           | 7200             |
| 3. Earth excavation. 1907590 cu. yds. (12135 cu. yds. per mile) @ 15 c.....              | 286138           |
| 4. Rock excavation. 5100 cu. yds. (32.44 cu. yds. per mile) @ 80 c.....                  | 4080             |
| 5. { Wooden-box culverts. 508300 ft. B.M. @ \$30 per M. . . . .                          | \$15249          |
| 5. { Iron-pipe culverts. 879840 lbs. @ 3c. per lb. . . . .                               | 26395            |
| 6. { Pile trestling. 4600 lin. ft. @ 35 c. per lin. ft. . . . .                          | 1610             |
| 6. { Timber trestling. 509300 ft. B.M. @ \$30 per M. . . . .                             | 15279            |
| 7. { Bridge masonry: 5520 cu. yds. @ \$8 per cu. yd. . . . .                             | 44160            |
| 7. { Bridges, iron, 100 spans, 2000000 lbs. @ 4 c. per lb. . . . .                       | 80000            |
| 8. Cattle-guards. . . . .                                                                | 8750             |
| 9. Ties (2640 per mile). 419813 (159.02 M.) @ 35 c. . . . .                              | 146935           |
| 10. Rails (70 lbs. per yd.): 110 tons per mile, 17492.2 tons (159.02 M.) @ \$26. . . . . | 384797           |
| 11. Rail sidings (70 lbs. per yd.): 110 tons per mile, 3300 tons (30 M.) @ \$26. . . . . | 85807            |
| 12. Switch timbers and ties. . . . .                                                     | 3300             |
| 13. Spikes: 5920 lbs. per mile, 1107040 (187 M.) @ 1.75. c. per lb. . . . .              | 19373            |
| 14. Splice-bars. 2635776 lbs. @ 1.35 c. per lb. . . . .                                  | 35583            |
| 15. Track-bolts (2 to joint (?)): 188458.3 lbs. @ 2.4 c. per lb. . . . .                 | 4520             |
| 16. Track-laying 187.2 miles @ \$500 per mile. . . . .                                   | 93600            |
| 17. Ballasting 2152 cu. yds. per mile, 402854 (187.2 M.) @ 60 c. . . . .                 | 241712           |
| 18. Turn-out and switch furnishings. . . . .                                             | 6450             |
| 19. Road-crossings, 68040 ft. B.M. @ \$30 per M. . . . .                                 | 2041             |
| 20. Section and tool-houses, 16 @ \$800. . . . .                                         | 12800            |
| 21. Water-stations. . . . .                                                              | 15000            |
| 22. Turn-tables, 6 @ \$800 . . . . .                                                     | 4800             |
| 23. Depots, grounds, and repair-shops. . . . .                                           | 78000            |
| 24. Terminal grounds and special land damages. . . . .                                   | 150000           |
| 25. Fencing, 314 miles (\$150 per mile). . . . .                                         | 47100            |
| 26. Engineering and office expenses (5% of \$1984458). . . . .                           | 99222            |
| 27. Interest on construction (3% of \$2083680). . . . .                                  | 62510            |
| 28. Rolling-stock (\$5000 per mile). . . . .                                             | 786000           |
| 29. Telegraph line: 157 miles @ \$200 per mile. . . . .                                  | 31400            |
|                                                                                          | <u>\$3060340</u> |

Average cost per mile ready for operation, \$19467.

Approximate cost of 130 miles from St. Cloud to Duluth, estimated at \$23000 per mile.

Approximate cost of entire line from Albert Lea to Duluth, 287.2 miles, \$6050340 (\$21060 per mile).



## PART II.

### RAILROAD ECONOMICS.

#### CHAPTER XVIII.

##### INTRODUCTION.

**363. The magnitude of railroad business.** The gross earnings of railroads for the year ending June 30, 1899, were over \$1,300,000,000. This is greater than the combined value of all the gold, silver, iron, wheat, and corn produced by the country. The following figures (to the nearest million of dollars) gives the value of various crops for 1899, according to the current U. S. Yearbook of Agriculture:

|             |     |             |      |
|-------------|-----|-------------|------|
| Gold.....   | 71  | Oats.....   | 198  |
| Silver..... | 33  | Hay.....    | 412  |
| Iron.....   | 245 | Coal.....   | 256  |
| Wheat.....  | 320 | Copper..... | 104  |
| Corn.....   | 629 | Lead.....   | 19   |
| Total ..... |     |             | 2287 |

About 929000 persons (about one eightieth of the population) were directly employed by the roads for a compensation of about \$523,000,000. Probably 3,000,000 to 4,000,000 people were supported by this. Beside all these, probably 5,000,000 employés were kept busy in occupations which are a more or less direct result of railroads, *e.g.*, locomotive- and car-shops, rail-mills, etc. We may therefore estimate that perhaps 20,000,000 people (or, say, one fourth of our population) are supported by railroads or by occupations which owe their chief existence to railroads.

The "number of passengers carried 1 mile" was 14,591,327,613. Calling the population of the United States 75,000,000 for round

numbers, it means an average ride of 195 miles for every man, woman, and child.

The "tons carried 1 mile" were 123,667,257,153, or nearly 1650 ton-miles per inhabitant. The payments made to the railroads averaged over \$17 per inhabitant.

Turning to a dark side of the picture, we find that the traffic was carried on at a cost of 7123 killed and 44620 injured. This averages one killed every hour and a quarter and one injured every twelve minutes. Of these large numbers, the "passengers" comprised but 239 and 3412 respectively. The remainder were employés and "others," the "others" consisting largely of "trespassers."

The actual bona-fide cost of the railroads of the country cannot be accurately computed (as will be shown later), but the capital, as represented by stocks and bonds, represents \$11,033,954,898, or about \$147 per inhabitant. This is roughly about one sixth of the total national wealth.

The above figures may give some idea of the magnitude of the interests involved in the operation of railroads. No single business in the country approaches it in capital involved, earnings, number of people affected, or effect on other business.

**364. Cost of transportation.** The importance of railroads may be also indicated by their power of creating cheap transportation. Less than one hundred years ago local famine and overabundant harvests within a radius of a few miles were not unknown. When the transportation of goods depended on actual portorage by human beings, as has been the case but recently in the Klondike, the transportation of 100 lbs. 20 miles might be considered an average day's labor. At \$1 per day, this equals \$1 per ton-mile. In 1899 the railroads transported freight at an *average* cost to the public of 0.724 c. per ton per mile, and the feeding of Europe with wheat from Manitoba has become a commercial possibility. In 1899 passengers paid an average charge of 1.925 c. per mile, and a trip of 1000 miles inside of 24 hours is now common.

**365. Study of railroad economics—its nature and limitations.** The multiplicity of the elements involved in most problems in railroad construction preclude the possibility of a solution which is demonstrably perfect. Barring out the comparatively few cases in this country where it is difficult to obtain *any* practicable location, it may be said that a comparatively

low order of talent will suffice to locate anywhere a railroad over which it is physically possible to run trains. It may be very badly located for obtaining business, the ruling grades may be excessive, the alignment may be very bad, and the road may be a hopeless financial failure, and yet trains *can* be run. Among the infinite number of possible locations of the road, the engineer must determine the route which will give the best railroad property for the least expenditure of money—the road whose earning capacity is so great that after paying the operating expenses and interests on the bonds, the surplus available for dividends or improvements is a maximum.

An unfortunate part of the problem is that even the blunders are not always readily apparent nor their magnitude. A defective dam or bridge will give way and every one realizes the failure, but a badly located railroad affects chiefly the finances of the enterprise by a series of leaks which are only perceptible and demonstrable by an expert, and even he can only say that certain changes would probably have a certain financial value.

**366. Outline of the engineer's duties.** The engineer must realize at the outset the nature and value of the conflicting interests which are involved in variable amount in each possible route.

(a) **The maximum of business must be obtained**, and yet it *may* happen that some of the business may only be obtained by an extravagant expenditure in building the line or by building a line very expensive to operate.

(b) **The ruling grades should be kept low**, and yet this *may* require a sacrifice in business obtained and also *may* cost more than it is worth.

(c) **The alignment should be made as favorable as possible**; favorable alignment reduces the future operating expenses, but it may require a very large immediate outlay.

(d) **The total cost must be kept within the amount at which the earnings will make it a profitable investment.**

(e) **The road must be completed and operated until the "normal" traffic is obtained and the road is self-supporting without exhausting the capital obtainable by the projectors**; for no matter how valuable the property may ultimately become, the projectors will lose nearly, if not quite, all they have invested if they lose control of the enterprise before it becomes a paying investment.

Each new route suggested makes a new combination of the above conflicting elements. The engineer must select a route by first eliminating all lines which are manifestly impracticable and then gradually narrowing the choice to the best routes whose advantages are so nearly equal that a closer detailed comparison is necessary.

The ruling grade and the details of alignment have a large influence on the operating expenses. A large part of this course of instruction therefore consists of a study of operating expenses under average normal conditions, and then a study of the effect on operating expenses of given changes in the alignment.

**367. Justification of such methods of computation.** It may be argued that the data on which these computations are based are so unreliable (because variable and to some extent non-computable) that no dependence can be based on the conclusions. This is true to the extent that it is useless to claim great precision in the computation of the value of any proposed change of alignment. Suppose, for example, it is computed that a given improvement in alignment will reduce the operating expenses of 20 trains per day by \$1000 per year. Suppose the change in alignment may be made for \$5000, which may be obtained at 5% interest. Even with large allowances for inaccuracy in the computation of the value, \$1000, it evidently will be better to incur an additional interest charge of \$250 than increase the annual operating expenses by \$1000. Moreover, since traffic is almost sure to increase (and interest charges are generally decreasing), the advantage of the improvement will only increase as time passes. On the other hand, if the improvement cannot be made except by an expenditure of, say, \$50000, the change would evidently be unjustifiable. When the interest on the first cost is practically equal to the annual operating value of the proposed improvement, there is evidently but little choice; no great harm can result from either decision, and the decision frequently will depend on the willingness to increase the total amount invested in the enterprise.

To express the above question more generally, in every computation of the operating value of a proposed improvement, it may always be shown that the true value lies somewhere between some maximum and some minimum. Closer calcula-

tions and more reliable data will narrow the range between these extreme values. According as the interest on the cost of the proposed improvement is greater or less than the mean of these limits, we may judge of its advisability. The range of the limits shows the uncertainty. If it lies outside of the limits there is no uncertainty, assuming that the limits have been properly determined. If well within the limits, either decision will answer unless other considerations determine the question. And so, although it is not often possible to obtain precise values, we may generally reach a conclusion which is unquestionable. Even under the most unfavorable circumstances, the computations, when made with the assistance of all the broad common sense and experience that can be brought to bear, will point to a decision which is much better than mere "judgment," which is responsible for very many glaring and costly railroad blunders. In short, Railroad Economics means the application of systematic methods of work *plus* experience and judgment, rather than a dependence on judgment unsystematically formed. It makes no pretense to furnishing mechanical rules by which all railroad problems may be solved by any one, but it does give a general method of applying principles by which an engineer of experience and judgment can apply his knowledge to better advantage. To the engineer of limited experience the methods are invaluable; without such methods of work his opinions are practically worthless; with them his conclusions are frequently more sound than the unsystematically formed judgments of a man with a glittering record. But the engineer of great experience may use these methods to form the best opinions which are obtainable, for he can apply his experience to make any necessary local modifications in the method of solution. The dangers lie in the extremes, either recklessly applying a rule on the basis of insufficient data to an unwarrantable extent, or, disgusted with such evident unreliability, neglecting altogether such systematic methods of work.

## CHAPTER XIX.

### THE PROMOTION OF RAILROAD PROJECTS.

368. **Method of formation of railroad corporations.** Many business enterprises, especially the smaller ones, are financed entirely by the use of money which is put into them directly in the form of stock or mere partnership interest. A railroad enterprise is frequently floated with a comparatively small financial expenditure on the part of the original promoters. The promoters become convinced that a railroad between *A* and *B*, passing through the intermediate towns of *C* and *D*, with others of less importance, will be a paying investment. They organize a company, have surveys made, obtain a charter, and then, being still better able (on account of the additional information obtained) to exploit the financial advantages of their scheme, they issue a prospectus and invite subscriptions to bonds. Sometimes a portion of these bonds are guaranteed, principal and interest, or perhaps the principal alone, by townships or by the national government. The cost of this preliminary work, although large in gross amount if the road is extensive, is yet but an insignificant proportion of the total amount involved. The proportionate amount that *can* be raised by means of bonds varies with the circumstances. In the early history of railroad building, when a road was projected into a new country where the traffic possibilities were great and there was absolutely no competition, the financial success of the enterprise would seem so assured that no difficulty would be experienced in raising from the sale of bonds all the money necessary to construct and equip the road. But the promoters (or stockholders) must furnish all money for the preliminary expenses, and must make up all deficiencies between the proceeds of the sale of the bonds and the capital needed for construction.

"In theory, stocks represent the property of the responsible owners of the road, and bonds are an encumbrance on that



property. According to this theory, a railroad enterprise should begin with an issue of stock somewhere near the value of the property to be created and no more bonds should be issued than are absolutely necessary to complete the enterprise. Now it is not denied that there are instances in which this theory is followed out. In New England, for example, as well as in some of the Southern States, there are a few roads represented wholly by stock or very lightly mortgaged. But this theory does not conform to the general history of railway construction in the United States, nor is it supported by the figures that appear in the summary. The truth is, railroads are built on borrowed capital, and the amount of stock that is issued represents in the majority of cases the difference between the actual cost of the undertaking and the confidence of the public expressed by the amount of bonds it is willing to absorb in the ultimate success of the venture." \*

"The same general law obtains and has always obtained throughout the world, that such properties (as railways) are always built on borrowed money up to the limit of what is regarded as the positive and certain minimum value. The risk only—the dubious margin which is dependent upon sagacity, skill, and good management—is assumed and held by the company proper who control and manage the property." †

**369. The two classes of financial interests—the security and profits of each.** From the above it may be seen that stocks, bonds, car-trust obligations, and even current liabilities represent railroad capital. The issue of the bonds "was one means of collecting the capital necessary to create the property against which the mortgage lies." The variation between these interests lies chiefly in the security and profits of each. The current liabilities are either discharged or, as frequently happens, they accumulate until they are funded and thus become a definite part of the railroad capital.

The growth of this tendency is shown in the following tabular form:

The bonded interest has greater security than the stock, but less profit. The interest on the bonds must be paid before any money can be disbursed as dividends. If the bond interest

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\* Henry C. Adams, Statistician, U. S. Int. Con. Commission.

† A. M. Wellington, *Economic Theory of Railway Location*.



| Railroads in the United States. | June 30, 1888.    |           | June 30, 1898.    |           |
|---------------------------------|-------------------|-----------|-------------------|-----------|
|                                 | Amount, millions. | Per cent. | Amount, millions. | Per cent. |
| Stocks. ....                    | 3864              | 47.5      | 5311              | 44.6      |
| Funded debt. ....               | 3869              | 47.6      | 5510              | 46.3      |
| Current liabilities, etc. ....  | 396               | 4.9       | 1087              | 9.1       |

is not paid, a receivership, and perhaps a foreclosure and sale of the road, is a probability, and in such case the stockholder's interests are frequently wiped out altogether. The bondholder's real profit is frequently very different from his nominal profit. He sometimes buys the bonds at a very considerable discount, which modifies the rate which the interest received bears to the amount really invested. Even the bondholder's security may suffer if his mortgage is a second (or fifth) mortgage, and the foreclosure sale fails to net sufficient to satisfy all previous claims.

On the other hand, the stockholder, who may have paid in but a small proportion of his subscription, *may*, if the venture is successful, receive a dividend which equals 50 or 100% of the money actually paid in, or, as before stated, his entire holdings may be entirely wiped out by a foreclosure sale. When the road is a great success and the dividends very large, additional issues of stock are generally made, which are distributed to the stockholders in proportion to their holdings, either gratuitously or at rates which give the stockholders a large advantage over outsiders. This is the process known as "watering." While it may sometimes be considered as a legitimate "salting down" of profits, it is frequently a cover for dishonest manipulation of the money market.

For the twelve years between 1887 and 1899 about *two thirds* of all the railroad stock in the United States paid *no* dividends, while of those that paid dividends the average rate varied from 4.96 to 5.74%. The year from June 30, 1898, to June 30, 1899, was the most prosperous year of the group, and yet nearly 60% of all railroad stock paid *no* dividend, and the average rate paid by those which paid at all was 4.96%. The total amount distributed in dividends was greater than ever before, but the average rate is the least of the above group because many roads, which had passed their dividends for many previous

years, distinguished themselves by declaring a dividend, even though small. During that same period but 13.35% of the stock paid over 6% interest. The total dividends paid amounted to but 2.01% of all the capital stock, while investments ordinarily are expected to yield from 4 to 6% (or more) according to the risk. Of course the effect of "watering" stock is to decrease the nominal rate of dividends, but there is no dodging the fact that, watered or not, even in that year of "good times," about 60% of all the stock paid *no* dividends. Unfortunately there are no accurate statistics showing how much of the stock of railroads represents actual paid-in capital and how much is "water." The great complication of railroad finances and the dishonest manipulation to which the finances of some railroads have been subjected would render such a computation practically worthless and hopelessly unreliable now.

During the year ending June 30, 1898 (which may in general be considered as a sample), 15.82% of the funded debt paid no interest. About one third of the funded debt paid between 4 and 5% interest, which is about the average which is paid.

The income from railroads (both interest on bonds and dividends on stock) may be shown graphically by diagrams, such as are given in the annual reports of the Interstate Commerce Commission. They show that while railroad investments are occasionally very profitable, the average return is less than that of ordinary investments *to the investors*. The *indirect* value of railroads in building up a section of country is almost incalculable and is worth many times the cost of the roads. It is a discouraging fact that very few railroads (old enough to have a history) have escaped the experience of a receivership, with the usual financial loss to the then stockholders. But there is probably not a railroad in existence which, however much a financial failure in itself, has not profited the community more than its cost.

**370. The small margin between profit and loss to projectors.** When a railroad is built entirely from the funds furnished by its promoters (or from the sale of stock) it will generally be a paying investment, although the rate of payment may be very small. The percentage of receipts that is demanded for actual operating expenses is usually about 67%. The remainder will usually pay a reasonable interest on the total capital involved. But the operating expenses are frequently 90 and even 100% of

the gross receipts. In such cases even the bondholders do not get their due and the stockholders have absolutely nothing. Therefore the stockholder's interest is very speculative. A comparatively small change in the business done (as is illustrated numerically in § 372) will not only wipe out altogether the dividend—taken from the last small percentage of the total receipts and which may equal 50% or more of the capital stock *actually paid in*—but it may even endanger the bondholders' security and cause them to foreclose their mortgage. In such a case the stockholders' interest is usually entirely lost. It does not alter the essential character of the above-stated relations that the stockholders sometimes protect themselves somewhat by buying bonds. By so doing they simply decrease their risk and also decrease the possible profit that might result from the investment of a given total amount of capital.

**371. Extent to which a railroad is a monopoly.** It is a popular fallacy that a railroad, when not subject to the direct competition of another road, has an absolute monopoly—that it controls “all the traffic there is” and that its income will be practically independent of the facilities afforded to the public. The growth of railroad traffic, like the use of the so-called necessities or luxuries of life, depends entirely on the supply and the cost (in money or effort) to obtain it. A large part of railroad traffic belongs to the unnecessary class—such as traveling for pleasure. Such traffic is very largely affected by mere matters of convenience, such as well-built stations, convenient terminals, smooth track, etc. The freight traffic is very largely dependent on the possibility of delivering manufactured articles or produce at the markets so that the *total* cost of production and transportation shall not exceed the total cost in that same market of similar articles obtained elsewhere. The creation of facilities so that a factory or mine may successfully compete with other factories or mines will develop such traffic. The receipts from such a traffic may render it possible to still further develop facilities which will in return encourage further business. On the other hand, even the partial withdrawal of such facilities may render it impossible for the factory or mine to compete successfully with rivals; the traffic furnished by them is completely cut off and the railroad (and indirectly the whole community) suffers correspondingly. The “strictly necessary” traffic is thus so small that few railroads could pay

their operating expenses from it. The dividends of a road come from the last comparatively small percentage of its revenue, and such revenue comes from the "unnecessary" traffic which must be coaxed and which is so easily affected by apparently insignificant "conveniences."

**372. Profit resulting from an increase in business done; loss resulting from a decrease.** In a subsequent chapter it will be shown that a large portion of the operating expenses are independent of small fluctuations in the business done and that the operating expenses are roughly two thirds of the gross revenue. Assume that by changes in the alignment the business obtained has been increased (or diminished) 10%. Assume for simplicity that the operating expenses on the revised track are the same as on the route originally planned; also that the cost of the track is the same and hence the fixed charges are assumed to be constant for all the cases considered. Assume the fixed charges to be 28%. The additional business, when carried in cars otherwise but partly filled will hardly increase the operating expenses by a measurable amount. When extra cars or extra trains are required, the cost will increase up to about 60% of the average cost per train mile. We may say that 10% increase may in general be carried at a rate of 40% of the average cost of the traffic. A reduction of 10% in traffic may be assumed to reduce expenses a similar amount. The effect of the change in business will therefore be as follows:

|                                | Business increased 10%.            | Business decreased 10%.            |
|--------------------------------|------------------------------------|------------------------------------|
| Operating exp. = 67            | $67(1 + 10\% \times 40\%) = 69.68$ | $67(1 - 10\% \times 40\%) = 64.32$ |
| Fixed charges = 28             | 28.00                              | 28.00                              |
| 95                             | 97.68                              | 92.32                              |
| Total income... 100            | Income..... 110.00                 | Income..... 90.00                  |
| Available for dividends..... 5 | Available for dividends..... 12.32 | Deficit..... 2.32                  |

In the one case the increase in business, which may often be obtained by judicious changes in the alignment or even by better management without changing the alignment, more than doubles the amount available for dividends. In the other case the profits are gone, and there is an absolute deficit. The above is a numerical illustration of the argument, previously

stated, of the small margin between profit and loss to the original projectors.

**373. Estimation of probable volume of traffic and of probable growth.** Since traffic and traffic facilities are mutually interdependent and since a large part of the normal traffic is merely potential until the road is built, it follows that the traffic of a road will not attain its normal volume until a considerable time after it is opened for operation. But the estimation even of this normal volume is a very uncertain problem. The estimate may be approached in three ways:

1st. The actual gross revenue derived by all the railroads in that section of the country (as determined by State or U. S. Gov. reports) may be divided by the total population of the section and thus the average annual expenditure per head of population may be determined. A determination of this value for each one of a series of years will give an idea of the normal rate of growth of the traffic. Multiplying this annual contribution by the population which may be considered as tributary gives a *valuation* of the possible traffic. Such an estimate is unreliable (*a*) because the *average* annual contribution may not fit that particular locality, (*b*) because it is very difficult to correctly estimate the number of the true tributary population especially when other railroads encroach more or less into the territory. Since a rough value of this sort may be readily determined, it has its value as a check, if for nothing else.

2d. The actual revenue obtained by some road whose circumstances are as nearly as possible identical with the road to be considered may be computed. The weak point consists in the assumption that the character of the two roads is identical or in incorrectly estimating the allowance to be made for observed differences. The method of course has its value as a check.

3d. A laborious calculation may be made from an actual study of the route—determining the possible output of all factories, mines, etc., the amount of farm produce and of lumber that might be shipped, with an estimate of probable passenger traffic based on that of like towns similarly situated. This method is the best when it is properly done, but there is always the danger of leaving out sources of income—both existent and that to be developed by traffic facilities, or, on the other hand, of overestimating the value of expected traffic. In the



following tabular form are shown the population, gross receipts, receipts per head of population, mileage, earnings per mile of line operated, and mileage per 10,000 of population for the whole United States. It should be noted that the values are only *averages*, that individual variations are large, and that only a very rough dependence may be placed on them as applied to any particular case.

| Year.   | Population<br>(estimated). | Gross<br>receipts. | Receipts<br>per head<br>of popu-<br>lation. | Mileage.† | Earnings<br>per mile<br>of line<br>operated. | Mileage<br>per<br>10,000<br>popula-<br>tion.‡ |
|---------|----------------------------|--------------------|---------------------------------------------|-----------|----------------------------------------------|-----------------------------------------------|
| 1888... | 60,100,000                 | \$910,621,220      | \$15.15                                     | 136,884   | \$6653                                       | 24.94                                         |
| 1889... | 61,450,000                 | 964,816,129        | 15.81                                       | 153,385   | 6290                                         | 25.67                                         |
| 1890... | *62,801,571                | 1051,877,632       | 16.75                                       | 156,404   | 6725                                         | 26.05                                         |
| 1891... | 64,150,000                 | 1096,761,395       | 17.10                                       | 161,275   | 6801                                         | 26.28                                         |
| 1892... | 65,500,000                 | 1171,407,343       | 17.89                                       | 162,397   | 7213                                         | 26.19                                         |
| 1893... | 68,850,000                 | 1220,751,874       | 18.26                                       | 169,780   | 7190                                         | 26.40                                         |
| 1894... | 68,200,000                 | 1073,361,797       | 15.74                                       | 175,691   | 6109                                         | 26.20                                         |
| 1895... | 69,550,000                 | 1075,371,462       | 15.46                                       | 177,746   | 6050                                         | 25.97                                         |
| 1896... | 70,900,000                 | 1150,169,376       | 16.22                                       | 181,983   | 6320                                         | 25.78                                         |
| 1897... | 72,350,000                 | 1122,089,773       | 15.53                                       | 183,284   | 6122                                         | 25.53                                         |
| 1898... | 73,600,000                 | 1247,325,621       | 16.95                                       | 184,618   | 6755                                         | 25.32                                         |
| 1899... | 74,950,000                 | 1313,610,118       | 17.53                                       | 187,535   | 7005                                         | 25.25                                         |
| 1900... | *76,295,220                | 1480,673,054       | 19.41                                       | 190,496   | 7776                                         | 24.96                                         |

\* Actual.

† Excludes a small percentage not reporting "gross receipts."

‡ Actual mileage.

The probable growth in traffic, after the traffic has once attained its normal volume, is a small but almost certain quantity. In the above tabular form this is indicated by the gradual growth in "receipts per head of population" from 1888 to 1893. Then the sudden drop due to the panic of 1893 is clearly indicated, and also the gradual growth in the last few years. Even in England, where the population has been nearly stationary for many years, the growth though small is unmistakable. On the other hand the growth in some of the Western States has been very large. For example, the gross earnings per head of population in the State of Iowa increased from \$1.42 in 1862 to \$10.00 in 1870, and to \$19.46 in 1884.

There will seldom be any justification in building to accommodate a larger business than what is "in sight." Even if it could be anticipated with certainty that a large increase in

business would come in ten years, there are many reasons why it would be unwise to build on a scale larger than that required for the business to be immediately handled. Even though it may cost more in the future to provide the added accommodations (*e.g.* larger terminals, engine-houses, etc.), the extra expense will be nearly if not quite offset by the interest saved by avoiding the larger outlay for a period of years which may often prove much longer than was expected. A still more important reason is the avoidance of uselessly sinking money at a time when every cent may be needed to insure the success of the enterprise as a whole.

**374. Probable number of trains per day. Increase with growth of traffic.** The number of passenger trains per day cannot be determined by dividing the total number of passengers estimated to be carried per day by the capacity of the cars that can be hauled by one engine. There are many small railroads, running three or four passenger trains per day each way, which do not carry as many passengers all told as are carried on one heavy train of a trunk line. But because the bulk of the passenger traffic, especially on such light-traffic roads, is "unnecessary" traffic (see § 371) and must be encouraged and coaxed, the trains must be run much more frequently than mere capacity requires. The minimum number of passenger trains per day on even the lightest-traffic road should be two. These need not necessarily be passenger trains exclusively. They may be mixed trains.

The number required for freight service may be kept more nearly according to the actual tonnage to be moved. At least one local freight will be required, and this is apt to be considerably within the capacity of the engine. Some very light-traffic roads have little else than local freight to handle, and on such there is less chance of economical management. Roads with heavy traffic can load up each engine quite accurately according to its hauling capacity and the resulting economy is great. Fluctuations in traffic are readily allowed for by adding on or dropping off one or more trains. Passenger trains must be run on regular schedule, full or empty. Freight trains are run by train-despatcher's orders. A few freight trains per day may be run on a nominal schedule, but all others will be run as extras. The criterion for an increase in the number of passenger trains is impossible to define by set rules. Since it should always



come before it is absolutely demanded by the train capacity being overtaxed, it may be said in general terms that a train should be added when it is believed that the consequent increase in facilities will cause an increase in traffic the value of which will equal or exceed the added expense of the extra train.

**375. Effect on traffic of an increase in facilities.** The term facilities here includes everything which facilitates the transport of articles from the door of the producer to the door of the consumer. As pointed out before, in many cases of freight transport, the reduction of facilities below a certain point will mean the entire loss of such traffic owing to local inability to successfully compete with more favored localities. Sometimes owing to a lack of facilities a railroad company feels compelled to pay the cartage or to make a corresponding reduction on what would normally be the freight rate. In competitive freight business such a method of procedure is a virtual necessity in order to retain even a respectable share of the business. Even though the railroad has no direct competitor, it must if possible enable its customers to meet their competitors on even terms. In passenger business the effect of facilities is perhaps even more marked. The pleasure travel will be largely cut down if not destroyed. It is on record that a railroad company once ordered the manager of a station restaurant to largely increase the attractions at that restaurant (as a method of attracting traffic) and agreed to pay the expected resulting loss. The net result was not only a large increase in railroad business (as was expected), but even an increase in the profits of the restaurant.

**376. Loss caused by inconvenient terminals and by stations far removed from business centers.** This is but a special case of the subject discussed just in the preceding paragraph. The competition once existing between the West Shore and the New York Central was hopeless for the West Shore from the start. The possession of a terminal at the Grand Central Station gave the New York Central an advantage over the West Shore with its inconvenient terminal at Weehawken which could not be compensated by any obtainable advantage by the West Shore. This is especially true of the passenger business. The through freight business passing through or terminating at New York is handled so generally by means of floats that the disadvantage in this respect is not so great. The

enormous expenditure (roughly \$10,000,000) made by the Pennsylvania R. R., on the Broad Street Station (and its approaches) in Philadelphia, a large part of which was made in crossing the Schuylkill River and running to City Hall Square, rather than retain their terminal in West Philadelphia, is an illustration of the policy of a great road on such a question. The fact that the original plan and expenditure has been very largely increased since the first construction proves that the management has not only approved the original large outlay, but saw the wisdom of making a very large increase in the expenditure.

The construction of great terminals is comparatively infrequent and seldom concerns the majority of engineers. But an engineer has frequently to consider the question of the location of a way station with reference to the business center of the town. The following points may (or may not) have to be considered, and the real question consists in striking a proper balance between conflicting considerations.

(1) During the early history of a railroad enterprise it is especially needful to avoid or at least postpone all expenditures which are not demonstrably justifiable.

(2) The ideal place for a railroad station is a location immediately contiguous to the business center of the town. The location of the station even one fourth of a mile from this may result in a loss of business. Increase this distance to one mile and the loss is very serious. Increase it to five miles and the loss approaches 100%.

(3) The cost of the ideal location and the necessary right of way may be a very large sum of money for the new enterprise. On the other hand the increase in property values and in the general prosperity of the town, caused by the railroad itself, will so enhance the value of a more convenient location that its cost at some future time will generally be extravagant if not absolutely prohibitory. The original location is therefore under ordinary conditions a finality.

(4) To some extent the railroad will cause a movement of the business center toward it, especially in the establishment of new business, factories, etc., but the disadvantages caused to business already established is permanent.

(5) In any attempt to compute the loss resulting from a location at a given distance from the business center it must be

recognized that each problem is distinct in itself and that any change or growth in the business of the town changes the amount of this loss.

The argument for locating the station at some distance from the center of the town may be based on (a) the cost of right of way, thus involving the question of a large initial outlay, (b) the cost of very expensive construction (e.g. bridges), again involving a large initial outlay, (c) the avoidance of excessive grade into and out of the town. It sometimes happens that a railroad is following a line which would naturally cause it to pass at a considerable elevation above (rarely below) the town. In this case there is to be considered not only the possible greater initial cost, but the even more important increase in operating cost due to the introduction of a very heavy grade. To study such a case, compute the annual increase in operating expenses due to the additional grade, curvature, and distance; add to this the annual interest on the increased initial cost (if any) and compare this sum with the estimated annual loss due to the inconvenient location. The estimation of the increase in operating expenses is discussed in a subsequent chapter. The loss of business due to inconvenient location can only be guessed at. Wellington says that at a distance of one mile the loss would average 25%, with upper and lower limits of 10 and 40%, depending on the keenness of the competition and other modifying circumstances. For each additional mile reduce 25% of the preceding value. While such estimates are grossly approximate, yet with the aid of sound judgment they are better than nothing and may be used to check gross errors.

**377. General principles which should govern the expenditure of money for railroad purposes.** It will be shown later that the elimination of grade, curvature, and distance have a positive money value; that the reduction of ruling grade is of far greater value; that the creation of facilities for the handling of a large traffic is of the highest importance and yet the added cost of these improvements is sometimes a large percentage of the cost of *some road* over which it would be physically possible to run trains between the termini.

The subsequent chapters will be largely devoted to a discussion of the value of these details, but the general principles governing the expenditure of money for such purposes may be stated as follows:

1. No money should be spent (beyond the unavoidable minimum) unless it may be shown that the addition is in itself a profitable investment. The additional sum may not wreck the enterprise and it may add something to the value of the road, but unless it adds more than the improvement costs it is not justifiable.

2. If it may be positively demonstrated that an improvement will be more valuable to the road than its cost, it should certainly be made even if the required capital is obtained with difficulty. This is all the more necessary if the neglect to do so will permanently hamper the road with an operating disadvantage which will only grow worse as the traffic increases.

3. This last principle has two exceptions: (*a*) the cost of the improvement may wreck the whole enterprise and cause a total loss to the original investors. For, unless the original promoters can build the road and operate it until its stock has a market value and the road is beyond immediate danger of a receivership, they are apt to lose the most if not all of their investment; (*b*) an improvement which is very costly although unquestionably wise may often be postponed by means of a cheap temporary construction. Cases in point are found at many of the changes of alignment of the Pennsylvania R. R., the N. Y., N. H. & H. R. R., and many others. While some of the cases indicate faulty original construction, at many of the places the original construction was wise, considering the then scanty traffic, and now the improvement is wise considering the great traffic.

## CHAPTER XX.

### OPERATING EXPENSES.

**378. Distribution of gross revenue.** When a railroad comprises but one single property, owned and operated by itself, the distribution of the gross revenue is a comparatively simple matter. The operating expenses then absorb about two thirds of the gross revenue; the fixed charges (chiefly the interest on the bonds) require about 25 or 30% more, leaving perhaps 3 to 8% (more or less) available for dividends. A recent report on the Fitchburg R. R. shows the following:

|                                                                   |                    |               |
|-------------------------------------------------------------------|--------------------|---------------|
| Operating expenses. ....                                          | \$5,083,571        | 69.1%         |
| Fixed charges. ....                                               | 1,567,640          | 21.3%         |
| Available for dividends, surplus, or permanent improvements. .... | 708,259            | 9.6%          |
| Total revenue. ....                                               | <u>\$7,359,470</u> | <u>100.0%</u> |

But the financial statements of a large majority of the railroad corporations are by no means so simple. The great consolidations and reorganizations of recent years have been effected by an exceedingly complicated system of leases and sub-leases, purchases, "mergers," etc., whose forms are various. Railroads in their corporate capacity frequently own stocks and bonds of other corporations (railroad properties and otherwise) and receive, as part of their income, the dividends (or bond interest) from the investments.

In consequence of this complication, the U. S. Interstate Commerce Commission presents a "condensed income account" of which the following is a sample (1899):

|                                                                            |                    |
|----------------------------------------------------------------------------|--------------------|
| Gross earnings from operation (received by station-agents, etc.) . . . . . | \$1,313,610,118    |
| Less operating expenses (fuel, wages, etc.) . . . . .                      | 856,968,999        |
| Income from operation. . . . .                                             | <u>456,641,119</u> |
| Income from other sources (lease of road, stocks, bonds, etc.) . . . . .   | 148,713,983        |
| Total income. . . . .                                                      | <u>605,355,102</u> |

|                                                                                    |             |
|------------------------------------------------------------------------------------|-------------|
| Total deductions from income (interest, rents for lease of road, taxes, etc.)..... | 441 200,289 |
| Net income. ....                                                                   | 164,154.813 |
| Total dividends (including "other payments")..                                     | 111,089,936 |
| Surplus from operations. ....                                                      | 53,064,877  |

In the above account an item of *income* (e.g. lease of road) reported by one road will be reported as a "deduction from income" by the road which leases the other.

The above statement may be reduced to an income account of all the railways *considered as one system*. We then have

|                                                                                                                                               |               |        |
|-----------------------------------------------------------------------------------------------------------------------------------------------|---------------|--------|
| Operating expenses. ....                                                                                                                      | \$856,968,999 |        |
| Salaries and maintenance of leased lines<br>(really operating expenses, but considered above as fixed charges against the leasing lines)..... | 595,192       |        |
|                                                                                                                                               | 857,564,191   | 64.1%  |
| Net interest and taxes.....                                                                                                                   | 295,098,014   | 22.0%  |
| Available for dividends, adjustments, and improvements. ....                                                                                  | 186,992,909   | 13.9%  |
|                                                                                                                                               | 1,339,655,114 | 100.0% |
| Gross earnings from operation.....                                                                                                            | 1,313,610,118 |        |
| Clear income from investments (i.e., the balance of intercorporate payments and receipts on corporate investments).....                       | 26,044,996    |        |
|                                                                                                                                               | 1,339,655,114 |        |

Of the \$186,992,909, the amount disbursed as dividends to outside stockholders (besides that paid to railroads in their corporate capacity) was \$94,273,796. This left a balance of \$92,719,113 "available for adjustments and improvements." Of this, part was spent in permanent improvements, part was advanced to cover deficits in the operation of weak lines and more than half was left as "surplus," i.e. working capital.

The percentages of the gross revenue which are devoted to operating expenses, fixed charges, and dividends are not *necessarily* an indication of creditable management or the reverse. Causes utterly beyond the control of the management, such as the local price of coal, may abnormally increase certain items of expense, while ruinous competition may cut down the gross revenue so that little or nothing is left for dividends. A favorable location will sometimes make a road prosperous



in spite of bad management. On the other hand, the highest grade of skill will fail to keep some roads out of the hands of a receiver.

**379. Fourfold distribution of operating expenses.** The distribution of operating expenses here used is copied from the method of the Interstate Commerce Commission. The aim is to divide the expenses into groups which are as mutually independent and distinct as possible—although, as will be seen later, a change in one item of expense will variously affect other items. The groups are:

|                                                                                                                                                                                                                                                                                                                                                                               | Average value. |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| 1. Maintenance of way and structures. ....                                                                                                                                                                                                                                                                                                                                    | 20.662%        |
| The values for five years have an extreme range of about 1.2%. The subdivisions of this group and of the others will be given later.                                                                                                                                                                                                                                          |                |
| 2. Maintenance of equipment. ....                                                                                                                                                                                                                                                                                                                                             | 16.892%        |
| Extreme range of 1.834%. The tendency has been for this item to grow larger, not only in absolute amount but in percentage of total expenditure.                                                                                                                                                                                                                              |                |
| 3. Conducting transportation. ....                                                                                                                                                                                                                                                                                                                                            | 57.793%        |
| This item has been growing <i>relatively</i> less. During (and immediately after) the panic of 1893, the maintenance of way and of equipment was made as small as possible, which made the cost of conducting transportation <i>relatively</i> larger. During the recent more prosperous years deficiencies of equipment have been made up, making this item relatively less. |                |
| 4. General expenses. ....                                                                                                                                                                                                                                                                                                                                                     | 4.653%         |
| A nearly constant item.                                                                                                                                                                                                                                                                                                                                                       |                |
|                                                                                                                                                                                                                                                                                                                                                                               | <hr/> 100.000% |

The above percentages represent the averages given by the reports for the five years from 1895 to 1899 inclusive.

**380. Operating expenses per train-mile.** The reports of the U. S. Interstate Commerce Commission give the average cost per train-mile for every railroad in the United States. Although there are wide variations in these values, it is remarkable that the very large majority of roads give values which agree to within a small range, and that within this range are found not only the great trunk lines with their enormous train mileage, but also roads with very light traffic.

In the following tabular form is shown a statement taken from the report for 1898 of ten of the longest railroads in the United States and, in comparison with them, a corresponding statement



regarding ten more roads selected *at random*, except in the respect that each had a mileage of less than 100 miles. Although the extreme variations are greater, yet there is no very marked difference in the general values for operating expenses per train-mile, or in the ratio of expenses to earnings. The averages for the ten long roads agree fairly well with the averages for the whole country, but there would be no trouble (as is shown by some of the individual cases) in finding another group of ten short roads giving either greater or less average values than those given. And yet the tendency to uniform values, regardless of the mileage, is very striking.

| No. in report. |                                 | Mileage. | Operating expenses per train-mile. | Ratio $\frac{\text{exp.}}{\text{earn.}}$ |
|----------------|---------------------------------|----------|------------------------------------|------------------------------------------|
|                | Whole United States. . . . .    | 186,396  | 0.956                              | 65.58                                    |
| 71             | Canadian Pacific. . . . .       | 6,568    | 0.854                              | 58.31                                    |
| 1465           | C., M. & St. P. . . . .         | 6,191    | 0.883                              | 58.94                                    |
| 1443           | C., B., & Q. . . . .            | 5,860    | 0.881                              | 60.87                                    |
| 1879           | Southern Pacific. . . . .       | 5,426    | 1.320                              | 58.70                                    |
| 1142           | Southern. . . . .               | 5,232    | 0.809                              | 65.32                                    |
| 1436           | Chicago & Northwestern. . . . . | 5,086    | 0.885                              | 63.35                                    |
| 1405           | A., T. & S. F. . . . .          | 4,565    | 0.917                              | 67.59                                    |
| 1560           | Northern Pacific. . . . .       | 4,524    | 1.177                              | 46.81                                    |
| 1495           | Great Northern. . . . .         | 3,860    | 1.101                              | 46.97                                    |
| 1264           | Illinois Central. . . . .       | 3,807    | .764                               | 63.56                                    |
|                | Average of ten . . . . .        |          | 0.969                              | 59.04                                    |

|      |                               |    |       |        |
|------|-------------------------------|----|-------|--------|
| 7    | Bennington & Rutland. . . . . | 59 | 0.582 | 71.42  |
| 105  | Mont. & Wells R. . . . .      | 44 | 0.828 | 83.96  |
| 167  | Balto. & Del. Bay. . . . .    | 45 | 1.098 | 102.83 |
| 234  | Cent. N. Y. & W. . . . .      | 63 | 0.451 | 91.17  |
| 888  | Man. & N. E. . . . .          | 99 | 0.739 | 54.49  |
| 1074 | Farmv. & Powh. . . . .        | 93 | 0.781 | 76.22  |
| 1284 | Lex. & East. . . . .          | 94 | 0.975 | 68.46  |
| 1540 | Manistique. . . . .           | 60 | 1.162 | 69.01  |
| 1812 | Wh. & Bl. River Val. . . . .  | 64 | .799  | 53.08  |
| 1979 | No. Pac. Coast. . . . .       | 88 | .769  | 66.58  |
|      | Average of ten . . . . .      |    | 0.819 | 73.72  |

The constancy of the average cost per train mile for several years past may be noted from the following tabular form.

The enforced economies after the panic of 1893 are well shown. The reduction generally took the form of a lowering of the standards of maintenance of way and of maintenance of

| Year.     | Average cost per train-mile. |
|-----------|------------------------------|
| 1890..... | 96.006                       |
| 1891..... | 95.707                       |
| 1892..... | 96.580                       |
| 1893..... | 97.272                       |
| 1894..... | 93.478                       |
| 1895..... | 91.829                       |
| 1896..... | 93.838                       |
| 1897..... | 92.918                       |
| 1898..... | 95.635                       |
| 1899..... | 98.390                       |
|           | 95.165                       |

equipment. The marked advance from 1897 to 1898 and to 1899 was largely caused by the necessity for restoring the roads to proper condition, replenishing worn-out equipment and providing additional equipment to handle the greatly increased volume of business.

In looking over the list, it may be noted that the cases where the operating expenses per train-mile and the ratio of expenses to earnings vary very greatly from the average are almost invariably those of the very small roads or of "junction roads" where the operating conditions are abnormal. For example, one little road, with a total length of 13 miles and total annual operating expenses of \$5312, spent but 22½¢. per train-mile, which precisely exhausted its earnings. As another abnormal case, a road 44 miles long spent \$3.81 per train-mile, which was nearly *fourteen* times its earnings. In another case a road 13 miles long earned \$7.76 per train-mile and spent \$6.03 (78%) on operating expenses, but the fixed charges were abnormal and the earnings were less than half the sum of the operating expenses and fixed charges. The *normal* case, even for the small road, is that the cost per train-mile and the ratio of operating expenses to earnings will agree fairly well with the average, and when there is a marked difference it is generally due to some abnormal conditions of expenses or of earning capacity.

**381. Reasons for uniformity in expenses per train-mile.** The chief reason is that, although on the heavy-traffic road everything is kept up on a finer scale, better roadbed, heavier rails, better rolling stock, more employees, better buildings, stations, and terminals, etc., yet the number of trains is so much greater that the divisor is just enough larger to make the average

cost about constant. This is but a general statement of a fact which will be discussed in detail under the different items of expense.

**382. Detailed classification of expenses with ratios to the total expense.** The Interstate Commerce Commission now publishes each year a classification with detailed summation for the cost of each item. These summations are made up from reports furnished by railroads which have (in the reports already made) represented about 94% of the total traffic handled. In the annexed tabular form (Table XX) are shown the percentages which each item bears to the total. The character of the changes from year to year in these ratios is very instructive and will be considered in the detailed discussion of the items which will follow.

Table XX is copied from the Interstate Commerce Commission report for 1899, pp. 88-90.

**383. Elements of the cost (with variations and tendencies) of the various items.** The I. C. C. report for the year ending June 30, 1895, was the first to include the distribution of expenses according to the present classification. The number of reports since then are too few to be of much value in determining the tendency to variation of the several items, and similar calculations made in previous years have by no means an equal reliability. Nevertheless the items as given are reliable and may be utilized, as far as any such computations are to be depended on, in estimating future expenses. A great deal of very interesting and instructive information may be derived from a study of the variations of these items, but the chief purpose of this discussion is to point out those elements of the cost of operating trains which may be affected by such changes of location as an engineer is able to make. There are some items of expense with which the engineer has not the slightest concern—nor will they be altered by any change in alignment or constructive detail which he may make. In the following discussion such items will be passed over with a brief discussion of the sub-items included.

#### MAINTENANCE OF WAY.

**384. Item 1. REPAIRS OF ROADWAY.** The item of repairs of roadway is very large—about half of the total cost of main-

tenance of way and structures. It includes the cost of frogs, switches, switch-stands, and interlocking signals. The distribution and laying of ties and rails, ballasting and tamping track, ditching, weeding, widening and protecting banks, the maintenance of snow-fences, dikes, and retaining walls, are also included. In short, any expense of maintaining the roadbed in condition which cannot be definitely assigned to one of the next few items will generally belong to this item—except perhaps those of item 10 (*q.v.*). The larger part of such items of expense is labor, and the variations will largely depend on the fluctuations in the wages of trackmen. Formerly these were much higher than now. About fifteen years ago they had dropped to what Wellington considered to be a permanent average of \$1.25 per day. In 1893 it had dropped to \$1.22, then in 1897 and 1898 to \$1.16. In 1899 it was raised to \$1.18.

In 1899 the *average* cost of this item per mile of main track was about \$480, but this figure, after all, is of but little value because, for the reason already given in general in §381, it will be found that the cost for any road varies almost exactly as the train-mileage and will average very closely to 11c. per train-mile, whether the traffic be heavy or light.

**385. Item 2. RENEWAL OF RAILS:** This item may be considered as having been withdrawn from the previous item simply because it is one of the largest of the single items and because its cost is very readily determined. It includes the cost of the rails, their inspection, and their delivery (but not their distribution). The item shows a large percentage of variation, the figures (percentage of total expenses) being 1.322, 1.391, 1.546, 1.444, and 1.499 by the last five reports. The drop from 1.546 in 1897 to 1.391 in 1898 was just 10%. These fluctuations are due first to that considerable fluctuation in the price of rails which railroads can hardly expect to escape, and secondly to variations in the standard of maintenance caused first by hard times, which are then followed by unusual expenditures in good times, or by the expenditures absolutely essential to restore the track to its former condition. The item includes all rails wherever used, whether on main track, siding, repair track, gravel track, on wharves or coal-docks, and even includes guard-rails. But it does *not* include any rail attachments such as joints, frogs, switches, etc. The rate of rail wear under various conditions has already been discussed in Chapter IX.

**386. Item 3. RENEWAL OF TIES.** As with the previous item, this item is simply a detachment from the general item, repairs of roadway. As with rails, the cost of laying and distributing the ties is not included, but the cost of tie-plates and tie-plugs, also chemical treatment for preservation, if such is used, is included in this item. While the cost will vary considerably between different roads on account of first cost, kind of wood, climate, etc., the item for any one road for a period of years cannot vary greatly, unless there is a marked change in the standard of maintenance. The actual cost of such work has already been discussed in Chapter VIII.

**387. Item 4. REPAIRS AND RENEWALS OF BRIDGES AND CULVERTS.** This item includes not only the maintenance cost of all bridges, trestles, viaducts, and culverts, but of all piers, abutments, riprapping, etc., necessary to maintain them, and even the cost of operating drawbridges. The locating engineer is not concerned with this item, except as he may consider that some distance which is to be added (or cut out) has the average number of culverts and bridges. With culverts and small bridges there would be little or no error in such an assumption, but if there were any large bridges on the portion of track under discussion, they would need special consideration.

**388. Items 5 to 10. REPAIRS AND RENEWALS OF FENCES, ROAD CROSSINGS, AND CATTLE-GUARDS—OF BUILDINGS AND FIXTURES—OF DOCKS AND WHARVES—OF TELEGRAPH PLANT; STATIONERY AND PRINTING; AND "OTHER EXPENSES."** These items in the aggregate amount to but 3% of the average cost per train-mile. The fluctuations have so small an effect on the average cost per train-mile that they may be neglected. In item 5 are included not only those things which are specifically mentioned, but also those structures which in general are not directly affected by the running of trains. For example, "road crossings" include not only the maintenance of highway crossings at grade, but also overhead highway crossings and whatever a railroad may have to pay for the maintenance of a bridge by which another railroad crosses it. On the other hand, the maintenance of a bridge by which a railroad crosses another road (highway or railroad) is charged to bridges. The effect (if any) of these items on any changes in construction which an engineer may make will be specifically discussed in the succeeding chapters.

## MAINTENANCE OF EQUIPMENT.

**389. Item 11. SUPERINTENDENCE.** This item includes those fixed charges in superintendence which do not fluctuate with small variations in business done. It includes the salaries of superintendent of motive power, master mechanic, master car-builder, foremen, etc., but does not include that of road foremen of engines nor enginemen. In a general way the item is proportional to the general scale of business of the road, but does not fluctuate with it.

**390. Item 12. REPAIRS AND RENEWALS OF LOCOMOTIVES.** This item must be studied by the locating engineer in order to determine the effect on locomotive repairs and renewals of an addition to distance (considered in Chapter XXI), the effect (chiefly in wheel wear) of a reduction in curvature (considered in Chapter XXII), or the effect of grade (considered in Chapter XXIII). In studying the effect of grade, the policy of adopting heavier locomotives and the effect of this on this item must also be considered. This item includes the expenses of work whose effect is supposed to last for an indefinite period. It does not include the expense of cleaning out boilers, packing cylinders, etc., which occurs regularly and which is charged to item 21, round-house men. It does include all current repairs, general overhauling, and even the replacement of old and worn-out locomotives by new ones to the extent of keeping up the original standard and number. Of course additions beyond this must be considered as so much increase in the original capital investment. As a locomotive becomes older the *annual* repair charge becomes a larger percentage on the first cost, and it may become as much as one fourth and even one third of the first cost. When a locomotive is in this condition it is usually consigned to the scrap-pile; the annual cost for maintenance becomes too large an item for its annual mileage. The effect on expenses of increasing the weight of engines is too complicated a problem to admit of precise solution, but certain elements of it may be readily computed. While the cost of repairs is greater for the heavier engines, the increase is only about one half as fast as the increase in weight—some of the sub-items not being increased at all.



**391. Items 13, 14, 15. REPAIRS AND RENEWALS OF PASSENGER CARS, OF FREIGHT CARS, AND OF WORK CARS.** As with engine repairs, the item excludes consumable supplies (oil, waste, illuminating oil or gas, ice, etc.), but includes in general all items necessary to maintain the cars up to the full standard of condition and number, and even to replace old worn-out cars by new. When, as is frequently the case with both cars and locomotives, the new rolling stock is larger, better, and of a higher standard than that which is replaced, the difference in cost should be added to capital investment. The chief concern of the locating engineer regarding this item is the effect on car repairs of additional distance, of variations in curvature (affecting wheel wear chiefly), and of grade (affecting the draft-gear and general wear and tear). These items will be considered under their proper heads in the following chapters.

**392. Items 16, 17, 18, and 19. REPAIRS AND RENEWALS OF MARINE EQUIPMENT—OF SHOP MACHINERY AND TOOLS; STATIONERY AND PRINTING; OTHER EXPENSES.** The location of the road along the line has no connection with the maintenance of marine equipment. The maintenance of shop machinery and tools can only be affected as the work of repairs of rolling stock fluctuates, and of course in a much smaller ratio. No change which an engineer can effect will have any appreciable influence on this item.

The other items are too small and have too little connection with location to be here discussed except as it may be considered that they vary with train mileage, which an engineer may influence (see Chapter XXIII, Grades).

#### CONDUCTING TRANSPORTATION.

**393. Item 20. SUPERINTENDENCE.** As with item 11, this item is not subject to minor fluctuations in business, but only varies with changes in the general scale of the business of the road.

**394. Item 21. ENGINE AND ROUND-HOUSE MEN.** This item includes the wages of engineers, firemen, and also all men employed around the engine-houses except those who are making such repairs as should be charged to maintenance of equipment (item 12). The item is a large one, but is only affected by one class of change of location—a difference in length of line. The



wages of the round-house men constitute but a small percentage of this item, and the wages of the enginemen vary almost directly as the mileage. On very short roads, where the number of round trips which may properly constitute a day's work is definitely limited and on which there is but little night or Sunday work, the wages may be practically by the day, and a variation in length of several hundred feet or even a few miles in the length of the road may make practically no difference in the wages paid. But on the larger roads, operated by divisions, on which (especially in freight work) there is no distinction of day or night, week day or Sunday, the varying length of divisions is equalized by calling them  $1\frac{1}{2}$  or  $1\frac{1}{4}$  runs, a "run" usually being considered as about 100 miles. The enginemen are then paid according to the number of runs made per month. The effect on this item of variations in distance is discussed more fully in Chapter XXI.

**395. Item 22. FUEL FOR LOCOMOTIVES.** The item includes the entire cost of the fuel until it is placed in the engine-tender. The cost therefore includes not only the first cost at the point of delivery to the road, but also the expense of hauling it over the road from the point of delivery to the various coaling stations and the cost of operating the coal-pockets from which it is loaded on to the tenders. Although the cost is fairly regular for any one road, it is exceedingly variable for different roads. Roads running through the coal regions can often obtain their coal for eighty or ninety cents per ton. Other roads far removed from the coal-mines have been compelled to pay six dollars per ton. In the three succeeding chapters there will be considered in detail the effect on fuel consumption of variations in location. It will be shown that fuel consumption is quite largely independent of distance and the number of cars hauled.

**396. Items 23, 24, and 25. WATER-SUPPLY; OIL, TALLOW, AND WASTE; OTHER SUPPLIES FOR LOCOMOTIVES:** The cost of the water-supply is quite largely a fixed charge except where it is supplied by municipalities at meter rates. The consumption of all these supplies will vary nearly as the engine-mileage.

**397. Item 26. TRAIN SERVICE.** This item is one of the largest single items and includes in general the wages of all the train-hands except the enginemen. As with enginemen, they are paid according to the number of runs. The item is therefore of importance to the locating engineer from the one

standpoint of distance, and even then only when the variation in distance which is considered will affect the classification of the run and therefore the rate of pay for that run.

**398. Item 27. TRAIN SUPPLIES AND EXPENSES.** These include the large list of consumable supplies such as lubricating oil, illuminating oil or gas, ice, fuel for heating, cleaning material, etc., which are used on the cars, and not on the locomotives. The consumption of some of these articles is chiefly a matter of time;—in other cases it is a function of the mileage.

**399. Items 28, 29, 30, and 31. SWITCHMEN, FLAGMEN, AND WATCHMEN; TELEGRAPH EXPENSES; STATION SERVICE; AND STATION SUPPLIES.** These items will be proportional to the general scale of business of the road, but are independent of small fluctuations in business. The main items are obvious from the titles. Many sub-items, which are very small or are of occasional or accidental occurrence, are also included under these items for lack of a better classification.

**400. Items 32, 33, and 34. SWITCHING CHARGES—BALANCE CAR MILEAGE—BALANCE; HIRE OF EQUIPMENT.** The first of these is a charge paid by a road to other corporations for switching done for the road. The locating engineer is not concerned with this item.

**CAR MILEAGE.** This is a charge paid by a road for the use of the cars (chiefly freight cars) of another road. To save the rehandling of freight at junctions the policy of running freight cars on to foreign roads is very extensively adopted. Since the foreign road receives (ultimately) its mileage proportion of the freight charge, it justly pays the home road a rate which is supposed to represent the value of the use of a freight car for so many miles. The foreign road then loads up the freight car with freight consigned to some point on the home road and sends it back, again paying mileage for the distance traveled on the foreign road, a proportional freight charge having been received for that service. By a clearing-house arrangement the various roads settle their debit and credit accounts with each other by the payment of a balance. Such is the simple theory. In practice the cars are not sent back to the home road at once, but wander off according to the local demand. As long as strict account is kept of the movements of every car and the home road is paid a charge which really covers the value of such service, no harm is done the home road except

that sometimes, when business has suddenly increased, the home road cannot get enough cars to handle its business. The value of a car is then abnormally above its ordinary value and the home road suffers for lack of the rolling stock which belongs to it. The charge being paid according to mileage, any variations of distance have a direct bearing on this item.

**HIRE OF EQUIPMENT.** This may refer to locomotives or cars which are hired for a special service, or, on very poor roads, it may refer to equipment, which is hired rather than purchased. The locating engineer has no concern with this item.

**401. Items 35, 36, and 37. LOSS AND DAMAGE; INJURIES TO PERSONS; CLEARING WRECKS.** These expenses are fortuitous and bear no absolute relation to road-mileage or train-mileage. While they depend largely on the standards of discipline on the road, even the best of roads have to pay some small proportion of their earnings to these items. The possible relation between curvature and accidents is discussed in Chapter XXII, but otherwise the locating engineer has no concern with these items.

**402. Items 38 to 53.** All of the remaining items (for a list of which see § 382) are of no concern to the locating engineer. They are either general expenses (such as taxes) or are special items (such as the operation of marine equipment) which will not be changed by variations in distance, curvature, or grades which a locating engineer may make. They will not therefore be further discussed.

## CHAPTER XXI.

### DISTANCE.

403. **Relation of distance to rates and expenses.** Rates are usually based on distance traveled, on the apparent hypotheses that each additional mile of distance adds its proportional amount not only to the service rendered but also to the expense of rendering it. Neither hypothesis is true. The value of the service of transporting a passenger or a ton of freight from *A* to *B* is a more or less uncertain gross amount depending on the necessities of the case and independent of the exact distance. Except for that very small part of passenger traffic which is undertaken for the mere pleasure of traveling the general object to be attained in either passenger or freight traffic is the transportation from *A* to *B*, however it is attained. A mile greater distance does not improve the service rendered; in fact, it consumes valuable time of the passengers and perhaps deteriorates the freight. From the standpoint of service rendered, the railroad which adopts a more costly construction and thereby saves a mile or more in the route between two places is thereby fairly entitled to additional compensation rather than have it cut down as it would be by a strict mileage rate. The actual value of the service rendered may therefore vary from an insignificant amount which is less than any reasonable charge (which therefore discourages such traffic) and its value in cases of necessity—a value which can hardly be measured in money. If the passenger charge between New York and Philadelphia were raised to \$5, \$10, or even \$20, there would still be some passengers who would pay it and go, because *to them* it would be worth \$5, \$10, or \$20, or even more. Therefore when they pay \$2.50 they are not paying what the service is worth to them. The service rendered cannot therefore be made a measure of the charge, nor is the service rendered proportional to the miles of distance.

The idea that the cost of transportation is proportional to

the distance is much more prevalent and is in some respects more justifiable, but it is still far from true. This is especially true of passenger service. The cost of transporting a single passenger is but little more than the cost of printing his ticket. Once aboard the train, it makes but little difference to the railroad whether he travels one mile or a hundred. Of course there are certain very large expenses due to the passenger traffic which must be paid for by a tariff which is rightfully demanded, but such expenses have no relation to the cost of an additional mile or so of distance inserted between stations. The same is true to a slightly less degree of the freight traffic. As shown later, the items of expense in the total cost of a train-mile, which are directly affected by a small increase in distance, are but a small proportion of the total cost.

**404. The conditions other than distance that affect the cost; reasons why rates are usually based on distance.** Curvature and minor grades have a considerable influence on the cost of transportation, as will be shown in detail in the next two chapters, but they are never considered in making rates. Ruling grades have a very large influence on the cost, but they are likewise disregarded in making rates. An accurate measure of the effect of these elements is difficult and complicated and would not be appreciated by the general public. Mere distance is easily calculated; the public is satisfied with such a method of calculation; and the railroads therefore adopt a tariff which pays expenses and profits even though the charges are not in accordance with the expenses or the service rendered.

An addition to the length of the line may (and generally does) involve curvature and grade as well as added distance. In this chapter is considered merely the effect of the added distance. The effect of grade and curvature must be considered separately, according to the methods outlined in succeeding chapters. The additional length considered is likewise assumed *not* to affect the business done nor the number of stations, but that it is a mere addition to length of track.

#### EFFECT OF DISTANCE ON OPERATING EXPENSES.

**405. Effect of slight changes in distance on maintenance of way.** With a few unimportant exceptions all the items of expense under maintenance of way and structures (see § 407)

will be increased directly as any increase in distance. This must certainly be true for items 1, 2, 3, and 5, which alone comprise about three fourths of the total expense for maintenance of way. If we assume that the proposed change of length involves no difference in the number of bridges, culverts, buildings, and fixtures, docks and wharves, we may consider items 4, 6, and 7 to be unaffected. This will generally be true for small changes in length, measured in feet. For larger differences, measured in miles, items 4 and 6 will vary nearly as the distance. The same may be said of items 9 and 10. The cost of maintaining the telegraph line will probably be increased about 60% of the unit cost. The effect of changes in distance on these various items of maintenance of way (as well as the other items of expense of a train-mile) will be tabulated in § 408.

**406. Effect on maintenance of equipment.** The relation between an increase in length of line and the expenses of items 11, 15, 17, 18, and 19 are quite indefinite. In some respects they would be unaffected by slight changes of distance. From other points of view there is no reason why the expenses should not be considered proportionate to the distance. For example, the added track will probably require as much work from the construction train as any other part of the road and is therefore responsible for as much of the "repairs and renewals of work-cars"—item 15. Fortunately all of these items are so small, even in the aggregate, that little error will be involved by either decision. It will therefore be assumed that these items are affected 100% for large additions in distance and but 50% for small additions.

Item 16 is evidently unaffected.

**Item 12.** Locomotives deteriorate (1) with age; (2) by expansion and contraction, especially of the fire-boxes, when fires are drawn and relighted; (3) on account of the strains due to stopping and starting; (4) the strains and wear of wheels due to curved track; (5) the additional stresses due to grade and change of grade: and (6) on account of the work of pulling on a straight level track. Observe that the first five causes have no direct relation to an addition of mere distance (the possible curvature or grade incident to the additional distance being a separate matter). How much of the total deterioration is due to the last cause? Wellington attacks this problem as follows: the records of engine-repair shops readily furnish



the proportionate cost of the repairs of boiler, running-gear, etc. An estimate is then made of the effect of each cause on each item. For example, the boiler is responsible for 20% of the repairs and renewals. Of this 7% (say one third) is assigned to "terminal service, getting up steam, making up trains," 4% to curvature and grades, 2% to "stopping and starting at way stations," and the other 7% to "distance on tangent between stations." The other items are treated similarly. Wellington says, "As this [subdivision of expenses] has been done with great care to get the best attainable authority for each (which it would occupy too much space to give in detail), the margin for possible error is not great enough to be of moment, although no absolute exactness can be claimed for it." His final estimate is that distance is responsible for 42% of the total cost of repairs and renewals. This value will therefore be used for all additional distances, great or small.

**Items 13 and 14.** The causes of deterioration of both passenger and freight cars may be classified exactly as above—omitting merely cause 2—the expansion and contraction due to firing. Considering that a large part of the repairs of freight cars is due to the draft-gear and brakes, which are affected chiefly by the heavy strains due to stopping and starting and to grades, while the repairs of wheels are largely due to the wear of wheels on curves, it is not surprising that he allows only 36% of the cost of repairs and renewals of freight cars to be due to straight distance. He made no direct estimate for passenger-cars, but points out the fact that the maintenance of the seats, furniture, and ornamentation make up much more than half the cost of passenger-car repairs. A large part of such deterioration is due to age and the weather, although that of the seats is largely a function of passenger wear and therefore of distance traveled. Although the items of deterioration in passenger cars is very different from those of freight cars yet if a similar calculation is made for passenger cars it will be found that the final figure is substantially the same as for freight cars and will here be so regarded.

**407. Effect on conducting transportation. Item 20.** This is evidently unaffected by small or even considerable additions to distance.

**Item 21.** Theoretically, train wages should vary as mileage. On the larger roads, where, especially in the freight service,



there is little or no distinction of day or night, week-day or Sunday, it is practically impossible to hire the trainmen to work between certain definite hours of the day and pay them accordingly, as is done with factory employees. As explained in Chapter XX, § 394, the system usually adopted of paying trainmen is such that small changes of distance (measured in feet) would not affect train wages. The wages of round-house men would not be affected under any conditions, and those of the engineers and of the trainmen (item 26) would not generally be affected unless the change of distance is very great—perhaps ten miles. Since items 21 and 26 are both very large, it will not do to ignore this item or to average it. The pay of round-house men is about 7% of item 21. We may therefore say that if the change in distance is so great that train wages will be affected item 21 will be affected 93% and item 26 will be affected 100%. For shorter changes of distance they will be unaffected.

**Item 22.** A surprisingly large percentage of the fuel consumed is not utilized in drawing a train along the road. Part of this loss is due to firing up, part is wasted when the engine is standing still, which is a large part of the total time. The policy of banking fires instead of drawing them reduces the injury resulting from great fluctuations in temperature, but the total coal consumed is about the same and we may therefore consider that almost a fireboxful of coal is wasted whether the fires are banked or drawn. The amount thus wasted (or at least *not* utilized in direct hauling) has been estimated at 5 to 10% of the whole consumption. Experiments\* have shown that an engine standing idle in a yard, protected from wind, well-jacketed, etc., will require from 25 to 32 lbs. of coal per hour simply to keep up steam. It has been found that the fastest express trains will lose *one fourth* of their total time between termini in stops, and freight trains on a single-track road will generally spend four hours per day on sidings. The waste of coal from this cause is estimated at 3 to 6% of the total consumption. The energy consumed in stopping and starting is very great. A train running 30 miles per hour has enough kinetic energy to move it on a level straight track more than two miles. Every time a train running at 30 miles per hour is stopped, enough energy is consumed by the brakes to run

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\* Wellington, *Economic Theory*, p. 207.

it from one to two miles. When starting, it will require an equal amount of work to restore that velocity, *in addition* to the ordinary resistances. It has been shown that on the Manhattan Elevated Railroad, where stops will average every three eighths of a mile, this cause alone will account for the consumption of nearly *three fourths* of the fuel. Of course on ordinary railroads the proportion is not nearly so great, but it is probably as much as 10 to 20% as an average figure. For a through express train making but few stops the figure would be small, except for the effect of "slow-downs." For suburban trains the proportion would be abnormally high. The fuel required to overcome the added resistances due to curvature and grade are of course exceedingly variable, depending on the particular alignment of the road considered. An approach to the truth may be made by considering the average curvature per mile for the roads of the United States and the average grades, and computing, by the methods given in subsequent chapters, the extra fuel consumed on account of such average conditions, and these items will apparently be responsible for 3% due to curvature and about 15% due to grades. Summarizing the above we have:

|                             |       |       |              |
|-----------------------------|-------|-------|--------------|
| Firing. ....                | 5 to  | 10%   |              |
| Wasted while still. ....    | 3 "   | 6%    |              |
| Stopping and starting. .... | 10 "  | 20%   |              |
| Average curvature. ....     | 3 "   | 3%    |              |
| Average grade. ....         | 15 "  | 15%   |              |
|                             | <hr/> | <hr/> |              |
|                             | 36    | 54    |              |
| Direct hauling. ....        | 64 "  | 46    | Average, 55% |
|                             | <hr/> | <hr/> |              |
|                             | 100   | 100   |              |

This shows that the addition of mere straight level distance would not increase the consumption of fuel more than 55% of the average consumption per mile.

**Items 23, 24, and 25.** If water is paid for by meter, the cost is strictly according to consumption, which would vary almost according to the number of engine-miles. When supplied from the company's own plant, as is usually the case, a slight increase will not appreciably affect the cost. Nothing is wasted during firing or while the engine is still. The use is therefore more nearly as the mileage, and the cost for an additional mile

may be considered as 50% of its average cost per train-mile. Items 24 and 25 will be considered similarly. Fortunately these items, whose variation with additional distance is somewhat obscure and variable, only aggregate a little over 1% of the cost of a train-mile and therefore a considerable percentage of error is of little or no importance.

**Item 26.** (See comments on item 21.)

**Item 27.** This item, as well as many other small items that follow, will be irregularly affected by a small increase in distance. It would appear equally wrong to say that they would be unaffected or to say that they will vary directly as the mileage. 50% will be allowed.

**Item 28.** The necessity for flagmen and watchmen varies in general as the mileage. An addition in distance is less apt to increase the number of switchmen. 50% of this item will be added.

**Item 29.** Telegraph expenses include the wages of operators (unaffected), and the special expenses due to offices and telegraph stations and to operating the line—the maintenance of the line being charged to item 8. This item will be but little affected, if at all, by additional distance, but 20% will be allowed. Items 30, 31, 32, and 34 are unaffected. Items 33, 35, 36, and 37 are affected 100%. Items 38 to 46 are unaffected.

The "general expenses" (items 46 to 53) will be unaffected.

**408. Estimate of total effect on expenses of small change in distance (measured in feet); estimate for distances measured in miles.** According to the accompanying compilation the cost of operating additional distance will be about 35% of the average cost per train-mile when the additional distance is small but will be about 56% if the additional distance is several miles. The figures may also be considered as the saving in the operating expenses resulting from a shortening of the line.

The average cost of a train-mile during the years from 1890 to 1899 varied from 91.8c. to 98.4c., with an average value of 95.2c. On this basis the above figures become 33.2 and 53.3 cents per train-mile respectively. Some trains run 365 days per year others but 313. The tendency is toward the larger figure and it will therefore be used in these calculations. The added cost per daily train per year for *each foot* of distance is

$$\frac{33.2 \times 365 \times 2}{5280} = 4.59c.$$



made of the above figures will not very greatly alter the final result. A numerical illustration of the value of saving distance will be given later.

#### EFFECT OF DISTANCE ON RECEIPTS.

**409. Classification of traffic.** There are various methods of classifying traffic, according to the use it is intended to make of the classification. The method here adopted will have reference to its competitive or non-competitive character and also to the method of division of the receipts on through traffic. Traffic may be classified first as "through" and "local"—through traffic being that traveling over two (or more) lines, no matter how short or non-competitive it may be; "local" traffic is that confined entirely to one road. A fivefold classification is however necessary—which is:

A. Non-competitive local—on one road with no choice of routes.

B. Non-competitive through—on two (or more) roads, but with no choice.

C. Competitive local—a choice of two (or more) routes, but the entire haul may be made on the home road.

D. Competitive through—direct competition between two or more routes each passing over two or more lines.

E. Semi-competitive through—a non-competitive haul on the home road and a competitive haul on foreign roads.

There are other possible combinations, but they all reduce to one of the above forms so far as their essential effect is concerned.

**410. Method of division of through rates between the roads run over.** Through rates are divided between the roads run over in proportion to the mileage. There may be terminal charges and possibly other more or less arbitrary deductions to be taken from the total amount received, but when the final division is made the remainder is divided according to the mileage. On account of this method of division and also because non-competitive rates are always fixed according to the distance, there results the unusual feature that, unlike curvature and grade, there is a compensating advantage in increased distance, which applies to all the above kinds of traffic except one (competitive local), and that the compensation is sometimes sufficient to make the added distance an actual

source of profit. It has just been proved that the cost of hauling a train an additional mile is only 35 to 56% of the average cost. Therefore in all non-competitive business (local or through) where the rate is according to the distance, there is an actual profit in all such added distance. In competitive local business, in which the rate is fixed by competition and has practically no relation to distance, any additional distance is dead loss. In competitive through business the profit or loss depends on the distances involved. This may best be demonstrated by examples.

411. Effect of a change in the length of the home road on its receipts from through competitive traffic. Suppose the home road is 100 miles long and the foreign road is 150 miles long. Then the home road will receive  $\frac{100}{100+150} = 40\%$  of the through rate.

Suppose the home road is lengthened 5 miles; then it will receive  $\frac{105}{105+150} = 41.176\%$  of the through rate. The traffic being competitive, the rate will be a fixed quantity regardless of this change of distance. By the first plan the rate received is 0.4% per mile; adding 5 miles, the rate for the original 100 miles may be considered *the same* as before; and that the additional 5 miles receive 1.176%, or 0.235% per mile. This is 59% of the original rate per mile, and since this is more than the cost per mile for the additional distance (see § 408), the added distance is evidently *in this case* a source of distinct profit. On the other hand, if the line is shortened 5 miles, it may be similarly shown that not only are the receipts lessened, but that the saving in operating expenses by the shorter distance is less than the reduction in receipts.

A second example will be considered to illustrate another phase. Suppose the home road is 200 miles long and the foreign road is 50 miles long. In this case the home road will receive

$\frac{200}{200+50} = 80\%$  of the through rate. Suppose the home road is

lengthened 5 miles; then it will receive  $\frac{205}{205+50} = 80.392\%$  of the through rate. By the first plan the rate received is 0.400% per mile; adding 5 miles, there is a surplus of 0.392, or 0.0784 per mile, which is but 19.6% of the original rate.



At this rate the extra distance evidently is not profitable, although it is not a dead loss—there is some compensation.

**412. The most advantageous conditions for roads forming part of a through competitive route.** From the above it may be seen that when a road is but a short link in a long competitive through route, an addition to its length will increase its receipts and increase them more than the addition to the operating expenses.

As the proportionate length of the home road increases the less will this advantage become, until at some proportion an increase in distance will just pay for itself. As the proportionate length grows greater the advantage becomes a disadvantage until, when the competitive haul is entirely on the home road, any increase in distance becomes a net loss without any compensation. It is therefore advantageous for a road to be a short link in a long competitive route; an increase in that link will be financially advantageous; if the total length is less than that of the competing line, the advantage is still greater, for then the rate received per mile will be greater.

**413. Effect of the variations in the length of haul and the classes of the business actually done.** The above distances refer to particular lengths of haul and are not necessarily the total lengths of the road. Each station on the road has traffic relations with an indefinite number of traffic points all over the country. The traffic between each station on the road and any other station in the country between which traffic may pass therefore furnishes a new combination, the effect of which will be an element in the total effect of a change of distance. In consequence of this, any *exact* solution of such a problem becomes impracticable, but a sufficiently accurate solution for all practical purposes is frequently obtainable. For it frequently happens that the great bulk of a road's business is non-competitive, or, on the other hand, it may be competitive-through, and that the proportion of one or more definite kinds of traffic is so large as to overshadow the other miscellaneous traffic. In such cases an approximate but sufficiently accurate solution is possible.

**414. General conclusions regarding a change in distance.** (a) In *all* non-competitive business (local and through) the added distance is actually profitable. Sometimes practically



all of the business of the road is non-competitive; a considerable proportion of it is always non-competitive.

(b) When the competitive local business is very large and the competitive through business has a very large average home haul compared with the foreign haul, the added distance is a source of loss. Such situations are unusual and are generally confined to trunk lines.

(c) The above may be still further condensed to the general conclusion that there is always *some* compensation for the added cost of operating an added length of line and that it frequently is a source of actual profit.

(d) There is, however, a limitation which should not be lost sight of. The above argument may be carried to the logical conclusion that, if added distance is profitable, the engineer should purposely lengthen the line. But added distance means added operating expenses. A sufficient tariff to meet these is a tax on the community—a tax which more or less discourages traffic. It is contrary to public policy to burden a community with an avoidable expense. But, on the other hand, a railroad is not a charitable organization, but a money-making enterprise, and cannot be expected to unduly load up its first cost in order that subsequent operating expenses may be unduly cheapened and the tariff unduly lowered. A common reason for increased distance is the saving of the first cost of a very expensive although shorter line.

(e) Finally, although there is a considerable and uncompensated loss resulting from curvature and grade which will justify a considerable expenditure to avoid them, there is by no means as much justification to incur additional expenditure to avoid distance. Of course needless lengthening should be avoided. A moderate expenditure to shorten the line may be justifiable, but large expenditures to decrease distance are never justifiable except when the great bulk of the traffic is exceedingly heavy and is competitive.

**415. Justification of decreasing distance to save time.** It should be recalled that the changes which an engineer may make which are physically or financially possible will ordinarily have but little effect on the time required for a trip. The time which can thus be saved will have practically no value for the freight business—at least any value which would justify changing the route. When there is a large directly competitive

passenger traffic between two cities (*e.g.* New York to Philadelphia) a difference of even 10 minutes in the time required for a run might have considerable financial importance, but such cases are comparatively rare. It may therefore be concluded that the value of the time saved by shortening distance will not ordinarily be a justification for increased expense to accomplish it.

#### 416. Effect of change of distance on the business done.

The above discussion is based on the assumption that the business done is unaffected by any proposed change in distance. If a proposed reduction in distance involves a loss of business obtained, it is almost certainly unwise. But if by increasing the distance the original cost of the road is decreased (because the construction is of less expensive character), and if the receipts are greater, and are increased still more by an increase in business done, then the change is probably wise. While it is almost impossible in a subject of such complexity to give a general rule, the following is generally safe: Adopt a route of such length that the annual traffic per mile of line is a maximum. This statement may be improved by allowing the element of original cost to enter and say, adopt a route of such length that the annual traffic per mile of line divided by the average cost per mile is a maximum. Even in the above the operating cost per mile, as affected by the curvature and grades on the various routes, does not enter, but any attempt to formulate a general rule which would allow for variable operating expenses would evidently be too complicated for practical application.

## CHAPTER XXII.

### CURVATURE.

**417. General objections to curvature.** In the popular mind curvature is one of the most objectionable features of railroad alignment. The cause of this is plain. The objectionable qualities are on the surface, and are apparent to the non-technical mind. They may be itemized as follows:

1. Curvature increases **operating expenses** by increasing (a) the required tractive force, (b) the wear and tear of roadbed and track, (c) the wear and tear of equipment, and (d) the required number of track-walkers and watchmen.

2. It may affect the **operation of trains** (a) by limiting the length of trains, and (b) by preventing the use of the heaviest types of engines.

3. It may affect **travel** (a) by the difficulty of making time, (b) on account of rough riding, and (c) on account of the apprehension of danger.

4. There is actually an increased **danger** of collision, derailment, or other form of accident.

Some of these objections are quite definite and their true value may be computed. Others are more general and vague and are usually exaggerated. These objections will be discussed in inverse order.

**418. Financial value of the danger of accident due to curvature.** At the outset it should be realized that in general the problem is *not* one of curvature *vs.* no curvature, but simply sharp curvature *vs.* easier curvature (the central angle remaining the same), or a greater or less percentage of elimination of the degrees of central angle. A straight road between termini is in general a financial (if not a physical) impossibility. The practical question is then, how much is the financial value of such diminution of danger that may result from such eliminations of curvature as an engineer is able to make?

In the year 1898 there were 2228 railroad accidents reported by the *Railroad Gazette*, whose lists of all accidents worth reporting are very complete. Of these a very large proportion clearly had no relation whatever to curvature. But suppose we assume that 50% (or 1114 accidents) were directly caused by curvature. Since there are approximately 200,000 curves on the railroads of the country, there was on the average an accident for every 179 curves during the year. Therefore we may say, according to the theory of probabilities, that the chances are even that an accident may happen on any particular curve in 179 years. This assumes all curves to be equally dangerous, which is not true, but we may temporarily consider it to be true. If, at the time of the construction of the road, \$1.00 were placed at compound interest at 5% for 179 years, it would produce in that time \$620.89 for each dollar saved, wherewith to pay all damages, while the amount necessary to eliminate that curvature, even if it were possible, would probably be several thousand dollars. The number of passengers carried one mile for one killed in 1898-99 was 61,051,580. If a passenger were to ride continuously at the rate of sixty miles per hour, day and night, year after year, he would need to ride for more than 116 years before he had covered such a mileage, and even then the probabilities of his death being due to curvature or to such a reduction of curvature as an engineer might accomplish are very small. Of course particular curves are often, for special reasons, a source of danger and justify the employment of special watchmen. They would also justify very large expenditures for thier elimination if possible. But as a general proposition it is evidently impossible to assign a definite money value to the danger of a serious accident happening on a particular curve which has no special elements of danger.

Another element of safety on curved track is that trait of human nature to exercise greater care where the danger is more apparent. Many accidents are on record which have been caused by a carelessness of locomotive engineers on a *straight* track when the extra watchfulness usually observed on a curved track would have avoided them.

**419. Effect of curvature on travel.** (a) **Difficulty in making time.** The growing use of transition curves has largely eliminated the necessity for reducing speed on curves, and even when the speed is reduced it is done so easily and quickly by means

of air-brakes that but little time is lost. If two parallel lines were competing sharply for passenger traffic, the handicap of sharp curvature on one road and easy curvature on the other might have a considerable financial value, but ordinarily the *mere reduction* of time due to sharp curvature will not have any computable financial value.

(b) **On account of rough riding.** Again, this is much reduced by the use of transition curves. Some roads suffer from a general reputation for crookedness, but in such cases the excessive curvature is practically unavoidable. This cause probably does have some effect in influencing competitive passenger traffic.

(c) **On account of the apprehension of danger.** This doubtless has its influence in deterring travel. The amount of its influence is hardly computable. When the track is in good condition and transition curves are used so that the riding is smooth, even the apprehension of danger will largely disappear.

Travel is doubtless more or less affected by curvature, but it is impossible to say how much. Nevertheless the engineer should not ordinarily give this item any financial weight whatsoever. Freight traffic (two thirds of the total) is unaffected by it. It chiefly affects that limited class of sharply competitive passenger traffic—a traffic of which most roads have not a trace.

**420. Effect on operation of trains.** (a) **Limiting the length of trains.** When curvature actually limits the length of trains, as is sometimes true, the objection is valid and serious. But this can generally be avoided. If a curve occurs on a ruling grade without a reduction of the grade sufficient to compensate for the curvature, then the resistance on that curve will be a maximum and that curve will limit the trains to even a less weight than that which may be hauled on the ruling grade. In such cases the unquestionably correct policy is to “compensate for curvature,” as explained later (see §§ 427, 428), and not allow such an objection to exist. It is *possible* for curvature to limit the length of trains even without the effect of grade. On the Hudson River R. R. the total net fall from Albany to New York is so small that it has practically no influence in determining grade. On the other hand, a considerable portion of the route follows a steep rocky river bank which is so crooked that much curvature is unavoidable and very sharp curvature

can only be avoided by very large expenditure. As a consequence sharp curvature has been used and the resistance on the curves is far greater than that of any fluctuations of grade which it was necessary to use. Or, at least, a comparatively small expenditure would suffice to cut down any grade so that its resistance would be less than that of some curve which could not be avoided except at an enormous cost. And as a result, since the length of trains is really limited by curvature, minor grades of 0.3 to 0.5% have been freely introduced which might be removed at comparatively small expense. The above case is very unusual. Low grades are usually associated with generally level country where curvature is easily avoided—as in the Camden and Atlantic R. R. Even in the extreme case of the Hudson River road the maximum curvature is only equivalent to a comparatively low ruling grade.

(b) **Preventing the use of the heaviest types of engines.** The validity of this objection depends somewhat on the degree of curvature and the detailed construction of the engine. While some types of engines might have difficulty on curves of extremely short radius, yet the objection is ordinarily invalid. This will best be appreciated when it is recalled that the "Consolidation" type was originally designed for use on the sharp curvature of the mountain divisions of the Lehigh Valley R. R., and that the type has been found so satisfactory that it has been extensively employed elsewhere. It should also be remembered that during the Civil War an immense traffic daily passed over a hastily constructed trestle near Petersburg, Va., the track having a radius of 50 feet. As a result of a test made at Renovo on the Philadelphia and Erie R. R. by Mr. Isaac Dripps, Gen. Mast. Mech., in 1875,\* it was claimed that a Consolidation engine encountered less resistance per ton than one of the "American" type. Whether the test was strictly reliable or not, it certainly demonstrated that there was no trouble in using these heavy engines on very sharp curvature, and we may therefore consider that, except in the most extreme cases, this objection has no force whatsoever.

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\* Seventh An. Rep. Am. Mast. Mech. Assn.







(2) At what degree of curvature is the total train resistance double its value on a tangent? Probably no one figure would be exact for all conditions. Train resistance varies with the velocity and with the various conditions of train loading even on a tangent, and it is by no means certain (or even probable) that the ratio would be exactly the same for all conditions. As an average figure we may say that a train running at average velocity on a  $10^\circ$  curve will encounter a resistance due to curvature of about 10 lbs. per ton, which is the average resistance found on a level tangent. On a  $10^\circ$  curve therefore the resistance is doubled.

(3) A train-mile costs about so much—approximately \$1.00. Doubling the tractive resistance will increase certain items of expenditure about so much. Their combined value is so much per cent of the cost of a train-mile. A mile of continuous  $10^\circ$  curve contains  $528^\circ$  of central angle. A mile of such track would add so much per cent to the average train-mile expenses and each degree of central angle is responsible for  $\frac{1}{528}$  of this increase. Since the increase is irrespective of radius and depends only on the degrees of central angle, we therefore say that each degree of central angle of a curve will add so much to the average operating expenses of a train-mile.

The "cost per train-mile" considered above should be considered as the cost of a mile of level tangent. If we for a moment consider that all the railroads of the country were made absolutely straight and level, it is apparent that the average cost per train-mile instead of being about 95c. would be somewhat less. The percentage should therefore be applied to this reduced value, but the net effect of this change would evidently be small.

**422. Effect of curvature on maintenance of way.** A very large proportion of the items of expense in a train-mile are absolutely unaffected by curvature. It will therefore simplify matters somewhat if we at once throw out all the unaffected items. Of the items of maintenance of way and structure all but the first three will be thrown out. Item 4 will be somewhat affected when bridges or trestles occur on a curve. But when it is considered what a very small percentage of this small item (2.378%) could be ascribed to curvature, since the very large majority of bridges and trestles are purposely made

straight, and since culverts, etc., are not affected, we may evidently ignore any variation in the item.

**Item 1. REPAIRS OF ROADWAY.** A very large proportion of the sub-items are absolutely unaffected. The care of embankments and slopes, ditching, weeding, etc., are evidently unaffected. The track labor on rails and ties and the work of surfacing will evidently be somewhat increased and yet it is very seldom that the length of a track section would be decreased simply on account of excessive curvature. But  $528^\circ$  per mile is an excessive amount of curvature. The average for the whole country is about  $30^\circ$  per mile, and there are very few instances of that amount of curvature ( $528^\circ$ ) in the length of a single mile. As before intimated, it is reasonable to assume that the *extra* work *per foot* on a  $20^\circ$  curve would be 10 times the extra work on a  $2^\circ$  curve, which verifies the general statement that the extra cost varies as the degrees of central angle. Considering how much of this item is independent of curvature and how little even the track labor is affected, it is possibly overstating the case to allow 25% increase for  $528^\circ$  of curvature in one mile.

**Item 2. RENEWALS OF RAILS.** Wellington says that some experiments made by himself and others made by Dr. Dudley agree in indicating that the rail wear on tangents may be considered as 1 lb. per yard per 10,000,000 tons duty, while the extra wear on curves would be  $\frac{1}{2}$  lb. per degree of curve per 10,000,000 tons duty. Therefore on a  $10^\circ$  curve the *extra* rail wear would be five times as great as on a tangent and the *increase* would therefore be 500%. On iron rails and on inferior steel rails the wear on tangents would be larger proportionally, and this is probably the reason for Wellington's adopting an average increase of but 300%, and this same figure will be adopted.

**Item 3. RENEWALS OF TIES.** Curvature affects ties by increasing the "rail cutting" and on account of the more frequent respiking, which "spike-kills" the ties even before they have decayed. Wellington estimates that a tie which will last nine years on a tangent will last but six years on a  $10^\circ$  curve. He adds 50% for tie renewals. He considers the decrease in tie life to be proportional to the degree of curve and therefore again verifies the general statement made above regarding the expense of curvature.

#### 423. Effect of curvature on maintenance of equipment.

Items 11, 16, 18, and 19 will be considered as unaffected.

**Item 12. REPAIRS AND RENEWALS OF LOCOMOTIVES.** Curvature affects locomotive repairs by increasing very largely the wear on tires and wheels, and also the wear and strain due to the additional power required. Wellington neglected the last cause since the resistance due to curvature is so small compared to that due to even a moderate grade. He further considered that only 30% of the items of engine repairs are affected at all by curvature, and that the effect of curvature and grades on these is only  $\frac{1}{3}$  or 10%, and that curvature is responsible for 60% of that, or, finally, that only 6% of engine repairs are caused by curvature as it exists. He then computed that the actual average curvature of railroads (about 30° per mile) is but  $\frac{1}{20}$  of the 600° (instead of 528°) in his standard mile. Therefore he said that 600° of curvature would increase engine repairs by  $20 \times 6\%$ , or 120%. He acknowledges that the reasoning is not conclusive. It apparently is weak in this respect: the resistance, and also the wear, is less per degree of curve on the sharper curves than on the easier. On this account, and also because 528° of curvature is considered the standard, rather than 600°, the estimate will be cut down to 100%. (Another method of computation will be substituted for this as soon as possible.)

**Items 13, 14, and 15.** For similar reasons the estimates for these items will be made 100%. The effect of curvature will apply to all cars about equally.

**Item 17.** The repairs and renewals of shop machinery and tools will not be increased more than 50% per mile for the additional repairs required of the above equipment.

**424. Effect of curvature on conducting transportation.** We may at once throw out all items except 22, 23, 24, and 25, a small part of 28, and possibly 35, 36, and 37. This last group has already been discussed in § 418; the aggregate of the three items is but 1.752%; curvature is responsible for only a small proportion of the item, and the reduction which an engineer is able to effect would be so small that we may neglect it.

**Item 28** is somewhat analogous to the above. Curvature does not affect a large part of the item, but an extreme case of curvature will occasionally require an extra watchman. Considering, however, that curvature does *not* in general require watch-

men, and that such cases are the unusual cases in mountainous regions where the curvature is unavoidable and not materially reducible, it would evidently be wrong to charge curvature in general with such an item, although there would be justification for it in individual cases. It will therefore be ignored.

**Items 22, 23, 24, and 25.** In § 407, Chapter XXI, the proportion of fuel assigned to direct hauling on a tangent is computed as amounting to about 55%. Since this direct resistance is assumed to be exactly doubled, we will charge 55% for fuel. There will evidently be no error worth considering in allowing the same proportionate amount as the charge for water, oil, waste, etc.

“General expenses,” items 47 to 53, are of course unaffected.

**425. Estimate of total effect per degree of central angle.** Compiling the above estimates we have the following tabulation:

TABLE XXII.—EFFECT ON OPERATING EXPENSES OF CHANGES  
IN CURVATURE.

| Item No.     | Normal average. | Per cent affected. | Cost per mile per cent. |
|--------------|-----------------|--------------------|-------------------------|
| 1            | 10.596          | 25                 | 2.649                   |
| 2            | 1.440           | 300                | 4.320                   |
| 3            | 3.093           | 50                 | 1.546                   |
| 4 }<br>10 }  | 5.533           | 0                  | 0                       |
|              | 20.662          | .....              | 8.515                   |
| 11           | .650            | 0                  | 0                       |
| 12           | 5.879           | 100                | 5.879                   |
| 13           | 2.209           | 100                | 2.209                   |
| 14           | 6.765           | 100                | 6.765                   |
| 15           | .155            | 100                | .155                    |
| 16           | .209            | 0                  | 0                       |
| 17           | .490            | 50                 | .245                    |
| 18           | .040            | 0                  | 0                       |
| 19           | .495            | 0                  | 0                       |
|              | 16.892          | .....              | 15.253                  |
| 20           | 1.761           | 0                  | 0                       |
| 21           | 9.781           | 0                  | 0                       |
| 22           | 9.681           | 55                 | 5.335                   |
| 23           | .671            | 55                 | .369                    |
| 24           | .376            | 55                 | .207                    |
| 25           | .184            | 55                 | .101                    |
| 26 }<br>46 } | 35.340          | 0                  | 0                       |
|              | 57.794          | .....              | 6.012                   |
| 47 }<br>53 } | 4.653           | 0                  | 0                       |
|              | 100.000         | .....              | 29.780                  |

According to it,  $528^\circ$  of curvature in one mile would increase the expenses of each train passing over it by  $29.78\%$  of the average cost of a train-mile, and according to the general principles laid down in § 421,  $1^\circ$  of central angle of any curve, no matter what the radius, will increase the expenses by  $\frac{1}{3\frac{1}{2}}\%$  of  $29.78\%$ , or  $.0564\%$  per degree. Therefore the cost per year per daily train each way is (at 95 c. per train-mile)

$$95 \times .0564\% \times 2 \times 365 = 39.11 \text{ c.}$$

As a simple illustration (a more extended one will be given later), suppose that by using greater freedom with regard to earthwork the crooked line sketched may be reduced to the simple curve shown and a curvature of, say,  $110^\circ$  may be reduced to, say,  $60^\circ$ .

Note that since the extreme tangents are identical, the saving in central angle results from the elimination of the reversed



FIG. 209.

curvature and of that part of the direct curvature necessary to balance the reversed curvature. Assume that there are six daily trains each way. Then the annual saving is

$$50^\circ \times .3911 \times 6 = \$117.33,$$

which at 5% would justify an expenditure of \$2346.60. If the extra cost of construction does not exceed this, the improvement is justifiable, and is made all the more so if the probabilities are great that the future traffic will largely exceed six trains per day. At the same time the warning regarding "discounting the future" with respect to expected traffic should not be neglected. The possible effect of change of distance has not been referred to in the above problem. In any case it is a distinct problem. According to the above sketch, the difference in distance is probably very slight, and considering the compensating character of extra distance, such small differences may usually be disregarded. The possible effect of change of grade will be discussed in the next chapter. Assuming that there is no difference to be considered on account of either grade or distance, the question hinges on the advisability of spending \$2346.60 for the improvement.

**426. Reliability and value of the above estimate.** It should be realized at the outset that no extreme accuracy is claimed for the above estimate. The effect of curvature is somewhat variable as well as uncertain, but such estimates have this great value. Vary the estimates of individual items as you please (within reason), and the final result is still about the same and may be used to guide the judgment. As an illustration, suppose that the item of renewals of rails is assumed to be affected 400% rather than 300%, the justifiable expenditure to avoid the curvature in the above case may similarly be computed as \$2460, an increase of less than 5%. But, after all, the real question is not whether the improvement is worth \$2346 or \$2460. The extra work involved may perhaps be done for \$500 or it may require \$10000. The above general method furnishes a criterion which, while not accurate, is so much better than a reliance on vague judgment that it should not be ignored.

#### COMPENSATION FOR CURVATURE.

**427. Reasons for compensation.** The effect of curvature on a grade is to increase the resistance by an amount which is equivalent to a material addition to that grade. On minor grades the addition is of little importance, but when the grade is nearly or quite the ruling grade of the road, then the additional resistance induced by a curve will make that curve a place of maxi-



imum resistance and the real maximum will be a "virtual grade" somewhat higher than the nominal maximum. If, in Fig. 210,

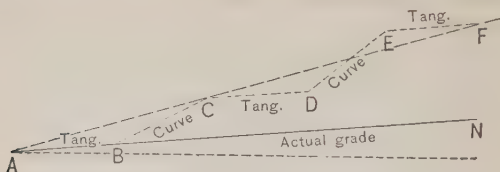


FIG. 210.

$AN$  represents an actual *uniform* grade consisting of tangents and curves, the "virtual grade" on curves at  $BC$  and  $DE$  may be represented by  $BC$  and  $DE$ . If  $BC$  and  $DE$  are very long, or if a stop becomes necessary on the curve, then the full disadvantage of the curve becomes developed. If the whole grade may be operated without stoppage, then, as elaborated further in the next chapter, the whole grade may be operated as if equal to the average grade,  $AF$ , which is better than  $BC$ , although much worse than  $AN$ . The process of "compensation" consists in reducing the grade on every curve by such an amount that the actual resistance on each curve, due to both curvature and grade, shall precisely equal the resistance on the tangent. The practical effect of such reduction is that the "virtual" grade is kept constant, while the nominal grade fluctuates.

One effect of this is that (see Fig. 211) instead of accomplish-

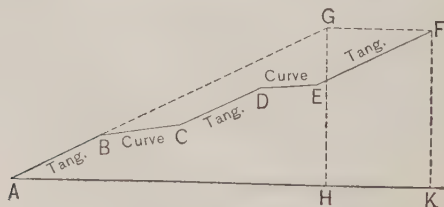


FIG. 211.

ing the vertical rise from  $A$  to  $G$  (i.e.,  $HG$ ) in the horizontal distance  $AH$ , it requires the horizontal distance  $AK$ . Such an addition to the horizontal distance can usually be obtained by proper development, and it should always be done on a ruling



grade. Of course it is possible that it will cost more to accomplish this than it is worth, but the engineer should be sure of this before allowing this virtual increase of the grade.

European engineers early realized the significance of unreduced curvature and the folly of laying out a uniform ruling grade regardless of the curvature encountered. Curve compensation is now quite generally allowed for in this country, but thousands of miles have been laid out without any compensation. A very common limitation of curvature and grade has been the alliterative figures  $6^\circ$  curvature and 60 feet per mile of grade, either singly or in combination. Assuming that the resistance on a  $6^\circ$  curve is equivalent to a 0.3% grade (15.84 feet per mile), then a  $6^\circ$  curve occurring on a 60-foot grade would develop more resistance than a 75-foot grade on a tangent. The "mountain cut-off" of the Lehigh Valley Railroad near Wilkesbarre is a fine example of a heavy grade compensated for curvature, and yet so laid out that the virtual grade is uniform from bottom to top, a distance of several miles.

428. The proper rate of compensation. This evidently is the rate of grade of which the resistance just equals the resistance due to the curve. But such resistance is variable. It is greater as the velocity is lower; it is generally about 2 lbs. per ton (equivalent to a 0.1% grade) per degree of curve when starting a train. On this account, the compensation for a curve which occurs at a known stopping-place for the heaviest trains should be 0.1% per degree of curve. The resistance is not even strictly proportional to the degree of curvature, although it is usually considered to be so. In fact most formulæ for curve resistance are based on such a relation. But if the experimentally determined resistances for low curvatures are applied to the excessive curvature of the New York Elevated road, for example, the rules become ridiculous. On this account the compensation *per degree of curve* may be made less on a sharp curve than on an easy curve. The compensation actually required for very fast trains is less than for slow trains, say 0.02 or 0.03% per degree of curve; but since the comparatively slow and heavy freight trains are the trains which are chiefly limited by ruling grade, the compensation must be made with respect to those trains. From 0.04 to 0.05% per degree is the rate of compensation most usually employed for average conditions. Curves which occur *below* a known stopping-place for *all* trains need

not be compensated, for the extra resistance of the curve will be simply utilized in place of brakes to stop the train. If a curve occurs just *above* a stopping-place, it is very serious and should be amply compensated. Of course the down-grade traffic need not be considered.

It sometimes happens that the ordinary rate of compensation will consume so much of the vertical height (especially if the curvature is excessive) that a steeper through grade must be adopted than was first computed, and then the trains might stall on the tangents rather than on the curves. In such cases a slight reduction in the rate of compensation might be justifiable. The proper rate of compensation can therefore be estimated from the following rules:

(1) On the upper side of a stopping-place for the heaviest trains compensate  $0.10\%$  per degree of curve.

(2) On the lower side of such a stopping-place do not compensate at all.

(3) Ordinarily compensate about  $0.05\%$  per degree of curve.

(4) Reduce this rate to  $0.04\%$  or even  $0.03\%$  per degree of curve if the grade on tangents must be increased to reach the required summit.

(5) Reduce the rate somewhat for curvature above  $8^\circ$  or  $10^\circ$ .

(6) Curves on minor grades need not be compensated.

**429. The limitations of maximum curvature.** What is the maximum degree of curvature which should be allowed on any road? The true answer is probably that there is no definite limit. It has been shown that sharp curvature does not prevent the use of the heaviest types of engines, and although a sharp curve unquestionably increases operating expenses, the increase is but one of degree with hardly any definite limit. The general character of the country and the gross capital available (or the probable earnings) are generally the true criterions.

A portion of the road from Denver to Leadville, Col., is an example of the necessity of considering sharp curvature. The traffic that might be expected on the line was so meagre and yet the general character of the country was so forbidding that a road built according to the usual standards would have cost very much more than the traffic could possibly pay for. The line as adopted cost about \$20,000 per mile, and yet in a stretch of 11.2 miles there are about 127 curves. One is a  $25^\circ$   $20'$  curve, twenty-four are  $24^\circ$  curves, twenty-five are  $20^\circ$  curves,

and seventy-two are sharper than  $10^\circ$ . If  $10^\circ$  had been made the limit (a rather high limit according to usual ideas), it is probable that the line would have been found impracticable (except with prohibitive grades) unless four or five times as much per mile had been spent on it, and this would have ruined the project financially.

For many years the main-line traffic of the Baltimore and Ohio R. R. has passed over a 300-foot curve ( $19^\circ 10'$ ) and a 400-foot curve ( $14^\circ 22'$ ) at Harper's Ferry. A few years ago some reduction was made in this by means of a tunnel, but the fact that such a road thought it wise to construct and operate such curves (and such illustrations on the heaviest-traffic roads are quite common) shows how foolish it is for an engineer to sacrifice money or (which is much more common) sacrifice gradients in order to reduce the *rate* of curvature on a road which at its best is but a second- or third-class road.

Of course such belittling of the effects of curvature may be (and sometimes is) carried to an extreme and cause an engineer to fail to give to curvature its due consideration. Degrees of central angle should always be reduced by all the ingenuity of the engineer, and should only be limited by the general relation between the financial and topographical conditions of the problem. Easy curvature is in general better than sharp curvature and should be adopted when it may be done at a small financial sacrifice, especially since it reduces distance generally and may even cut down the initial cost of that section of the road. But large financial expenditures are rarely, if ever, justifiable where the net result is a mere increase in radius without a reduction in central angle. An analysis of the changes which have been so extensively made during late years on the Penn. R. R. and the N. Y., N. H. & H. R. R. will show invariably a reduction of distance, or of central angle, or both, and perhaps incidentally an increase in radius of curvature. There are but few, if any, cases where the sole object to be attained by the improvement is a mere increase in radius.

## CHAPTER XXIII.

### GRADE.

430. **Two distinct effects of grade.** The effects of grade on train expenses are of two distinct kinds; one possible effect is very costly and should be limited even at considerable expenditure; the other is of comparatively little importance, its cost being slight. As long as the length of the train is not limited the occurrence of a grade on a road simply means that the engine is required to develop so many foot-pounds of work in raising the train so many feet of vertical height. For example, if a freight train weighing 600 tons (1,200,000 lbs.) climbs a hill 50 feet high, the engine performs an *additional* work of creating 60,000,000 foot-pounds of potential energy. If this height is surmounted in 2 miles and in 6 minutes of actual time (20 miles per hour), the *extra* work is 10,000,000 foot-pounds per minute, or about 303 horse-power. But the disadvantages of such a rise are always largely compensated. Except for the fact that one terminus of a road is generally higher than the other every up grade is followed, more or less directly, by a down grade which is operated partly by the potential energy acquired during the previous climb. But when we consider the trains running in both directions even the difference of elevation of the terminus is largely neutralized. If we could eliminate frictional resistances and particularly the use of brakes, the *net* effect of minor grades on the operation of minor grades in both directions would be zero. Whatever was lost on any up grade would be regained on a succeeding down grade, or at any rate on the return trip. On the very lowest grades (the limits of which are defined later) we may consider this to be literally true, viz., that nothing is lost by their presence; whatever is temporarily lost in climbing them is either immediately regained on a subsequent light down grade or is regained on the return trip. If a stop is required at the bottom of a sag, there is a net and uncompensated loss of energy.

On the other hand, if the length of trains is limited by the grade, it will require more trains to handle a given traffic. The receipts from the traffic are a definite sum. The cost of handling it will be nearly in proportion to the number of trains. Anticipating a more complete discussion, it may be said as an example that increasing the ruling grade from 1.20% (63.36 feet per mile) to 1.55% (81.84 feet per mile—an increase of about 18.5 feet per mile) will be sufficient to increase the required number of trains for a given gross traffic about 25%, i.e., five trains will be required to handle the traffic which four trains would have handled before at a cost slightly more than four-fifths as much. The effect of this on dividends may readily be imagined.

**431. Application to the movement of trains of the laws of accelerated motion.** When a train starts from rest and acquires its normal velocity, it overcomes not only the usual tangent resistances (and perhaps curve and grade resistances), but it also performs work in storing into the train a vast fund of kinetic energy. This work is not lost, for every foot-pound of such energy may later be utilized in overcoming resistances, provided it is not wasted by the action of train-brakes. If for a moment we consider that a train runs without any friction, then, when running at a velocity of  $v$  feet per second, it possesses a kinetic energy which would raise it to a height  $h$  feet, when  $h = \frac{v^2}{2g}$ , in which  $g$  is the acceleration of gravity—32.16. Assuming that the engine is exerting just enough energy to overcome the frictional resistances, the train would climb a grade until the train was raised  $h$  feet above the point where its velocity was  $v$ . When it had climbed a height  $h'$  (less than  $h$ ) it would have a velocity  $v_1 = \sqrt{2g(h-h')}$ . As a numerical illustration, assume  $v = 30$  miles per hour = 44 feet per second. Then  $h = \frac{v^2}{2g} = 30.1$  feet, and assuming that the engine was exerting just enough force to overcome the rolling resistances on a level, the kinetic energy in the train would carry it for two miles up a grade of 15 feet per mile, or half a mile up a grade of 60 feet per mile. When the train had climbed 20 feet, there would still be 10.1 feet left and its velocity would be  $v_1 = \sqrt{2g(10.1)} = 25.49$  feet per second = 17.4 miles per hour. These figures, however, must be slightly modified on account of the weight and the revolving

action of the wheels, which form a considerable percentage of the total weight of the train. When train velocity is being acquired, part of the work done is spent in imparting the energy of rotation to the driving-wheels and various truck-wheels of the train. Since these wheels run on the rails and must turn as the train moves, their rotative kinetic energy is just as effective—as far as it goes—in becoming transformed back into useful work. The proportion of this energy to the total kinetic energy has already been demonstrated (see Chapter XV, § 347). The value of this correction is variable, but an average value of 5% has been adopted for use in the accompanying tabular form (Table XXIII), in which is given the corrected “velocity head” corresponding to various velocities in miles per hour. The table is computed from the following formula:

$$\text{Velocity head} = \frac{v^2 \text{ in ft. per sec.}}{64.32} = \frac{1.4667v \text{ in m. per h.}}{64.32} = 0.03344$$

adding 5% for the rotative kinetic energy of the wheels, 0.00167

The corrected velocity head therefore equals 0.03511

Part of the figures of Table XXIII were obtained by interpolation and the final *hundredth* may be in error by one unit but it may readily be shown that the final hundredth is of no practicable importance. It is also true that the chief use made of this table is with velocities much less than 50 miles per hour. Corresponding figures may be obtained for higher velocities, if desired, by multiplying the figure for *half* the velocity by *four*.

**432. Construction of a virtual profile.** The following simple demonstration will be made on the basis that the ordinary tractive resistances and also the tractive force of the locomotive are independent of velocity. For a considerable range of velocity which includes the most common freight-train velocities this assumption is so nearly correct that the method will give an approximately correct result, but for higher velocities and for more accurate results a more complicated method (given later) must be used. The following demonstration will serve well as a preliminary to the more accurate method. It may best be illustrated by considering a simple numerical example.

Assuming that a train is passing *A* (see Fig. 212), running at 30 miles per hour. Assume that the throttle is not changed or any brakes applied, but that the engine continues to exert the



TABLE XXIII.—VELOCITY HEAD (REPRESENTING THE KINETIC ENERGY) OF TRAINS MOVING AT VARIOUS VELOCITIES.

| Vel.<br>mi.<br>hr. | 0.0   | 0.1   | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   | 0.9   |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 10                 | 3.51  | 3.58  | 3.65  | 3.72  | 3.79  | 3.87  | 3.95  | 4.02  | 4.10  | 4.17  |
| 11                 | 4.25  | 4.33  | 4.41  | 4.49  | 4.57  | 4.65  | 4.73  | 4.81  | 4.89  | 4.97  |
| 12                 | 5.06  | 5.15  | 5.23  | 5.32  | 5.41  | 5.50  | 5.58  | 5.67  | 5.75  | 5.84  |
| 13                 | 5.93  | 6.02  | 6.12  | 6.21  | 6.31  | 6.40  | 6.50  | 6.59  | 6.69  | 6.78  |
| 14                 | 6.88  | 6.98  | 7.08  | 7.19  | 7.29  | 7.39  | 7.49  | 7.60  | 7.70  | 7.80  |
| 15                 | 7.90  | 8.00  | 8.11  | 8.22  | 8.33  | 8.44  | 8.55  | 8.66  | 8.77  | 8.88  |
| 16                 | 8.99  | 9.10  | 9.21  | 9.32  | 9.43  | 9.55  | 9.67  | 9.79  | 9.91  | 10.03 |
| 17                 | 10.15 | 10.27 | 10.39 | 10.51 | 10.63 | 10.75 | 10.87 | 10.99 | 11.12 | 11.25 |
| 18                 | 11.38 | 11.50 | 11.63 | 11.76 | 11.89 | 12.02 | 12.15 | 12.28 | 12.41 | 12.55 |
| 19                 | 12.68 | 12.81 | 12.95 | 13.08 | 13.22 | 13.35 | 13.49 | 13.63 | 13.77 | 13.91 |
| 20                 | 14.05 | 14.19 | 14.33 | 14.47 | 14.61 | 14.75 | 14.89 | 15.04 | 15.19 | 15.34 |
| 21                 | 15.49 | 15.64 | 15.79 | 15.94 | 16.09 | 16.24 | 16.39 | 16.54 | 16.69 | 16.84 |
| 22                 | 17.00 | 17.15 | 17.30 | 17.46 | 17.62 | 17.78 | 17.94 | 18.10 | 18.26 | 18.42 |
| 23                 | 18.58 | 18.74 | 18.90 | 19.06 | 19.22 | 19.38 | 19.55 | 19.72 | 19.89 | 20.06 |
| 24                 | 20.23 | 20.40 | 20.57 | 20.74 | 20.91 | 21.08 | 21.25 | 21.42 | 21.59 | 21.77 |
| 25                 | 21.95 | 22.12 | 22.30 | 22.48 | 22.66 | 22.84 | 23.02 | 23.20 | 23.38 | 23.56 |
| 26                 | 23.74 | 23.92 | 24.10 | 24.28 | 24.46 | 24.65 | 24.84 | 25.03 | 25.22 | 25.41 |
| 27                 | 25.60 | 25.79 | 25.98 | 26.17 | 26.36 | 26.55 | 26.74 | 26.93 | 27.13 | 27.33 |
| 28                 | 27.53 | 27.73 | 27.93 | 28.13 | 28.33 | 28.53 | 28.73 | 28.93 | 29.13 | 29.33 |
| 29                 | 29.53 | 29.73 | 29.93 | 30.13 | 30.34 | 30.55 | 30.76 | 30.97 | 31.18 | 31.39 |
| 30                 | 31.60 | 31.81 | 32.02 | 32.23 | 32.44 | 32.65 | 32.86 | 33.08 | 33.30 | 33.52 |
| 31                 | 33.74 | 33.96 | 34.18 | 34.40 | 34.62 | 34.84 | 35.06 | 35.28 | 35.50 | 35.72 |
| 32                 | 35.95 | 36.17 | 36.39 | 36.62 | 36.85 | 37.08 | 37.31 | 37.54 | 37.77 | 38.00 |
| 33                 | 38.23 | 38.46 | 38.69 | 38.92 | 39.15 | 39.38 | 39.62 | 39.86 | 40.10 | 40.34 |
| 34                 | 40.58 | 40.82 | 41.06 | 41.30 | 41.54 | 41.78 | 42.02 | 42.26 | 42.51 | 42.76 |
| 35                 | 43.01 | 43.26 | 43.51 | 43.76 | 44.01 | 44.26 | 44.51 | 44.76 | 45.01 | 45.26 |
| 36                 | 45.51 | 45.76 | 46.01 | 46.26 | 46.52 | 46.78 | 47.04 | 47.30 | 47.56 | 47.82 |
| 37                 | 48.08 | 48.34 | 48.60 | 48.86 | 49.12 | 49.38 | 49.64 | 49.91 | 50.18 | 50.45 |
| 38                 | 50.72 | 50.99 | 51.26 | 51.53 | 51.80 | 52.07 | 52.34 | 52.61 | 52.88 | 53.15 |
| 39                 | 53.42 | 53.69 | 53.96 | 54.23 | 54.51 | 54.79 | 55.07 | 55.35 | 55.63 | 55.91 |
| 40                 | 56.19 | 56.47 | 56.75 | 57.03 | 57.31 | 57.59 | 57.87 | 58.16 | 58.45 | 58.74 |
| 41                 | 59.03 | 59.32 | 59.61 | 59.90 | 60.19 | 60.48 | 60.77 | 61.06 | 61.35 | 61.64 |
| 42                 | 61.94 | 62.23 | 62.52 | 62.82 | 63.12 | 63.42 | 63.72 | 64.02 | 64.32 | 64.62 |
| 43                 | 64.92 | 65.22 | 65.52 | 65.82 | 66.12 | 66.43 | 66.74 | 67.05 | 67.36 | 67.67 |
| 44                 | 67.98 | 68.29 | 68.60 | 68.91 | 69.22 | 69.53 | 69.84 | 70.15 | 70.46 | 70.78 |
| 45                 | 71.10 | 71.42 | 71.74 | 72.06 | 72.38 | 72.70 | 73.02 | 73.34 | 73.66 | 73.98 |
| 46                 | 74.30 | 74.62 | 74.94 | 75.26 | 75.59 | 75.92 | 76.25 | 76.58 | 76.91 | 77.24 |
| 47                 | 77.57 | 77.90 | 78.23 | 78.56 | 78.89 | 79.22 | 79.55 | 79.89 | 80.23 | 80.57 |
| 48                 | 80.91 | 81.25 | 81.59 | 81.93 | 82.27 | 82.61 | 82.95 | 83.29 | 83.63 | 83.97 |
| 49                 | 84.32 | 84.66 | 85.00 | 85.34 | 85.69 | 86.04 | 86.39 | 86.74 | 87.09 | 87.44 |
| 50                 | 87.79 | 88.14 | 88.49 | 88.85 | 89.20 | 89.55 | 89.91 | 90.26 | 90.61 | 90.97 |

same draw-bar pull. At *A* its "velocity head" is that due to 30 miles per hour, or 31.60 feet. At *B* it has gained 40 feet more, and its velocity is that due to a velocity head of 71.60 feet, or slightly over 45 miles per hour. At *B'* its velocity is again 30 miles per hour and velocity head 31.60 feet. At *C* the velocity head is but 6.60 feet and the velocity about 13.7 miles per hour.



As the train runs from *C* to *D* its velocity increases to 30 miles at *C'* and to over 45 miles per hour at *D*. At *E* the velocity again becomes 30 miles per hour. Although there will be some slight modifications of the above figures in actual practice, yet the above is not a fanciful theoretical sketch. Thousands of just such undulations of grade are daily operated in such a way, without disturbing the throttle or applying brakes, and the draw-bar pull, if measured by a dynamometer, would be found to be practically constant. Of course the above case assumes that

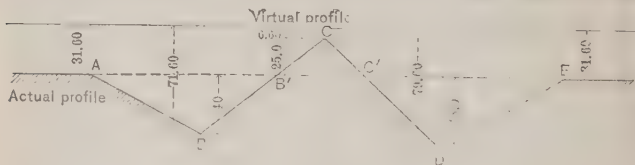


FIG. 212.

there are no stoppages and that the speed through the sags is not so great that safety requires the application of brakes. Observe that the "virtual profile" is here a straight line -- as it always is when the draw-bar pull is constant. The virtual profile (in this case as well as in every other case, illustrations of which will follow) is found by adding to the actual profile at any point an ordinate which represents the "velocity head" due to the velocity of the train at that point.

As another case, assume that a train is climbing the grade *AB* and exerting a pull just sufficient to maintain a constant velocity

up that grade. Then *A'B'* (parallel to *AB*) is the virtual profile, *AA'* representing the velocity head. At a stop being required at *C*, steam is shut off and brakes are applied at *B*, and the velocity head *BB'* reduces to zero at *C*. The train

starts from *C*, and at *D* attains a velocity corresponding to the ordinate *DD'*. At *D* the throttle may be slightly closed so that the velocity will be uniform and the virtual grade is *D'E'*, parallel to *DE*.

From the above it may be seen that a virtual profile has the following properties:

(a) When the velocity is *uniform*, the virtual profile is parallel with the actual.

(b) When the velocity is increasing the profiles are separating; when decreasing the profiles are approaching.

(c) When the velocity is zero the profiles coincide.

**433. Use, value, and possible misuse.** The essential feature respecting grades is the demand on the locomotive. From the foregoing it may readily be seen that the ruling grade of a road is not necessarily the steepest nominal grade. When a grade may be operated by momentum, i.e., when every train has an opportunity to take "a run at the hill," it may become a very harmless grade and not limit the length of trains, while another grade, actually much less, which occurs at a stopping-place for the heaviest trains, will require such extra exertion to get trains started that it may be the worst place on the road. Therefore the true way to consider the value of the grade at any critical place on the road is to construct a virtual profile for that section of the road. The required length of such a profile is variable, but in general may be said to be limited by points on each side of the critical section at which the velocity is definite, as at a stopping-place (velocity zero), or a long heavy grade where it is the minimum permissible, say 10 or 15 miles per hour.

Since the velocities of different trains vary, each train will have its own virtual profile at any particular place. The fast passenger trains are generally unaffected, practically. The requirement of high average speed necessitates the use of powerful engines, and grades which would stall a heavy freight will only cause a momentary and harmless reduction of speed of the fast passenger train.

A possible misuse of virtual profiles lies in the chance that a station or railroad grade crossing may be subsequently located on a heavy grade that was designed to be operated by momentum. But this should not be used as an argument against the employment of a virtual profile. The virtual profile shows the *actual state of the case* and only points out the necessity, if an unexpected requirement for a full stoppage of trains at a critical point has developed, of changing the location (if a station), or of changing the grade by regrading or by using an overhead crossing. Examples of such modifications are given in Chapter XXIV, The Improvement of Old Lines.

**434. Undulatory grades. Advantages.** Money can generally be saved by adopting an actual profile which is not strictly uniform—the matter of compensation for curvature being here

ignored. Its effect on the operation of trains is harmless provided the sag or hump is not too great. In Fig. 214 the undulatory grade may actually be operated as a uniform grade  $AG$ . The sag at  $C$  must be considered as a sag, even though  $BC$  is actually an up grade. But the engine is supposed to be working

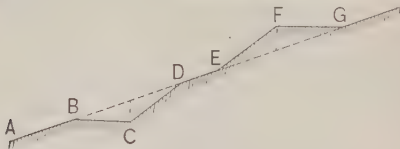


FIG. 214.

hard enough to carry a train at uniform velocity up a grade  $AG$ . Therefore it *gains* in velocity from  $B$  to  $C$ , and from  $C$  to  $D$  loses an equal amount. It may even be proven that the *time* required to pass the sag will be slightly less than the time required to run the uniform grade.

**Disadvantages.** The hump at  $F$  is dangerous in that, if the velocity at  $E$  is not equal to that corresponding to the extra velocity-head ordinate at  $F$ , the train will be stalled before reaching  $F$ . In practice there should be considerable margin. Any train should have a velocity of at least 10 miles per hour in passing any summit. This corresponds to a velocity head of 3.51 feet. An extra heavy head wind, slippery rails, etc., may use up any smaller margin and stall the train. If the grade  $AG$  is a ruling grade, then *no* hump should be allowed under any circumstances. For the heaviest trains are supposed to be so made up that the engine will *just* haul them up the ruling grades—of course with some margin for safety. Any increase of this grade, however short, would probably stall the train.

**Safe limits.** It is quite possible to have a sag so deep that it is not safe to allow freight trains to rush through them without the use of brakes. The use of brakes of course adds a distinct element of cost. To illustrate: If a freight train is running at a velocity of 20 miles per hour (velocity head 14.05 feet) and encounters a sag of 25 feet, the velocity head at the bottom of the sag will be 39.05 feet, which corresponds to a velocity of 33.3 miles per hour. This approaches the limit of safe speed for freight trains, and certainly passes the limit for trains not equipped with air-brakes and automatic couplers.

The term "safe limits" as used here, refers to the limits within which a freight train may be safely operated without the application of brakes or varying the work of the engine. Of course much greater undulations are frequently necessary and are safely operated, but it should be remembered that they add a distinct element to the cost of operating trains and that they must not be considered as harmless or that they should be introduced unless really necessary.

#### MINOR GRADES.

**435. Basis of cost of minor grades.** The basis of the computation of this least objectionable form of grade is as follows: The resistance encountered by a train on a level straight track is somewhat variable, depending on the velocity and the number and character of the cars, but for average velocities we may consider that 10 lbs. per ton is a reasonable figure. This value agrees fairly well with the results of some dynamometer tests made by Mr. P. H. Dudley, using a passenger train of 313 tons running at about 50 miles per hour. It also agrees with Searles's formulæ (based on experiments) for the resistance of a freight train with 40 cars running 25 miles per hour. Ten pounds per ton is the grade resistance of a 0.5% grade, or that of 26.4 feet per mile. On the above basis, a 0.5% grade will just double the tractive resistance on a level straight track. We may compute, as in the previous chapter, the cost of doubling the tractive resistance for one mile. But since the extra resistance is due to lifting the train through 26.4 feet of elevation, we may divide the extra cost of a mile of 0.5% grade by 26.4 and we have the cost of *one foot* of difference of elevation, and then (disregarding the limiting effect of grades) we may say that this cost of one foot of difference of elevation will be independent of the rate of grade. There are, however, limitations to this general proposition which will be developed in the next section.

**436. Classification of minor grades.** These are classified with reference to their effect on the operation of trains. In the first class are grades which may be operated without changing the work of the engine and which have practically no other effect than a harmless fluctuation of the velocity. But a grade which belongs to this class when considering a fast passenger train will belong to another class when considering a slow but heavy

freight train. And since it is the slow heavy freight trains which must be chiefly considered, a grade will usually be classified with respect to them. The limit of class A (the harmless class) therefore depends on the maximum allowable speed. The effect of a sag on speed will depend on the vertical feet of drop rather than on the rate of grade, for with the engine working as usual on even a light down grade a train

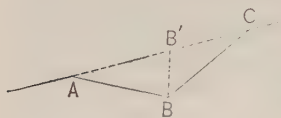


FIG. 215.

would soon exceed permissible speed. Assume that a freight train runs at an average speed of 15 miles per hour with a minimum of 10 miles and a permissible maximum of 30 miles per hour. Assume that a train runs up the grade at A with a uniform velocity of 15 miles per hour, i.e., the engine is working so that the velocity *would* be uniform to C. How much sag ( $BB'$ ) can there be without the speed exceeding 30 miles per hour?

|                                          |       |
|------------------------------------------|-------|
| Velocity head for 30 miles per hour..... | 31.60 |
| “ “ “ 15 “ “ “ .....                     | 7.90  |
| The drop $BB'$ will therefore be.....    | 23.70 |

While each case must be figured by itself, considering the probable velocity of approach and the maximum permissible velocity, we may say that a sag of *about* 24 feet will ordinarily mark the limit of this class. With a higher velocity of approach even this limit will be much reduced.

The classification therefore applies to sags and humps and to the vertical feet of drop or climb which are involved, rather than to grade *per se*. The practical application of these principles is necessarily confined to humps or sags which are possibly removable and does not apply to the long grades which are essential to connect predetermined points of the route—grades which are irreducible except by development and which must be studied as ruling grades (see §§ 440–445).

The application therefore consists in the comparative study of two proposed grades, noting the relative energy required to operate them and the probable cost. The depth in feet saved would be the maximum difference between the grades, and the classification will depend on the necessary method of operating the trains.

The next classification (*B*) applies to drops so deep that steam must be shut off when descending the grade, while the work required of the engine when ascending the opposite grade is correspondingly increased. The loss is not so serious as in the next case, but the inability of the engine to work continuously may result in a failure to accumulate sufficient kinetic energy to carry the train over a succeeding summit.

The third class (*C*) includes the grades so long that brakes must be applied to prevent excessive velocity. The loss involved is very heavy; the brakes require power for their application, they wear the brake-shoes and wheel-tires, they destroy kinetic or potential energy which had previously been created, while the tax on the locomotive on the corresponding ascending grade is very great. The ascending grade may or may not be a ruling grade.

**437. Effect on operating expenses.** As in Chapter XXII we may at once throw out a large proportion of the items of expense of an average train-mile. In "maintenance of way and structures" items 4 to 10 are evidently unaffected.

**Item 1. REPAIRS OF ROADWAY.** It is very plain that a large proportion of the sub-items are absolutely unaffected by minor grades. In fact it is a little difficult to ascribe any definite increase to any sub-item. The rail wear is somewhat increased and this will have some effect on the trackwork, but on the other hand the increased grade sometimes results in better drainage and therefore less work to keep the track in condition. Wellington allows 5% increase as a "liberal estimate" for class *C*, and no increase for the other classes.

**Item 2. RENEWALS OF RAILS.** Observations of rail wear on heavy grades show that it is much greater than on level tangents. But usually such heavy grades are operated by shorter trains or with the help of pusher engines, and the proportion of engine tonnage to the total is much greater than is ordinarily the case. And since an engine has much greater effect on rail wear than cars, particularly on account of the use of sand, an excess of engine tonnage would have a marked effect. But such circumstances would inevitably accompany ruling grades and not minor grades. Nevertheless the effect of the use of sand on up grades and the possible skidding of wheels on down grades will wear the rails somewhat. Even the possible slipping of the drivers, although sand is not used,



will wear the rails. Wellington allows 10% increase for class *C* and 5% for class *B*.

**Item 3. RENEWALS OF TIES.** The added wear of ties might be considered proportional to that of the rails except that, as in the case of the roadbed in general, the better drainage secured by the grade will tend to increase the life of the ties. Wellington makes the estimate the same as for item 1, 5% for class *C* and no increase for the other classes.

**Maintenance of equipment.** Items 11, 16, 17, 18, and 19 are evidently unaffected. Items 12 to 15. The chief sub-items of increase will evidently be the repairs and renewals of wheels and brake-shoes both for locomotives and cars. In the case of cars the draw-bar is apt to suffer from severe alternate compression and extension due to push and pull. The locomotive mechanism will suffer somewhat from the extra demands on it, and the boiler on account of the intermittent character of the demands on it. It would seem as if such effects would be quite large, but an examination of the comparative records of engine and car repairs on mountain divisions and on comparatively level divisions shows no such difference as might be expected. On this account Wellington cuts down these items to 4% for class *C* and 1% for class *B*.

**Conducting transportation.** As in Chapter XXI, § 407, since the resistance is assumed to be doubled, we may take the same figure (55%) as the cost of the fuel for climbing the 26.4 feet. But the total cost of both the rise and fall is to be considered. In class *B*, although steam is shut off, heat (and fuel) is wasted by mere radiation. This has been estimated (Chapter XXI, § 407) as about 5%. Therefore we may allow 60% for class *B*. For class *C* we must allow in addition the energy spent in applying brakes, which we may assume as 5% more, making 65%. Items 23, 24, and 25 may be estimated similarly. The other items under this head as well as General Expenses are evidently unaffected.

**438. Estimate of the cost of one foot of change of elevation.** Collecting these estimates, we have the accompanying tabular form, showing that the percentage of increase for operating grades of class *B* or class *C* will be 6.77% and 8.56%, respectively. On the basis of an average cost of 95 c. per train-mile the additional cost for the 26.4 feet in one mile would be 6.43 c. and 8.13 c., or 0.24 c. and 0.31 c. per foot. For each train per



day each way per year the value per foot of difference of elevation is:

$$\text{For class } B: 2 \times 365 \times \$0.0024 = \$1.75;$$

$$\text{" " } C: 2 \times 365 \times \$0.0031 = \$2.26.$$

It will frequently happen that a grade must be considered as belonging to class *C* for heavy freight trains, and that it belongs to class *B* or even class *A* for other trains. If no Sunday trains are run, 313 should be used instead of 365 as a multiplier in the above equations.

439. Operating value of the removal of a hump in a grade. As a simple illustration of the above, suppose that the irregular grade *ABCD* may be cut down to the uniform grade *AD*, either by direct lowering of the track or by a modification of alignment. If a freight train, running to the *left*, passes *C* at a velocity of 15 miles per hour (velocity head, 7.90 feet) and drops

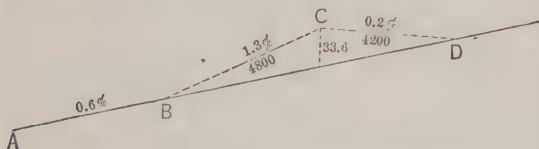


FIG. 216.

down to *B* (a vertical fall of 62.40 feet) without shutting off steam, its velocity head at *B* would be 70.30 feet and its velocity 44.8 miles per hour, which is inadmissible. Therefore the hump certainly does not belong to class *A*, for freight trains. Suppose that steam is shut off when passing *C*. Then, if we consider the average value of 0.5% as the grade which is equivalent to the normal rolling resistances, we may consider the 1.3% grade as an 0.8% grade, down which the train passes without resistance. An 0.8% grade for 4800 feet would be a drop of 38.40 feet, and the velocity head at *B* would be  $7.90 + 38.40 = 46.30$  feet, and the velocity would be 36.3 miles per hour, which may or may not be considered as inadmissible for freight trains. According to the decision the grade belongs to class *C* or to class *B*. For trains moving to the *right*, or up grade, there is no definite criterion, but a grade of 1.3% is a severe tax on a locomotive, even if it is not a ruling grade. Ignoring its possible

TABLE XXIV.—EFFECT ON OPERATING EXPENSES OF CHANGES IN GRADE.

| No.   | Item (abbreviated).*                     | Normal average. | Class B.          |               | Class C.          |               |
|-------|------------------------------------------|-----------------|-------------------|---------------|-------------------|---------------|
|       |                                          |                 | Per cent affected | Cost per mile | Per cent affected | Cost per mile |
| 1     | Roadway.....                             | 10.596          | 0                 | 0             | 5                 | .53           |
| 2     | Rails.....                               | 1.440           | 5                 | .07           | 10                | .14           |
| 3     | Ties.....                                | 3.093           | 0                 | .07           | 5                 | .15           |
| 4-10  | Bridges, buildings, etc.                 | 5.533           | 0                 | 0             | 0                 | 0             |
|       | Maintenance of way.                      | 20.662          | .....             | .07           | .....             | .82           |
| 11    | Superintendence.....                     | .650            | 0                 | 0             | 0                 | 0             |
| 12    | Repairs locomotives....                  | 5.879           | 1                 | .06           | 4                 | .23           |
| 13    | Repairs pass. cars....                   | 2.209           | 1                 | .02           | 4                 | .09           |
| 14    | Repairs freight cars....                 | 6.765           | 1                 | .07           | 4                 | .27           |
| 15    | Repairs work cars.....                   | .155            | 1                 | .00           | 4                 | .06           |
| 16    | Marine equipment.....                    | .209            | 0                 | 0             | 0                 | 0             |
| 17    | Shops.....                               | .490            | 0                 | 0             | 0                 | 0             |
| 18    | Stat. and printing.....                  | .040            | 0                 | 0             | 0                 | 0             |
| 19    | Other expenses.....                      | .495            | 0                 | 0             | 0                 | 0             |
|       | Main. of equip.....                      | 16.892          | .....             | .15           | .....             | .65           |
| 20    | Superintendence.....                     | 1.761           | 0                 | 0             | 0                 | 0             |
| 21    | Enginemmen.....                          | 9.781           | 0                 | 0             | 0                 | 0             |
| 22    | Fuel.....                                | 9.681           | 60                | 5.81          | 65                | 6.29          |
| 23    | Water.....                               | .671            | 60                | .40           | 65                | .44           |
| 24    | Oil, etc.....                            | .376            | 60                | .23           | 65                | .24           |
| 25    | Other supplies.....                      | .184            | 60                | .11           | 65                | .12           |
| 26-46 | Train service, station service, etc..... | 35.340          | 0                 | 0             | 0                 | 0             |
|       | Conducting transp..                      | 57.794          | .....             | 6.55          | .....             | 7.09          |
| 47-53 | General expenses.....                    | 4.653           | 0                 | 0             | 0                 | 0             |
|       |                                          | 100.000         | .....             | 6.77          | .....             | 8.56          |

\* For full title of item see Table XX.

limiting effect (which is a separate matter), the value of the 33.6 feet is evidently

and  $33.6 \times \$2.26 = \$75.94$  per daily train for class *C*

$33.6 \times \$1.75 = \$58.80$  “ “ “ “ “ “ *B*.

Assuming that there are six daily trains each way for which the grade would be classified as *C*, and four others which could operate the grade as a “*B*” grade, the total annual cost would be

$$6 \times \$75.94 = \$455.64$$

$$4 \times \$58.80 = \$235.20$$

$$\underline{\$690.84}$$

This annual cost, capitalized at 5%, equals \$13817, which is the justifiable expenditure to avoid the hump. Assuming that the cut would involve 300000 cubic yards, at 20 c. per cubic yard, it would cost \$60000 to make the through cut. On the above basis the cut would not be justifiable, but a small part of such cutting would so reduce the hump that it would not belong to class *C* for any trains and it might even be reduced to class *A*. Of course other solutions are possible. A slightly different route may be chosen from *B* to *D*, involving a different distance, different curvature, and a marked reduction in the hump. The effect of all such changes must be combined and their *net* effect determined.

#### RULING GRADES.

**440. Definition.** Ruling grades are those which limit the weight of the train of cars which may be hauled by one engine. The subject of "pusher grades" will be considered later. For the present it will suffice to say that on all well-designed roads the large majority of the grades on any one division are kept below some limit which is considered the ruling grade. If a heavier grade is absolutely necessary no special expense will be made to keep it below a rate where the resistance is twice (or possibly three times) the resistance on the ruling grade, and then the trains can be hauled unbroken up these few special grades with the help of one (or two) pusher engines. So far as limitation of train length is concerned, these pusher grades are no worse than the regular ruling grades and, except for the expense of operating the pusher engines (which is a separate matter), they are not appreciably more expensive than any ruling grade. As before stated, the engineer cannot alter very greatly the general level of the road when the general route has been decided on. He may remove sags or humps, or he may lower the natural grade of the route by development in order to bring the grade within the adopted limit of ruling grade. The financial value of removing sags and humps has been considered. It now remains to determine the financial relation between the lowest permissible ruling grade and the money which may profitably be spent to secure it.

**441. Choice of ruling grade.** It is of course impracticable for an engine to drop off or pick up cars according to the grades

which may be encountered along the line. A train load is made up at one terminus of a division and must run to the other terminus. Excluding from consideration any short but steep grades which may *always* be operated by momentum, and also all pusher grades, the maximum grade on that division is the ruling grade.

It will evidently be economy to reduce the few grades which naturally would be a little higher than the great majority of others until such a large amount of grade is at some uniform limit that a reduction at all these places would cost more than it is worth. The precise determination of this limit is practically impossible, but an approximate value may be at once determined from a general survey of the route. The distance apart of the termini of the division into their difference of elevation is a first trial figure for the rate of the grade. If a grade even approximately uniform is impossible owing to the elevations of predetermined intermediate points, the worst place may be selected and the natural grade of that part of the route determined. If this grade is much steeper than the general run of the natural grades, it may be policy to reduce it by development or to boldly plan to operate that place as a pusher grade. The choice of possible grades thus has large limitations, and it justifies very close study to determine the best combination of grades and pusher grades. When the choice has narrowed down to two limits, the lower of which may be obtained by the expenditure of a definite extra sum, the choice may be readily computed, as will be developed.

**442. Maximum train load on any grade.** The tractive power of a locomotive has been discussed in Chap. XV, § 322. The *net* train load which may be placed behind any engine is the difference between the weight of the engine itself and the *gross* load which can be handled under the given circumstances, with a given weight on the drivers. Since the design of locomotives is so variable, it is impracticable to show in tabular form the power of all kinds of locomotives on all grades. In Table XXV are given the tractive powers of locomotives of a wide range of types and weights and with various ratios of adhesion. They may be accepted as typical figures and will serve to compute the effect of variations of grade on train load. In Table XXVI is given the total train resistance in pounds per ton for various grades and for various values of track resistances. By a combination

TABLE XXV.—TRACTIVE POWER OF VARIOUS TYPES OF LOCOMOTIVES AT VARIOUS RATES OF ADHESION.

| Kind.        | Gauge. | Total weight of engine and tender. |       | Weight of engine only. | Weight on drivers. | Tractive power when rate of adhesion is |               |               |
|--------------|--------|------------------------------------|-------|------------------------|--------------------|-----------------------------------------|---------------|---------------|
|              |        | lbs.                               | tons. |                        |                    | $\frac{1}{4}$                           | $\frac{2}{3}$ | $\frac{1}{2}$ |
| American...  | Nar.   | 19,000                             | 24.5  | 32,000                 | 22,000             | 5,500                                   | 4,350         | 4,100         |
| "            | "      | 80,000                             | 40    | 49,000                 | 32,000             | 8,000                                   | 7,200         | 6,400         |
| Mogul.....   | "      | 81,000                             | 40.5  | 51,000                 | 42,000             | 10,500                                  | 9,450         | 8,400         |
| 10-wheel...  | "      | 87,000                             | 43.5  | 55,000                 | 42,000             | 10,500                                  | 9,450         | 8,400         |
| Consol....   | "      | 62,000                             | 31    | 39,000                 | 34,000             | 8,500                                   | 7,650         | 6,800         |
| "            | "      | 144,000                            | 72    | 94,000                 | 84,000             | 21,000                                  | 18,900        | 16,800        |
| American...  | Stand. | 104,000                            | 52    | 62,000                 | 40,000             | 10,000                                  | 9,000         | 8,000         |
| "Chautau"... | "      | 314,000                            | 157.3 | 196,000                | 99,400             | 24,850                                  | 22,365        | 19,880        |
| Mogul....    | "      | 206,000                            | 103   | 126,000                | 106,400            | 26,600                                  | 23,940        | 21,280        |
| 10-wheel...  | "      | 276,000                            | 138   | 176,510                | 127,010            | 31,752                                  | 28,577        | 25,402        |
| Consol....   | "      | 214,000                            | 107   | 120,000                | 106,000            | 26,500                                  | 23,850        | 21,200        |
| "            | "      | 324,800                            | 162.4 | 204,800                | 181,200            | 45,300                                  | 40,770        | 36,240        |

of these two tables the net train load on any grade under given conditions may be quickly determined. For example, an ordinary consolidation engine having a weight of 106000 pounds on the drivers (see Table XXV) will have a tractive force of 26500 pounds under fair conditions of track, when the adhesion ratio is  $\frac{1}{4}$ . When climbing slowly up a grade of 1.30% the tractive resistance will be about 32 pounds per ton if the rolling-stock and track are fair—assuming a tractive resistance on a level of 6 pounds per ton. Dividing 26500 by 32 we have 828 tons, the gross train load. Subtracting 107 tons, the weight of the engine and tender in working order, we have 721 tons, the *net* load. Incidentally we may note that, cutting down the grade to 0.90% (a reduction of only 21.12 feet per mile), the resistance per ton is reduced to 24 pounds and the gross train load is increased to 1104 tons and the net load to 997 tons—an increase of about 38%.

As another numerical example, consider a contractor's locomotive (not referred to in Table XXV), a light four-wheel-connected-tank narrow-gauge engine, with a total weight of 12000 pounds, all on the drivers. On the rough temporary track used by contractors the tractive ratio may be as low as  $\frac{1}{5}$ . The tractive adhesion should therefore be taken as 2400 pounds. Assume that the grade when hauling "empties" is 4.7% and

that the tractive resistance on such a track *on a level* is 10 pounds per ton. By Table XXVI, the total train resistance is therefore (by interpolation) 104 pounds per ton.  $2400 \div 104 = 23$  tons; subtracting the weight of the engine we have 17 tons, the net load of empty cars—perhaps twenty cars weighing 1700 pounds per car.

In general, and to compute accurately the train load under conditions not exactly given in the tables, the maximum train load may be computed according to the following rule:

The maximum load behind an engine on any grade may be found by multiplying the weight on the drivers by the ratio of adhesion and dividing this by the sum of the grade and tractive resistances per ton; this gives the gross load, from which the weight of the engine and tender must be subtracted to find the net load.

**443. Proportion of the traffic affected by the ruling grade.** Some very light traffic roads are not so fortunate as to have a traffic which will be largely affected by the rate of the ruling grade. When passenger traffic is light, and when, for the sake of encouraging traffic, more frequent trains are run than are required from the standpoint of engine capacity, it may happen that no passenger trains are really limited by any grade on the road—i.e., an extra passenger car *could* be added if needed. The maximum grade then has no worse effect (for passenger trains) than to cause a harmless reduction of speed at a few points. The local freight business is frequently affected in practically the same way. All coal, mineral, or timber roads are affected by the rate of ruling grade as far as such traffic is concerned. Likewise the through business in general merchandise, especially of the heavy traffic roads, will generally be affected by the rate of ruling grade. Therefore in computing the effect of ruling grade, the total number of trains on the road should not ordinarily be considered, but only the trains to which cars are added, until the limit of the hauling power of the engine on the ruling grades is reached.

**444. Financial value of increasing the train load.** The gross receipts for transporting a given amount of freight is a definite sum regardless of the number of train loads. The cost of a train mile is practically constant. If it were exactly so, the saving in operating expenses would be strictly proportional to the number of trains saved. How will the cost per train



TABLE XXVI.—TOTAL TRAIN RESISTANCE PER TON (OF 2000 POUNDS) ON VARIOUS GRADES.

| Grade.         |                | When tractive resistance on a level in pounds per ton is |    |    |    |    | Grade.         |                | When tractive resistance on a level in pounds per ton is |     |     |     |     |
|----------------|----------------|----------------------------------------------------------|----|----|----|----|----------------|----------------|----------------------------------------------------------|-----|-----|-----|-----|
| Rate per cent. | Feet per mile. | 6                                                        | 7  | 8  | 9  | 10 | Rate per cent. | Feet per mile. | 6                                                        | 7   | 8   | 9   | 10  |
| 0.00           | 0.00           | 6                                                        | 7  | 8  | 9  | 10 | 2.00           | 105.60         | 46                                                       | 47  | 48  | 49  | 50  |
| .05            | 2.64           | 7                                                        | 8  | 9  | 10 | 11 | .05            | 108.24         | 47                                                       | 48  | 49  | 50  | 51  |
| .10            | 5.28           | 8                                                        | 9  | 10 | 11 | 12 | .10            | 110.88         | 48                                                       | 49  | 50  | 51  | 52  |
| .15            | 7.92           | 9                                                        | 10 | 11 | 12 | 13 | .15            | 113.52         | 49                                                       | 50  | 51  | 52  | 53  |
| .20            | 10.56          | 10                                                       | 11 | 12 | 13 | 14 | .20            | 116.16         | 50                                                       | 51  | 52  | 53  | 54  |
| 0.25           | 13.20          | 11                                                       | 12 | 13 | 14 | 15 | 2.25           | 118.80         | 51                                                       | 52  | 53  | 54  | 55  |
| .30            | 15.84          | 12                                                       | 13 | 14 | 15 | 16 | .30            | 121.44         | 52                                                       | 53  | 54  | 55  | 56  |
| .35            | 18.48          | 13                                                       | 14 | 15 | 16 | 17 | .35            | 124.08         | 53                                                       | 54  | 55  | 56  | 57  |
| .40            | 21.12          | 14                                                       | 15 | 16 | 17 | 18 | .40            | 126.72         | 54                                                       | 55  | 56  | 57  | 58  |
| .45            | 23.76          | 15                                                       | 16 | 17 | 18 | 19 | .45            | 129.36         | 55                                                       | 56  | 57  | 58  | 59  |
| 0.50           | 26.40          | 16                                                       | 17 | 18 | 19 | 20 | 2.50           | 132.00         | 56                                                       | 57  | 58  | 59  | 60  |
| .55            | 29.04          | 17                                                       | 18 | 19 | 20 | 21 | .55            | 134.64         | 57                                                       | 58  | 59  | 60  | 61  |
| .60            | 31.68          | 18                                                       | 19 | 20 | 21 | 22 | .60            | 137.28         | 58                                                       | 59  | 60  | 61  | 62  |
| .65            | 34.32          | 19                                                       | 20 | 21 | 22 | 23 | .65            | 139.92         | 59                                                       | 60  | 61  | 62  | 63  |
| .70            | 36.96          | 20                                                       | 21 | 22 | 23 | 24 | .70            | 142.56         | 60                                                       | 61  | 62  | 63  | 64  |
| 0.75           | 39.60          | 21                                                       | 22 | 23 | 24 | 25 | 2.75           | 145.20         | 61                                                       | 62  | 63  | 64  | 65  |
| .80            | 42.24          | 22                                                       | 23 | 24 | 25 | 26 | .80            | 147.84         | 62                                                       | 63  | 64  | 65  | 66  |
| .85            | 44.88          | 23                                                       | 24 | 25 | 26 | 27 | .85            | 150.48         | 63                                                       | 64  | 65  | 66  | 67  |
| .90            | 47.52          | 24                                                       | 25 | 26 | 27 | 28 | .90            | 153.12         | 64                                                       | 65  | 66  | 67  | 68  |
| 0.95           | 50.16          | 25                                                       | 26 | 27 | 28 | 29 | .95            | 155.76         | 65                                                       | 66  | 67  | 68  | 69  |
| 1.00           | 52.80          | 26                                                       | 27 | 28 | 29 | 30 | 3.00           | 158.40         | 66                                                       | 67  | 68  | 69  | 70  |
| .05            | 55.44          | 27                                                       | 28 | 29 | 30 | 31 | .05            | 161.04         | 67                                                       | 68  | 69  | 70  | 71  |
| .10            | 58.08          | 28                                                       | 29 | 30 | 31 | 32 | .10            | 163.68         | 68                                                       | 69  | 70  | 71  | 72  |
| .15            | 60.72          | 29                                                       | 30 | 31 | 32 | 33 | .15            | 166.32         | 69                                                       | 70  | 71  | 72  | 73  |
| .20            | 63.36          | 30                                                       | 31 | 32 | 33 | 34 | .20            | 168.96         | 70                                                       | 71  | 72  | 73  | 74  |
| 1.25           | 66.00          | 31                                                       | 32 | 33 | 34 | 35 | 3.25           | 171.60         | 71                                                       | 72  | 73  | 74  | 75  |
| .30            | 68.64          | 32                                                       | 33 | 34 | 35 | 36 | .30            | 174.24         | 72                                                       | 73  | 74  | 75  | 76  |
| .35            | 71.28          | 33                                                       | 34 | 35 | 36 | 37 | .35            | 176.88         | 73                                                       | 74  | 75  | 76  | 77  |
| .40            | 73.92          | 34                                                       | 35 | 36 | 37 | 38 | .40            | 179.52         | 74                                                       | 75  | 76  | 77  | 78  |
| .45            | 76.56          | 35                                                       | 36 | 37 | 38 | 39 | .45            | 182.16         | 75                                                       | 76  | 77  | 78  | 79  |
| 1.50           | 79.20          | 36                                                       | 37 | 38 | 39 | 40 | 3.50           | 184.80         | 76                                                       | 77  | 78  | 79  | 80  |
| .55            | 81.84          | 37                                                       | 38 | 39 | 40 | 41 | 4.00           | 211.20         | 86                                                       | 87  | 88  | 89  | 90  |
| .60            | 84.48          | 38                                                       | 39 | 40 | 41 | 42 | 4.50           | 237.60         | 96                                                       | 97  | 98  | 99  | 100 |
| .65            | 87.12          | 39                                                       | 40 | 41 | 42 | 43 | 5.00           | 264.00         | 106                                                      | 107 | 108 | 109 | 110 |
| .70            | 89.76          | 40                                                       | 41 | 42 | 43 | 44 | 5.50           | 290.40         | 116                                                      | 117 | 118 | 119 | 120 |
| 1.75           | 92.40          | 41                                                       | 42 | 43 | 44 | 45 | 6.00           | 316.80         | 126                                                      | 127 | 128 | 129 | 130 |
| .80            | 95.04          | 42                                                       | 43 | 44 | 45 | 46 | 6.50           | 343.20         | 136                                                      | 137 | 138 | 139 | 140 |
| .85            | 97.68          | 43                                                       | 44 | 45 | 46 | 47 | 7.00           | 369.60         | 146                                                      | 147 | 148 | 149 | 150 |
| .90            | 100.32         | 44                                                       | 45 | 46 | 47 | 48 | 8.00           | 422.40         | 166                                                      | 167 | 168 | 169 | 170 |
| 1.95           | 102.96         | 45                                                       | 46 | 47 | 48 | 49 | 9.00           | 475.20         | 186                                                      | 187 | 188 | 189 | 190 |
| 2.00           | 105.60         | 46                                                       | 47 | 48 | 49 | 50 | 10.00          | 528.00         | 206                                                      | 207 | 208 | 209 | 210 |

mile vary when by a reduction in ruling grade more cars are handled in one train than before? First, compute the effect



of increasing the train load so that one less engine will handle the traffic, or, for example, that an engine can haul 11 cars instead of 10 or 44 instead of 40—that 10 engines will do the work for which 11 engines would be required with the steeper grade. What will be the relative cost of running 10 heavy trains rather than 11 lighter trains, or, rather, what will be the extra cost of the extra engine?

Since the gross traffic to be handled is assumed to be the same, the number of cars required to handle it will also be the same whatever the number of trains, and the effect of those cars on the wear and tear of track, etc., will evidently be constant. The locomotive, on account of the greater concentration of loading of the driver wheels, damages the track (in proportion to its tonnage) much more than the cars. It has been estimated that the locomotive is responsible for one half of the track wear. Such an estimate is verified by the wear of rails on steep tracks around coal-mines where standard cars are hauled by cables. If we assume that 50% of Items 2 and 3 and of that part of Item 1 which varies with tonnage is due to the locomotives, then the *extra* expense caused by the extra engine will be 50% of Items 2 and 3 and 50% of 25% of Item 1. The other items of maintenance of way are unaffected except that truss bridges, trestles, and the maintenance of a few buildings will be slightly affected by the extra locomotive. But the actual effect is quite indefinite and is evidently very small.

**Maintenance of equipment:** Engine repairs will evidently be affected according to the mileage. Throughout the ruling grade of the road (by whichever system of grades) the engines (assumed of uniform style) are working at their utmost capacity. On the lighter grades and level sections the engines will have easier work when the cars are fewer and this will have a tendency to reduce engine repairs. Suppose that by decreasing the number of cars 10% on the easy grades the engine repairs on each engine are reduced 2%. There is little or no justification for estimating the reduction to be more than this. Then on the ten engines the saving is 20% of the average charge for 1 engine. Suppose that by decreasing the number of cars 20% on the easy grades the engine repairs are reduced 4%, on the five engines they are reduced 20% again. In either case the net added cost due to the extra engine would be but 80% of the average cost. While the above estimate is but a guess,

yet it is very evident that the extra cost for this item is but little less than the normal charge.

Car repairs will be reduced by a decrease in the number of cars per train. The average draw-bar pull will be less, the wear and tear due to stoppage and starting will be less. This is the one item in which an increased number of trains for the same tonnage is an actual advantage. The saving per car is evidently greater when 4 trains are increased to 5 than when 10 trains are increased to 11; but the saving per train added on is constant. Wellington estimates the saving to be 10%. His basis of calculation is somewhat different, but it reduces to the same thing. The estimate applies chiefly to Item 14 and to Item 13 in so far as passenger trains are affected by ruling grade. The other items of maintenance of equipment are but little, if any, affected.

**Conducting transportation.** Items 20, 21, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 45, and 46 may be considered as varying according to the train mileage. While some of them seem to have but little direct connection with train mileage, yet if a road increases its traffic from 10 trains a day to 20 trains a day all of these items seem to increase in due proportion.

Fuel, etc., for locomotives (Items 22-25) will increase nearly as the engine mileage. In either case the engines work to the limit of their capacity on the ruling grades. In either case the loss of heat due to radiation is the same. But the engines with the lighter trains work a little easier on the light or level grades. By the same course of reasoning as was given regarding engine repairs the fuel saving from the normal requirement for the extra engine will be about the same no matter whether there is an addition of one engine in 5 or 10. The saving in fuel will be assumed at 25% of the normal consumption, or rather that the use of the extra engine adds 75% of the normal charge for fuel. The same estimate applies to items 23, 24, and 25.

Car mileage is unaffected. Items 38 to 44 will be considered as unaffected, also the general expenses.

**445. Operating value of a reduction in the rate of the ruling grade.** Collecting the above estimates, we have Table XXVII. To this must be added something for the capital cost of the extra engine. Assume that it costs \$10000 and that its mileage life is 800000 miles. This makes an average charge of 1.25 c. per mile. Of course the cost of operation, maintenance, and repairs

is included in the tabulated expense.  $54.82\%$  of 95 c. = 52.08 c.  
Adding 1.25 c., we have 53.33 c.

TABLE XXVII.—COST OF AN ADDITIONAL TRAIN TO HANDLE  
A GIVEN TRAFFIC.

| No.   | Item (abbreviated).            | Normal average. | Per cent affected. | Cost per cent. |
|-------|--------------------------------|-----------------|--------------------|----------------|
| 1     | Roadway.....                   | 10.596          | 12.5               | 1.32           |
| 2     | Rails.....                     | 1.440           | 50                 | .72            |
| 3     | Ties.....                      | 3.093           | 50                 | 1.55           |
| 4-10  | Bridges, buildings, etc.....   | 5.533           | 0                  | 0              |
|       | Maintenance of way.....        | 20.662          | .....              | 3.59           |
| 11    | Superintendence.....           | .650            | 0                  | 0              |
| 12    | Repairs of locomotives.....    | 5.879           | 80                 | 4.70           |
| 13    | Repairs of passenger-cars..... | 2.209           | 5                  | .11            |
| 14    | Repairs of freight-cars.....   | 6.765           | 10                 | .68            |
| 15-19 | Miscellaneous.....             | 1.389           | 0                  | 0              |
|       | Maintenance of equipment....   | 16.892          | .....              | 3.91           |
| 20    | Superintendence.....           | 1.761           | 100                | 1.76           |
| 21    | Enginemen.....                 | 9.781           | 100                | 9.78           |
| 22-25 | Fuel, etc.....                 | 10.912          | 75                 | 8.18           |
| 26-32 | Train service, etc.....        | 24.285          | 100                | 24.28          |
| 33    | Car mileage.....               | 2.094           | 0                  | 0              |
| 34-37 | Damages, etc.....              | 2.085           | 100                | 2.09           |
| 38-44 | Miscellaneous.....             | 5.646           | 0                  | 0              |
| 45-46 | Stationery, etc.....           | 1.229           | 100                | 1.23           |
|       | Conducting transportation....  | 57.793          | .....              | 47.32          |
| 47-53 | General expenses.....          | 4.653           | 0                  | 0              |
|       |                                | 100.000         | .....              | 54.82          |

As a practical application of the above figures, assume that on a constructed and operated road the ruling grade on a 100-mile division is  $1.6\%$ ; the actual traffic affected by ruling grade is 8 daily trains with a net load of 552 tons or 4416 tons. It is found that with an expenditure of \$400000 the ruling grade may be reduced to  $1.2\%$ . Will it pay? At  $1.2\%$  grade the net load behind an 80-ton consolidation engine, with 48 tons on the drivers, adhesion  $\frac{1}{4}$ , is 720 tons. The traffic (4416 tons) may therefore be hauled by 6 engines, the balance, less than 100 tons, being taken care of by lighter trains not affected by the ruling grade. Since the *additional* cost of the engine drawing lighter trains is 53 c. per mile, the saving by reducing from 8 engines to 6 is that due to 2 engines. The annual saving is therefore  $2 \times \$0.5333 \times 100 \times 365 = \$38930.90$ , which capital-

ized at  $5\% = \$778618$ . This shows that if the improvement can be accomplished for \$400000 it is worth while.

As in other similar problems, it must be reiterated that although there are some more or less uncertain elements in the above estimates, yet with a considerable margin for error in individual items the value of the whole improvement will not be very greatly altered and the estimate will be infinitely better than an indefinite reliance on vague "judgment." Of course certain items in the above estimates are somewhat variable and should be altered to fit the particular case to be computed.

#### PUSHER GRADES

**446. General principles underlying the use of pusher engines.** On nearly all roads there are some grades which are greatly in excess of the general average rate of grade and these heavy grades cannot usually be materially reduced without an expenditure which is excessive and beyond the financial capacity of the road. If no pusher engines are used, the length of all heavy trains is limited by these grades. The financial value of the reduction of such ruling grades has already been shown. But in the operation of pusher grades there is incurred the additional cost of pusher-engine service, for a pusher engine must run *twice* over the grade for each train which is assisted. It is possible for this additional expense to equal or even exceed the advantage to be gained. In any case it means the adoption of the lesser of two evils, or the adoption of the more economical method. A simple example will illustrate the point. Assume that at one point on the road there is a grade of 1.9%, which is five miles long. Assume that all other grades are less than 0.92%. If pushers are not to be used the net capacity of a 107-ton consolidation engine with 53 tons on the drivers, assuming  $\frac{9}{40}$  adhesion and 6 pounds per ton for normal resistance, will be 435 tons, and that will be the maximum weight of train allowable. By using pusher engines on this one 5-mile grade the train load is at once doubled and the number of trains cut down one half. This double load, 870 tons, can easily be hauled by one engine up the 0.92% grades. As a rough comparison, free from details and allowances, we may say:

(a) 10 trains per day over a 100-mile division, 435 tons net per train, will require 1000 engine miles daily.

(b) 5 trains per day handling the same traffic, 870 tons net per train, with  $2 \times 5 \times 5$  pusher-engine miles, will require  $(5 \times 100) + (2 \times 5 \times 5) = 550$  engine miles daily. There is thus a large saving in the number of engine miles and also in the number of the engines required for the work. Moreover, the engines are working to the limit of their capacity for a much larger proportion of the time, and their work is therefore more economically done. The work of overcoming the normal resistances of so many loaded cars over so many miles of track and of lifting so many tons up the gross differences of elevation of predetermined points of the line is approximately the same whatever the exact route, and if the grades are so made that fewer engines working more constantly can accomplish the work as well as more engines which are not hard worked for a considerable proportion of the time, the economy is very apparent and unquestionable. Wellington expresses it concisely: "It is a truth of the first importance that the objection to high gradients is not the work which the engines have to do on them, but it is the work which they do *not* do when they thunder over the track with a light train behind them, from end to end of a division, in order that the needed power may be at hand at a few scattered points where alone it is needed."

447. **Balance of grades for pusher service.** In the above illustration the "through" grade and the "pusher" grade are "balanced" for the use of one equal pusher. It is therefore evident that if some intermediate grade (such as 1.4%) were permitted, it could only be operated by (a) making it the ruling grade and cutting down all train loads from 870 tons to 594 tons, or (b) operating it as a pusher grade, although with a loss of economy, since two engines would have much more power than necessary. The proper plan in such a case would be to strive to reduce the 1.4% grade to 0.92%, or, if that seemed impracticable, to attempt to get an operating advantage at the expense of an increase of the 1.4% grade to anything short of 1.9%. For the increase in *rate* of grade would cost almost nothing, and some advantage might be obtained which would practically compensate for the introduction of a pusher grade. *Another possible* solution would be to operate the 1.9% with two pushers, adopt a corresponding grade for use with one pusher and a corresponding ruling grade for through trains. With the above

data these three grades would be 1.90%, 1.27%, and 0.54%, obtained as follows:

Traction power of three engines =  $106000 \times \frac{9}{40} \times 3 = 71550$  pounds.

Resistance on 1.9% grade =  $6 + (20 \times 1.9) = 44$  lbs. per ton.

$71550 \div 44 = 1626 =$  gross load in tons.

$1626 - (3 \times 107) = 1305 =$  net load in tons.

$1305 \div (2 \times 107) = 1519 =$  gross load on the one-pusher grade.

Traction power of two engines = 47700 lbs.

$47700 \div 1519 = 31.40 =$  possible tractive force in lbs. per ton.

$(31.40 - 6) \div 20 = 1.27\% =$  permissible grade for one pusher.

$1305 \div 107 = 1412 =$  gross load on the through grade.

Traction power of one engine = 23850 lbs.

$23850 \div 1412 = 16.89 =$  possible tractive force in lbs. per ton.

$(16.89 - 6) \div 20 = 0.54\% =$  permissible through grade.

It should be realized that, assuming the accuracy of the normal resistance (6 lbs.) and the normal adhesion ( $\frac{9}{40}$ ) and with the use of 107-ton locomotives with 53 tons on the drivers, the above figures are *precisely* what is required for hauling with one, two, and three engines. Other types of engines, other values for resistance and adhesion will vary considerably the gross load in tons which may be hauled up those grades, but starting with 0.54% as a through grade, the corresponding values for one and for two pushers would vary but slightly from those given. To show the tendency of these variations, the corresponding values have been computed as follows:

| Adhesion.      | Resistance per ton. | Load on drivers. | Through grade. | One-pusher grade. | Two-pusher grade. |
|----------------|---------------------|------------------|----------------|-------------------|-------------------|
| $\frac{9}{40}$ | 6 lbs.              | 53 tons.         | 0.54%          | 1.27%             | 1.90%             |
| $\frac{9}{40}$ | 7 "                 | 53 "             | .54%           | 1.31%             | 1.96%             |
| $\frac{9}{40}$ | 6 "                 | 53 "             | .54%           | 1.28%             | 1.93%             |
| $\frac{9}{40}$ | 6 "                 | 53 "             | .54%           | 1.26%             | 1.86%             |
| $\frac{9}{40}$ | 7 "                 | 53 "             | .54%           | 1.29%             | 1.92%             |

The above form shows that *increasing* the resistance per ton and *decreasing* the adhesion have opposite effects on altering the ratio of these grades, and as a storm, for example, would increase the resistance and decrease the adhesion, the changes in the ratio would be compensating although the absolute reduction in train load might be considerable.



In Table XXVIII is shown a series of "balanced" grades on which a given *net* train load may be operated by means of one or two pusher engines. For example, assuming a track resistance of 6 pounds per ton, a consolidation engine of the type shown in the table can haul a train weighing 977 tons (exclusive of the engine) up a grade of  $0.80\%$ . If this is the maximum *through* grade, pusher grades as high as  $1.70\%$  for one pusher, or  $2.46\%$  for two pushers, may be introduced and the same *net* load may be hauled up these grades.

The ratios of pusher grade to through grade, as given in Table XXVIII, are exactly true only for the conditions named as to weight and type of engine, ratio of adhesion, and normal track resistance. But a little comparative study of the two halves of Table XXVIII and of the tabular form given on page 483 will show that although the net load which can be hauled on any grade varies considerably with the normal track resistance and also with the ratio of adhesion, yet the *ratios* of through to pusher grade, for either one or two pushers, varies but slightly with ordinary changes in these conditions. Therefore when the precise conditions are unknown or variable, the figures of Table XXVIII may be considered as applicable to any ordinary practice, especially for preliminary computations. For final calculations on any proposed ruling grade and pusher grade, the whole problem should be worked out on the principles outlined above and on the basis of the best data obtainable.

*Problem:* If the *through* ruling grade for the road has been established at  $1.12\%$ , what pusher grades are permissible?  
*Answer:* Interpolating in Table XXVIII, we may employ a grade of  $2.22\%$  if the track and road-bed are to be such that a tractive resistance of 6 pounds per ton can be expected. With a poorer track, the normal resistance assumed as 8 pounds per ton, the rate is raised to  $2.27\%$ . The *increase* in rate of pusher grade with increase of resistance is due to the fact that the net load hauled is less—so much less that on the pusher grade a larger part of the adhesion is available to overcome a grade resistance.

**448. Operation of pusher engines.** The maximum efficiency in operating pusher engines is obtained when the pusher engine is kept constantly at work, and this is facilitated when the pusher grade is as long as possible, i.e., when the heavy grades and the great bulk of the difference of elevation to be surmounted is



TABLE XXVIII.—BALANCED GRADES FOR ONE, TWO, AND THREE ENGINES.

*Basis.*—Through and pusher engines alike; consolidation type; total weight, 107 tons; weight on drivers, 53 tons; adhesion, 4%, giving a tractive force for each engine of 23850 lbs.; normal track resistance 6 (also 8) lbs. per ton.

| Through grade. | Track resistance, 6 lbs.                     |                                               |              | Track resistance, 8 lbs.                     |                                               |              |
|----------------|----------------------------------------------|-----------------------------------------------|--------------|----------------------------------------------|-----------------------------------------------|--------------|
|                | Net load for one engine in tons (2000 lbs.). | Corresponding pusher grade for same net load. |              | Net load for one engine in tons (2000 lbs.). | Corresponding pusher grade for same net load. |              |
|                |                                              | One pusher.                                   | Two pushers. |                                              | One pusher.                                   | Two pushers. |
| Level.         | 3868 tons                                    | 0.28%                                         | 0.55%        | 2874 tons                                    | 0.37%                                         | 0.72%        |
| 0.10%          | 2874 "                                       | 0.47%                                         | 0.82%        | 2278 "                                       | 0.56%                                         | 0.98%        |
| 0.20%          | 2278 "                                       | 0.66%                                         | 1.08%        | 1880 "                                       | 0.71%                                         | 1.23%        |
| 0.30%          | 1880 "                                       | 0.84%                                         | 1.33%        | 1596 "                                       | 0.92%                                         | 1.47%        |
| 0.40%          | 1596 "                                       | 1.02%                                         | 1.57%        | 1384 "                                       | 1.09%                                         | 1.70%        |
| 0.50%          | 1384 "                                       | 1.19%                                         | 1.80%        | 1218 "                                       | 1.27%                                         | 1.92%        |
| 0.60%          | 1218 "                                       | 1.37%                                         | 2.02%        | 1085 "                                       | 1.44%                                         | 2.14%        |
| 0.70%          | 1085 "                                       | 1.54%                                         | 2.24%        | 977 "                                        | 1.60%                                         | 2.36%        |
| 0.80%          | 977 "                                        | 1.70%                                         | 2.46%        | 887 "                                        | 1.77%                                         | 2.56%        |
| 0.90%          | 887 "                                        | 1.87%                                         | 2.66%        | 810 "                                        | 1.93%                                         | 2.76%        |
| 1.00%          | 810 "                                        | 2.03%                                         | 2.86%        | 745 "                                        | 2.09%                                         | 2.96%        |
| 1.10%          | 745 "                                        | 2.19%                                         | 3.06%        | 688 "                                        | 2.24%                                         | 3.15%        |
| 1.20%          | 688 "                                        | 2.34%                                         | 3.25%        | 638 "                                        | 2.40%                                         | 3.33%        |
| 1.30%          | 638 "                                        | 2.50%                                         | 3.43%        | 594 "                                        | 2.55%                                         | 3.51%        |
| 1.40%          | 594 "                                        | 2.65%                                         | 3.61%        | 555 "                                        | 2.70%                                         | 3.68%        |
| 1.50%          | 555 "                                        | 2.80%                                         | 3.78%        | 521 "                                        | 2.85%                                         | 3.85%        |
| 1.60%          | 521 "                                        | 2.95%                                         | 3.95%        | 489 "                                        | 2.99%                                         | 4.02%        |
| 1.70%          | 489 "                                        | 3.09%                                         | 4.12%        | 461 "                                        | 3.13%                                         | 4.17%        |
| 1.80%          | 461 "                                        | 3.23%                                         | 4.27%        | 435 "                                        | 3.27%                                         | 4.33%        |
| 1.90%          | 435 "                                        | 3.37%                                         | 4.43%        | 411 "                                        | 3.42%                                         | 4.49%        |
| 2.00%          | 411 "                                        | 3.52%                                         | 4.59%        | 390 "                                        | 3.55%                                         | 4.63%        |
| 2.10%          | 390 "                                        | 3.65%                                         | 4.73%        | 370 "                                        | 3.68%                                         | 4.78%        |
| 2.20%          | 370 "                                        | 3.78%                                         | 4.88%        | 352 "                                        | 3.81%                                         | 4.92%        |
| 2.30%          | 352 "                                        | 3.91%                                         | 5.02%        | 335 "                                        | 3.94%                                         | 5.05%        |
| 2.40%          | 335 "                                        | 4.04%                                         | 5.15%        | 319 "                                        | 4.07%                                         | 5.19%        |
| 2.50%          | 319 "                                        | 4.17%                                         | 5.29%        | 304 "                                        | 4.20%                                         | 5.32%        |

at one place. For example, a pusher grade of three miles followed by a comparatively level stretch of three miles and then by another pusher grade of two miles cannot all be operated as cheaply as a continuous pusher grade of five miles. Either the two grades must be operated as a continuous grade of eight miles (sixteen pusher miles per trip) or else as two short pusher grades, in which case there would be a very great loss of time and a difficulty in so arranging the schedules that a train need

not wait for a pusher or the pushers need not waste too much time in idleness waiting for trains. If the level stretch were imperative, the two grades would probably be operated as one but an effort should be made to bring the grades together. It is not necessary to bring the trains to a stop to uncouple the pusher engine, but a stop is generally made for coupling on, and the actual cost in loss of energy and in wear and tear of stopping and starting a heavy train is as great as the cost of running an engine light for several miles.

There are two ways in which it is possible to economize in the use of pusher engines. (a) When the traffic of a road is so very light that a pusher engine will not be kept reasonably busy on the pusher grade it may be worth while to place a siding long enough for the longest trains both at top and bottom of the pusher grade and then take up the train in sections. Perhaps the worst objection to this method is the time lost while the engine runs the extra mileage, but with such very light traffic roads a little time more or less is of small consequence. On light traffic roads this method of surmounting a heavy grade will be occasionally adopted even if pushers are never used. If the traffic is fluctuating, the method has the advantage of only requiring such operation when it is needed and avoiding the purchase and operation of a pusher engine which has but little to do and which might be idle for a considerable proportion of the year. (b) The second possible method of economizing is only practicable when a pusher grade begins or ends at or near a station yard where switching-engines are required. In such cases there is a possible economy in utilizing the switching engines as pushers, especially when the work in each class is small, and thus obtain a greater useful mileage. But such cases are special and generally imply small traffic.

A telegraph-station at top and bottom of a pusher grade is generally indispensable to effective and safe operation.

**449. Length of a pusher grade.** The virtual length of the pusher grade, as indicated by the mileage of the pusher engine, is always somewhat in excess of the true length of the grade as shown on the profile, and sometimes the excess length is very great. If a station is located on a lower grade within a mile or so of the top or bottom of a pusher grade, it will ordinarily be advisable to couple or uncouple at or near the station since the telegraph-station, switching, and signaling may

more economically operated at a regular station. If the extra engine is coupled on ahead of the through engine (as is sometimes required by law for passenger trains) the uncoupling at the top of the grade may be accomplished by running the assistant engine ahead at greater speed after it is uncoupled, and, after running it on a siding, clearing the track for the train. But this requires considerable extra track at the top of the grade. Therefore, when estimating the length of the pusher grade, the most desirable position for the terminal sidings must be studied and the length determined accordingly rather than by measuring the mere length of the grade on the profile. Of course these odd distances are always *cross*; the coupling or uncoupling should not be done while on the grade.

**450. Cost of pusher-engine service.** The cost evidently depends partly on the mileage run, while some items are wholly independent of the mileage. A pusher engine, when working on grades where the conditions are fairly favorable, will accomplish a mileage of 100 to 125 miles per day, and this is about equal to that of an ordinary freight engine. Therefore such items as wages which are independent of mileage will be assumed to cost as much per mile as they do for ordinary train service. If the mileage is less than this, an extra allowance should be made.

The effect of a pusher engine on maintenance of way may be considered to be the same as that produced by an additional engine, as developed in § 444. The same allowance (3.59%) will therefore be made. The cost of repairs and renewals of locomotives may be estimated the same as for other engines. Wages for engine and round-house men will be the same. There is certainly no ground for considering that the cost of fuel and other engine supplies can be materially less than the usual figures. On the return trip down the grade the engine runs almost without steam (after getting started), but, on the other hand, the engine works hard when climbing up the grade. The cost of switchmen, etc., and telegraph expenses (Items 28 and 29) will evidently add their full quota. Collecting these items, we have 36.27% or 34.46 c. for each mile run. Adding, as in § 445, 1.25 c. as interest charge on the cost of the engine, we have 35.71 c. Then each mile of the incline will cost twice this or 71.42 c. for a round trip, or  $71.42 \times 365 = \$260$  per year per mile of incline per daily train needing assistance.

TABLE XXIX.—ITEMS OF THE COST PER MILE OF A PUSHER ENGINE.

| No.   | Items.                       | Normal average. | Per cent affected. | Cost per engine mile, per cent. |
|-------|------------------------------|-----------------|--------------------|---------------------------------|
| 1     | Repairs of roadway .....     | 10.596          | 12.5               | 1.32                            |
| 2     | Renewals of rails .....      | 1.440           | 50                 | .72                             |
| 3     | Renewals of ties .....       | 3.093           | 50                 | 1.55                            |
| 12    | Repairs of locomotives ..... | 5.879           | 100                | 5.88                            |
| 21    | Enginemen .....              | 9.781           | 100                | 9.78                            |
| 22-25 | Engine supplies .....        | 10.912          | 100                | 10.91                           |
| 28    | Switchmen, etc. ....         | 4.136           | 100                | 4.14                            |
| 29    | Telegraph .....              | 1.974           | 100                | 1.97                            |
|       |                              |                 |                    | 36.27                           |

## 451. Numerical comparison of pusher and through grades.

In § 445 the computation was made of the desirability of reducing a 1.6% ruling grade to a 1.2% grade. Suppose it is found that by keeping the 1.6% grades as pusher grades having a total length of 20 miles on a 100 mile division, the other grades may be reduced to a grade not exceeding 0.713% (the corresponding through grade) for an expenditure of \$200000. Will it pay? The saving by cutting down trains from 8 to 4, computed as before, would be (see § 445)  $4 \times \$0.5333 \times 100 \times 365 = \$77862$ . But this saving is only accomplished by the employment of pushers making four round trips over 20 miles of pusher grade at a cost of  $4 \times 20 \times \$260 = \$20800$ .

The net annual saving is therefore \$57062, which when capitalized at 5% = \$1,141,240.

The above estimate probably has this defect. The total daily pusher-engine mileage is but  $2 \times 4 \times 20 = 160$ , scarcely work enough for two pushers. Unless the pusher grades were bunched into two groups of about 10 miles each, two pusher engines could not do the work. If the number of trains was much larger, then the above method of calculation would be more exact even though the 20 miles of pusher grade was divided among four or five different grades. Therefore with the above data the annual cost of the pusher service would probably be much more—perhaps twice as much—and the annual saving about \$36000, which would justify an expenditure of \$720000. But even this would very amply justify the assumed expenditure of \$200000 which would accomplish this result.

The above computation is but an illustration of the general

truth which has been previously stated. In spite of the uncertainties and the variations of many items in the above estimates it will generally be possible to make a computation which will show unquestionably, as in the above instance, what is the best and the most economical method of procedure. When the capitalized valuations of both methods are so nearly equal that a proper choice is more difficult, the question will frequently be determined by the relative ease of raising additional capital.

#### BALANCE OF GRADES FOR UNEQUAL TRAFFIC.

**452. Nature of the subject.** It sometimes happens, as when a road runs into a mountainous country for the purpose of hauling therefrom the natural products of lumber or minerals, that the heavy grades are all in one direction—that the whole line consists of a more or less unbroken climb having perhaps a few comparatively level stretches, but no down grade (except possibly a slight sag) in the direction of the general up grade. With such lines this present topic has no concern. But the majority of railroads have termini at nearly the same level (500 feet in 500 miles has no practical effect on grade) and consist of up and down grades in nearly equal amounts and rates. The general rate of ruling grade is determined by the character of the country and the character and financial backing of the road to be built. It is always possible to reduce the grade at some point by “development” or in general by the expenditure of more money. It has been tacitly assumed in the previous discussions that when the ruling grade has been determined all grades in either direction are cut down to that limit. If the traffic in both directions were the same this would be the proper policy and sometimes is so. But it has developed, especially on the great east and west trunk lines, that the *weight* of the eastbound freight traffic is enormously greater than that of the westbound—that westbound trains consist very largely of “empties” and that an engine which could haul twenty loaded cars up a given grade in eastbound traffic could haul the same cars empty up a much higher grade when running west. As an illustration of the large disproportion which may exist, the eastbound ton-mileage on the P. R. R. between the years 1851 and 1885 was 3.7 times the westbound ton-mileage. Between the years 1876 and 1880 the ratio rose to more than 4.5 to 1.

On such a basis it is as important and necessary to obtain, say, a 0.6% ruling grade against the eastbound traffic as to have, say, a 1.0% grade against the westbound traffic. This is the basis of the following discussion. It now remains to estimate the probable ratio of the traffic in the two directions and from that to determine the proper "balance" of the opposite ruling grades.

**453. Computation of the theoretical balance.** Assume first, for simplicity, that the exact business in either direction is accurately known. A little thought will show the truth of the following statements.

1. The locomotive and passenger-car traffic in both directions is equal.

2. Except as a road may carry emigrants, the passenger traffic in both directions is equal. Of course there are innumerable individual instances in which the return trip is made by another route, but it is seldom if ever that there is any marked tendency to uniformity in this. Considering that a car load of, say, 50 passengers at 150 pounds apiece weigh but 7500 pounds, which is  $\frac{1}{4}$  of the 45000 pounds which the car may weigh, even a considerable variation in the number of passengers will not appreciably affect the hauling of cars on grades. On parlor-cars and sleepers the ratio of live load to dead load (say 20 passengers, 3000 pounds, and the car, 75000 pounds) is even more insignificant. The effect of passenger traffic on balance of grades may therefore be disregarded.

3. Empty cars have a greater resistance *per ton* than loaded cars. Therefore in computing the hauling capacity of a locomotive hauling so many tons of "empties," a larger figure must be used for the ordinary tractive resistances—say two pounds per ton greater.

4. Owing to greater or less imperfections of management a small percentage of cars will run empty or but partly full in the direction of greatest traffic.

5. Freight having great bulk and weight (such as grain, lumber, coal, etc.) is run from the rural districts toward the cities and manufacturing districts.

6. The return traffic—manufactured products—although worth as much or more, do not weigh as much.

As a simple numerical illustration assume that the weight of the cars is 40% and the live load 60% of the total load when



the cars are "full"—although not loaded to their absolute limit of capacity. Assume that the relative weight of live load to be hauled in the other direction is but  $\frac{1}{3}$ . Then the gross trainload (exclusive of the locomotive) is  $40 + (\frac{1}{3} \times 60) = 60\%$  of the load in the other direction. Assume that the grade against the heaviest traffic is  $0.9\%$ . An 80-ton engine with 48 tons on the drivers,  $\frac{1}{4}$  adhesion, normal tractive resistance 6 pounds per ton, will haul a train of 920 tons up that grade. Of this load the cars are assumed to weigh  $40\%$ , or 368 tons, and the live load 552 tons. On the return trip the weight of the cars with their load is but  $920 \times 60\% = 552$  tons, or with the engine 632 tons. This could be hauled up a  $1.60\%$  grade, assuming that the resistance was the same per ton. But  $\frac{2}{3}$  of the return cars must be figured as empty; they make an added resistance of 2 pounds per ton; these cars weigh  $\frac{2}{3}$  of 368 tons, or 245.33 tons. The balance of the train weighs  $632 - 245.33 = 386.67$  tons. Then we have

$$245.33 \times 8 = 1962.67$$

$$386.67 \times 6 = 2320.$$

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$$4282.67 \text{ pounds,}$$

which is the tractive force required for rolling resistances, etc. Subtracting this from the total adhesion, 24000 pounds, we have left 19717.33 pounds available for grade, or 31.2 pounds per ton, which corresponds to a  $1.56\%$  grade, which is the proper balance of grade *under the above conditions*.

**454. Computation of relative traffic.** Some of the principal elements have already been referred to, but in addition the following facts should be considered.

(a) The greatest disparity in traffic occurs through the handling of large amounts of coal, lumber, iron ore, grain, etc. On roads which handle but little of these articles or on which for local reasons coal is hauled one way and large shipments of grain the other way the disparity will be less and will perhaps be insignificant.

(b) A marked change in the development of the country may, and often does, cause a marked difference in the disparity of traffic. The heaviest traffic (in mere weight) is always toward manufacturing regions and away from agricultural regions. But when a region, from being purely agricultural or mineral, be-



comes largely manufacturing, or when a manufacturing region develops an industry which will cause a growth of heavy freight traffic from it, a marked change in the relative freight movement will be the result.

(c) Very great fluctuations in the relative traffic may be expected for prolonged intervals.

(d) An estimate of the relative traffic may be formed by the same general method used in computing the total traffic of the road (see § 373, Chap. XIX) or by noting the relative traffic on existing roads which may be assumed to have practically the same traffic as the proposed road will obtain.

## CHAPTER XXIV.

### THE IMPROVEMENT OF OLD LINES.

**455. Classification of improvements.** The improvements here considered are only those of alignment—horizontal and vertical. Strictly there is no definite limit, either in kind or magnitude, to the improvements which may be made. But since a railroad cannot ordinarily obtain money, even for improvements, to an amount greater than some small proportion of the previously invested capital, it becomes doubly necessary to expend such money to the greatest possible advantage. It has been previously shown that securing additional business and increasing the train load are the two most important factors in decreasing dividends. After these, and of far less importance, come reductions of curvature, reductions of distance (frequently of doubtful policy, see Chap. XXI, § 414), and elimination of sags and humps. These various improvements will be briefly discussed.

(a) **Securing additional business.** It is not often possible by any small modification of alignment to materially increase the business of a road. The cases which do occur are usually those in which a gross error of judgment was committed during the original construction. For instance, in the early history of railroad construction many roads were largely aided by the towns through which the road passed, part of the money necessary for construction being raised by the sale of bonds, which were assumed or guaranteed and subsequently paid by the towns. Such aid was often demanded and exacted by the promoters. Instances are not unknown where a failure to come to an agreement has caused the promoters to deliberately pass by the town at a distance of some miles, to the mutual disadvantage of the road and the town. If the town subsequently grew in spite of this disadvantage, the *annual* loss of business might readily amount to more than the original sum in dispute.

Such an instance would be a legitimate opportunity for study of the advisability of a re-location.

As another instance (the original location being justifiable) a railroad might have been located along the bank of a considerable river too wide to be crossed except at considerable expense. When originally constructed the enterprise would not justify the two extra bridges needed to reach the town. A growth in prosperity and in the business obtainable might subsequently make such extra expense a profitable investment.

(b) **Increasing the train load.** On account of its importance this will be separately considered in § 458 *et seq.*

(c) **Reductions in curvature and distance and the elimination of sags and humps.** The financial value of these improvements has already been discussed in Chapters XXI, XXII, and XXIII. Such improvements are constantly being made by all progressive roads. The need for such changes occurs in some cases because the original location was very faulty, the revised location being no more expensive than the original, and in other cases because the original location was the best that was then financially possible and because the present expanded business will justify a change.

(d) **Changing the location of stations or of passing sidings.** The station may sometimes be re-located so as to bring it nearer to the business center and thus increase the business done. But the principal reasons for re-locating stations or passing sidings is that starting trains may have an easier grade on which to overcome the additional resistances of starting. Such change will be discussed in detail in § 460.

**456. Advantages of re-locations.** There are certain undoubted advantages possessed by the engineer who is endeavoring to improve an old line.

(a) The gross traffic to be handled is definitely known.

(b) The actual cost per train-mile for that road (which may differ very greatly from the average) is also known, and therefore the value of the proposed improvement can be more accurately determined.

(c) The actual performance of such locomotives as are used on the road may be studied at leisure and more reliable data may be obtained for the computations.

**457. Disadvantages of re-locations.** The disadvantages are generally more apparent and frequently appear practically

insuperable—more so than they prove to be on closer inspection. (a) It frequently means the abandonment of a greater or less length of old line and the construction of new line. At first thought it might seem as if a change of line such as would permit an increase of train-load of 50 or perhaps 100% could never be obtained, or at least that it could not be done except at an impracticable expense. On the contrary a change of 10% of the old line is frequently all that is necessary to reduce the grades so that the train-loads hauled by one engine may be nearly if not quite doubled. And when it is considered that the cost of a road to sub-grade is generally not more than one-third of the total cost of construction and equipment per mile, it becomes plain that an expenditure of but a small percentage of the original outlay, expended where it will do the most good, will often suffice to increase enormously the earning capacity.

(b) One of the most difficult matters is to convince the financial backers of the road that the proposed improvement will be justifiable. The cause is simple. The disadvantages of the original construction lie in the large increase of certain items of expense which are necessary to handle a given traffic. And yet the fact that the expenditures are larger than they need be are only apparent to the expert, and the fact that a saving may be made is considered to be largely a matter of opinion until it is demonstrated by actual trial. On the other hand the cost of the proposed changes is definite, and the very fact that the road has been uneconomically worked and is in a poor financial condition makes it difficult to obtain money for improvements.

(c) The legal right to abandon a section of operated line and thus reduce the value of some adjoining property has sometimes been successfully attacked. A common instance would be that of a factory which was located adjoining the right of way for convenience of transportation facilities. The abandonment of that section of the right of way would probably be fatal to the successful operation of the factory. The objection may be largely eliminated by the maintenance of the old right of way as a long siding (although the business of the factory might not be worth it), but it is not always so easy of solution, and this phase of the question must always be considered.

## REDUCTION OF VIRTUAL GRADE.

**458. Obtaining data for computations.** As developed in the last chapter (§§ 432-434) the real object to be attained is the reduction of the *virtual grade*. The method of comparing grade under various assumed conditions was there discussed. When the road is still "on paper" some such method is all that is possible; but when the road is in actual operation the *virtual grade* of the road at various critical points, with the rolling stock actually in use, may be determined by a simple test and the effect of a proposed change may be reliably computed. Bearing in mind the general principle that the *virtual grade line* is the locus of points determined by adding to the actual grade profile ordinates equal to the velocity head of the train, it only becomes necessary to measure the velocity at various points. Since the velocity is *not* usually uniform, its precise determination at any instant is almost impossible, but it will generally be found to be sufficiently precise to assume the velocity to be uniform for a short distance, and then observe the time required to pass that short space. Suppose that an ordinary watch is used and the time taken to the nearest second. At 30 miles per hour, the velocity is 44 feet per second. To obtain the time to within 1%, the time would need to be 100 seconds and the space 4400 feet. But with variable velocity there would be too great error in assuming the velocity as uniform for 4400 feet or for the time of 100 seconds. Using a stop watch registering fifths of a second, a 1% accuracy would require but 20 seconds and a space of 880 feet, at 30 miles per hour. Wellington suggests that the space be made 293 feet 4 inches, or  $\frac{1}{18}$  of a mile; then the speed in miles per hour equals  $200 \div s$ , in which  $s$  is the time in seconds required to traverse the 293' 4". For instance, suppose the time required to pass the interval is 12.5 seconds.  $\frac{1}{18}$  mile in 12.5 seconds = one mile in 225 seconds, or 16 miles per hour. But likewise  $200 \div 12.5 = 16$ , the required velocity. The following features should be noted when obtaining data for the computations:

(a) All critical grades on the road should be located and their profiles obtained — by a survey if necessary.

(b) At the bottom and top of all long grades (and perhaps at intermediate points if the grades are very long) spaces of known

length (preferably  $293\frac{1}{3}$  feet) should be measured off and marked by flags, painted boards, or any other serviceable targets.

(c) Provided with a stop-watch marking fifths of seconds the observer should ride on the trains affected by these grades and note the exact interval of time required to pass these spaces. If the space is  $293\frac{1}{3}$  feet, the velocity in miles per hour  $= 200 \div$  interval in seconds. In general,

$$V = \frac{\text{distance in feet} \times 3600}{\text{time in seconds} \times 5280} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (147)$$

(d) Since these critical grades are those which require the greatest tax on the power of the locomotive, the conditions under which the locomotive is working must be known—i.e., the steam pressure, point of cut-off, and position of the throttle. Economy of coal consumption as well as efficient working at high speeds requires that steam be used expansively (using an early cut-off), and even that the throttle be partly closed; but when an engine is slowly climbing up a maximum grade with a full load it is not exerting its maximum tractive power unless it has its maximum steam pressure, wide-open throttle, and is cutting off nearly at full stroke. These data must therefore be obtained so as to know whether the engine is developing at a critical place all the tractive force of which it is capable. The condition of the track (wet and slippery or dry) and the approximate direction and force of the wind should be noted with sufficient accuracy to judge whether the test has been made under ordinary conditions rather than under conditions which are exceptionally favorable or unfavorable.

(e) The train-loading should be obtained as closely as possible. Of course the dead weight of the cars is easily found, and the records of the freight department will usually give the live load with all sufficient accuracy.

**459. Use of the data obtained.** A very brief inspection of the results, freed from refined calculations or uncertainties, will demonstrate the following truths:

(a) If, on a uniform grade, the velocity increases, it shows that, under those conditions of engine working, the load is less than the engine can handle on that grade

(b) If the velocity decreases, it shows that the load is greater than the engine can handle on an indefinite length of such

grade. It shows that such a grade is being operated by momentum. From the rate of decrease of velocity the maximum practicable length of such a grade (starting with a given velocity) may be easily computed.

(c) By combining results under different conditions of grade but with practically the same engine working, the tractive power of the engine may be determined (according to the principles previously demonstrated) for any grade and velocity. For example: On an examination of the profile of a division of a road the maximum grade was found to be 1.62% (85.54 feet per mile). At the bottom and near the top of this grade two lengths of 293' 4" are laid off. The distance between the centers of these lengths is 6000 feet. A freight train moving up the grade is timed at  $9\frac{3}{5}$  seconds on the lower stretch and  $7\frac{3}{5}$  seconds on the upper. These times correspond to  $\frac{200}{9.4}$  and  $\frac{200}{7.6}$  or 21.3 and 26.3 miles per hour respectively. It is at once observed that the velocity has increased and that the engine could draw even a heavier load up such a grade for an indefinite distance. How much heavier might the load be?

For simplicity we will assume that the conditions were normal, neither exceptionally favorable nor unfavorable, and that the engine was worked to its maximum capacity. The engine is a "consolidation" weighing 128700 pounds, with 112600 pounds on the drivers. The train-load behind the engine consists of ten loaded cars weighing 465 tons and eleven empties weighing 183 tons, thus making a total train-weight of 712 tons. Applying Eq. 140, we find that the *additional* force which the engine has actually exerted per ton in increasing the velocity from 21.3 to 26.3 miles per hour in a distance of 6000 feet is

$$P = \frac{70.224}{6000} (26.3^2 - 21.3^2) = 2.78 \text{ pounds per ton}$$

The grade resistance on a 1.62% grade is 32.4 pounds per ton. With an average train resistance of seven pounds per ton the total necessary pull for uniform velocity would be 39.4 pounds. But the engine is actually exerting an additional pull of 2.78 pounds per ton. Evidently its total load might be increased proportionately, i.e., the total train-load might equal

$$712 \times \frac{2.78 + 39.4}{39.4} = 762 \text{ tons.}$$



This shows that 50 tons additional might have been loaded on, say, three more empties or one loaded car and one empty. An overload of a few tons would easily be made up by a very slight reduction in the velocity.

The above calculation should of course be considered simply as a "single observation." The performance of the same engine on the same grade (as well as on many other grades) on succeeding days should also be noted. It may readily happen that variations in the condition of the track or of the handling of the engine may make considerable variation in the results of the several calculations, but when the work is properly done it is always possible to draw definite and very positive deductions.

**46c. Reducing the starting grade at stations.** The resistance to starting a train is augmented from two causes: (*a*) the tractive resistances are usually about 20 pounds per ton instead of, say, 6 pounds, and (*b*) the inertia resistance must be overcome. The inertia resistance of a freight train (see § 347) which is expected to attain a velocity of 15 miles per hour in a distance of 1000 feet is (see Eq. 140)

$$P = \frac{70 \cdot 224}{1000} (15^2 - 0) = 15.8 \text{ pounds per ton, which is the equivalent of a } 0.79\% \text{ grade.}$$

Adding this to a grade which nearly or quite equals the ruling grade, it virtually creates a new and higher ruling grade. Of course that additional force can be greatly reduced at the expense of slower acceleration, but even this cannot be done indefinitely, and an acceleration to only 15 miles per hour in 1000 feet is as slow as should be allowed for. With perhaps 14 pounds per ton additional tractive resistance, we have about 30 pounds per ton additional equivalent

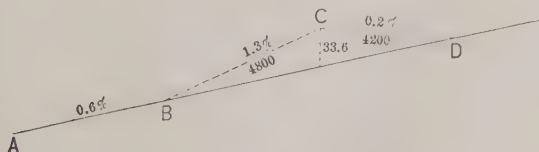


FIG. 217.

lent to a 1.5% grade. Instances are known where it has proven wise to create a *hump* (in what was otherwise a uniform grade)

at a station. The effect of this on high-speed passenger trains moving *up* the grade would be merely to reduce their speed very slightly. No harm is done to trains moving *down* the grade. Freight trains moving *up* the grade and intending to stop at the station will merely have their velocity reduced as they approach the station and will actually save part of the wear and tear otherwise resulting from applying brakes. When the trains start they are assisted by the short down grade just where they need assistance most. Even if the grade *is* still an up grade, the pull required at starting is *less* than that required on the uniform grade by an amount equal to 20 times the difference of the grade in per cent.

## APPENDIX.

### THE ADJUSTMENTS OF INSTRUMENTS.

THE accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are so apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects, which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument-maker or repairer.

A WARNING is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the real error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental *defects* which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

DO NOT DISTURB THE ADJUSTING-SCREWS ANY MORE THAN NECESSARY. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mech-

anism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions. Jarring diminishes the friction between the parts and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment made by unskillful hands may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated:

(a) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(b) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(c) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the unadjusted instrument. Illustrations of all these cases will be given under their proper heads.

#### ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower clamp loose, swing the instrument so that the plate-bubbles are parallel to the lines of opposite leveling-screws. Level up until both bubbles are central. Swing the instrument  $180^\circ$ . If the bubbles again settle at the center, the adjustment is perfect. If either bubble does not settle in the center, move the leveling-screws until the bubble is *half-way* back to the center. Then, before touching the adjusting-screws, note carefully the position of the bubbles and observe whether the bubbles always settle at the *same* place in the tube, no matter to what position the in-

strument may be rotated. When the instrument is so leveled, the axis is truly vertical and the discrepancies between this constant position of the bubbles and the centers of the tubes measure the errors of adjustment. By means of the adjusting-screws bring each bubble to the center of the tube. If this is done so skillfully that the true level of the instrument is not disturbed, the bubbles should settle in the center for all positions of the instrument. Under unskillful hands, two or more such trials may be necessary.

When the plates are not horizontal, the measured angle is greater than the true horizontal angle by the difference between the measured angle and its projection on a horizontal plane. When this angle of inclination is small, the difference is insignificant. Therefore when the plate-bubbles are *very nearly* in adjustment, the error of measurement of horizontal angles may be far within the lowest unit of measurement used. A *small* error of adjustment of the plate-bubble *perpendicular* to the telescope will affect the horizontal angles by only a small proportion of the error, which will be perhaps imperceptible. Vertical angles will be affected by the same insignificant amount. A *small* error of adjustment of the plate-bubble *parallel* to the telescope will affect horizontal angles very slightly, but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in what positions in the tubes the bubbles will remain fixed for all positions of azimuth and then keeping the bubbles adjusted to these positions, for the axis is then truly vertical. It will often save time to work in this way temporarily rather than to stop to make the adjustments. This should especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary regardless of whether the telescope is revolved with the upper plate loose and the lower plate clamped or whether the whole instrument is revolved, the plates being clamped together. If there is any appreciable difference, it shows that the two vertical axes or "centers" of the plates are not concentric. This may be due to cheap and faulty construction or to the excessive wear that may be sometimes observed in an old instrument originally well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular to the vertical axis of the instrument.* This is best tested by using a long plumb-line, so placed that the telescope must be pointed upward at an angle of about  $45^\circ$  to sight at the top of the plumb-line and downward about the same amount, if possible, to sight at the lower end. The vertical axis of the transit must be made truly vertical. Sight at the upper part of the line, clamping the horizontal plates. Swing the telescope down and see if the cross-wire again bisects the cord. If so, the adjustment is *probably* perfect (a conceivable exception will be

noted later); if not, raise or lower one end of the axis by means of the adjusting-screws, placed at the top of one of the standards, until the cross-wire will bisect the cord both at top and bottom. The plumb-bob may be steadied, if necessary, by hanging it in a pail of water. As many telescopes cannot be focused on an object nearer than 6 or 8 feet from the telescope, this method requires a long plumb-line swung from a high point, which may be inconvenient.

Another method is to set up the instrument about 10 feet from a high wall. After leveling, sight at some convenient mark high up on the wall. Swing the telescope down and make a mark (when working alone some convenient natural mark may generally be found) low down on the wall. Plunge the telescope and revolve the instrument about its vertical axis and again sight at the upper mark. Swing down to the lower mark. If the wire again bisects it, the adjustment is perfect. If not, fix a point *half-way* between the two positions of the lower mark. The plane of this point, the upper point, and the center of the instrument is truly vertical. Adjust the axis to these upper and lower points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the revolving axis of the telescope.* With the instrument level and the telescope nearly horizontal point at some well-defined point at a distance of 200 feet or more. Plunge the telescope and establish a point in the opposite direction. Turn the whole instrument about the vertical axis until it again points at the first mark. Again plunge to "direct position" (*i.e.*, with the level-tube under the telescope). If the vertical cross-wire again points at the second mark, the adjustment is perfect. If not, the error is *one-fourth* of the distance between the two positions of the second mark. Loosen the capstan screw on one side of the telescope and tighten it on the other side until the vertical wire is set at the one-fourth mark. Turn the whole instrument by means of the tangent screw until the vertical wire is *midway* between the two positions of the second mark. Plunge the telescope. If the adjusting has been skillfully done, the cross-wire should come exactly to the first mark. As an "erecting eyepiece" reinverts an image already inverted, the ring carrying the cross-wires must be moved in the *same* direction as the *apparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the practice of producing a line by plunging the telescope, but when this is required to be done with great accuracy it is always better to obtain the forward point by reversion (as described above for making the test) and take the *mean* of the two forward points. Horizontal and vertical angles are practically unaffected by *small* errors of this adjustment, unless, in the case of horizontal angles, the vertical angles to the points observed are very different.

Unnecessary motion of the adjusting-screws may sometimes be avoided by carefully establishing the forward point on line by repeated reversions of the instrument, and thus determining by repeated trials the exact amount of the error. *Differences* in the amount of error determined would be evidence of inaccuracy in manipulating the instrument, and would show that an adjustment based on the first trial would *probably* prove unsatisfactory.

The 2d and 3d adjustments are mutually dependent. If either adjustment is badly out, the other adjustment cannot be made except as follows:

(a) The second adjustment can be made regardless of the third when the lines to the high point and the low point make *equal* angles with the horizontal.

(b) The third adjustment can be made regardless of the second when the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and especially when the error of either adjustment is small, no trouble will be found in perfecting either adjustment on account of a small error in the other adjustment.

If the test for the second adjustment is made by means of the plumb-line and the vertical cross-wire intersects the line at all points as the telescope is raised or lowered, it not only demonstrates at once the accuracy of that adjustment, but also shows that the third adjustment is either perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center of the tube when the line of collimation is horizontal.* The line of collimation should coincide with the optical axis of the telescope. If the object-glass and eyepiece have been properly centered, the previous adjustment will have brought the vertical cross-wire to the center of the field of view. The horizontal cross-wire should also be brought to the center of the field of view, and the bubble should be adjusted to it.

a. *Peg method.* Set up the transit at one end of a nearly level stretch of about 300 feet. Clamp the telescope with its bubble in the center. Drive a stake vertically under the eyepiece of the transit, and another about 300 feet away. Observe the height of the center of the eyepiece (the telescope being level) above the stake (calling it *a*); observe the reading of the rod when held on the other stake (calling it *b*); take the instrument to the other stake and set it up so that the eyepiece is



vertically over the stake, observing the height,  $c$ ; take a reading on the first stake, calling it  $d$ . If this adjustment is perfect then

$$a - d = b - c,$$

$$\text{or} \quad (a - d) - (b - c) = 0.$$

$$\text{Call} \quad (a - d) - (b - c) = 2m.$$

When  $m$  is positive, the line points downward;

“  $m$  “ negative, “ “ “ upward.

To adjust: if the line points *up*, sight the horizontal cross-wire (by moving the vertical tangent screw) at a point which is  $m$  lower, then adjust the bubble so that it is in the center.

By taking several independent values for  $a$ ,  $b$ ,  $c$ , and  $d$ , a mean value for  $m$  is obtained, which is more reliable and which may save much unnecessary working of the adjusting-screws.

*b. Using an auxiliary level.* When a carefully adjusted level is at hand, this adjustment may sometimes be more easily made by setting up the transit and level, so that their lines of collimation are as nearly as possible at the same height. If a point may be found which is half a mile or more away and which is on the horizontal cross-wire of the level, the horizontal cross-wire of the transit may be pointed directly at it, and the bubble adjusted accordingly. Any slight difference in the heights of the lines of collimation of the transit and level (say  $\frac{1}{4}$ " ) may almost be disregarded at a distance of  $\frac{1}{2}$  mile or more, or, if the difference of level would have an appreciable effect, even this may be practically eliminated by making an estimated allowance when sighting at the distant point. Or, if a distant point is not available, a level-rod with target may be used at a distance of (say) 300 feet, making allowance for the carefully determined difference of elevation of the two lines of collimation.

*5. Zero of vertical circle.* When the line of collimation is truly horizontal and the vertical axis is truly vertical, the reading of the vertical circle should be  $0^\circ$ . If the arc is adjustable, it should be brought to  $0^\circ$ . If it is not adjustable, the *index error* should be observed, so that it may be applied to all readings of vertical angles.

#### ADJUSTMENTS OF THE WYE LEVEL.

*1. To make the line of collimation coincide with the center of the rings.* Point the intersection of the cross-wires at some

well-defined point which is at a considerable distance. The instrument need not be level, which allows much greater liberty in choosing a convenient point. The vertical axis should be clamped, and the clips over the wyes should be loosened and raised. Rotate the telescope in the wyes. The intersection of the cross-wires should be continually on the point. If it is not, it requires adjustment. Rotate the telescope  $180^\circ$  and adjust *one-half* of the error by means of the capstan-headed screws that move the cross-wire ring. It should be remembered that, with an erecting telescope, on account of the inversion of the image, the ring should be moved in the direction of the *apparent* error. Adjust the other half of the error with the leveling-screws. Then rotate the telescope  $90^\circ$  from its usual position, sight accurately at the point, and then rotate  $180^\circ$  from that position and adjust any error as before. It may require several trials, but it is necessary to adjust the ring until the intersection of the cross-wires will remain on the point for any position of rotation.

If such a test is made on a very distant point and again on a point only 10 or 15 feet from the instrument, the adjustment may be found correct for one point and incorrect for the other. This indicates that the object-slide is improperly centered. Usually this defect can only be corrected by an instrument-maker. If the difference is very small it may be ignored, but the adjustment should then be made on a point which is at about the mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotated, it indicates that the eyepiece is improperly adjusted. This defect is likewise usually corrected only by the maker. It does not interfere with instrumental accuracy, but it usually causes the intersection of the cross-wires to be eccentric with the field of view.

2. *To make the axis of the level-tube parallel to the line of collimation.* Raise the clips as far as possible. Swing the level so that it is parallel to a pair of opposite leveling-screws and clamp it. Bring the bubble to the middle of the tube by means of the leveling-screws. Take the telescope out of the wyes and replace it end for end, using *extreme care* that the wyes are not jarred by the action. If the bubble does not come to the center, correct *one-half* of the error by the vertical adjusting-screws at one end of the bubble. Correct the other half by the leveling-screws. Test the work by again changing the telescope end for end in the wyes.

Care should be taken while making this adjustment to see

that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope  $180^\circ$ . If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

#### ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope  $180^\circ$ . If it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws. Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.* The method of adjustment is identical with that for the transit (No. 4, p. 505) except that the cross-wire must be

adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit; i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described. If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker. The error may be due to any one of several causes, which are

(a) faulty centering of object-slide;

(b) faulty centering of eyepiece;

(c) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis. If the error is only just perceptible, it will not probably cause any error in the work.



## EXPLANATORY NOTE ON THE USE OF THE TABLES.

The logarithms here given are "five-place," but the last figure sometimes has a special mark over it (*e.g.*,  $\bar{6}$ ) which indicates that one-half a unit in the last place should be *added*. For example

| the value       | includes all values between      |
|-----------------|----------------------------------|
| .69586          | .6958575000 + and .6958624999... |
| .6958 $\bar{6}$ | .6958625000 + and .6958674999... |

The maximum error in any one value therefore does not exceed one-quarter of a fifth-place unit.

When adding or subtracting such logarithms allow a half-unit for such a sign. For example

|                 |                 |                 |
|-----------------|-----------------|-----------------|
| .69586          | .69586          | .6958 $\bar{6}$ |
| .10841          | .1084 $\bar{1}$ | .1084 $\bar{1}$ |
| .1294 $\bar{7}$ | .1294 $\bar{7}$ | .1294 $\bar{7}$ |
| <hr/>           | <hr/>           | <hr/>           |
| .9337 $\bar{4}$ | .93375          | .9337 $\bar{5}$ |

All other logarithmic operations are performed as usual and are supposed to be understood by the student.

TABLE I.—RADII OF CURVES.

| Deg | 0°      |         | 1°      |         | 2°      |         | 3°      |         | Deg |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|-----|
| Min | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Min |
| 0   | ∞       | ∞       | 5729.6  | 3.75813 | 2864.9  | 3.45711 | 1910.1  | 3.28105 | 0   |
| 1   | 343775  | 5.53627 | 5635.7  | .75095  | 2841.3  | .45351  | 1899.5  | .27864  | 1   |
| 2   | 171387  | 5.23524 | 5544.8  | .74389  | 2818.0  | .44993  | 1889.1  | .27625  | 2   |
| 3   | 114592  | 5.05915 | 5456.8  | .73694  | 2795.1  | .44639  | 1878.8  | .27387  | 3   |
| 4   | 85944   | 4.93421 | 5371.6  | .73010  | 2772.5  | .44287  | 1868.6  | .27151  | 4   |
| 5   | 68755   | 4.87837 | 5288.9  | .72336  | 2750.4  | .43939  | 1858.5  | .26915  | 5   |
| 6   | 57296   | 4.77512 | 5208.8  | 3.71673 | 2728.5  | 3.43593 | 1848.5  | 3.26681 | 6   |
| 7   | 49111   | .69117  | 5131.0  | .71020  | 2707.0  | .43249  | 1838.6  | .26448  | 7   |
| 8   | 42972   | .63318  | 5055.6  | .70377  | 2685.9  | .42909  | 1828.8  | .26217  | 8   |
| 9   | 38197   | .58203  | 4982.3  | .69743  | 2665.1  | .42571  | 1819.1  | .25986  | 9   |
| 10  | 34377   | .53627  | 4911.2  | .69118  | 2644.6  | .42235  | 1809.6  | .25757  | 10  |
| 11  | 31252   | 4.49488 | 4842.0  | 3.68502 | 2624.4  | 3.41903 | 1800.1  | 3.25529 | 11  |
| 12  | 28648   | .45769  | 4774.7  | .67895  | 2604.5  | .41572  | 1790.7  | .25303  | 12  |
| 13  | 26444   | .42283  | 4709.3  | .67296  | 2584.9  | .41245  | 1781.5  | .25077  | 13  |
| 14  | 24555   | .39014  | 4645.7  | .66705  | 2565.6  | .40918  | 1772.3  | .24853  | 14  |
| 15  | 22918   | .36018  | 4583.8  | .66122  | 2546.6  | .40597  | 1763.2  | .24629  | 15  |
| 16  | 21486   | 4.33215 | 4523.4  | 3.65547 | 2527.9  | 3.40276 | 1754.2  | 3.24407 | 16  |
| 17  | 20222   | .30582  | 4464.7  | .64979  | 2509.5  | .39958  | 1745.3  | .24186  | 17  |
| 18  | 19099   | .28100  | 4407.5  | .64419  | 2491.3  | .39642  | 1736.5  | .23967  | 18  |
| 19  | 18093   | .25752  | 4351.7  | .63865  | 2473.4  | .39329  | 1727.8  | .23748  | 19  |
| 20  | 17189   | .23524  | 4297.3  | .63319  | 2455.7  | .39017  | 1719.1  | .23530  | 20  |
| 21  | 16370   | 4.21405 | 4244.2  | 3.62780 | 2438.3  | 3.38708 | 1710.6  | 3.23314 | 21  |
| 22  | 15626   | .19385  | 4192.5  | .62247  | 2421.1  | .38401  | 1702.1  | .23098  | 22  |
| 23  | 14947   | .17454  | 4142.0  | .61720  | 2404.2  | .38097  | 1693.7  | .22884  | 23  |
| 24  | 14324   | .15606  | 4092.7  | .61200  | 2387.5  | .37794  | 1685.4  | .22670  | 24  |
| 25  | 13751   | .13833  | 4044.5  | .60686  | 2371.0  | .37494  | 1677.2  | .22458  | 25  |
| 26  | 13222   | 4.12130 | 3997.5  | 3.60178 | 2354.8  | 3.37195 | 1669.1  | 3.22247 | 26  |
| 27  | 12732   | .10491  | 3951.5  | .59676  | 2338.8  | .36899  | 1661.0  | .22037  | 27  |
| 28  | 12278   | .08911  | 3906.6  | .59180  | 2323.0  | .36604  | 1653.0  | .21827  | 28  |
| 29  | 11854   | .07387  | 3862.7  | .58689  | 2307.4  | .36312  | 1645.1  | .21619  | 29  |
| 30  | 11459   | .05915  | 3819.8  | .58204  | 2292.0  | .36021  | 1637.3  | .21412  | 30  |
| 31  | 11090   | 4.04491 | 3777.9  | 3.57724 | 2276.8  | 3.35733 | 1629.5  | 3.21206 | 31  |
| 32  | 10743   | .03112  | 3736.8  | .57250  | 2261.9  | .35446  | 1621.8  | .21000  | 32  |
| 33  | 10417   | .01776  | 3696.6  | .56780  | 2247.1  | .35162  | 1614.2  | .20796  | 33  |
| 34  | 10111   | 4.00479 | 3657.3  | .56316  | 2232.5  | .34879  | 1606.7  | .20593  | 34  |
| 35  | 9822    | 3.99221 | 3618.8  | .55856  | 2218.1  | .34598  | 1599.2  | .20390  | 35  |
| 36  | 9549.3  | 3.97997 | 3581.1  | 3.55401 | 2203.9  | 3.34318 | 1591.8  | 3.20189 | 36  |
| 37  | 9291.3  | .96807  | 3544.2  | .54951  | 2189.8  | .34041  | 1584.5  | .19988  | 37  |
| 38  | 9046.7  | .95649  | 3508.0  | .54506  | 2176.0  | .33765  | 1577.2  | .19789  | 38  |
| 39  | 8814.8  | .94521  | 3472.6  | .54065  | 2162.3  | .33491  | 1570.0  | .19590  | 39  |
| 40  | 8594.4  | .93421  | 3437.9  | .53629  | 2148.8  | .33219  | 1562.9  | .19392  | 40  |
| 41  | 8384.8  | 3.92349 | 3403.8  | 3.53197 | 2135.4  | 3.32949 | 1555.8  | 3.19195 | 41  |
| 42  | 8185.2  | .91302  | 3370.5  | .52769  | 2122.3  | .32680  | 1548.8  | .18999  | 42  |
| 43  | 7994.8  | .90281  | 3337.7  | .52345  | 2109.2  | .32412  | 1541.9  | .18804  | 43  |
| 44  | 7813.1  | .89282  | 3305.7  | .51925  | 2096.4  | .32147  | 1535.0  | .18610  | 44  |
| 45  | 7639.5  | .88306  | 3274.2  | .51510  | 2083.7  | .31883  | 1528.2  | .18417  | 45  |
| 46  | 7473.4  | 3.87352 | 3243.3  | 3.51098 | 2071.1  | 3.31621 | 1521.4  | 3.18224 | 46  |
| 47  | 7314.4  | .86418  | 3213.0  | .50691  | 2058.7  | .31360  | 1514.7  | .18032  | 47  |
| 48  | 7162.0  | .85533  | 3183.2  | .50287  | 2046.5  | .31101  | 1508.1  | .17842  | 48  |
| 49  | 7015.9  | .84608  | 3154.0  | .49886  | 2034.4  | .30843  | 1501.5  | .17652  | 49  |
| 50  | 6875.6  | .83731  | 3125.4  | .49490  | 2022.4  | .30587  | 1495.0  | .17462  | 50  |
| 51  | 6740.7  | 3.82871 | 3097.2  | 3.49097 | 2010.6  | 3.30332 | 1488.5  | 3.17274 | 51  |
| 52  | 6611.1  | .82027  | 3069.6  | .48707  | 1998.9  | .30079  | 1482.1  | .17087  | 52  |
| 53  | 6486.4  | .81200  | 3042.4  | .48321  | 1987.3  | .29827  | 1475.7  | .16900  | 53  |
| 54  | 6366.3  | .80338  | 3015.7  | .47939  | 1975.9  | .29577  | 1469.4  | .16714  | 54  |
| 55  | 6250.5  | .79591  | 2989.5  | .47559  | 1964.6  | .29328  | 1463.2  | .16529  | 55  |
| 56  | 6138.9  | 3.78809 | 2963.7  | 3.47183 | 1953.5  | 3.29081 | 1457.0  | 3.16344 | 56  |
| 57  | 6031.2  | .78040  | 2938.4  | .46811  | 1942.4  | .28835  | 1450.8  | .16161  | 57  |
| 58  | 5927.2  | .77285  | 2913.5  | .46441  | 1931.5  | .28590  | 1444.7  | .15978  | 58  |
| 59  | 5826.8  | .76542  | 2889.0  | .46075  | 1920.7  | .28347  | 1438.7  | .15796  | 59  |
| 60  | 5729.6  | .75813  | 2864.9  | .45711  | 1910.1  | .28105  | 1432.7  | .15615  | 60  |



TABLE I.—RADII OF CURVES.

| Deg | 4°      |         | 5°      |         | 6°      |         | 7°      |         | Deg |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|-----|
| Min | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Radius. | Log R   | Min |
| 0   | 1432.7  | 3.15615 | 1146.3  | 3.05929 | 955.37  | 2.98017 | 819.02  | 2.91329 | 0   |
| 1   | 1426.7  | .15434  | 1142.5  | .05784  | 952.72  | .97896  | 817.08  | .91226  | 1   |
| 2   | 1420.8  | .15255  | 1138.7  | .05640  | 950.09  | .97776  | 815.14  | .91123  | 2   |
| 3   | 1415.0  | .15076  | 1134.9  | .05497  | 947.48  | .97657  | 813.22  | .91021  | 3   |
| 4   | 1409.2  | .14897  | 1131.2  | .05354  | 944.88  | .97537  | 811.30  | .90918  | 4   |
| 5   | 1403.5  | .14720  | 1127.5  | .05211  | 942.29  | .97418  | 809.40  | .90816  | 5   |
| 6   | 1397.8  | 3.14543 | 1123.8  | 3.05069 | 939.72  | 2.97300 | 807.50  | 2.90714 | 6   |
| 7   | 1392.1  | .14387  | 1120.2  | .04928  | 937.16  | .97181  | 805.61  | .90612  | 7   |
| 8   | 1386.5  | .14191  | 1116.5  | .04787  | 934.62  | .97063  | 803.73  | .90511  | 8   |
| 9   | 1380.9  | .14017  | 1112.9  | .04646  | 932.09  | .96945  | 801.86  | .90410  | 9   |
| 10  | 1375.4  | .13843  | 1109.3  | .04506  | 929.57  | .96828  | 800.00  | .90309  | 10  |
| 11  | 1369.9  | 3.13669 | 1105.8  | 3.04366 | 927.07  | 2.96711 | 798.14  | 2.90208 | 11  |
| 12  | 1364.5  | .13497  | 1102.2  | .04227  | 924.58  | .96594  | 796.30  | .90107  | 12  |
| 13  | 1359.1  | .13325  | 1098.7  | .04088  | 922.10  | .96478  | 794.46  | .90007  | 13  |
| 14  | 1353.8  | .13154  | 1095.2  | .03949  | 919.64  | .96361  | 792.63  | .89907  | 14  |
| 15  | 1348.4  | .12983  | 1091.7  | .03811  | 917.19  | .96246  | 790.81  | .89807  | 15  |
| 16  | 1343.2  | 3.12813 | 1088.3  | 3.03674 | 914.75  | 2.96130 | 789.00  | 2.89708 | 16  |
| 17  | 1338.0  | .12644  | 1084.8  | .03537  | 912.33  | .96015  | 787.20  | .89608  | 17  |
| 18  | 1332.8  | .12475  | 1081.4  | .03400  | 909.92  | .95900  | 785.41  | .89509  | 18  |
| 19  | 1327.6  | .12307  | 1078.1  | .03264  | 907.52  | .95785  | 783.62  | .89410  | 19  |
| 20  | 1322.5  | .12140  | 1074.7  | .03128  | 905.13  | .95671  | 781.84  | .89312  | 20  |
| 21  | 1317.5  | 3.11974 | 1071.3  | 3.02992 | 902.76  | 2.95557 | 780.07  | 2.89213 | 21  |
| 22  | 1312.4  | .11808  | 1068.0  | .02857  | 900.40  | .95443  | 778.31  | .89115  | 22  |
| 23  | 1307.4  | .11642  | 1064.7  | .02723  | 898.05  | .95330  | 776.55  | .89017  | 23  |
| 24  | 1302.5  | .11477  | 1061.4  | .02589  | 895.71  | .95217  | 774.81  | .88919  | 24  |
| 25  | 1297.6  | .11313  | 1058.2  | .02455  | 893.39  | .95104  | 773.07  | .88821  | 25  |
| 26  | 1292.7  | 3.11150 | 1054.9  | 3.02322 | 891.08  | 2.94991 | 771.34  | 2.88724 | 26  |
| 27  | 1287.9  | .10987  | 1051.7  | .02189  | 888.78  | .94879  | 769.61  | .88627  | 27  |
| 28  | 1283.1  | .10825  | 1048.5  | .02056  | 886.49  | .94767  | 767.90  | .88530  | 28  |
| 29  | 1278.3  | .10663  | 1045.3  | .01924  | 884.21  | .94655  | 766.19  | .88433  | 29  |
| 30  | 1273.6  | .10502  | 1042.1  | .01792  | 881.95  | .94544  | 764.49  | .88337  | 30  |
| 31  | 1268.9  | 3.10341 | 1039.0  | 3.01661 | 879.69  | 2.94433 | 762.80  | 2.88241 | 31  |
| 32  | 1264.2  | .10182  | 1035.9  | .01530  | 877.45  | .94322  | 761.11  | .88145  | 32  |
| 33  | 1259.6  | .10022  | 1032.8  | .01400  | 875.22  | .94212  | 759.43  | .88049  | 33  |
| 34  | 1255.0  | .09864  | 1029.7  | .01270  | 873.00  | .94101  | 757.76  | .87953  | 34  |
| 35  | 1250.4  | .09705  | 1026.6  | .01140  | 870.80  | .93991  | 756.10  | .87858  | 35  |
| 36  | 1245.9  | 3.09548 | 1023.5  | 3.01010 | 868.60  | 2.93882 | 754.44  | 2.87762 | 36  |
| 37  | 1241.4  | .09391  | 1020.5  | .00882  | 866.41  | .93772  | 752.80  | .87668  | 37  |
| 38  | 1236.9  | .09234  | 1017.5  | .00753  | 864.24  | .93663  | 751.16  | .87573  | 38  |
| 39  | 1232.5  | .09079  | 1014.5  | .00625  | 862.07  | .93554  | 749.52  | .87478  | 39  |
| 40  | 1228.1  | .08923  | 1011.5  | .00497  | 859.92  | .93446  | 747.89  | .87384  | 40  |
| 41  | 1223.7  | 3.08789 | 1008.6  | 3.00370 | 857.78  | 2.93337 | 746.27  | 2.87290 | 41  |
| 42  | 1219.4  | .08614  | 1005.6  | .00242  | 855.65  | .93229  | 744.66  | .87196  | 42  |
| 43  | 1215.1  | .08461  | 1002.7  | 3.00116 | 853.53  | .93122  | 743.06  | .87102  | 43  |
| 44  | 1210.8  | .08308  | 999.76  | 2.99989 | 851.42  | .93014  | 741.46  | .87008  | 44  |
| 45  | 1206.6  | .08155  | 996.87  | .99863  | 849.32  | .92907  | 739.86  | .86915  | 45  |
| 46  | 1202.4  | 3.08003 | 993.99  | 2.99738 | 847.23  | 2.92800 | 738.28  | 2.86822 | 46  |
| 47  | 1198.2  | .07852  | 991.13  | .99613  | 845.15  | .92693  | 736.70  | .86729  | 47  |
| 48  | 1194.0  | .07701  | 988.28  | .99488  | 843.08  | .92587  | 735.13  | .86636  | 48  |
| 49  | 1189.9  | .07550  | 985.45  | .99363  | 841.02  | .92480  | 733.56  | .86544  | 49  |
| 50  | 1185.8  | .07400  | 982.64  | .99239  | 838.97  | .92374  | 732.01  | .86451  | 50  |
| 51  | 1181.7  | 3.07251 | 979.84  | 2.99115 | 836.93  | 2.92269 | 730.45  | 2.86359 | 51  |
| 52  | 1177.7  | .07102  | 977.06  | .98992  | 834.90  | .92163  | 728.91  | .86267  | 52  |
| 53  | 1173.6  | .06954  | 974.29  | .98869  | 832.89  | .92058  | 727.37  | .86175  | 53  |
| 54  | 1169.7  | .06806  | 971.54  | .98745  | 830.88  | .91953  | 725.84  | .86084  | 54  |
| 55  | 1165.7  | .06658  | 968.81  | .98624  | 828.88  | .91849  | 724.31  | .85992  | 55  |
| 56  | 1161.8  | 3.06511 | 966.09  | 2.98501 | 826.89  | 2.91744 | 722.79  | 2.85901 | 56  |
| 57  | 1157.9  | .06365  | 963.39  | .98380  | 824.91  | .91640  | 721.28  | .85810  | 57  |
| 58  | 1154.0  | .06219  | 960.70  | .98258  | 822.93  | .91536  | 719.77  | .85719  | 58  |
| 59  | 1150.1  | .06074  | 958.03  | .98137  | 820.97  | .91433  | 718.27  | .85629  | 59  |
| 60  | 1146.3  | .05929  | 955.37  | .98017  | 819.02  | .91329  | 716.78  | .85538  | 60  |

TABLE I.—RADII OF CURVES.

| Deg. | 8°      |              | 9°      |              | 10°     |              | 11°     |              |
|------|---------|--------------|---------|--------------|---------|--------------|---------|--------------|
| Min. | Radius. | Log <i>R</i> | Radius. | Log <i>R</i> | Radius. | Log <i>R</i> | Radius. | Log <i>R</i> |
| 0    | 716.78  | 2.85538      | 637.27  | 2.80432      | 573.69  | 2.75867      | 521.67  | 2.71739      |
| 1    | 715.29  | .85448       | 636.10  | .80352       | 572.73  | .75795       | 520.88  | .71674       |
| 2    | 713.81  | .85358       | 634.93  | .80272       | 571.78  | .75723       | 520.10  | .71608       |
| 3    | 712.34  | .85268       | 633.76  | .80192       | 570.84  | .75651       | 519.32  | .71543       |
| 4    | 710.87  | .85178       | 632.60  | .80113       | 569.90  | .75579       | 518.54  | .71478       |
| 5    | 709.40  | .85089       | 631.44  | .80033       | 568.96  | .75508       | 517.76  | .71413       |
| 6    | 707.95  | 2.85000      | 630.29  | 2.79954      | 568.02  | 2.75436      | 516.99  | 2.71348      |
| 7    | 706.49  | .84911       | 629.14  | .79874       | 567.09  | .75365       | 516.21  | .71283       |
| 8    | 705.05  | .84822       | 627.99  | .79795       | 566.16  | .75293       | 515.44  | .71218       |
| 9    | 703.61  | .84733       | 626.85  | .79716       | 565.23  | .75222       | 514.68  | .71153       |
| 10   | 702.17  | .84644       | 625.71  | .79637       | 564.31  | .75151       | 513.91  | .71088       |
| 11   | 700.75  | 2.84556      | 624.58  | 2.79558      | 563.38  | 2.75080      | 513.15  | 2.71024      |
| 12   | 699.33  | .84468       | 623.45  | .79480       | 562.47  | .75009       | 512.38  | .70959       |
| 13   | 697.91  | .84380       | 622.32  | .79401       | 561.55  | .74938       | 511.63  | .70895       |
| 14   | 696.50  | .84292       | 621.20  | .79323       | 560.64  | .74868       | 510.87  | .70831       |
| 15   | 695.09  | .84204       | 620.09  | .79245       | 559.73  | .74798       | 510.11  | .70767       |
| 16   | 693.70  | 2.84117      | 618.97  | 2.79167      | 558.82  | 2.74727      | 509.36  | 2.70702      |
| 17   | 692.30  | .84029       | 617.87  | .79089       | 557.92  | .74657       | 508.61  | .70638       |
| 18   | 690.91  | .83942       | 616.76  | .79011       | 557.02  | .74587       | 507.86  | .70575       |
| 19   | 689.53  | .83855       | 615.66  | .78934       | 556.12  | .74517       | 507.12  | .70511       |
| 20   | 688.16  | .83768       | 614.56  | .78856       | 555.23  | .74447       | 506.38  | .70447       |
| 21   | 686.78  | 2.83682      | 613.47  | 2.78779      | 554.34  | 2.74377      | 505.64  | 2.70383      |
| 22   | 685.42  | .83595       | 612.38  | .78702       | 553.45  | .74307       | 504.90  | .70320       |
| 23   | 684.06  | .83509       | 611.30  | .78625       | 552.56  | .74238       | 504.16  | .70257       |
| 24   | 682.70  | .83423       | 610.21  | .78548       | 551.68  | .74168       | 503.42  | .70193       |
| 25   | 681.35  | .83337       | 609.14  | .78471       | 550.80  | .74099       | 502.69  | .70130       |
| 26   | 680.01  | 2.83251      | 608.06  | 2.78395      | 549.92  | 2.74030      | 501.96  | 2.70067      |
| 27   | 678.67  | .83166       | 606.99  | .78318       | 549.05  | .73961       | 501.23  | .70004       |
| 28   | 677.34  | .83080       | 605.93  | .78242       | 548.17  | .73892       | 500.51  | .69941       |
| 29   | 676.01  | .82995       | 604.86  | .78165       | 547.30  | .73823       | 499.78  | .69878       |
| 30   | 674.69  | .82910       | 603.80  | .78089       | 546.44  | .73754       | 499.06  | .69815       |
| 31   | 673.37  | 2.82825      | 602.75  | 2.78013      | 545.57  | 2.73685      | 498.34  | 2.69752      |
| 32   | 672.06  | .82740       | 601.70  | .77938       | 544.71  | .73617       | 497.62  | .69690       |
| 33   | 670.75  | .82656       | 600.65  | .77862       | 543.86  | .73548       | 496.91  | .69627       |
| 34   | 669.45  | .82571       | 599.61  | .77786       | 543.00  | .73480       | 496.19  | .69565       |
| 35   | 668.15  | .82487       | 598.57  | .77711       | 542.15  | .73412       | 495.48  | .69503       |
| 36   | 666.86  | 2.82403      | 597.53  | 2.77636      | 541.30  | 2.73343      | 494.77  | 2.69440      |
| 37   | 665.57  | .82319       | 596.50  | .77561       | 540.45  | .73275       | 494.07  | .69378       |
| 38   | 664.29  | .82235       | 595.47  | .77486       | 539.61  | .73207       | 493.36  | .69316       |
| 39   | 663.01  | .82152       | 594.44  | .77411       | 538.76  | .73140       | 492.66  | .69254       |
| 40   | 661.74  | .82068       | 593.42  | .77336       | 537.92  | .73072       | 491.96  | .69192       |
| 41   | 660.47  | 2.81985      | 592.40  | 2.77261      | 537.09  | 2.73004      | 491.26  | 2.69131      |
| 42   | 659.21  | .81902       | 591.38  | .77187       | 536.25  | .72937       | 490.56  | .69069       |
| 43   | 657.95  | .81819       | 590.37  | .77112       | 535.42  | .72869       | 489.86  | .69007       |
| 44   | 656.69  | .81736       | 589.36  | .77038       | 534.59  | .72802       | 489.17  | .68946       |
| 45   | 655.45  | .81653       | 588.36  | .76964       | 533.77  | .72735       | 488.48  | .68884       |
| 46   | 654.20  | 2.81571      | 587.36  | 2.76890      | 532.94  | 2.72668      | 487.79  | 2.68823      |
| 47   | 652.96  | .81489       | 586.36  | .76816       | 532.12  | .72601       | 487.10  | .68762       |
| 48   | 651.73  | .81406       | 585.36  | .76742       | 531.30  | .72534       | 486.42  | .68701       |
| 49   | 650.50  | .81324       | 584.37  | .76669       | 530.49  | .72467       | 485.73  | .68640       |
| 50   | 649.27  | .81243       | 583.38  | .76595       | 529.67  | .72401       | 485.05  | .68579       |
| 51   | 648.05  | 2.81161      | 582.40  | 2.76522      | 528.86  | 2.72334      | 484.37  | 2.68518      |
| 52   | 646.84  | .81079       | 581.42  | .76449       | 528.05  | .72267       | 483.69  | .68457       |
| 53   | 645.63  | .80998       | 580.44  | .76376       | 527.25  | .72201       | 483.02  | .68396       |
| 54   | 644.42  | .80917       | 579.47  | .76303       | 526.44  | .72135       | 482.34  | .68335       |
| 55   | 643.22  | .80836       | 578.49  | .76230       | 525.64  | .72069       | 481.67  | .68275       |
| 56   | 642.02  | 2.80755      | 577.53  | 2.76157      | 524.84  | 2.72003      | 481.00  | 2.68214      |
| 57   | 640.83  | .80674       | 576.56  | .76084       | 524.05  | .71937       | 480.33  | .68154       |
| 58   | 639.64  | .80593       | 575.60  | .76012       | 523.25  | .71871       | 479.67  | .68094       |
| 59   | 638.45  | .80513       | 574.64  | .75939       | 522.46  | .71805       | 479.00  | .68033       |
| 60   | 637.27  | .80432       | 573.69  | .75867       | 521.67  | .71739       | 478.34  | .67973       |

TABLE I.—RADII OF CURVES.

| Deg. | Radius. | Log R   | Deg. | Radius. | Log R   | Deg. | Radius. | Log R   | Deg. | Radius | Log R   |
|------|---------|---------|------|---------|---------|------|---------|---------|------|--------|---------|
| 12°  | 478.34  | 2.67973 | 14°  | 410.28  | 2.61307 | 16°  | 359.26  | 2.55541 | 21°  | 274.37 | 2.43833 |
| 2    | 477.02  | .67853  | 2    | 409.31  | .61205  | 5    | 357.42  | .55317  | 10   | 272.23 | .43494  |
| 4    | 475.71  | .67734  | 4    | 408.34  | .61102  | 10   | 355.59  | .55094  | 20   | 270.13 | .43157  |
| 6    | 474.40  | .67614  | 6    | 407.38  | .61000  | 15   | 353.77  | .54872  | 30   | 268.06 | .42823  |
| 8    | 473.10  | .67495  | 8    | 406.42  | .60898  | 20   | 351.98  | .54652  | 40   | 266.02 | .42492  |
| 10   | 471.81  | 2.67376 | 10   | 405.47  | 2.60796 | 25   | 350.21  | .54432  | 50   | 264.02 | .42163  |
| 12   | 470.53  | .67258  | 12   | 404.53  | .60694  | 30   | 348.45  | 2.54214 | 22°  | 262.04 | 2.41837 |
| 14   | 469.25  | .67140  | 14   | 403.58  | .60593  | 35   | 346.71  | .53997  | 10   | 260.10 | .41513  |
| 16   | 467.98  | .67022  | 16   | 402.65  | .60492  | 40   | 344.99  | .53780  | 20   | 258.18 | .41192  |
| 18   | 466.72  | .66905  | 18   | 401.71  | .60391  | 45   | 343.29  | .53565  | 30   | 256.29 | .40873  |
| 20   | 465.46  | 2.66788 | 20   | 400.78  | 2.60291 | 50   | 341.60  | .53351  | 40   | 254.43 | .40557  |
| 22   | 464.21  | .66671  | 22   | 399.86  | .60190  | 55   | 339.93  | .53138  | 50   | 252.60 | .40243  |
| 24   | 462.97  | .66555  | 24   | 398.94  | .60090  | 17°  | 338.27  | 2.52927 | 23°  | 250.79 | 2.39931 |
| 26   | 461.73  | .66439  | 26   | 398.02  | .59990  | 5    | 336.64  | .52716  | 10   | 249.01 | .39622  |
| 28   | 460.50  | .66323  | 28   | 397.11  | .59891  | 10   | 335.01  | .52506  | 20   | 247.26 | .39315  |
| 30   | 459.28  | 2.66207 | 30   | 396.20  | 2.59791 | 15   | 333.41  | .52297  | 30   | 245.53 | .39010  |
| 32   | 458.06  | .66092  | 32   | 395.30  | .59692  | 20   | 331.82  | .52090  | 40   | 243.82 | .38707  |
| 34   | 456.85  | .65977  | 34   | 394.40  | .59593  | 25   | 330.24  | .51883  | 50   | 242.14 | .38407  |
| 36   | 455.65  | .65863  | 36   | 393.50  | .59494  | 30   | 328.66  | 2.51677 | 24°  | 240.48 | 2.38109 |
| 38   | 454.45  | .65748  | 38   | 392.61  | .59396  | 35   | 327.13  | .51472  | 10   | 238.85 | .37813  |
| 40   | 453.26  | 2.65634 | 40   | 391.72  | 2.59298 | 40   | 325.60  | .51269  | 20   | 237.24 | .37519  |
| 42   | 452.07  | .65521  | 42   | 390.84  | .59199  | 45   | 324.09  | .51066  | 30   | 235.65 | .37227  |
| 44   | 450.89  | .65407  | 44   | 389.96  | .59102  | 50   | 322.59  | .50864  | 40   | 234.08 | .36937  |
| 46   | 449.72  | .65294  | 46   | 389.08  | .59004  | 55   | 321.10  | .50663  | 50   | 232.54 | .36649  |
| 48   | 448.56  | .65181  | 48   | 388.21  | .58907  | 18°  | 319.62  | 2.50464 | 25°  | 231.01 | 2.36363 |
| 50   | 447.40  | 2.65069 | 50   | 387.34  | 2.58809 | 5    | 318.18  | .50265  | 30   | 226.55 | .35517  |
| 52   | 446.24  | .64957  | 52   | 386.48  | .58713  | 10   | 316.71  | .50067  | 26°  | 222.27 | .34688  |
| 54   | 445.09  | .64845  | 54   | 385.62  | .58616  | 15   | 315.28  | .49869  | 30   | 218.15 | .33875  |
| 56   | 443.95  | .64733  | 56   | 384.77  | .58519  | 20   | 313.86  | .49673  | 27°  | 214.18 | 2.33078 |
| 58   | 442.81  | .64622  | 58   | 383.91  | .58423  | 25   | 312.45  | .49478  | 30   | 210.36 | .32296  |
| 13°  | 441.68  | 2.64511 | 15°  | 383.06  | 2.58327 | 30   | 311.06  | 2.49234 | 28°  | 206.68 | .31529  |
| 2    | 440.56  | .64400  | 2    | 382.22  | .58231  | 35   | 309.67  | .49030  | 30   | 203.13 | .30776  |
| 4    | 439.44  | .64290  | 4    | 381.38  | .58135  | 40   | 308.30  | .48898  | 29°  | 199.70 | 2.30037 |
| 6    | 438.33  | .64180  | 6    | 380.54  | .58040  | 45   | 306.95  | .48706  | 30   | 196.38 | .29310  |
| 8    | 437.22  | .64070  | 8    | 379.71  | .57945  | 50   | 305.60  | .48515  | 30°  | 193.19 | .28597  |
| 10   | 436.12  | 2.63960 | 10   | 378.88  | 2.57850 | 55   | 304.27  | .48325  | 30   | 190.09 | .27896  |
| 12   | 435.02  | .63851  | 12   | 378.05  | .57755  | 19°  | 302.94  | 2.48130 | 31°  | 187.10 | 2.27207 |
| 14   | 433.93  | .63742  | 14   | 377.23  | .57661  | 5    | 301.63  | .47948  | 32   | 181.40 | .25863  |
| 16   | 432.84  | .63633  | 16   | 376.41  | .57566  | 10   | 300.33  | .47760  | 33   | 176.05 | .24563  |
| 18   | 431.76  | .63524  | 18   | 375.60  | .57472  | 15   | 299.04  | .47573  | 34   | 171.02 | .23303  |
| 20   | 430.69  | 2.63416 | 20   | 374.79  | 2.57378 | 20   | 297.77  | .47388  | 35   | 166.28 | .22083  |
| 22   | 429.62  | .63308  | 22   | 373.98  | .57284  | 25   | 296.50  | .47202  | 36   | 161.80 | 2.20899 |
| 24   | 428.56  | .63201  | 24   | 373.17  | .57191  | 30   | 295.25  | 2.47018 | 37   | 157.58 | .19749  |
| 26   | 427.50  | .63093  | 26   | 372.37  | .57097  | 35   | 294.00  | .46835  | 38   | 153.58 | .18633  |
| 28   | 426.44  | .62986  | 28   | 371.57  | .57004  | 40   | 292.77  | .46652  | 39   | 149.79 | .17547  |
| 30   | 425.40  | 2.62879 | 30   | 370.78  | 2.56911 | 45   | 291.55  | .46471  | 40   | 146.19 | .16492  |
| 32   | 424.35  | .62773  | 32   | 369.99  | .56819  | 50   | 290.33  | .46289  | 41   | 142.77 | 2.15464 |
| 34   | 423.32  | .62666  | 34   | 369.20  | .56726  | 55   | 289.13  | .46109  | 42   | 139.52 | .14464  |
| 36   | 422.28  | .62560  | 36   | 368.42  | .56634  | 20°  | 287.94  | 2.45930 | 43   | 136.43 | .13489  |
| 38   | 421.26  | .62454  | 38   | 367.64  | .56542  | 5    | 286.76  | .45751  | 44   | 133.47 | .12539  |
| 40   | 420.23  | 2.62349 | 40   | 366.86  | 2.56450 | 10   | 285.58  | .45573  | 45   | 130.66 | .11613  |
| 42   | 419.22  | .62243  | 42   | 366.09  | .56358  | 15   | 284.42  | .45396  | 46   | 127.97 | 2.10709 |
| 44   | 418.20  | .62138  | 44   | 365.31  | .56266  | 20   | 283.27  | .45219  | 47   | 125.39 | .09827  |
| 46   | 417.19  | .62034  | 46   | 364.55  | .56175  | 25   | 282.12  | .45044  | 48   | 122.93 | .08965  |
| 48   | 416.19  | .61929  | 48   | 363.78  | .56084  | 30   | 280.99  | 2.44869 | 49   | 120.57 | .08124  |
| 50   | 415.19  | 2.61825 | 50   | 363.02  | 2.55993 | 35   | 279.86  | .44694  | 50   | 118.31 | .07302  |
| 52   | 414.20  | .61721  | 52   | 362.26  | .55902  | 40   | 278.75  | .44521  | 52   | 114.06 | 2.05713 |
| 54   | 413.21  | .61617  | 54   | 361.51  | .55812  | 45   | 277.64  | .44348  | 54   | 110.13 | .04192  |
| 56   | 412.23  | .61514  | 56   | 360.76  | .55721  | 50   | 276.54  | .44176  | 56   | 106.50 | .02736  |
| 58   | 411.25  | .61410  | 58   | 360.01  | .55631  | 55   | 275.45  | .44004  | 58   | 103.13 | .01340  |
| 14°  | 410.28  | 2.61307 | 16°  | 359.26  | 2.55541 | 21°  | 274.37  | 2.43833 | 60   | 100.00 | 2.00000 |

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS  
FOR A 1° CURVE.

| Δ   | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. | Δ   | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. | Δ   | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. |
|-----|-------------|---------------------|-----------------------|-----|-------------|---------------------|-----------------------|-----|-------------|---------------------|-----------------------|
| 1°  | 50.00       | 0.218               | 100.00                | 11° | 551.70      | 26.500              | 1098.3                | 21° | 1061.9      | 97.58               | 2088.3                |
| 10' | 58.34       | 0.297               | 116.67                | 10  | 560.11      | 27.313              | 1114.9                | 10  | 1070.6      | 99.15               | 2104.7                |
| 20  | 66.67       | 0.388               | 133.33                | 20  | 568.53      | 28.137              | 1131.5                | 20  | 1079.2      | 100.75              | 2121.1                |
| 30  | 75.01       | 0.491               | 150.00                | 30  | 576.95      | 28.974              | 1148.1                | 30  | 1087.8      | 102.35              | 2137.5                |
| 40  | 83.34       | 0.606               | 166.66                | 40  | 585.36      | 29.824              | 1164.7                | 40  | 1096.4      | 103.97              | 2153.9                |
| 50  | 91.68       | 0.733               | 183.33                | 50  | 593.79      | 30.686              | 1181.2                | 50  | 1105.1      | 105.60              | 2170.3                |
| 2°  | 100.01      | 0.873               | 199.99                | 12° | 602.21      | 31.561              | 1197.8                | 22° | 1113.7      | 107.24              | 2186.8                |
| 10  | 108.35      | 1.024               | 216.66                | 10  | 610.64      | 32.447              | 1214.4                | 10  | 1122.4      | 108.90              | 2203.2                |
| 20  | 116.68      | 1.188               | 233.32                | 20  | 619.07      | 33.347              | 1231.0                | 20  | 1131.0      | 110.57              | 2219.7                |
| 30  | 125.02      | 1.364               | 249.98                | 30  | 627.50      | 34.259              | 1247.5                | 30  | 1139.7      | 112.25              | 2236.1                |
| 40  | 133.36      | 1.552               | 266.65                | 40  | 635.93      | 35.183              | 1264.1                | 40  | 1148.4      | 113.95              | 2252.5                |
| 50  | 141.70      | 1.752               | 283.31                | 50  | 644.37      | 36.120              | 1280.7                | 50  | 1157.0      | 115.66              | 2268.9                |
| 3°  | 150.04      | 1.964               | 299.97                | 13° | 652.81      | 37.069              | 1297.2                | 23° | 1165.7      | 117.38              | 2284.4                |
| 10  | 158.38      | 2.188               | 316.63                | 10  | 661.25      | 38.031              | 1313.8                | 10  | 1174.4      | 119.12              | 2301.0                |
| 20  | 166.72      | 2.425               | 333.29                | 20  | 669.70      | 39.006              | 1330.3                | 20  | 1183.1      | 120.87              | 2317.3                |
| 30  | 175.06      | 2.674               | 349.95                | 30  | 678.15      | 39.993              | 1346.9                | 30  | 1191.8      | 122.63              | 2333.6                |
| 40  | 183.40      | 2.934               | 366.61                | 40  | 686.60      | 40.992              | 1363.4                | 40  | 1200.5      | 124.41              | 2349.9                |
| 50  | 191.74      | 3.207               | 383.27                | 50  | 695.06      | 42.004              | 1380.0                | 50  | 1209.2      | 126.20              | 2366.2                |
| 4°  | 200.08      | 3.492               | 399.92                | 14° | 703.51      | 43.029              | 1396.5                | 24° | 1217.9      | 128.00              | 2382.5                |
| 10  | 208.43      | 3.796               | 416.58                | 10  | 711.97      | 44.066              | 1413.1                | 10  | 1226.6      | 129.82              | 2398.8                |
| 20  | 216.77      | 4.099               | 433.24                | 20  | 720.44      | 45.116              | 1429.6                | 20  | 1235.3      | 131.65              | 2415.1                |
| 30  | 225.12      | 4.421               | 449.89                | 30  | 728.90      | 46.178              | 1446.2                | 30  | 1244.0      | 133.50              | 2431.4                |
| 40  | 233.47      | 4.755               | 466.54                | 40  | 737.37      | 47.253              | 1462.7                | 40  | 1252.8      | 135.36              | 2447.7                |
| 50  | 241.81      | 5.100               | 483.20                | 50  | 745.85      | 48.341              | 1479.2                | 50  | 1261.5      | 137.23              | 2464.0                |
| 5°  | 250.16      | 5.459               | 499.85                | 15° | 754.32      | 49.441              | 1495.7                | 25° | 1270.2      | 139.11              | 2480.2                |
| 10  | 258.51      | 5.829               | 516.50                | 10  | 762.80      | 50.554              | 1512.3                | 10  | 1279.0      | 141.01              | 2496.5                |
| 20  | 266.86      | 6.211               | 533.15                | 20  | 771.28      | 51.679              | 1528.8                | 20  | 1287.7      | 142.93              | 2512.8                |
| 30  | 275.21      | 6.606               | 549.80                | 30  | 779.77      | 52.818              | 1545.3                | 30  | 1296.5      | 144.85              | 2529.0                |
| 40  | 283.57      | 7.013               | 566.44                | 40  | 788.26      | 53.969              | 1561.8                | 40  | 1305.3      | 146.79              | 2545.3                |
| 50  | 291.92      | 7.432               | 583.09                | 50  | 796.75      | 55.132              | 1578.3                | 50  | 1314.0      | 148.75              | 2561.5                |
| 6°  | 300.28      | 7.863               | 599.73                | 16° | 805.25      | 56.309              | 1594.8                | 26° | 1322.8      | 150.71              | 2577.8                |
| 10  | 308.64      | 8.307               | 616.38                | 10  | 813.75      | 57.498              | 1611.3                | 10  | 1331.6      | 152.69              | 2594.0                |
| 20  | 316.99      | 8.762               | 633.02                | 20  | 822.25      | 58.699              | 1627.8                | 20  | 1340.4      | 154.69              | 2610.3                |
| 30  | 325.35      | 9.230               | 649.66                | 30  | 830.76      | 59.914              | 1644.3                | 30  | 1349.2      | 156.70              | 2626.5                |
| 40  | 333.71      | 9.710               | 666.30                | 40  | 839.27      | 61.141              | 1660.8                | 40  | 1358.0      | 158.72              | 2642.7                |
| 50  | 342.08      | 10.202              | 682.94                | 50  | 847.78      | 62.381              | 1677.3                | 50  | 1366.8      | 160.76              | 2658.9                |
| 7°  | 350.44      | 10.707              | 699.57                | 17° | 856.30      | 63.634              | 1693.8                | 27° | 1375.6      | 162.81              | 2675.1                |
| 10  | 358.81      | 11.224              | 716.21                | 10  | 864.82      | 64.900              | 1710.3                | 10  | 1384.4      | 164.87              | 2691.3                |
| 20  | 367.17      | 11.753              | 732.84                | 20  | 873.35      | 66.178              | 1726.8                | 20  | 1393.2      | 166.95              | 2707.5                |
| 30  | 375.54      | 12.294              | 749.47                | 30  | 881.88      | 67.470              | 1743.2                | 30  | 1402.0      | 169.04              | 2723.7                |
| 40  | 383.91      | 12.847              | 766.10                | 40  | 890.41      | 68.774              | 1759.7                | 40  | 1410.9      | 171.15              | 2739.9                |
| 50  | 392.28      | 13.412              | 782.73                | 50  | 898.95      | 70.091              | 1776.2                | 50  | 1419.7      | 173.27              | 2756.1                |
| 8°  | 400.66      | 13.991              | 799.36                | 18° | 907.49      | 71.421              | 1792.6                | 28° | 1428.6      | 175.41              | 2772.3                |
| 10  | 409.03      | 14.582              | 815.99                | 10  | 916.03      | 72.764              | 1809.1                | 10  | 1437.4      | 177.55              | 2788.4                |
| 20  | 417.41      | 15.184              | 832.61                | 20  | 924.58      | 74.119              | 1825.5                | 20  | 1446.3      | 179.72              | 2804.6                |
| 30  | 425.79      | 15.799              | 849.23                | 30  | 933.13      | 75.488              | 1842.0                | 30  | 1455.1      | 181.89              | 2820.7                |
| 40  | 434.17      | 16.426              | 865.85                | 40  | 941.69      | 76.869              | 1858.4                | 40  | 1464.0      | 184.08              | 2836.9                |
| 50  | 442.55      | 17.066              | 882.47                | 50  | 950.25      | 78.264              | 1874.8                | 50  | 1472.9      | 186.29              | 2853.0                |
| 9°  | 450.93      | 17.717              | 899.09                | 19° | 958.81      | 79.671              | 1891.3                | 29° | 1481.8      | 188.51              | 2869.2                |
| 10  | 459.32      | 18.381              | 915.70                | 10  | 967.38      | 81.092              | 1907.8                | 10  | 1490.7      | 190.74              | 2885.3                |
| 20  | 467.71      | 19.058              | 932.31                | 20  | 975.96      | 82.525              | 1924.2                | 20  | 1499.6      | 192.99              | 2901.4                |
| 30  | 476.10      | 19.746              | 948.92                | 30  | 984.53      | 83.972              | 1940.6                | 30  | 1508.5      | 195.25              | 2917.6                |
| 40  | 484.49      | 20.447              | 965.53                | 40  | 993.12      | 85.431              | 1957.1                | 40  | 1517.4      | 197.53              | 2933.7                |
| 50  | 492.88      | 21.161              | 982.14                | 50  | 1001.70     | 86.904              | 1973.5                | 50  | 1526.3      | 199.82              | 2949.8                |
| 10° | 501.28      | 21.886              | 998.74                | 20° | 1010.29     | 88.389              | 1989.9                | 30° | 1535.3      | 202.12              | 2965.9                |
| 10  | 509.68      | 22.624              | 1015.35               | 10  | 1018.89     | 89.888              | 2006.3                | 10  | 1544.2      | 204.44              | 2982.0                |
| 20  | 518.08      | 23.375              | 1031.95               | 20  | 1027.49     | 91.399              | 2022.7                | 20  | 1553.1      | 206.77              | 2998.1                |
| 30  | 526.48      | 24.138              | 1048.54               | 30  | 1036.09     | 92.924              | 2039.1                | 30  | 1562.1      | 209.12              | 3014.2                |
| 40  | 534.89      | 24.913              | 1065.14               | 40  | 1044.70     | 94.462              | 2055.5                | 40  | 1571.0      | 211.48              | 3030.2                |
| 50  | 543.29      | 25.700              | 1081.73               | 50  | 1053.31     | 96.013              | 2071.9                | 50  | 1580.0      | 213.86              | 3046.3                |
| 11° | 551.70      | 26.500              | 1098.33               | 21° | 1061.93     | 97.577              | 2088.3                | 31° | 1589.0      | 216.25              | 3062.4                |



TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS  
FOR A 1° CURVE.

| $\Delta$ | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. | $\Delta$ | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. | $\Delta$ | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. |
|----------|-------------|---------------------|-----------------------|----------|-------------|---------------------|-----------------------|----------|-------------|---------------------|-----------------------|
| 31°      | 1589.0      | 216.25              | 3062.4                | 41°      | 2142.2      | 387.38              | 4013.1                | 51°      | 2732.9      | 618.39              | 4933.4                |
| 10       | 1598.0      | 218.66              | 3078.4                | 10       | 2151.7      | 390.71              | 4028.7                | 10       | 2743.1      | 622.81              | 4948.4                |
| 20       | 1606.9      | 221.08              | 3094.5                | 20       | 2161.2      | 394.06              | 4044.3                | 20       | 2753.4      | 627.24              | 4963.4                |
| 30       | 1615.9      | 223.51              | 3110.5                | 30       | 2170.8      | 397.43              | 4059.9                | 30       | 2763.7      | 631.69              | 4978.4                |
| 40       | 1624.9      | 225.96              | 3126.6                | 40       | 2180.3      | 400.82              | 4075.5                | 40       | 2773.9      | 636.16              | 4993.4                |
| 50       | 1633.9      | 228.42              | 3142.6                | 50       | 2189.9      | 404.22              | 4091.1                | 50       | 2784.2      | 640.66              | 5008.4                |
| 32°      | 1643.0      | 230.90              | 3158.6                | 42°      | 2199.4      | 407.64              | 4106.6                | 52°      | 2794.5      | 645.17              | 5023.4                |
| 10       | 1652.0      | 233.39              | 3174.6                | 10       | 2209.0      | 411.07              | 4122.2                | 10       | 2804.9      | 649.70              | 5038.4                |
| 20       | 1661.0      | 235.90              | 3190.6                | 20       | 2218.6      | 414.52              | 4137.7                | 20       | 2815.2      | 654.25              | 5053.4                |
| 30       | 1670.0      | 238.43              | 3206.6                | 30       | 2228.1      | 417.99              | 4153.3                | 30       | 2825.6      | 658.83              | 5068.3                |
| 40       | 1679.1      | 240.96              | 3222.6                | 40       | 2237.7      | 421.48              | 4168.8                | 40       | 2835.9      | 663.42              | 5083.3                |
| 50       | 1688.1      | 243.52              | 3238.6                | 50       | 2247.3      | 424.98              | 4184.3                | 50       | 2846.3      | 668.03              | 5098.2                |
| 33°      | 1697.2      | 246.08              | 3254.6                | 43°      | 2257.0      | 428.50              | 4199.8                | 53°      | 2856.7      | 672.66              | 5113.1                |
| 10       | 1706.3      | 248.66              | 3270.6                | 10       | 2266.6      | 432.04              | 4215.3                | 10       | 2867.1      | 677.32              | 5128.0                |
| 20       | 1715.3      | 251.26              | 3286.6                | 20       | 2276.2      | 435.59              | 4230.8                | 20       | 2877.5      | 681.99              | 5142.9                |
| 30       | 1724.4      | 253.87              | 3302.5                | 30       | 2285.9      | 439.16              | 4246.3                | 30       | 2888.0      | 686.68              | 5157.8                |
| 40       | 1733.5      | 256.50              | 3318.5                | 40       | 2295.6      | 442.75              | 4261.8                | 40       | 2898.4      | 691.40              | 5172.7                |
| 50       | 1742.6      | 259.14              | 3334.4                | 50       | 2305.2      | 446.35              | 4277.3                | 50       | 2908.9      | 696.13              | 5187.6                |
| 34°      | 1751.7      | 261.80              | 3350.4                | 44°      | 2314.9      | 449.98              | 4292.7                | 54°      | 2919.4      | 700.89              | 5202.4                |
| 10       | 1760.8      | 264.47              | 3366.3                | 10       | 2324.6      | 453.62              | 4308.2                | 10       | 2929.9      | 705.66              | 5217.3                |
| 20       | 1770.0      | 267.16              | 3382.2                | 20       | 2334.3      | 457.27              | 4323.6                | 20       | 2940.4      | 710.46              | 5232.1                |
| 30       | 1779.1      | 269.86              | 3398.2                | 30       | 2344.1      | 460.95              | 4339.0                | 30       | 2951.0      | 715.28              | 5246.9                |
| 40       | 1788.2      | 272.58              | 3414.1                | 40       | 2353.8      | 464.64              | 4354.5                | 40       | 2961.5      | 720.11              | 5261.7                |
| 50       | 1797.4      | 275.31              | 3430.0                | 50       | 2363.5      | 468.35              | 4369.9                | 50       | 2972.1      | 724.97              | 5276.5                |
| 35°      | 1808.6      | 278.05              | 3445.9                | 45°      | 2373.3      | 472.08              | 4385.3                | 55°      | 2982.7      | 729.85              | 5291.3                |
| 10       | 1815.7      | 280.82              | 3461.8                | 10       | 2383.1      | 475.82              | 4400.7                | 10       | 2993.3      | 734.76              | 5306.1                |
| 20       | 1824.9      | 283.60              | 3477.7                | 20       | 2392.8      | 479.59              | 4416.1                | 20       | 3003.9      | 739.68              | 5320.9                |
| 30       | 1834.1      | 286.39              | 3493.5                | 30       | 2402.6      | 483.37              | 4431.4                | 30       | 3014.5      | 744.62              | 5335.6                |
| 40       | 1843.3      | 289.20              | 3509.4                | 40       | 2412.4      | 487.16              | 4446.8                | 40       | 3025.2      | 749.59              | 5350.4                |
| 50       | 1852.5      | 292.02              | 3525.3                | 50       | 2422.3      | 490.98              | 4462.2                | 50       | 3035.8      | 754.57              | 5365.1                |
| 36°      | 1861.7      | 294.86              | 3541.1                | 46°      | 2432.1      | 494.82              | 4477.5                | 56°      | 3046.5      | 759.58              | 5379.8                |
| 10       | 1870.9      | 297.72              | 3557.0                | 10       | 2441.9      | 498.67              | 4492.8                | 10       | 3057.2      | 764.61              | 5394.5                |
| 20       | 1880.1      | 300.59              | 3572.8                | 20       | 2451.8      | 502.54              | 4508.2                | 20       | 3067.9      | 769.66              | 5409.2                |
| 30       | 1889.4      | 303.47              | 3588.6                | 30       | 2461.7      | 506.42              | 4523.5                | 30       | 3078.7      | 774.73              | 5423.9                |
| 40       | 1898.6      | 306.37              | 3604.5                | 40       | 2471.5      | 510.33              | 4538.8                | 40       | 3089.4      | 779.83              | 5438.6                |
| 50       | 1907.9      | 309.29              | 3620.3                | 50       | 2481.4      | 514.25              | 4554.1                | 50       | 3100.2      | 784.94              | 5453.3                |
| 37°      | 1917.1      | 312.22              | 3636.1                | 47°      | 2491.3      | 518.20              | 4569.4                | 57°      | 3110.9      | 790.08              | 5467.9                |
| 10       | 1926.4      | 315.17              | 3651.9                | 10       | 2501.2      | 522.16              | 4584.7                | 10       | 3121.7      | 795.24              | 5482.5                |
| 20       | 1935.7      | 318.13              | 3667.7                | 20       | 2511.2      | 526.13              | 4599.9                | 20       | 3132.6      | 800.42              | 5497.2                |
| 30       | 1945.0      | 321.11              | 3683.5                | 30       | 2521.1      | 530.13              | 4615.2                | 30       | 3143.4      | 805.62              | 5511.8                |
| 40       | 1954.3      | 324.11              | 3699.3                | 40       | 2531.1      | 534.15              | 4630.4                | 40       | 3154.2      | 810.85              | 5526.4                |
| 50       | 1963.6      | 327.12              | 3715.0                | 50       | 2541.0      | 538.18              | 4645.7                | 50       | 3165.1      | 816.10              | 5541.0                |
| 38°      | 1972.9      | 330.15              | 3730.8                | 48°      | 2551.0      | 542.23              | 4660.9                | 58°      | 3176.0      | 821.37              | 5555.6                |
| 10       | 1982.2      | 333.19              | 3746.5                | 10       | 2561.0      | 546.30              | 4676.1                | 10       | 3186.9      | 826.66              | 5570.2                |
| 20       | 1991.5      | 336.25              | 3762.3                | 20       | 2571.0      | 550.39              | 4691.3                | 20       | 3197.8      | 831.98              | 5584.7                |
| 30       | 2000.9      | 339.32              | 3778.0                | 30       | 2581.0      | 554.50              | 4706.5                | 30       | 3208.8      | 837.31              | 5599.3                |
| 40       | 2010.2      | 342.41              | 3793.8                | 40       | 2591.1      | 558.63              | 4721.7                | 40       | 3219.7      | 842.67              | 5613.8                |
| 50       | 2019.6      | 345.52              | 3809.5                | 50       | 2601.1      | 562.77              | 4736.9                | 50       | 3230.7      | 848.06              | 5628.3                |
| 39°      | 2029.0      | 348.64              | 3825.2                | 49°      | 2611.2      | 566.94              | 4752.1                | 59°      | 3241.7      | 853.46              | 5642.8                |
| 10       | 2038.4      | 351.78              | 3840.9                | 10       | 2621.2      | 571.12              | 4767.3                | 10       | 3252.7      | 858.89              | 5657.3                |
| 20       | 2047.8      | 354.94              | 3856.6                | 20       | 2631.3      | 575.32              | 4782.4                | 20       | 3263.7      | 864.34              | 5671.8                |
| 30       | 2057.2      | 358.11              | 3872.3                | 30       | 2641.4      | 579.54              | 4797.5                | 30       | 3274.8      | 869.82              | 5686.3                |
| 40       | 2066.6      | 361.29              | 3888.0                | 40       | 2651.5      | 583.78              | 4812.7                | 40       | 3285.8      | 875.32              | 5700.8                |
| 50       | 2076.0      | 364.50              | 3903.6                | 50       | 2661.6      | 588.04              | 4827.8                | 50       | 3296.9      | 880.84              | 5715.2                |
| 40°      | 2085.4      | 367.72              | 3919.3                | 50°      | 2671.8      | 592.32              | 4842.9                | 60°      | 3308.0      | 886.38              | 5729.7                |
| 10       | 2094.9      | 370.95              | 3935.0                | 10       | 2681.9      | 596.62              | 4858.0                | 10       | 3319.1      | 891.95              | 5744.1                |
| 20       | 2104.3      | 374.20              | 3950.6                | 20       | 2692.1      | 600.93              | 4873.1                | 20       | 3330.3      | 897.54              | 5758.5                |
| 30       | 2113.8      | 377.47              | 3966.3                | 30       | 2702.3      | 605.27              | 4888.2                | 30       | 3341.4      | 903.15              | 5772.9                |
| 40       | 2123.3      | 380.76              | 3981.9                | 40       | 2712.5      | 609.62              | 4903.2                | 40       | 3352.6      | 908.79              | 5787.3                |
| 50       | 2132.7      | 384.08              | 3997.5                | 50       | 2722.7      | 614.00              | 4918.3                | 50       | 3363.8      | 914.45              | 5801.7                |
| 41°      | 2142.2      | 387.38              | 4013.1                | 51°      | 2732.9      | 618.39              | 4933.4                | 61°      | 3375.0      | 920.14              | 5816.0                |

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS  
FOR A 1° CURVE.

| Δ   | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. | Δ   | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. | Δ   | Tang.<br>T. | Ext.<br>Dist.<br>E. | Long<br>Chord<br>L.C. |
|-----|-------------|---------------------|-----------------------|-----|-------------|---------------------|-----------------------|-----|-------------|---------------------|-----------------------|
| 61° | 3375.0      | 920.14              | 5816.0                | 71° | 4086.9      | 1308.2              | 6654.4                | 81° | 4893.6      | 1805.3              | 7442.2                |
| 10  | 3386.3      | 925.85              | 5830.4                | 10  | 4099.5      | 1315.5              | 6668.0                | 10  | 4908.0      | 1814.7              | 7454.9                |
| 20  | 3397.5      | 931.58              | 5844.7                | 20  | 4112.1      | 1322.9              | 6681.6                | 20  | 4922.5      | 1824.1              | 7467.4                |
| 30  | 3408.8      | 937.34              | 5859.1                | 30  | 4124.8      | 1330.3              | 6695.1                | 30  | 4937.0      | 1833.6              | 7480.0                |
| 40  | 3420.1      | 943.12              | 5873.4                | 40  | 4137.4      | 1337.7              | 6708.6                | 40  | 4951.5      | 1843.1              | 7492.5                |
| 50  | 3431.4      | 948.92              | 5887.7                | 50  | 4150.1      | 1345.1              | 6722.1                | 50  | 4966.1      | 1852.6              | 7505.1                |
| 62° | 3442.7      | 954.75              | 5902.0                | 72° | 4162.8      | 1352.6              | 6735.6                | 82° | 4980.7      | 1862.2              | 7518.0                |
| 10  | 3454.1      | 960.60              | 5916.3                | 10  | 4175.6      | 1360.1              | 6749.1                | 10  | 4995.4      | 1871.8              | 7530.5                |
| 20  | 3465.4      | 966.48              | 5930.5                | 20  | 4188.4      | 1367.6              | 6762.5                | 20  | 5010.0      | 1881.5              | 7543.0                |
| 30  | 3476.8      | 972.39              | 5944.8                | 30  | 4201.2      | 1375.2              | 6776.0                | 30  | 5024.8      | 1891.2              | 7555.5                |
| 40  | 3488.2      | 978.31              | 5959.0                | 40  | 4214.0      | 1382.8              | 6789.4                | 40  | 5039.5      | 1900.9              | 7568.2                |
| 50  | 3499.7      | 984.27              | 5973.3                | 50  | 4226.8      | 1390.4              | 6802.8                | 50  | 5054.3      | 1910.7              | 7580.7                |
| 63° | 3511.1      | 990.24              | 5987.5                | 73° | 4239.7      | 1398.0              | 6816.3                | 83° | 5069.2      | 1920.5              | 7593.2                |
| 10  | 3522.6      | 996.24              | 6001.7                | 10  | 4252.6      | 1405.7              | 6829.6                | 10  | 5084.0      | 1930.4              | 7605.8                |
| 20  | 3534.1      | 1002.3              | 6015.9                | 20  | 4265.6      | 1413.5              | 6843.0                | 20  | 5099.0      | 1940.3              | 7618.3                |
| 30  | 3545.6      | 1008.3              | 6030.0                | 30  | 4278.5      | 1421.2              | 6856.4                | 30  | 5113.9      | 1950.3              | 7630.9                |
| 40  | 3557.2      | 1014.4              | 6044.2                | 40  | 4291.5      | 1429.0              | 6869.7                | 40  | 5128.9      | 1960.2              | 7643.6                |
| 50  | 3568.7      | 1020.5              | 6058.4                | 50  | 4304.6      | 1436.8              | 6883.1                | 50  | 5143.9      | 1970.3              | 7655.4                |
| 64° | 3580.3      | 1026.6              | 6072.5                | 74° | 4317.6      | 1444.6              | 6896.4                | 84° | 5159.0      | 1980.4              | 7667.7                |
| 10  | 3591.9      | 1032.8              | 6086.6                | 10  | 4330.7      | 1452.5              | 6909.7                | 10  | 5174.1      | 1990.5              | 7680.0                |
| 20  | 3603.5      | 1039.0              | 6100.7                | 20  | 4343.8      | 1460.4              | 6923.0                | 20  | 5189.3      | 2000.6              | 7692.2                |
| 30  | 3615.1      | 1045.2              | 6114.8                | 30  | 4356.9      | 1468.4              | 6936.2                | 30  | 5204.4      | 2010.8              | 7704.4                |
| 40  | 3626.8      | 1051.4              | 6128.9                | 40  | 4370.1      | 1476.4              | 6949.5                | 40  | 5219.7      | 2021.1              | 7717.1                |
| 50  | 3638.5      | 1057.7              | 6143.0                | 50  | 4383.3      | 1484.4              | 6962.8                | 50  | 5234.9      | 2031.4              | 7729.2                |
| 65° | 3650.2      | 1063.9              | 6157.1                | 75° | 4396.5      | 1492.4              | 6976.0                | 85° | 5250.3      | 2041.7              | 7741.1                |
| 10  | 3661.9      | 1070.2              | 6171.1                | 10  | 4409.8      | 1500.5              | 6989.2                | 10  | 5265.6      | 2052.1              | 7754.1                |
| 20  | 3673.7      | 1076.6              | 6185.2                | 20  | 4423.1      | 1508.6              | 7002.4                | 20  | 5281.0      | 2062.5              | 7766.6                |
| 30  | 3685.4      | 1082.9              | 6199.2                | 30  | 4436.4      | 1516.7              | 7015.6                | 30  | 5296.4      | 2073.0              | 7778.7                |
| 40  | 3697.2      | 1089.3              | 6213.2                | 40  | 4449.7      | 1524.9              | 7028.8                | 40  | 5311.9      | 2083.5              | 7790.9                |
| 50  | 3709.0      | 1095.7              | 6227.2                | 50  | 4463.1      | 1533.1              | 7041.9                | 50  | 5327.4      | 2094.1              | 7803.1                |
| 66° | 3720.9      | 1102.2              | 6241.2                | 76° | 4476.5      | 1541.4              | 7055.0                | 86° | 5343.0      | 2104.7              | 7815.1                |
| 10  | 3732.7      | 1108.6              | 6255.2                | 10  | 4489.9      | 1549.7              | 7068.2                | 10  | 5358.6      | 2115.3              | 7827.7                |
| 20  | 3744.6      | 1115.1              | 6269.1                | 20  | 4503.4      | 1558.0              | 7081.3                | 20  | 5374.2      | 2126.0              | 7839.9                |
| 30  | 3756.5      | 1121.7              | 6283.1                | 30  | 4516.9      | 1566.3              | 7094.4                | 30  | 5389.9      | 2136.7              | 7851.1                |
| 40  | 3768.5      | 1128.2              | 6297.0                | 40  | 4530.4      | 1574.7              | 7107.5                | 40  | 5405.6      | 2147.5              | 7863.1                |
| 50  | 3780.4      | 1134.8              | 6310.9                | 50  | 4544.0      | 1583.1              | 7120.5                | 50  | 5421.4      | 2158.4              | 7876.1                |
| 67° | 3792.4      | 1141.4              | 6324.8                | 77° | 4557.6      | 1591.6              | 7133.6                | 87° | 5437.2      | 2169.2              | 7888.1                |
| 10  | 3804.4      | 1148.0              | 6338.7                | 10  | 4571.2      | 1600.1              | 7146.6                | 10  | 5453.1      | 2180.2              | 7900.0                |
| 20  | 3816.4      | 1154.7              | 6352.6                | 20  | 4584.8      | 1608.6              | 7159.6                | 20  | 5469.0      | 2191.1              | 7912.1                |
| 30  | 3828.4      | 1161.3              | 6366.4                | 30  | 4598.5      | 1617.1              | 7172.6                | 30  | 5484.9      | 2202.2              | 7924.2                |
| 40  | 3840.5      | 1168.1              | 6380.3                | 40  | 4612.2      | 1625.7              | 7185.6                | 40  | 5500.9      | 2213.2              | 7936.3                |
| 50  | 3852.6      | 1174.8              | 6394.1                | 50  | 4626.0      | 1634.4              | 7198.6                | 50  | 5517.0      | 2224.3              | 7948.3                |
| 68° | 3864.7      | 1181.6              | 6408.0                | 78° | 4639.8      | 1643.0              | 7211.6                | 88° | 5533.1      | 2235.5              | 7960.1                |
| 10  | 3876.8      | 1188.4              | 6421.8                | 10  | 4653.6      | 1651.7              | 7224.5                | 10  | 5549.2      | 2246.7              | 7972.7                |
| 20  | 3889.0      | 1195.2              | 6435.6                | 20  | 4667.4      | 1660.5              | 7237.4                | 20  | 5565.4      | 2258.0              | 7984.8                |
| 30  | 3901.2      | 1202.0              | 6449.4                | 30  | 4681.3      | 1669.2              | 7250.4                | 30  | 5581.6      | 2269.3              | 7996.6                |
| 40  | 3913.4      | 1208.9              | 6463.1                | 40  | 4695.2      | 1678.1              | 7263.3                | 40  | 5597.8      | 2280.6              | 8008.3                |
| 50  | 3925.6      | 1215.8              | 6476.9                | 50  | 4709.2      | 1686.9              | 7276.1                | 50  | 5614.2      | 2292.0              | 8020.1                |
| 69° | 3937.9      | 1222.7              | 6490.6                | 79° | 4723.2      | 1695.8              | 7289.0                | 89° | 5630.5      | 2303.5              | 8031.1                |
| 10  | 3950.2      | 1229.7              | 6504.4                | 10  | 4737.2      | 1704.7              | 7301.9                | 10  | 5646.9      | 2315.0              | 8043.1                |
| 20  | 3962.5      | 1236.7              | 6518.1                | 20  | 4751.2      | 1713.7              | 7314.7                | 20  | 5663.4      | 2326.6              | 8055.5                |
| 30  | 3974.8      | 1243.7              | 6531.8                | 30  | 4765.3      | 1722.7              | 7327.5                | 30  | 5679.9      | 2338.2              | 8067.7                |
| 40  | 3987.2      | 1250.8              | 6545.5                | 40  | 4779.4      | 1731.7              | 7340.3                | 40  | 5696.4      | 2349.8              | 8079.9                |
| 50  | 3999.5      | 1257.9              | 6559.1                | 50  | 4793.6      | 1740.8              | 7353.1                | 50  | 5713.0      | 2361.5              | 8091.1                |
| 70° | 4011.9      | 1265.0              | 6572.8                | 80° | 4808.7      | 1749.9              | 7365.9                | 90° | 5729.7      | 2373.3              | 8103.1                |
| 10  | 4024.4      | 1272.1              | 6586.4                | 10  | 4822.0      | 1759.0              | 7378.7                | 10  | 5746.3      | 2385.1              | 8114.4                |
| 20  | 4036.8      | 1279.3              | 6600.1                | 20  | 4836.2      | 1768.2              | 7391.4                | 20  | 5763.1      | 2397.0              | 8126.0                |
| 30  | 4049.3      | 1286.5              | 6613.7                | 30  | 4850.5      | 1777.4              | 7404.1                | 30  | 5779.9      | 2408.9              | 8138.1                |
| 40  | 4061.8      | 1293.7              | 6627.3                | 40  | 4864.8      | 1786.7              | 7416.8                | 40  | 5796.7      | 2420.9              | 8150.0                |
| 50  | 4074.4      | 1300.9              | 6640.9                | 50  | 4879.2      | 1796.0              | 7429.5                | 50  | 5813.6      | 2432.9              | 8161.1                |
| 71° | 4086.9      | 1308.2              | 6654.4                | 81° | 4893.6      | 1805.3              | 7442.2                | 91° | 5830.5      | 2444.9              | 8173.1                |

TABLE III.—SWITCH LEADS AND DISTANCES.

LEAD-RAILS CIRCULAR THROUGHOUT; GAUGE 4' 8½". See § 262.

| Frog No. (n). | Frog Angle (F). | Lead (L) (Eq. 79). | Chord (QT) (Eq. 77). | Radius of Lead-rails (r, Eq. 78). | Log r.  | Degree of Curve (d). | Frog No. (n). |
|---------------|-----------------|--------------------|----------------------|-----------------------------------|---------|----------------------|---------------|
| 4             | 14° 15' 00"     | 37.67              | 37.38                | 150.67                            | 2.17801 | 38° 46'              | 4             |
| 4.5           | 12 40 59        | 42.37              | 42.12                | 190.69                            | .28032  | 30 24                | 4.5           |
| 5             | 11 25 16        | 47.08              | 46.85                | 235.42                            | .37183  | 24 32                | 5             |
| 5.5           | 10 23 20        | 51.79              | 51.58                | 284.85                            | .45462  | 20 13                | 5.5           |
| 6             | 9 31 38         | 56.50              | 56.30                | 339.00                            | .53020  | 16 58                | 6             |
| 6.5           | 8 47 51         | 61.21              | 61.03                | 397.85                            | .59972  | 14 26                | 6.5           |
| 7             | 8 10 16         | 65.92              | 65.75                | 461.42                            | .66409  | 12 26                | 7             |
| 7.5           | 7 37 41         | 70.62              | 70.47                | 529.69                            | .72402  | 10 50                | 7.5           |
| 8             | 7 09 10         | 75.33              | 75.19                | 602.67                            | .78007  | 9 31                 | 8             |
| 8.5           | 6 43 59         | 80.04              | 79.90                | 680.36                            | .83273  | 8 26                 | 8.5           |
| 9             | 6 21 35         | 84.75              | 84.62                | 762.75                            | .88238  | 7 31                 | 9             |
| 9.5           | 6 01 32         | 89.46              | 89.33                | 849.85                            | .92934  | 6 45                 | 9.5           |
| 10            | 5 43 29         | 94.17              | 94.05                | 941.67                            | 2.97389 | 6 05                 | 10            |
| 10.5          | 5 27 09         | 98.87              | 98.76                | 1038.19                           | 3.01627 | 5 32                 | 10.5          |
| 11            | 5 12 18         | 103.58             | 103.47               | 1139.42                           | .05668  | 5 02                 | 11            |
| 11.5          | 4 58 45         | 108.29             | 108.19               | 1245.36                           | .09529  | 4 38                 | 11.5          |
| 12            | 4 46 19         | 113.00             | 112.90               | 1356.00                           | 3.13226 | 4 14                 | 12            |

TURNOUTS WITH STRAIGHT POINT-RAILS AND STRAIGHT FROG-RAILS; GAUGE 4' 8½". See § 265.

| Frog No. (n). | Switch Point Angle (a). | L'gth of Switch Point (DN). | L'gth of Str'g't Frog-rail (F). | Lead (L) (Eq. 90). | Chord (ST) (Eq. 88). | Radius of Lead-rails (r, Eq. 87). | Log r.  | Degree of Curve (d). | Frog No. (n). |
|---------------|-------------------------|-----------------------------|---------------------------------|--------------------|----------------------|-----------------------------------|---------|----------------------|---------------|
| 4             | 3° 40'                  | 7.5                         | 1.50                            | 32.20              | 23.09                | 125.21                            | 2.09764 | 47° 05'              | 4             |
| 4.5           | 3 40                    | 7.5                         | 1.69                            | 34.29              | 25.03                | 159.25                            | .20208  | 36 36                | 4.5           |
| 5             | 2 45                    | 10.0                        | 1.87                            | 41.85              | 29.88                | 197.65                            | .29589  | 29 22                | 5             |
| 5.5           | 2 45                    | 10.0                        | 2.06                            | 44.16              | 32.03                | 240.44                            | .38100  | 24 00                | 5.5           |
| 6             | 1 50                    | 15.0                        | 2.25                            | 56.00              | 38.66                | 288.09                            | .45953  | 19 59                | 6             |
| 6.5           | 1 50                    | 15.0                        | 2.44                            | 58.84              | 41.34                | 340.19                            | .53172  | 16 54                | 6.5           |
| 7             | 1 50                    | 15.0                        | 2.62                            | 61.65              | 43.98                | 397.65                            | .59950  | 14 27                | 7             |
| 7.5           | 1 50                    | 15.0                        | 2.81                            | 64.36              | 46.50                | 460.00                            | .66276  | 12 29                | 7.5           |
| 8             | 1 50                    | 15.0                        | 3.00                            | 67.04              | 48.99                | 527.91                            | .72256  | 10 52                | 8             |
| 8.5           | 1 50                    | 15.0                        | 3.19                            | 69.60              | 51.38                | 600.94                            | .77883  | 9 33                 | 8.5           |
| 9             | 1 50                    | 15.0                        | 3.37                            | 72.20              | 53.80                | 681.16                            | .83325  | 8 25                 | 9             |
| 9.5           | 1 50                    | 15.0                        | 3.56                            | 74.70              | 56.11                | 767.11                            | .88486  | 7 28                 | 9.5           |
| 10            | 1 50                    | 15.0                        | 3.75                            | 77.04              | 58.28                | 858.14                            | .93356  | 6 41                 | 10            |
| 10.5          | 1 50                    | 15.0                        | 3.94                            | 79.51              | 60.57                | 959.00                            | 2.98182 | 5 59                 | 10.5          |
| 11            | 1 50                    | 15.0                        | 4.12                            | 81.82              | 62.69                | 1065.52                           | 3.02756 | 5 23                 | 11            |
| 11.5          | 1 50                    | 15.0                        | 4.31                            | 84.09              | 64.78                | 1180.16                           | 3.07194 | 4 51                 | 11.5          |
| 12            | 1 50                    | 15.0                        | 4.50                            | 86.16              | 66.67                | 1299.93                           | 3.11392 | 4 24                 | 12            |

TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES (F).

| Frog No. (n). | Frog Angle (F). | Nat. sin F. | Nat. cos F. | Log sin F. | Log cos F. | Log cot F. | Log vers F. | Frog No. (n). |
|---------------|-----------------|-------------|-------------|------------|------------|------------|-------------|---------------|
| 4             | 14° 15' 00"     | .24615      | .96923      | 9.39120    | 9.98642    | 10.59522   | 8.48811     | 4             |
| 4.5           | 12 40 49        | .21951      | .97561      | .34145     | .98927     | .64782     | .38721      | 4.5           |
| 5             | 11 25 16        | .19802      | .98020      | .29670     | .99131     | .69461     | .29670      | 5             |
| 5.5           | 10 23 20        | .18033      | .98360      | .25606     | .99282     | .73675     | .21467      | 5.5           |
| 6             | 9 31 38         | .16552      | .98621      | .21884     | .99397     | .77513     | .13966      | 6             |
| 6.5           | 8 47 51         | .15294      | .98823      | .18453     | .99486     | .81033     | .07058      | 6.5           |
| 7             | 8 10 16         | .14213      | .98985      | .15268     | .99557     | .84288     | 8.00655     | 7             |
| 7.5           | 7 37 41         | .13274      | .99115      | .12301     | .99614     | .87313     | 7.94691     | 7.5           |
| 8             | 7 09 10         | .12452      | .99222      | .09522     | .99660     | .90138     | .89110      | 8             |
| 8.5           | 6 43 59         | .11724      | .99310      | .06909     | .99699     | .92790     | .83864      | 8.5           |
| 9             | 6 21 35         | .11077      | .99385      | .04442     | .99732     | .95289     | .78915      | 9             |
| 9.5           | 6 01 32         | .10497      | .99448      | 9.02107    | .99759     | .97652     | .74232      | 9.5           |
| 10            | 5 43 29         | .09975      | .99501      | 8.99891    | .99783     | 10.99892   | .69788      | 10            |
| 10.5          | 5 27 09         | .09502      | .99548      | .97781     | .99803     | 11.02021   | .65560      | 10.5          |
| 11            | 5 12 18         | .09072      | .99588      | .95770     | .99820     | .04050     | .61528      | 11            |
| 11.5          | 4 58 45         | .08679      | .99623      | .93848     | .99836     | .05987     | .57676      | 11.5          |
| 12            | 4 46 19         | .08319      | .99653      | 8.92007    | 9.99849    | 11.07842   | 7.53986     | 12            |



TABLE IV.—ELEMENTS OF TRANSITION CURVES.

| Point. | Total<br>Central Angle<br>$\phi$ | Nat. sin $\phi$ | Nat. cos $\phi$ | Log sin $\phi$ | Log cos $\phi$ | Log vers $\phi$ | $x$    | Log $x$ | $y$     |
|--------|----------------------------------|-----------------|-----------------|----------------|----------------|-----------------|--------|---------|---------|
| 1      | 0° 0' 30"                        | .0022           | <i>one</i>      | 7.33878        | 10.00000       | 4.37654         | 0.027  | 8.43568 | 25.000  |
| 2      | 0 22 30                          | .0085           | <i>one</i>      | 7.81590        | 9.99999        | 5.33678         | 0.136  | 9.13465 | 50.000  |
| 3      | 0 45 00                          | .0131           | .9999           | 8.11692        | 9.99999        | 5.93284         | 0.382  | 9.58181 | 74.999  |
| 4      | 1 15 00                          | .0218           | .9997           | 8.33375        | 9.99989        | 6.37653         | 0.818  | 9.91280 | 99.995  |
| 5      | 1 52 30                          | .0327           | .9994           | 8.51489        | 9.99975        | 6.72869         | 1.500  | 0.17602 | 124.985 |
| 6      | 2 37 30                          | .0458           | .9989           | 8.68085        | 9.99954        | 7.02091         | 2.481  | 0.39467 | 149.966 |
| 7      | 3 30 00                          | .0610           | .9981           | 8.78567        | 9.99919        | 7.27072         | 3.817  | 0.58171 | 174.930 |
| 8      | 4 30 00                          | .0784           | .9969           | 8.89464        | 9.99866        | 7.48892         | 5.561  | 0.74514 | 199.870 |
| 9      | 5 37 30                          | .0980           | .9952           | 8.99130        | 9.99790        | 7.68262         | 7.792  | 0.89104 | 224.772 |
| 10     | 6 52 30                          | .1197           | .9928           | 9.07810        | 9.99685        | 7.85675         | 10.489 | 1.02061 | 249.623 |

## 0° 30'-per-25-foot spiral.

Deflections from the tangent at the point occupied when the instrument is at—

| sighting<br>at | Q        | 1         | 2          | 3          | 4          | 5          | 6          | 7          | 8          | 9          | 10         |
|----------------|----------|-----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Q              | 0° 0' 0" | 0° 3' 45" | 0° 13' 07" | 0° 27' 30" | 0° 46' 52" | 1° 11' 15" | 1° 40' 37" | 2° 15' 00" | 2° 54' 22" | 3° 38' 45" | 4° 28' 10" |
| 1              | 0 3 45   | 0 0 00    | 0 07 30    | 0 20 37    | 0 38 45    | 1 01 52    | 1 30 00    | 2 03 07    | 2 41 15    | 3 24 22    | 4 12 30    |
| 2              | 0 9 22   | 0 7 30    | 0 00 00    | 0 11 15    | 0 28 07    | 0 50 00    | 1 16 52    | 1 48 45    | 2 25 37    | 3 07 30    | 3 54 22    |
| 3              | 0 17 30  | 0 16 52   | 0 11 15    | 0 00 00    | 0 15 00    | 0 35 37    | 1 01 15    | 1 31 52    | 2 07 30    | 2 48 07    | 3 33 45    |
| 4              | 0 28 07  | 0 28 45   | 0 24 22    | 0 15 00    | 0 00 00    | 0 18 45    | 0 43 07    | 1 12 30    | 1 46 52    | 2 26 15    | 3 10 37    |
| 5              | 0 41 15  | 0 43 07   | 0 40 00    | 0 31 52    | 0 18 45    | 0 00 00    | 0 22 30    | 0 50 37    | 1 23 45    | 2 01 52    | 2 45 00    |
| 6              | 0 56 52  | 1 00 00   | 0 58 07    | 0 51 15    | 0 39 22    | 0 22 30    | 0 00 00    | 0 28 15    | 0 58 07    | 1 35 00    | 2 16 52    |
| 7              | 1 15 00  | 1 19 22   | 1 18 45    | 1 13 07    | 1 02 30    | 0 46 52    | 0 26 15    | 0 00 00    | 0 30 00    | 1 05 37    | 1 46 15    |
| 8              | 1 35 37  | 1 41 15   | 1 41 52    | 1 37 30    | 1 28 07    | 1 13 45    | 0 54 22    | 0 30 00    | 0 00 00    | 0 33 45    | 1 13 07    |
| 9              | 1 58 45  | 2 05 37   | 2 07 30    | 2 04 22    | 1 56 15    | 1 43 07    | 1 25 00    | 1 01 52    | 0 33 45    | 0 00 00    | 0 37 30    |
| 10             | 2 24 20  | 2 32 30   | 2 35 37    | 2 33 45    | 2 26 52    | 2 15 00    | 1 58 07    | 1 36 15    | 1 09 22    | 0 37 30    | 0 00 00    |

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

| Point. | Total<br>Central angle<br>$\phi$ | Nat. sin $\phi$ | Nat. cos $\phi$ | Log sin $\phi$ | Log cos $\phi$ | Log vers $\phi$ | $x$    | Log $x$ | $y$     |
|--------|----------------------------------|-----------------|-----------------|----------------|----------------|-----------------|--------|---------|---------|
| 1      | 0° 15'                           | .0043           | one             | 7.63981        | 9.99999        | 4.97860         | .055   | 8.73672 | 25.000  |
| 2      | 0 45                             | .0131           | .9999           | 8.11692        | 9.99996        | 5.55484         | .273   | 9.43616 | 49.999  |
| 3      | 1 30                             | .0262           | .9996           | 8.41792        | 9.99985        | 6.55468         | .763   | 9.88352 | 74.994  |
| 4      | 2 30                             | .0436           | .9990           | 8.63968        | 9.99958        | 6.97853         | 1.636  | 0.21378 | 99.979  |
| 5      | 3 45                             | .0654           | .9978           | 8.81560        | 9.99907        | 7.33063         | 2.990  | 0.47697 | 124.942 |
| 6      | 5 15                             | .0915           | .9938           | 8.96143        | 9.99817        | 7.62274         | 4.900  | 0.69548 | 149.865 |
| 7      | 7 00                             | .1218           | .9835           | 9.08689        | 9.99675        | 7.87288         | 7.628  | 0.85241 | 174.722 |
| 8      | 9 00                             | .1564           | .9677           | 9.19433        | 9.99462        | 8.08021         | 11.107 | 1.04559 | 199.479 |
| 9      | 11 15                            | .1951           | .9808           | 9.29023        | 9.99157        | 8.28365         | 15.502 | 1.19039 | 224.690 |
| 10     | 13 45                            | .2377           | .9713           | 9.37600        | 9.98737        | 8.45724         | 20.913 | 1.32041 | 248.497 |

1° per 25-foot spiral.

Deflections from the tangent at the point occupied when the instrument is at —

| sighting<br>at | $\phi$   | 1          | 2          | 3         | 4          | 5          | 6          | 7          | 8          | 9          | 10         |
|----------------|----------|------------|------------|-----------|------------|------------|------------|------------|------------|------------|------------|
| $\phi$         | 0° 0' 0" | 0° 07' 30" | 0° 26' 15" | 0° 55' 0" | 1° 33' 45" | 2° 22' 30" | 3° 21' 15" | 4° 30' 00" | 5° 48' 47" | 7° 17' 34" | 8° 56' 22" |
| 1              | 0 07 30  | 0 00 00    | 0 15 00    | 0 41 15   | 1 17 30    | 2 03 45    | 3 00 00    | 4 06 15    | 5 22 30    | 6 48 47    | 8 25 05    |
| 2              | 0 18 45  | 0 15 00    | 0 00 00    | 0 22 30   | 0 56 15    | 1 40 00    | 2 33 45    | 3 37 30    | 4 51 15    | 6 15 02    | 7 48 49    |
| 3              | 0 35 00  | 0 33 45    | 0 22 30    | 0 00 00   | 0 30 00    | 1 11 15    | 2 02 30    | 3 03 45    | 4 15 00    | 5 36 15    | 7 07 32    |
| 4              | 0 56 15  | 0 57 30    | 0 48 45    | 0 30 00   | 0 00 00    | 0 37 30    | 1 26 15    | 2 25 00    | 3 33 45    | 4 52 30    | 6 21 17    |
| 5              | 1 22 30  | 1 26 15    | 1 20 00    | 1 03 45   | 0 37 30    | 0 00 00    | 0 45 00    | 1 41 15    | 2 47 30    | 4 03 45    | 5 30 00    |
| 6              | 1 53 45  | 2 00 00    | 1 56 15    | 1 42 30   | 1 18 45    | 0 45 00    | 0 00 00    | 0 52 30    | 1 56 15    | 3 10 00    | 4 33 45    |
| 7              | 2 30 00  | 2 38 45    | 2 37 30    | 2 26 15   | 2 05 00    | 1 33 45    | 0 52 30    | 0 00 00    | 2 11 15    | 3 11 15    | 3 32 30    |
| 8              | 3 11 13  | 3 22 30    | 3 23 45    | 3 15 00   | 2 56 15    | 2 27 30    | 1 48 45    | 1 00 00    | 0 00 00    | 1 07 30    | 2 26 15    |
| 9              | 3 57 26  | 4 11 13    | 4 14 58    | 4 08 45   | 3 52 30    | 3 26 15    | 2 50 00    | 2 03 45    | 1 07 30    | 0 00 00    | 1 15 00    |
| 10             | 4 48 38  | 5 04 55    | 5 11 11    | 5 07 28   | 4 53 43    | 4 30 00    | 3 56 15    | 3 12 30    | 2 18 45    | 1 15 00    | 0 00 00    |

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

| Point. | Total<br>Central Angle<br>$\phi$ | Nat. sin $\phi$ | Nat. cos $\phi$ | Log sin $\phi$ | Log cos $\phi$ | Log vers $\phi$ | $x$    | Log $x$ | $y$     |
|--------|----------------------------------|-----------------|-----------------|----------------|----------------|-----------------|--------|---------|---------|
| 1      | 0° 30'                           | .0087           | .9999           | 7.94084        | 9.99998        | 5.58066         | 0.109  | 9.03771 | 25.000  |
| 2      | 1 30                             | .0262           | .9996           | 8.41792        | 9.99985        | 6.53488         | 0.545  | 9.73670 | 49.996  |
| 3      | 3 00                             | .0523           | .9986           | 8.71880        | 9.99940        | 7.13687         | 1.527  | 0.18388 | 74.977  |
| 4      | 5 00                             | .0871           | .9962           | 8.94029        | 8.99834        | 7.58039         | 3.271  | 0.51468 | 99.916  |
| 5      | 7 30                             | .1305           | .9914           | 9.11570        | 9.99627        | 7.93232         | 5.992  | 0.77762 | 124.767 |
| 6      | 10 30                            | .1822           | .9832           | 9.26063        | 9.99265        | 8.22389         | 9.903  | 0.99579 | 149.459 |
| 7      | 14 00                            | .2419           | .9703           | 9.38367        | 9.98690        | 8.47282         | 15.208 | 1.13207 | 173.890 |
| 8      | 18 00                            | .3090           | .9510           | 9.48998        | 9.97820        | 8.68669         | 22.099 | 1.34437 | 197.922 |
| 9      | 22 30                            | .3827           | .9239           | 9.58284        | 9.96561        | 8.88150         | 30.752 | 1.48787 | 221.376 |
| 10     | 27 30                            | .4617           | .8870           | 9.66440        | 9.94793        | 9.05303         | 41.317 | 1.61613 | 244.034 |

## 2°-per-35-foot spiral.

—sight—  
ing  
at—

Deflections from the tangent at the point occupied when the instrument is at—

| $\phi$     | 1          | 2          | 3          | 4          | 5          | 6          | 7          | 8           | 9           | 10          |
|------------|------------|------------|------------|------------|------------|------------|------------|-------------|-------------|-------------|
| 0° 00' 00" | 0° 15' 00" | 0° 52' 30" | 1° 50' 00" | 3° 07' 30" | 4° 45' 00" | 6° 42' 30" | 9° 00' 05" | 11° 37' 45" | 14° 35' 30" | 17° 53' 25" |
| 0 15 00    | 0 00 00    | 0 30 00    | 1 22 30    | 2 35 00    | 4 07 30    | 6 00 01    | 8 12 34    | 10 45 10    | 13 37 52    | 16 50 42    |
| 0 37 30    | 0 30 00    | 0 00 00    | 0 45 00    | 1 52 30    | 3 20 00    | 5 07 30    | 7 15 02    | 9 42 36     | 12 30 15    | 15 38 00    |
| 1 10 00    | 1 07 30    | 0 45 00    | 0 00 00    | 1 00 00    | 2 22 30    | 4 05 00    | 6 07 30    | 8 30 04     | 11 12 39    | 14 15 20    |
| 1 52 30    | 1 55 00    | 1 37 30    | 1 00 00    | 0 00 00    | 1 15 00    | 2 52 30    | 4 50 00    | 7 07 30     | 9 45 05     | 12 42 42    |
| 2 45 00    | 2 52 30    | 2 40 00    | 2 07 30    | 1 15 00    | 0 00 00    | 1 30 00    | 3 22 30    | 5 35 00     | 8 07 30     | 11 00 06    |
| 3 47 28    | 3 59 59    | 3 52 30    | 3 25 00    | 2 37 30    | 1 30 00    | 0 00 00    | 1 45 00    | 3 52 30     | 6 20 00     | 9 07 30     |
| 4 59 53    | 5 17 26    | 5 14 58    | 4 52 30    | 4 10 00    | 3 07 30    | 1 45 00    | 0 00 00    | 2 00 00     | 4 22 30     | 7 05 00     |
| 6 22 15    | 6 44 50    | 6 47 24    | 6 29 58    | 5 52 30    | 4 55 00    | 3 37 30    | 2 00 00    | 0 00 00     | 2 15 00     | 4 52 30     |
| 7 54 30    | 8 22 08    | 8 29 45    | 8 17 21    | 7 44 55    | 6 52 30    | 5 40 00    | 4 07 30    | 2 15 00     | 0 00 00     | 2 30 00     |
| 9 36 35    | 10 09 17   | 10 22 00   | 10 14 40   | 9 47 18    | 8 59 54    | 7 52 30    | 6 25 00    | 4 37 30     | 2 30 00     | 0 00 00     |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.                 |
|-----|--------|------|------|------|------|------|------|------|------|------|-----------------------|
| 100 | 00 000 | 043  | 087  | 130  | 173  | 216  | 260  | 303  | 346  | 389  |                       |
| 101 | 432    | 475  | 518  | 561  | 604  | 646  | 689  | 732  | 775  | 817  | 43 43 42 41           |
| 102 | 860    | 902  | 945  | 987  | *030 | *072 | *114 | *157 | *199 | *241 | 1 4.3 4.3 4.2 4.1     |
| 103 | 01 283 | 326  | 368  | 410  | 452  | 494  | 536  | 578  | 619  | 661  | 2 8.7 8.6 8.4 8.2     |
| 104 | 703    | 745  | 787  | 828  | 870  | 911  | 953  | 994  | *036 | *077 | 3 13.0 12.9 12.6 12.3 |
| 105 | 02 119 | 160  | 201  | 243  | 284  | 325  | 366  | 407  | 448  | 489  | 4 17.4 17.2 16.8 16.5 |
| 106 | 530    | 571  | 612  | 653  | 694  | 735  | 775  | 816  | 857  | 898  | 5 21.7 21.5 21.0 20.8 |
| 107 | 938    | 979  | *019 | *060 | *100 | *141 | *181 | *221 | *262 | *302 | 6 26.1 25.8 25.2 24.6 |
| 108 | 03 342 | 382  | 422  | 463  | 503  | 543  | 583  | 623  | 663  | 703  | 7 30.4 30.1 29.4 28.7 |
| 109 | 742    | 782  | 822  | 862  | 901  | 941  | 981  | *020 | *060 | *100 | 8 34.8 34.4 33.6 32.8 |
|     |        |      |      |      |      |      |      |      |      |      | 9 39.1 38.7 37.8 36.9 |
| 110 | 04 139 | 178  | 218  | 257  | 297  | 336  | 375  | 415  | 454  | 493  | 40 40 39 38           |
| 111 | 532    | 571  | 610  | 649  | 688  | 727  | 766  | 805  | 844  | 883  | 1 4.0 4.0 3.9 3.8     |
| 112 | 922    | 960  | 999  | *038 | *076 | *115 | *154 | *192 | *231 | *269 | 2 8.1 8.0 7.8 7.6     |
| 113 | 05 308 | 346  | 384  | 423  | 461  | 499  | 538  | 576  | 614  | 652  | 3 12.1 12.0 11.7 11.4 |
| 114 | 690    | 728  | 766  | 804  | 842  | 880  | 918  | 956  | 994  | *032 | 4 16.2 16.0 15.6 15.2 |
| 115 | 06 070 | 107  | 145  | 183  | 220  | 258  | 296  | 333  | 371  | 408  | 5 20.2 20.0 19.5 19.0 |
| 116 | 446    | 483  | 520  | 558  | 595  | 632  | 670  | 707  | 744  | 781  | 6 24.3 24.0 23.4 22.8 |
| 117 | 818    | 855  | 893  | 930  | 967  | *004 | *040 | *077 | *114 | *151 | 7 28.3 28.0 27.3 26.6 |
| 118 | 07 188 | 225  | 261  | 298  | 335  | 372  | 408  | 445  | 481  | 518  | 8 32.4 32.0 31.2 30.4 |
| 119 | 554    | 591  | 627  | 664  | 700  | 737  | 773  | 809  | 845  | 882  | 9 36.4 36.0 35.1 34.2 |
| 120 | 918    | 954  | 990  | *026 | *062 | *098 | *134 | *170 | *206 | *242 | 37 37 36 35           |
| 121 | 08 278 | 314  | 350  | 386  | 422  | 457  | 493  | 529  | 564  | 600  | 1 3.7 3.7 3.6 3.5     |
| 122 | 836    | 871  | 907  | 942  | 978  | 1013 | 1049 | 1084 | 1120 | 1155 | 2 7.5 7.4 7.2 7.0     |
| 123 | 990    | *026 | *061 | *096 | *131 | *166 | *202 | *237 | *272 | *307 | 3 11.2 11.1 10.8 10.5 |
| 124 | 09 342 | 377  | 412  | 447  | 482  | 517  | 552  | 586  | 621  | 656  | 4 15.0 14.8 14.4 14.0 |
| 125 | 691    | 725  | 760  | 795  | 830  | 864  | 899  | 933  | 968  | *002 | 5 18.7 18.5 18.0 17.5 |
| 126 | 10 037 | 071  | 106  | 140  | 174  | 209  | 243  | 277  | 312  | 346  | 6 22.5 22.2 21.6 21.0 |
| 127 | 380    | 414  | 448  | 483  | 517  | 551  | 585  | 619  | 653  | 687  | 7 26.2 25.9 25.2 24.5 |
| 128 | 721    | 755  | 789  | 822  | 856  | 890  | 924  | 958  | 991  | *025 | 8 30.0 29.6 28.8 28.0 |
| 129 | 11 059 | 092  | 126  | 160  | 193  | 227  | 260  | 294  | 327  | 361  | 9 33.7 33.3 32.4 31.5 |
| 130 | 394    | 427  | 461  | 494  | 528  | 561  | 594  | 627  | 661  | 694  | 34 34 33 32           |
| 131 | 727    | 760  | 793  | 826  | 859  | 892  | 925  | 958  | 991  | *024 | 1 3.4 3.4 3.3 3.2     |
| 132 | 12 057 | 090  | 123  | 156  | 189  | 221  | 254  | 287  | 320  | 352  | 2 6.9 6.8 6.6 6.4     |
| 133 | 385    | 418  | 450  | 483  | 515  | 548  | 580  | 613  | 645  | 678  | 3 10.3 10.2 9.9 9.6   |
| 134 | 710    | 743  | 775  | 807  | 840  | 872  | 904  | 937  | 969  | *001 | 4 13.8 13.6 13.2 12.8 |
| 135 | 13 033 | 065  | 097  | 130  | 162  | 194  | 226  | 258  | 290  | 322  | 5 17.2 17.0 16.5 16.0 |
| 136 | 354    | 386  | 417  | 449  | 481  | 513  | 545  | 577  | 608  | 640  | 6 20.7 20.4 19.8 19.2 |
| 137 | 672    | 703  | 735  | 767  | 798  | 830  | 862  | 893  | 925  | 956  | 7 24.1 23.8 23.1 22.4 |
| 138 | 988    | *019 | *051 | *082 | *113 | *145 | *176 | *207 | *239 | *270 | 8 27.6 27.2 26.4 25.6 |
| 139 | 14 301 | 332  | 364  | 395  | 426  | 457  | 488  | 519  | 550  | 582  | 9 31.0 30.6 29.7 28.8 |
| 140 | 613    | 644  | 675  | 706  | 736  | 767  | 798  | 829  | 860  | 891  | 31 31 30 29           |
| 141 | 922    | 952  | 983  | *014 | *045 | *075 | *106 | *137 | *167 | *198 | 1 3.1 3.1 3.0 2.9     |
| 142 | 15 229 | 259  | 290  | 320  | 351  | 381  | 412  | 442  | 473  | 503  | 2 6.3 6.2 6.0 5.8     |
| 143 | 533    | 564  | 594  | 624  | 655  | 685  | 715  | 745  | 776  | 806  | 3 9.4 9.3 9.0 8.7     |
| 144 | 836    | 866  | 896  | 926  | 956  | 987  | *017 | *047 | *077 | *107 | 4 12.6 12.4 12.0 11.6 |
| 145 | 16 137 | 166  | 196  | 226  | 256  | 286  | 316  | 346  | 376  | 405  | 5 15.7 15.5 15.0 14.5 |
| 146 | 435    | 465  | 494  | 524  | 554  | 584  | 613  | 643  | 672  | 702  | 6 18.9 18.6 18.0 17.4 |
| 147 | 731    | 761  | 791  | 820  | 849  | 879  | 908  | 938  | 967  | 997  | 7 22.0 21.7 21.0 20.3 |
| 148 | 17 026 | 055  | 085  | 114  | 143  | 172  | 202  | 231  | 260  | 289  | 8 25.2 24.8 24.0 23.2 |
| 149 | 318    | 348  | 377  | 406  | 435  | 464  | 493  | 522  | 551  | 580  | 9 28.3 27.9 27.0 26.1 |
| 150 | 609    | 638  | 667  | 696  | 725  | 753  | 782  | 811  | 840  | 869  |                       |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.                 |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.             |
|-----|--------|-----|------|------|------|------|------|------|------|------|-------------------|
| 150 | 17 609 | 638 | 667  | 696  | 725  | 753  | 782  | 811  | 840  | 869  |                   |
| 151 | 897    | 926 | 955  | 984  | *012 | *041 | *070 | *098 | *127 | *156 | .1 2.9 2.8 2.7    |
| 152 | 18 134 | 213 | 241  | 270  | 298  | 327  | 355  | 384  | 412  | 440  | .2 5.8 5.6 5.4    |
| 153 | 469    | 497 | 526  | 554  | 582  | 611  | 639  | 667  | 695  | 724  | .3 8.7 8.4 8.1    |
| 154 | 752    | 780 | 808  | 836  | 864  | 893  | 921  | 949  | 977  | *005 | .4 11.6 11.2 10.8 |
| 155 | 19 033 | 061 | 089  | 117  | 145  | 173  | 201  | 229  | 256  | 284  | .5 14.5 14.0 13.5 |
| 156 | 312    | 340 | 368  | 396  | 423  | 451  | 479  | 507  | 534  | 562  | .6 17.4 16.8 16.2 |
| 157 | 590    | 617 | 645  | 673  | 700  | 728  | 755  | 783  | 810  | 838  | .7 20.3 19.6 18.9 |
| 158 | 865    | 893 | 920  | 948  | 975  | *003 | *030 | *057 | *085 | *112 | .8 23.2 22.4 21.6 |
| 159 | 20 139 | 167 | 194  | 221  | 249  | 276  | 303  | 330  | 357  | 385  | .9 26.1 25.2 24.3 |
| 160 | 412    | 439 | 466  | 493  | 520  | 547  | 574  | 601  | 628  | 655  |                   |
| 161 | 682    | 709 | 736  | 763  | 790  | 817  | 844  | 871  | 898  | 924  | .1 2.6 2.6        |
| 162 | 951    | 978 | *005 | *032 | *058 | *085 | *112 | *139 | *165 | *192 | .2 5.3 5.2        |
| 163 | 21 219 | 245 | 272  | 298  | 325  | 352  | 378  | 405  | 431  | 458  | .3 7.9 7.8        |
| 164 | 484    | 511 | 537  | 564  | 590  | 616  | 643  | 669  | 695  | 722  | .4 10.6 10.4      |
| 165 | 748    | 774 | 801  | 827  | 853  | 880  | 906  | 932  | 958  | 984  | .5 13.2 13.0      |
| 166 | 22 011 | 037 | 063  | 089  | 115  | 141  | 167  | 193  | 219  | 245  | .6 15.9 15.6      |
| 167 | 271    | 297 | 323  | 349  | 375  | 401  | 427  | 453  | 479  | 505  | .7 18.5 18.2      |
| 168 | 531    | 557 | 582  | 608  | 634  | 660  | 686  | 711  | 737  | 763  | .8 21.2 20.8      |
| 169 | 788    | 814 | 840  | 865  | 891  | 917  | 942  | 968  | 994  | *019 | .9 23.8 23.4      |
| 170 | 23 045 | 070 | 096  | 121  | 147  | 172  | 198  | 223  | 249  | 274  |                   |
| 171 | 299    | 325 | 350  | 375  | 401  | 426  | 451  | 477  | 502  | 527  | .1 2.5 2.5 2.4    |
| 172 | 553    | 578 | 603  | 628  | 653  | 679  | 704  | 729  | 754  | 779  | .2 5.1 5.0 4.8    |
| 173 | 804    | 829 | 855  | 880  | 905  | 930  | 955  | 980  | *005 | *030 | .3 7.6 7.5 7.2    |
| 174 | 24 065 | 080 | 105  | 129  | 154  | 179  | 201  | 229  | 254  | 279  | .4 10.2 10.0 9.6  |
| 175 | 304    | 328 | 353  | 378  | 403  | 427  | 452  | 477  | 502  | 526  | .5 12.7 12.5 12.0 |
| 176 | 551    | 576 | 600  | 625  | 650  | 674  | 699  | 723  | 748  | 773  | .6 15.3 15.0 14.4 |
| 177 | 797    | 822 | 846  | 871  | 895  | 920  | 944  | 968  | 993  | *017 | .7 17.8 17.5 16.8 |
| 178 | 25 042 | 066 | 091  | 115  | 139  | 164  | 188  | 212  | 237  | 261  | .8 20.4 20.0 19.2 |
| 179 | 285    | 309 | 334  | 358  | 382  | 406  | 430  | 455  | 479  | 503  | .9 22.9 22.5 21.6 |
| 180 | 527    | 551 | 575  | 599  | 623  | 647  | 672  | 696  | 720  | 744  |                   |
| 181 | 768    | 792 | 816  | 840  | 863  | 887  | 911  | 935  | 959  | 983  | .1 2.3 2.3        |
| 182 | 26 007 | 031 | 055  | 078  | 102  | 126  | 150  | 174  | 197  | 221  | .2 4.7 4.6        |
| 183 | 245    | 269 | 292  | 316  | 340  | 363  | 387  | 411  | 434  | 458  | .3 7.0 6.9        |
| 184 | 482    | 505 | 529  | 552  | 576  | 599  | 623  | 646  | 670  | 693  | .4 9.4 9.2        |
| 185 | 717    | 740 | 764  | 787  | 811  | 834  | 858  | 881  | 904  | 928  | .5 11.7 11.5      |
| 186 | 951    | 974 | 998  | *021 | *044 | *068 | *091 | *114 | *137 | *161 | .6 14.1 13.8      |
| 187 | 27 184 | 207 | 230  | 254  | 277  | 300  | 323  | 346  | 369  | 392  | .7 16.4 16.1      |
| 188 | 416    | 439 | 462  | 485  | 508  | 531  | 554  | 577  | 600  | 623  | .8 18.9 18.4      |
| 189 | 646    | 669 | 692  | 715  | 738  | 761  | 784  | 806  | 829  | 852  | .9 21.1 20.7      |
| 190 | 875    | 898 | 921  | 944  | 966  | 989  | *012 | *035 | *058 | *080 |                   |
| 191 | 28 103 | 126 | 149  | 171  | 194  | 217  | 239  | 262  | 285  | 307  | .1 2.2 2.2 2.1    |
| 192 | 330    | 352 | 375  | 398  | 420  | 443  | 465  | 488  | 510  | 533  | .2 4.5 4.4 4.3    |
| 193 | 555    | 578 | 600  | 623  | 645  | 668  | 690  | 713  | 735  | 758  | .3 6.7 6.6 6.4    |
| 194 | 780    | 802 | 825  | 847  | 869  | 892  | 914  | 936  | 959  | 981  | .4 9.0 8.8 8.6    |
| 195 | 29 003 | 025 | 048  | 070  | 092  | 114  | 137  | 159  | 181  | 203  | .5 11.2 11.0 10.7 |
| 196 | 225    | 248 | 270  | 292  | 314  | 336  | 358  | 380  | 402  | 424  | .6 13.5 13.2 12.9 |
| 197 | 446    | 468 | 490  | 512  | 534  | 556  | 578  | 600  | 622  | 644  | .7 15.7 15.4 15.0 |
| 198 | 666    | 688 | 710  | 732  | 754  | 776  | 798  | 820  | 841  | 863  | .8 18.0 17.6 17.2 |
| 199 | 885    | 907 | 929  | 950  | 972  | 994  | *016 | *038 | *059 | *081 | .9 20.2 19.8 19.3 |
| 200 | 30 103 | 124 | 146  | 168  | 190  | 211  | 233  | 254  | 276  | 298  |                   |
| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.             |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |      |
|-----|--------|-----|------|------|------|------|------|------|------|------|-------|------|------|
| 200 | 30 103 | 124 | 146  | 168  | 190  | 211  | 233  | 254  | 276  | 298  |       |      |      |
| 201 |        | 319 | 341  | 363  | 384  | 406  | 427  | 449  | 470  | 492  | 22 21 |      |      |
| 202 |        | 535 | 556  | 578  | 599  | 621  | 642  | 664  | 685  | 707  | .1    | 2.2  | 2.1  |
| 203 |        | 749 | 771  | 792  | 813  | 835  | 856  | 878  | 899  | 920  | .2    | 4.4  | 4.2  |
| 204 |        | 963 | 984  | *005 | *027 | *048 | *069 | *090 | *112 | *133 | .3    | 6.6  | 6.3  |
| 205 | 31 175 | 196 | 217  | 239  | 260  | 281  | 302  | 323  | 344  | 365  | .4    | 8.8  | 8.4  |
| 206 |        | 386 | 408  | 429  | 450  | 471  | 492  | 513  | 534  | 555  | .5    | 11.0 | 10.5 |
| 207 |        | 597 | 618  | 639  | 660  | 681  | 702  | 722  | 743  | 764  | .6    | 13.2 | 12.6 |
| 208 |        | 806 | 827  | 848  | 869  | 890  | 910  | 931  | 952  | 973  | .7    | 15.4 | 14.7 |
| 209 | 32 014 | 035 | 056  | 077  | 097  | 118  | 139  | 160  | 180  | 201  | .8    | 17.6 | 16.8 |
|     |        |     |      |      |      |      |      |      |      |      | .9    | 19.8 | 18.9 |
| 210 |        | 222 | 242  | 263  | 284  | 304  | 325  | 346  | 366  | 387  | 20 20 |      |      |
| 211 |        | 428 | 449  | 469  | 490  | 510  | 531  | 551  | 572  | 592  | .1    | 2.0  | 2.0  |
| 212 |        | 633 | 654  | 674  | 695  | 715  | 736  | 756  | 776  | 797  | .2    | 4.1  | 4.0  |
| 213 |        | 838 | 858  | 878  | 899  | 919  | 940  | 960  | 980  | *001 | .3    | 6.1  | 6.0  |
| 214 | 33 041 | 061 | 082  | 102  | 122  | 142  | 163  | 183  | 203  | 223  | .4    | 8.2  | 8.0  |
| 215 |        | 244 | 264  | 284  | 304  | 324  | 344  | 365  | 385  | 405  | .5    | 10.2 | 10.0 |
| 216 |        | 445 | 465  | 485  | 505  | 525  | 546  | 566  | 586  | 606  | .6    | 12.3 | 12.0 |
| 217 |        | 646 | 666  | 686  | 706  | 726  | 746  | 766  | 786  | 806  | .7    | 14.3 | 14.0 |
| 218 |        | 845 | 865  | 885  | 905  | 925  | 945  | 965  | 985  | *004 | .8    | 16.4 | 16.0 |
| 219 | 34 044 | 064 | 084  | 104  | 124  | 143  | 163  | 183  | 203  | 222  | .9    | 18.4 | 18.0 |
| 220 |        | 242 | 262  | 281  | 301  | 321  | 341  | 360  | 380  | 400  | 19 19 |      |      |
| 221 |        | 439 | 459  | 478  | 498  | 518  | 537  | 557  | 576  | 596  | .1    | 1.9  | 1.9  |
| 222 |        | 635 | 655  | 674  | 694  | 714  | 733  | 752  | 772  | 791  | .2    | 3.9  | 3.8  |
| 223 |        | 830 | 850  | 869  | 889  | 908  | 928  | 947  | 966  | 986  | .3    | 5.8  | 5.7  |
| 224 | 35 025 | 044 | 063  | 083  | 102  | 121  | 141  | 160  | 179  | 199  | .4    | 7.8  | 7.6  |
| 225 |        | 218 | 237  | 257  | 276  | 295  | 314  | 334  | 353  | 372  | .5    | 9.7  | 9.5  |
| 226 |        | 411 | 430  | 449  | 468  | 487  | 507  | 526  | 545  | 564  | .6    | 11.7 | 11.4 |
| 227 |        | 602 | 621  | 641  | 660  | 679  | 698  | 717  | 736  | 755  | .7    | 13.6 | 13.3 |
| 228 |        | 793 | 812  | 831  | 850  | 869  | 888  | 907  | 926  | 945  | .8    | 15.6 | 15.2 |
| 229 |        | 983 | *002 | *021 | *040 | *059 | *078 | *097 | *116 | *135 | .9    | 17.5 | 17.1 |
| 230 | 36 173 | 191 | 210  | 229  | 248  | 267  | 286  | 305  | 323  | 342  | 18 18 |      |      |
| 231 |        | 361 | 380  | 399  | 417  | 436  | 455  | 474  | 492  | 511  | .1    | 1.8  | 1.8  |
| 232 |        | 549 | 567  | 586  | 605  | 623  | 642  | 661  | 679  | 698  | .2    | 3.7  | 3.6  |
| 233 |        | 735 | 754  | 773  | 791  | 810  | 828  | 847  | 866  | 884  | .3    | 5.5  | 5.4  |
| 234 |        | 921 | 940  | 958  | 977  | 996  | *014 | *033 | *051 | *070 | .4    | 7.4  | 7.2  |
| 235 | 37 107 | 125 | 143  | 162  | 180  | 199  | 217  | 236  | 254  | 273  | .5    | 9.2  | 9.0  |
| 236 |        | 291 | 309  | 328  | 346  | 364  | 383  | 401  | 420  | 438  | .6    | 11.1 | 10.8 |
| 237 |        | 475 | 493  | 511  | 530  | 548  | 566  | 584  | 603  | 621  | .7    | 12.9 | 12.6 |
| 238 |        | 657 | 676  | 694  | 712  | 730  | 749  | 767  | 785  | 803  | .8    | 14.8 | 14.4 |
| 239 |        | 840 | 858  | 876  | 894  | 912  | 930  | 948  | 967  | 985  | .9    | 16.6 | 16.2 |
| 240 | 38 021 | 039 | 057  | 075  | 093  | 111  | 129  | 147  | 165  | 183  | 17 17 |      |      |
| 241 |        | 201 | 219  | 237  | 255  | 273  | 291  | 309  | 327  | 345  | .1    | 1.7  | 1.7  |
| 242 |        | 381 | 399  | 417  | 435  | 453  | 471  | 489  | 507  | 525  | .2    | 3.5  | 3.4  |
| 243 |        | 560 | 578  | 596  | 614  | 632  | 650  | 667  | 685  | 703  | .3    | 5.2  | 5.1  |
| 244 |        | 739 | 757  | 774  | 792  | 810  | 828  | 845  | 863  | 881  | .4    | 7.0  | 6.8  |
| 245 |        | 916 | 934  | 952  | 970  | 987  | *005 | *023 | *040 | *058 | .5    | 8.7  | 8.5  |
| 246 | 39 093 | 111 | 129  | 146  | 164  | 181  | 199  | 217  | 234  | 252  | .6    | 10.5 | 10.2 |
| 247 |        | 269 | 287  | 305  | 322  | 340  | 357  | 375  | 392  | 410  | .7    | 12.2 | 11.9 |
| 248 |        | 445 | 462  | 480  | 497  | 515  | 532  | 550  | 567  | 585  | .8    | 14.0 | 13.6 |
| 249 |        | 620 | 637  | 655  | 672  | 689  | 707  | 724  | 742  | 759  | .9    | 15.7 | 15.3 |
| 250 |        | 794 | 811  | 828  | 846  | 863  | 881  | 898  | 915  | 933  |       |      |      |
|     |        |     |      |      |      |      |      |      |      | 950  |       |      |      |
| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |      |      |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.        |
|-----|--------|------|------|------|------|------|------|------|------|------|--------------|
| 250 | 39 794 | 811  | 828  | 846  | 863  | 881  | 898  | 915  | 933  | 950  |              |
| 251 | 967    | 984  | *002 | *019 | *036 | *054 | *071 | *088 | *105 | *123 |              |
| 252 | 40 140 | 157  | 174  | 191  | 209  | 226  | 243  | 260  | 277  | 295  | 17 17        |
| 253 | 312    | 329  | 346  | 363  | 380  | 398  | 415  | 432  | 449  | 466  | .1 1.7 1.7   |
| 254 | 483    | 500  | 517  | 534  | 551  | 569  | 586  | 603  | 620  | 637  | .2 3.5 3.4   |
| 255 | 654    | 671  | 688  | 705  | 722  | 739  | 756  | 773  | 790  | 807  | .3 5.2 5.1   |
| 256 | 824    | 841  | 858  | 875  | 892  | 908  | 925  | 942  | 959  | 976  | .4 7.0 6.8   |
| 257 | 993    | *010 | *027 | *044 | *061 | *077 | *094 | *111 | *128 | *145 | .5 8.7 8.5   |
| 258 | 41 162 | 179  | 195  | 212  | 229  | 246  | 263  | 279  | 296  | 312  | .6 10.5 10.2 |
| 259 | 330    | 346  | 363  | 380  | 397  | 413  | 430  | 447  | 464  | 480  | .7 12.2 11.9 |
| 260 | 497    | 514  | 530  | 547  | 564  | 581  | 597  | 614  | 631  | 647  | .8 14.0 13.6 |
| 261 | 664    | 680  | 697  | 714  | 730  | 747  | 764  | 780  | 797  | 813  | .9 15.7 15.3 |
| 262 | 830    | 846  | 863  | 880  | 896  | 913  | 929  | 946  | 962  | 979  |              |
| 263 | 995    | *012 | *028 | *045 | *061 | *078 | *094 | *111 | *127 | *144 |              |
| 264 | 42 160 | 177  | 193  | 209  | 225  | 242  | 259  | 275  | 292  | 308  | 16 16        |
| 265 | 324    | 341  | 357  | 373  | 390  | 406  | 423  | 439  | 455  | 472  | .1 1.6 1.6   |
| 266 | 488    | 504  | 521  | 537  | 553  | 569  | 586  | 602  | 618  | 635  | .2 3.3 3.2   |
| 267 | 651    | 667  | 683  | 700  | 716  | 732  | 748  | 765  | 781  | 797  | .3 4.9 4.8   |
| 268 | 813    | 829  | 846  | 862  | 878  | 894  | 910  | 927  | 943  | 959  | .4 6.6 6.4   |
| 269 | 975    | 991  | *007 | *023 | *040 | *056 | *072 | *088 | *104 | *120 | .5 8.2 8.0   |
| 270 | 43 136 | 152  | 168  | 184  | 200  | 216  | 233  | 249  | 265  | 281  | .6 9.9 9.6   |
| 271 | 297    | 313  | 329  | 345  | 361  | 377  | 393  | 409  | 425  | 441  | .7 11.5 11.2 |
| 272 | 457    | 473  | 489  | 505  | 520  | 536  | 552  | 568  | 584  | 600  | .8 13.2 12.8 |
| 273 | 616    | 632  | 648  | 664  | 680  | 695  | 711  | 727  | 743  | 759  | .9 14.8 14.4 |
| 274 | 775    | 791  | 806  | 822  | 838  | 854  | 870  | 886  | 901  | 917  |              |
| 275 | 933    | 949  | 965  | 980  | 996  | *012 | *028 | *043 | *059 | *075 |              |
| 276 | 44 091 | 106  | 122  | 138  | 154  | 169  | 185  | 201  | 216  | 232  | 15 15        |
| 277 | 248    | 263  | 279  | 295  | 310  | 326  | 342  | 357  | 373  | 389  | .1 1.5 1.5   |
| 278 | 404    | 420  | 435  | 451  | 467  | 482  | 498  | 513  | 529  | 545  | .2 3.1 3.0   |
| 279 | 560    | 576  | 591  | 607  | 622  | 638  | 653  | 669  | 685  | 700  | .3 4.6 4.5   |
| 280 | 716    | 731  | 747  | 762  | 778  | 793  | 809  | 824  | 839  | 855  | .4 6.2 6.0   |
| 281 | 870    | 886  | 901  | 917  | 932  | 948  | 963  | 978  | 994  | *009 | .5 7.7 7.5   |
| 282 | 45 025 | 040  | 055  | 071  | 086  | 102  | 117  | 132  | 148  | 163  | .6 9.3 9.0   |
| 283 | 178    | 194  | 209  | 224  | 240  | 255  | 270  | 286  | 301  | 316  | .7 10.8 10.5 |
| 284 | 332    | 347  | 362  | 377  | 393  | 408  | 423  | 438  | 454  | 469  | .8 12.4 12.0 |
| 285 | 481    | 499  | 515  | 530  | 545  | 560  | 576  | 591  | 606  | 621  | .9 13.9 13.5 |
| 286 | 636    | 652  | 667  | 682  | 697  | 712  | 727  | 743  | 758  | 773  |              |
| 287 | 788    | 803  | 818  | 833  | 848  | 864  | 879  | 894  | 909  | 924  |              |
| 288 | 939    | 954  | 969  | 984  | 999  | *014 | *029 | *044 | *059 | *075 |              |
| 289 | 46 090 | 105  | 120  | 135  | 150  | 165  | 180  | 195  | 210  | 225  |              |
| 290 | 240    | 255  | 269  | 284  | 299  | 314  | 329  | 344  | 359  | 374  | 14 14        |
| 291 | 389    | 404  | 419  | 434  | 449  | 464  | 479  | 493  | 508  | 523  | .1 1.4 1.4   |
| 292 | 538    | 553  | 568  | 583  | 597  | 612  | 627  | 642  | 657  | 672  | .2 2.9 2.8   |
| 293 | 687    | 701  | 716  | 731  | 746  | 761  | 775  | 790  | 805  | 820  | .3 4.3 4.2   |
| 294 | 834    | 849  | 864  | 879  | 894  | 908  | 923  | 938  | 952  | 967  | .4 5.8 5.6   |
| 295 | 982    | 997  | *011 | *026 | *041 | *055 | *070 | *085 | *100 | *114 | .5 7.2 7.0   |
| 296 | 47 129 | 144  | 158  | 173  | 188  | 202  | 217  | 232  | 246  | 261  | .6 8.7 8.4   |
| 297 | 275    | 290  | 305  | 319  | 334  | 348  | 363  | 378  | 392  | 407  | .7 10.1 9.8  |
| 298 | 421    | 436  | 451  | 465  | 480  | 494  | 509  | 523  | 538  | 552  | .8 11.6 11.2 |
| 299 | 567    | 581  | 596  | 610  | 625  | 639  | 654  | 668  | 683  | 697  | .9 13.0 12.6 |
| 300 | 712    | 726  | 741  | 755  | 770  | 784  | 799  | 813  | 828  | 842  |              |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.        |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 300 | 47 712 | 726  | 741  | 755  | 770  | 784  | 799  | 813  | 828  | 842  |       |
| 301 | 856    | 871  | 885  | 900  | 914  | 928  | 943  | 957  | 972  | 986  |       |
| 302 | 48 000 | 015  | 029  | 044  | 058  | 072  | 087  | 101  | 115  | 130  |       |
| 303 | 144    | 158  | 173  | 187  | 201  | 216  | 230  | 244  | 259  | 273  |       |
| 304 | 287    | 301  | 316  | 330  | 344  | 358  | 373  | 387  | 401  | 415  |       |
| 305 | 430    | 444  | 458  | 472  | 487  | 501  | 515  | 529  | 543  | 558  |       |
| 306 | 572    | 586  | 600  | 614  | 629  | 643  | 657  | 671  | 685  | 699  |       |
| 307 | 714    | 728  | 742  | 756  | 770  | 784  | 798  | 812  | 827  | 841  |       |
| 308 | 855    | 869  | 883  | 897  | 911  | 925  | 939  | 953  | 967  | 982  |       |
| 309 | 996    | *010 | *024 | *038 | *052 | *066 | *080 | *094 | *108 | *122 |       |
| 310 | 49 136 | 150  | 164  | 178  | 192  | 206  | 220  | 234  | 248  | 262  |       |
| 311 | 276    | 290  | 304  | 318  | 332  | 346  | 359  | 373  | 387  | 401  |       |
| 312 | 415    | 429  | 443  | 457  | 471  | 485  | 499  | 513  | 526  | 540  |       |
| 313 | 554    | 568  | 582  | 596  | 610  | 624  | 637  | 651  | 665  | 679  |       |
| 314 | 693    | 707  | 720  | 734  | 748  | 762  | 776  | 789  | 803  | 817  |       |
| 315 | 831    | 845  | 858  | 872  | 886  | 900  | 913  | 927  | 941  | 955  |       |
| 316 | 968    | 982  | 996  | *010 | *023 | *037 | *051 | *065 | *078 | *092 |       |
| 317 | 50 106 | 119  | 133  | 147  | 160  | 174  | 188  | 201  | 215  | 229  |       |
| 318 | 242    | 256  | 270  | 283  | 297  | 311  | 324  | 338  | 352  | 365  |       |
| 319 | 379    | 392  | 406  | 420  | 433  | 447  | 460  | 474  | 488  | 501  |       |
| 320 | 515    | 528  | 542  | 555  | 569  | 583  | 596  | 610  | 623  | 637  |       |
| 321 | 650    | 664  | 677  | 691  | 704  | 718  | 731  | 745  | 758  | 772  |       |
| 322 | 785    | 799  | 812  | 826  | 839  | 853  | 866  | 880  | 893  | 907  |       |
| 323 | 920    | 933  | 947  | 960  | 974  | 987  | *001 | *014 | *027 | *041 |       |
| 324 | 51 054 | 068  | 081  | 094  | 108  | 121  | 135  | 148  | 161  | 175  |       |
| 325 | 188    | 201  | 215  | 228  | 242  | 255  | 268  | 282  | 295  | 309  |       |
| 326 | 322    | 335  | 348  | 361  | 375  | 388  | 401  | 415  | 428  | 441  |       |
| 327 | 455    | 468  | 481  | 494  | 508  | 521  | 534  | 547  | 561  | 574  |       |
| 328 | 587    | 600  | 614  | 627  | 640  | 653  | 667  | 680  | 693  | 706  |       |
| 329 | 719    | 733  | 746  | 759  | 772  | 785  | 798  | 812  | 825  | 838  |       |
| 330 | 851    | 864  | 877  | 891  | 904  | 917  | 930  | 943  | 956  | 969  |       |
| 331 | 983    | 996  | *009 | *022 | *035 | *048 | *061 | *074 | *087 | *100 |       |
| 332 | 52 114 | 127  | 140  | 153  | 166  | 179  | 192  | 205  | 218  | 231  |       |
| 333 | 244    | 257  | 270  | 283  | 296  | 309  | 322  | 335  | 348  | 361  |       |
| 334 | 374    | 387  | 400  | 413  | 426  | 439  | 452  | 465  | 478  | 491  |       |
| 335 | 504    | 517  | 530  | 543  | 556  | 569  | 582  | 595  | 608  | 621  |       |
| 336 | 634    | 647  | 660  | 672  | 685  | 698  | 711  | 724  | 737  | 750  |       |
| 337 | 763    | 776  | 789  | 801  | 814  | 827  | 840  | 853  | 866  | 879  |       |
| 338 | 891    | 904  | 917  | 930  | 943  | 956  | 968  | 981  | 994  | *007 |       |
| 339 | 53 020 | 033  | 045  | 058  | 071  | 084  | 097  | 109  | 122  | 135  |       |
| 340 | 148    | 160  | 173  | 186  | 199  | 211  | 224  | 237  | 250  | 262  |       |
| 341 | 275    | 288  | 301  | 313  | 326  | 339  | 352  | 364  | 377  | 390  |       |
| 342 | 402    | 415  | 428  | 440  | 453  | 466  | 478  | 491  | 504  | 516  |       |
| 343 | 529    | 542  | 554  | 567  | 580  | 592  | 605  | 618  | 630  | 643  |       |
| 344 | 656    | 668  | 681  | 693  | 706  | 719  | 731  | 744  | 756  | 769  |       |
| 345 | 782    | 794  | 807  | 819  | 832  | 845  | 857  | 870  | 882  | 895  |       |
| 346 | 907    | 920  | 932  | 945  | 958  | 970  | 983  | 995  | *008 | *020 |       |
| 347 | 54 033 | 045  | 058  | 070  | 083  | 095  | 108  | 120  | 133  | 145  |       |
| 348 | 158    | 170  | 183  | 195  | 208  | 220  | 232  | 245  | 257  | 270  |       |
| 349 | 282    | 295  | 307  | 320  | 332  | 344  | 357  | 369  | 382  | 394  |       |
| 350 | 407    | 419  | 431  | 444  | 456  | 469  | 481  | 493  | 506  | 518  |       |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.           |
|-----|--------|------|------|------|------|------|------|------|------|------|-----------------|
| 350 | 54 407 | 419  | 431  | 444  | 456  | 469  | 481  | 493  | 506  | 518  |                 |
| 351 | 530    | 543  | 555  | 568  | 580  | 592  | 605  | 617  | 629  | 642  | $\overline{12}$ |
| 352 | 654    | 666  | 679  | 691  | 703  | 716  | 728  | 740  | 753  | 765  | .1 1.2          |
| 353 | 777    | 790  | 802  | 814  | 826  | 839  | 851  | 863  | 876  | 888  | .2 2.5          |
| 354 | 900    | 912  | 925  | 937  | 949  | 961  | 974  | 986  | 998  | *010 | .3 3.7          |
| 355 | 55 023 | 035  | 047  | 059  | 071  | 084  | 096  | 108  | 120  | 133  | .4 5.0          |
| 356 | 145    | 157  | 169  | 181  | 194  | 206  | 218  | 230  | 242  | 254  | .5 6.2          |
| 357 | 267    | 279  | 291  | 303  | 315  | 327  | 340  | 352  | 364  | 376  | .6 7.5          |
| 358 | 388    | 400  | 412  | 424  | 437  | 449  | 461  | 473  | 485  | 497  | .7 8.7          |
| 359 | 509    | 521  | 533  | 545  | 558  | 570  | 582  | 594  | 606  | 618  | .8 10.0         |
| 360 | 630    | 642  | 654  | 666  | 678  | 690  | 702  | 714  | 726  | 738  | .9 11.2         |
| 361 | 750    | 762  | 775  | 787  | 799  | 811  | 823  | 835  | 847  | 859  | $\overline{12}$ |
| 362 | 871    | 883  | 895  | 907  | 919  | 931  | 943  | 955  | 966  | 978  | .1 1.2          |
| 363 | 990    | *002 | *014 | *026 | *038 | *050 | *062 | *074 | *086 | *098 | .2 2.4          |
| 364 | 56 110 | 122  | 134  | 146  | 158  | 170  | 181  | 193  | 205  | 217  | .3 3.6          |
| 365 | 229    | 241  | 253  | 265  | 277  | 288  | 300  | 312  | 324  | 336  | .4 4.8          |
| 366 | 348    | 360  | 372  | 383  | 395  | 407  | 419  | 431  | 443  | 455  | .5 6.0          |
| 367 | 466    | 478  | 490  | 502  | 514  | 525  | 537  | 549  | 561  | 573  | .6 7.2          |
| 368 | 585    | 596  | 608  | 620  | 632  | 643  | 655  | 667  | 679  | 691  | .7 8.4          |
| 369 | 702    | 714  | 726  | 738  | 749  | 761  | 773  | 785  | 796  | 808  | .8 9.6          |
| 370 | 820    | 832  | 843  | 855  | 867  | 879  | 890  | 902  | 914  | 925  | .9 10.8         |
| 371 | 937    | 949  | 961  | 972  | 984  | 996  | *007 | *019 | *031 | *042 | $\overline{11}$ |
| 372 | 57 054 | 066  | 077  | 089  | 101  | 112  | 124  | 136  | 147  | 159  | .1 1.1          |
| 373 | 171    | 182  | 194  | 206  | 217  | 229  | 240  | 252  | 264  | 275  | .2 2.3          |
| 374 | 287    | 299  | 310  | 322  | 333  | 345  | 357  | 368  | 380  | 391  | .3 3.4          |
| 375 | 403    | 414  | 426  | 438  | 449  | 461  | 472  | 484  | 495  | 507  | .4 4.6          |
| 376 | 519    | 530  | 542  | 553  | 565  | 576  | 588  | 599  | 611  | 622  | .5 5.7          |
| 377 | 634    | 645  | 657  | 668  | 680  | 691  | 703  | 714  | 726  | 737  | .6 6.9          |
| 378 | 749    | 760  | 772  | 783  | 795  | 806  | 818  | 829  | 841  | 852  | .7 8.0          |
| 379 | 864    | 875  | 887  | 898  | 909  | 921  | 932  | 944  | 955  | 967  | .8 9.2          |
| 380 | 978    | 990  | *001 | *012 | *024 | *035 | *047 | *058 | *069 | *081 | .9 10.3         |
| 381 | 58 092 | 104  | 115  | 126  | 138  | 149  | 161  | 172  | 183  | 195  | $\overline{11}$ |
| 382 | 206    | 217  | 229  | 240  | 252  | 263  | 274  | 286  | 297  | 308  | .1 1.1          |
| 383 | 320    | 331  | 342  | 354  | 365  | 376  | 388  | 399  | 410  | 422  | .2 2.2          |
| 384 | 433    | 444  | 455  | 467  | 478  | 489  | 501  | 512  | 523  | 535  | .3 3.3          |
| 385 | 546    | 557  | 568  | 580  | 591  | 602  | 613  | 625  | 636  | 647  | .4 4.4          |
| 386 | 658    | 670  | 681  | 692  | 703  | 715  | 726  | 737  | 748  | 760  | .5 5.5          |
| 387 | 771    | 782  | 793  | 804  | 816  | 827  | 838  | 849  | 861  | 872  | .6 6.6          |
| 388 | 883    | 894  | 905  | 916  | 928  | 939  | 950  | 961  | 972  | 984  | .7 7.7          |
| 389 | 995    | *006 | *017 | *028 | *039 | *050 | *062 | *073 | *084 | *095 | .8 8.8          |
| 390 | 59 106 | 117  | 128  | 140  | 151  | 162  | 173  | 184  | 195  | 206  | .9 9.9          |
| 391 | 217    | 229  | 240  | 251  | 262  | 273  | 284  | 295  | 306  | 317  | $\overline{10}$ |
| 392 | 328    | 339  | 351  | 362  | 373  | 384  | 395  | 406  | 417  | 428  | .1 1.0          |
| 393 | 439    | 450  | 461  | 472  | 483  | 494  | 505  | 516  | 527  | 538  | .2 2.1          |
| 394 | 549    | 560  | 571  | 582  | 593  | 604  | 615  | 626  | 637  | 648  | .3 3.1          |
| 395 | 659    | 670  | 681  | 692  | 703  | 714  | 725  | 736  | 747  | 758  | .4 4.2          |
| 396 | 769    | 780  | 791  | 802  | 813  | 824  | 835  | 846  | 857  | 868  | .5 5.2          |
| 397 | 879    | 890  | 901  | 912  | 923  | 933  | 944  | 955  | 966  | 977  | .6 6.3          |
| 398 | 988    | 999  | *010 | *021 | *032 | *043 | *053 | *064 | *075 | *086 | .7 7.3          |
| 399 | 50 097 | 108  | 119  | 130  | 141  | 151  | 162  | 173  | 184  | 195  | .8 8.4          |
| 400 | 206    | 217  | 227  | 238  | 249  | 260  | 271  | 282  | 293  | 303  | .9 9.4          |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.           |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|-----|-----|------|------|------|------|------|------|--------|
| 400 | 60 206 | 217 | 227 | 238 | 249  | 260  | 271  | 282  | 293  | 303  |        |
| 401 | 314    | 325 | 336 | 347 | 357  | 368  | 379  | 390  | 401  | 412  |        |
| 402 | 422    | 433 | 444 | 455 | 466  | 476  | 487  | 498  | 509  | 519  |        |
| 403 | 530    | 541 | 552 | 563 | 573  | 584  | 595  | 606  | 616  | 627  |        |
| 404 | 638    | 649 | 659 | 670 | 681  | 692  | 702  | 713  | 724  | 735  | .1 1.1 |
| 405 | 745    | 756 | 767 | 777 | 788  | 799  | 810  | 820  | 831  | 842  | .2 2.2 |
| 406 | 852    | 863 | 874 | 884 | 895  | 906  | 916  | 927  | 938  | 949  | .3 3.3 |
| 407 | 959    | 970 | 981 | 991 | *002 | *013 | *023 | *034 | *044 | *055 | .4 4.4 |
| 408 | 61 066 | 076 | 087 | 098 | 108  | 119  | 130  | 140  | 151  | 161  | .5 5.5 |
| 409 | 172    | 183 | 193 | 204 | 215  | 225  | 236  | 246  | 257  | 268  | .6 6.6 |
| 410 | 278    | 289 | 299 | 310 | 320  | 331  | 342  | 352  | 363  | 373  | .7 7.7 |
| 411 | 384    | 394 | 405 | 416 | 426  | 437  | 447  | 458  | 468  | 479  | .8 8.8 |
| 412 | 489    | 500 | 511 | 521 | 532  | 542  | 553  | 563  | 574  | 584  | .9 9.9 |
| 413 | 595    | 605 | 616 | 626 | 637  | 647  | 658  | 668  | 679  | 689  |        |
| 414 | 700    | 710 | 721 | 731 | 742  | 752  | 763  | 773  | 784  | 794  |        |
| 415 | 805    | 815 | 825 | 836 | 846  | 857  | 867  | 878  | 888  | 899  |        |
| 416 | 909    | 920 | 930 | 940 | 951  | 961  | 972  | 982  | 993  | *003 | .1 1.0 |
| 417 | 62 013 | 024 | 034 | 045 | 055  | 065  | 076  | 086  | 097  | 107  | .2 2.1 |
| 418 | 117    | 128 | 138 | 149 | 159  | 169  | 180  | 190  | 200  | 211  | .3 3.1 |
| 419 | 221    | 232 | 242 | 252 | 263  | 273  | 283  | 294  | 304  | 314  | .4 4.2 |
| 420 | 325    | 335 | 345 | 356 | 366  | 376  | 387  | 397  | 407  | 418  | .5 5.2 |
| 421 | 428    | 438 | 449 | 459 | 469  | 480  | 490  | 500  | 510  | 521  | .6 6.3 |
| 422 | 531    | 541 | 552 | 562 | 572  | 582  | 593  | 603  | 613  | 624  | .7 7.3 |
| 423 | 634    | 644 | 654 | 665 | 675  | 685  | 695  | 706  | 716  | 726  | .8 8.4 |
| 424 | 736    | 747 | 757 | 767 | 777  | 788  | 798  | 808  | 818  | 828  | .9 9.4 |
| 425 | 839    | 849 | 859 | 869 | 879  | 890  | 900  | 910  | 920  | 931  |        |
| 426 | 941    | 951 | 961 | 971 | 981  | 992  | *002 | *012 | *022 | *032 |        |
| 427 | 63 043 | 053 | 063 | 073 | 083  | 093  | 104  | 114  | 124  | 134  | .1 1.0 |
| 428 | 144    | 154 | 164 | 175 | 185  | 195  | 205  | 215  | 225  | 235  | .2 2.0 |
| 429 | 245    | 256 | 266 | 276 | 286  | 296  | 306  | 316  | 326  | 336  | .3 3.0 |
| 430 | 347    | 357 | 367 | 377 | 387  | 397  | 407  | 417  | 427  | 437  | .4 4.0 |
| 431 | 447    | 458 | 468 | 478 | 488  | 498  | 508  | 518  | 528  | 538  | .5 5.0 |
| 432 | 548    | 558 | 568 | 578 | 588  | 598  | 608  | 618  | 628  | 639  | .6 6.0 |
| 433 | 649    | 659 | 669 | 679 | 689  | 699  | 709  | 719  | 729  | 739  | .7 7.0 |
| 434 | 749    | 759 | 769 | 779 | 789  | 799  | 809  | 819  | 829  | 839  | .8 8.0 |
| 435 | 849    | 859 | 869 | 879 | 889  | 899  | 909  | 919  | 928  | 938  | .9 9.0 |
| 436 | 948    | 958 | 968 | 978 | 988  | 998  | *008 | *018 | *028 | *038 |        |
| 437 | 64 048 | 058 | 068 | 078 | 088  | 098  | 107  | 117  | 127  | 137  |        |
| 438 | 147    | 157 | 167 | 177 | 187  | 197  | 207  | 217  | 226  | 236  |        |
| 439 | 246    | 256 | 266 | 276 | 286  | 296  | 306  | 315  | 325  | 335  | .1 9   |
| 440 | 345    | 355 | 365 | 375 | 384  | 394  | 404  | 414  | 424  | 434  | .2 0.9 |
| 441 | 444    | 453 | 463 | 473 | 483  | 493  | 503  | 512  | 522  | 532  | .3 1.9 |
| 442 | 542    | 552 | 562 | 571 | 581  | 591  | 601  | 611  | 621  | 630  | .4 2.8 |
| 443 | 640    | 650 | 660 | 670 | 679  | 689  | 699  | 709  | 718  | 728  | .5 3.8 |
| 444 | 738    | 748 | 758 | 767 | 777  | 787  | 797  | 806  | 816  | 826  | .6 4.7 |
| 445 | 836    | 846 | 855 | 865 | 875  | 885  | 894  | 904  | 914  | 923  | .7 5.7 |
| 446 | 933    | 943 | 953 | 962 | 972  | 982  | 992  | *001 | *011 | *021 | .8 6.6 |
| 447 | 65 031 | 040 | 050 | 060 | 069  | 079  | 089  | 098  | 108  | 118  | .9 7.6 |
| 448 | 128    | 137 | 147 | 157 | 166  | 176  | 186  | 195  | 205  | 215  | .9 8.5 |
| 449 | 224    | 234 | 244 | 253 | 263  | 273  | 282  | 292  | 302  | 311  |        |
| 450 | 321    | 331 | 340 | 350 | 360  | 369  | 379  | 389  | 398  | 408  |        |
| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|------|------|------|------|------|------|------|------|------|--------|
| 450 | 65 321 | 331  | 340  | 350  | 360  | 369  | 379  | 389  | 398  | 406  |        |
| 451 | 417    | 427  | 437  | 446  | 456  | 466  | 475  | 485  | 494  | 504  |        |
| 452 | 514    | 523  | 533  | 542  | 552  | 562  | 571  | 581  | 590  | 600  | 10     |
| 453 | 610    | 619  | 629  | 638  | 648  | 657  | 667  | 677  | 686  | 696  | 1.0    |
| 454 | 705    | 715  | 724  | 734  | 744  | 753  | 763  | 772  | 782  | 791  | 2.0    |
| 455 | 801    | 810  | 820  | 830  | 839  | 849  | 858  | 868  | 877  | 887  | 3.0    |
| 456 | 896    | 906  | 915  | 925  | 934  | 944  | 953  | 963  | 972  | 982  | 4.0    |
| 457 | 991    | *001 | *010 | *020 | *029 | *038 | *048 | *058 | *067 | *077 | 5.0    |
| 458 | 66 086 | 096  | 105  | 115  | 124  | 134  | 143  | 153  | 162  | 172  | 6.0    |
| 459 | 181    | 190  | 200  | 209  | 219  | 228  | 238  | 247  | 257  | 266  | 7.0    |
| 460 | 276    | 285  | 294  | 304  | 313  | 323  | 332  | 342  | 351  | 360  | 8.0    |
| 461 | 370    | 379  | 389  | 398  | 408  | 417  | 426  | 436  | 445  | 455  | 9.0    |
| 462 | 464    | 473  | 483  | 492  | 502  | 511  | 520  | 530  | 539  | 548  |        |
| 463 | 558    | 567  | 577  | 586  | 595  | 605  | 614  | 623  | 633  | 642  |        |
| 464 | 652    | 661  | 670  | 680  | 689  | 698  | 708  | 717  | 726  | 736  |        |
| 465 | 745    | 754  | 764  | 773  | 782  | 792  | 801  | 810  | 820  | 829  |        |
| 466 | 838    | 848  | 857  | 866  | 876  | 885  | 894  | 904  | 913  | 922  |        |
| 467 | 931    | 941  | 950  | 959  | 969  | 978  | 987  | 996  | *006 | *015 | .1 0.9 |
| 468 | 67 024 | 034  | 043  | 052  | 061  | 071  | 080  | 089  | 099  | 108  | .2 1.9 |
| 469 | 117    | 126  | 136  | 145  | 154  | 163  | 173  | 182  | 191  | 200  | .3 2.8 |
| 470 | 210    | 219  | 228  | 237  | 246  | 256  | 265  | 274  | 283  | 293  | .4 3.8 |
| 471 | 302    | 311  | 320  | 329  | 339  | 348  | 357  | 366  | 376  | 385  | .5 4.7 |
| 472 | 394    | 403  | 412  | 421  | 431  | 440  | 449  | 458  | 467  | 477  | .6 5.7 |
| 473 | 486    | 495  | 504  | 513  | 523  | 532  | 541  | 550  | 559  | 568  | .7 6.6 |
| 474 | 578    | 587  | 596  | 605  | 614  | 623  | 633  | 642  | 651  | 660  | .8 7.6 |
| 475 | 669    | 678  | 687  | 697  | 706  | 715  | 724  | 733  | 742  | 751  | .9 8.5 |
| 476 | 760    | 770  | 779  | 788  | 797  | 806  | 815  | 824  | 833  | 842  |        |
| 477 | 852    | 861  | 870  | 879  | 888  | 897  | 906  | 915  | 924  | 933  |        |
| 478 | 943    | 952  | 961  | 970  | 979  | 988  | 997  | *006 | *015 | *024 |        |
| 479 | 68 033 | 042  | 051  | 060  | 070  | 079  | 088  | 097  | 106  | 115  | .1 0.9 |
| 480 | 124    | 133  | 142  | 151  | 160  | 169  | 178  | 187  | 196  | 205  | .2 1.8 |
| 481 | 214    | 223  | 232  | 241  | 250  | 259  | 268  | 277  | 286  | 295  | .3 2.7 |
| 482 | 304    | 313  | 322  | 331  | 340  | 349  | 358  | 367  | 376  | 385  | .4 3.6 |
| 483 | 394    | 403  | 412  | 421  | 430  | 439  | 448  | 457  | 466  | 475  | .5 4.5 |
| 484 | 484    | 493  | 502  | 511  | 520  | 529  | 538  | 547  | 556  | 565  | .6 5.4 |
| 485 | 574    | 583  | 592  | 601  | 610  | 619  | 628  | 637  | 646  | 654  | .7 6.3 |
| 486 | 663    | 672  | 681  | 690  | 699  | 708  | 717  | 726  | 735  | 744  | .8 7.2 |
| 487 | 753    | 762  | 770  | 779  | 788  | 797  | 806  | 815  | 824  | 833  | .9 8.1 |
| 488 | 842    | 851  | 860  | 868  | 877  | 886  | 895  | 904  | 913  | 922  |        |
| 489 | 931    | 940  | 948  | 957  | 966  | 975  | 984  | 993  | *002 | *010 |        |
| 490 | 69 019 | 028  | 037  | 046  | 055  | 064  | 073  | 081  | 090  | 099  | .1 0.8 |
| 491 | 108    | 117  | 126  | 134  | 143  | 152  | 161  | 170  | 179  | 187  | .2 1.7 |
| 492 | 196    | 205  | 214  | 223  | 232  | 240  | 249  | 258  | 267  | 276  | .3 2.5 |
| 493 | 284    | 293  | 302  | 311  | 320  | 328  | 337  | 346  | 355  | 364  | .4 3.4 |
| 494 | 372    | 381  | 390  | 399  | 408  | 416  | 425  | 434  | 443  | 451  | .5 4.2 |
| 495 | 460    | 469  | 478  | 487  | 495  | 504  | 513  | 522  | 530  | 529  | .6 5.1 |
| 496 | 548    | 557  | 565  | 574  | 583  | 592  | 600  | 609  | 618  | 627  | .7 5.9 |
| 497 | 635    | 644  | 653  | 662  | 670  | 679  | 688  | 697  | 705  | 714  | .8 6.8 |
| 498 | 723    | 731  | 740  | 749  | 758  | 767  | 775  | 784  | 792  | 801  | .9 7.6 |
| 499 | 810    | 819  | 827  | 836  | 845  | 853  | 862  | 871  | 879  | 888  |        |
| 500 | 897    | 905  | 914  | 923  | 931  | 940  | 949  | 958  | 966  | 975  |        |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|------|------|------|------|------|------|------|------|------|--------|
| 500 | 69 897 | 905  | 914  | 923  | 931  | 940  | 949  | 958  | 966  | 975  |        |
| 501 | 984    | 992  | *001 | *010 | *018 | *027 | *036 | *044 | *053 | *061 | 9      |
| 502 | 70 070 | 079  | 087  | 095  | 105  | 113  | 122  | 131  | 139  | 148  | .1 0.9 |
| 503 | 157    | 165  | 174  | 182  | 191  | 200  | 208  | 217  | 226  | 234  | .2 1.8 |
| 504 | 243    | 251  | 260  | 269  | 277  | 286  | 294  | 303  | 312  | 320  | .3 2.7 |
| 505 | 329    | 337  | 346  | 355  | 363  | 372  | 380  | 389  | 398  | 406  | .4 3.6 |
| 506 | 415    | 423  | 432  | 441  | 449  | 458  | 466  | 475  | 483  | 491  | .5 4.5 |
| 507 | 501    | 509  | 518  | 526  | 535  | 542  | 552  | 560  | 569  | 578  | .6 5.4 |
| 508 | 586    | 595  | 603  | 612  | 620  | 629  | 637  | 646  | 654  | 663  | .7 6.3 |
| 509 | 672    | 680  | 689  | 697  | 706  | 714  | 723  | 731  | 740  | 748  | .8 7.2 |
| 510 | 757    | 765  | 774  | 782  | 791  | 799  | 808  | 816  | 825  | 833  | .9 8.1 |
| 511 | 842    | 850  | 859  | 867  | 876  | 884  | 893  | 901  | 910  | 918  |        |
| 512 | 927    | 935  | 944  | 952  | 961  | 969  | 978  | 986  | 995  | *003 | 8      |
| 513 | 71 011 | 020  | 028  | 037  | 045  | 054  | 062  | 071  | 079  | 088  | .1 0.8 |
| 514 | 096    | 105  | 113  | 121  | 130  | 138  | 147  | 155  | 164  | 172  | .2 1.7 |
| 515 | 180    | 189  | 197  | 206  | 214  | 223  | 231  | 239  | 248  | 256  | .3 2.5 |
| 516 | 265    | 273  | 282  | 290  | 298  | 307  | 315  | 324  | 332  | 340  | .4 3.4 |
| 517 | 349    | 357  | 366  | 374  | 382  | 391  | 399  | 408  | 416  | 424  | .5 4.2 |
| 518 | 433    | 441  | 449  | 458  | 466  | 475  | 483  | 491  | 500  | 508  | .6 5.1 |
| 519 | 516    | 525  | 533  | 542  | 550  | 558  | 567  | 575  | 583  | 592  | .7 5.9 |
| 520 | 600    | 608  | 617  | 625  | 633  | 642  | 650  | 659  | 667  | 675  | .8 6.8 |
| 521 | 684    | 692  | 700  | 709  | 717  | 725  | 734  | 742  | 750  | 758  | .9 7.6 |
| 522 | 767    | 775  | 783  | 792  | 800  | 808  | 817  | 825  | 833  | 841  |        |
| 523 | 850    | 858  | 867  | 875  | 883  | 891  | 900  | 908  | 916  | 925  | 8      |
| 524 | 933    | 941  | 949  | 958  | 966  | 974  | 983  | 991  | 999  | *007 | .1 0.8 |
| 525 | 72 016 | 024  | 032  | 040  | 049  | 057  | 065  | 074  | 082  | 090  | .2 1.6 |
| 526 | 098    | 107  | 115  | 123  | 131  | 140  | 148  | 156  | 164  | 173  | .3 2.4 |
| 527 | 181    | 189  | 197  | 206  | 214  | 222  | 230  | 238  | 247  | 255  | .4 3.2 |
| 528 | 263    | 271  | 280  | 288  | 296  | 304  | 312  | 321  | 329  | 337  | .5 4.0 |
| 529 | 345    | 354  | 362  | 370  | 378  | 386  | 395  | 403  | 411  | 419  | .6 4.8 |
| 530 | 427    | 436  | 444  | 452  | 460  | 468  | 476  | 485  | 493  | 501  | .7 5.6 |
| 531 | 509    | 517  | 526  | 534  | 542  | 550  | 558  | 566  | 575  | 583  | .8 6.4 |
| 532 | 591    | 599  | 607  | 615  | 624  | 632  | 640  | 648  | 656  | 664  | .9 7.2 |
| 533 | 672    | 681  | 689  | 697  | 705  | 713  | 721  | 729  | 738  | 746  |        |
| 534 | 754    | 762  | 770  | 778  | 786  | 795  | 803  | 811  | 819  | 827  | 7      |
| 535 | 835    | 843  | 851  | 859  | 868  | 876  | 884  | 892  | 900  | 908  | .1 0.7 |
| 536 | 916    | 924  | 932  | 941  | 949  | 957  | 965  | 973  | 981  | 989  | .2 1.5 |
| 537 | 997    | *005 | *013 | *021 | *030 | *038 | *046 | *054 | *062 | *070 | .3 2.2 |
| 538 | 73 078 | 086  | 094  | 102  | 110  | 118  | 126  | 134  | 143  | 151  | .4 3.0 |
| 539 | 159    | 167  | 175  | 183  | 191  | 199  | 207  | 215  | 223  | 231  | .5 3.7 |
| 540 | 239    | 247  | 255  | 263  | 271  | 279  | 287  | 295  | 303  | 311  | .6 4.5 |
| 541 | 319    | 328  | 336  | 344  | 352  | 360  | 368  | 376  | 384  | 392  | .7 5.2 |
| 542 | 400    | 408  | 416  | 424  | 432  | 440  | 448  | 456  | 464  | 472  | .8 6.0 |
| 543 | 480    | 488  | 496  | 504  | 512  | 520  | 528  | 536  | 544  | 552  | .9 6.7 |
| 544 | 560    | 568  | 576  | 584  | 592  | 600  | 608  | 615  | 623  | 631  |        |
| 545 | 639    | 647  | 655  | 663  | 671  | 679  | 687  | 695  | 703  | 711  | 7      |
| 546 | 719    | 727  | 735  | 743  | 751  | 759  | 767  | 775  | 783  | 791  | .1 0.7 |
| 547 | 798    | 806  | 814  | 822  | 830  | 838  | 846  | 854  | 862  | 870  | .2 1.5 |
| 548 | 878    | 886  | 894  | 902  | 909  | 917  | 925  | 933  | 941  | 949  | .3 2.2 |
| 549 | 957    | 965  | 973  | 981  | 989  | 997  | *004 | *012 | *020 | *028 | .4 3.0 |
| 550 | 74 036 | 044  | 052  | 060  | 068  | 075  | 083  | 091  | 099  | 107  | .5 3.7 |
|     |        |      |      |      |      |      |      |      |      |      | .6 4.5 |
|     |        |      |      |      |      |      |      |      |      |      | .7 5.2 |
|     |        |      |      |      |      |      |      |      |      |      | .8 6.0 |
|     |        |      |      |      |      |      |      |      |      |      | .9 6.7 |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|-----|-----|------|------|------|------|------|------|--------|
| 550 | 74 036 | 044 | 052 | 060 | 068  | 075  | 083  | 091  | 099  | 107  |        |
| 551 | 115    | 123 | 131 | 139 | 146  | 154  | 162  | 170  | 176  | 186  |        |
| 552 | 194    | 202 | 209 | 217 | 225  | 233  | 241  | 249  | 257  | 264  |        |
| 553 | 272    | 280 | 288 | 296 | 304  | 312  | 319  | 327  | 335  | 343  |        |
| 554 | 351    | 359 | 366 | 374 | 382  | 390  | 396  | 406  | 413  | 421  |        |
| 555 | 429    | 437 | 445 | 453 | 460  | 468  | 476  | 484  | 492  | 499  |        |
| 556 | 507    | 515 | 523 | 531 | 538  | 546  | 554  | 562  | 570  | 577  |        |
| 557 | 585    | 593 | 601 | 609 | 616  | 624  | 632  | 640  | 648  | 655  |        |
| 558 | 663    | 671 | 679 | 687 | 694  | 702  | 710  | 718  | 725  | 733  | .1 0.8 |
| 559 | 741    | 749 | 756 | 764 | 772  | 780  | 788  | 795  | 803  | 811  | .2 1.6 |
| 560 | 819    | 826 | 834 | 842 | 850  | 857  | 865  | 873  | 881  | 888  | .3 2.4 |
| 561 | 896    | 904 | 912 | 919 | 927  | 935  | 942  | 950  | 958  | 966  | .4 3.2 |
| 562 | 973    | 981 | 989 | 997 | *004 | *012 | *020 | *027 | *035 | *043 | .5 4.0 |
| 563 | 75 051 | 058 | 066 | 074 | 081  | 089  | 097  | 105  | 112  | 120  | .6 4.8 |
| 564 | 128    | 135 | 143 | 151 | 158  | 166  | 174  | 182  | 189  | 197  | .7 5.6 |
| 565 | 205    | 212 | 220 | 228 | 235  | 243  | 251  | 258  | 266  | 274  | .8 6.4 |
| 566 | 281    | 289 | 297 | 304 | 312  | 320  | 327  | 335  | 343  | 350  | .9 7.2 |
| 567 | 358    | 366 | 373 | 381 | 389  | 396  | 404  | 412  | 419  | 427  |        |
| 568 | 435    | 442 | 450 | 458 | 465  | 473  | 480  | 488  | 496  | 503  |        |
| 569 | 511    | 519 | 526 | 534 | 541  | 549  | 557  | 564  | 572  | 580  |        |
| 570 | 587    | 595 | 602 | 610 | 618  | 625  | 633  | 641  | 648  | 656  |        |
| 571 | 663    | 671 | 679 | 686 | 694  | 701  | 709  | 717  | 724  | 732  | .1 0.7 |
| 572 | 739    | 747 | 755 | 762 | 770  | 777  | 785  | 792  | 800  | 808  | .2 1.5 |
| 573 | 815    | 823 | 830 | 838 | 846  | 853  | 861  | 868  | 876  | 883  | .3 2.2 |
| 574 | 891    | 899 | 906 | 914 | 921  | 928  | 936  | 944  | 951  | 959  | .4 3.0 |
| 575 | 967    | 974 | 982 | 989 | 997  | *004 | *012 | *019 | *027 | *034 | .5 3.7 |
| 576 | 76 042 | 050 | 057 | 065 | 072  | 080  | 087  | 095  | 102  | 110  | .6 4.5 |
| 577 | 117    | 125 | 132 | 140 | 147  | 155  | 162  | 170  | 178  | 185  | .7 5.2 |
| 578 | 193    | 200 | 208 | 215 | 223  | 230  | 238  | 245  | 253  | 260  | .8 6.0 |
| 579 | 268    | 275 | 283 | 290 | 298  | 305  | 313  | 320  | 328  | 335  | .9 6.7 |
| 580 | 343    | 350 | 358 | 365 | 372  | 380  | 387  | 395  | 402  | 410  |        |
| 581 | 417    | 425 | 432 | 440 | 447  | 455  | 462  | 470  | 477  | 485  |        |
| 582 | 492    | 500 | 507 | 514 | 522  | 529  | 537  | 544  | 552  | 559  |        |
| 583 | 567    | 574 | 582 | 589 | 596  | 604  | 611  | 619  | 626  | 634  |        |
| 584 | 641    | 648 | 656 | 663 | 671  | 678  | 686  | 693  | 700  | 708  |        |
| 585 | 715    | 723 | 730 | 738 | 745  | 752  | 760  | 767  | 775  | 782  |        |
| 586 | 790    | 797 | 804 | 812 | 819  | 827  | 834  | 841  | 849  | 856  |        |
| 587 | 864    | 871 | 878 | 886 | 893  | 901  | 908  | 915  | 922  | 930  | .1 0.7 |
| 588 | 937    | 945 | 952 | 960 | 967  | 974  | 982  | 989  | 997  | *004 | .2 1.4 |
| 589 | 77 011 | 019 | 026 | 033 | 041  | 048  | 055  | 063  | 070  | 078  | .3 2.1 |
| 590 | 085    | 092 | 100 | 107 | 114  | 122  | 129  | 136  | 144  | 151  | .4 2.8 |
| 591 | 158    | 166 | 173 | 181 | 188  | 195  | 203  | 210  | 217  | 225  | .5 3.5 |
| 592 | 232    | 239 | 247 | 254 | 261  | 269  | 276  | 283  | 291  | 298  | .6 4.2 |
| 593 | 305    | 313 | 320 | 327 | 335  | 342  | 349  | 356  | 364  | 371  | .7 4.9 |
| 594 | 378    | 386 | 393 | 400 | 408  | 415  | 422  | 430  | 437  | 444  | .8 5.6 |
| 595 | 451    | 459 | 466 | 473 | 481  | 488  | 495  | 503  | 510  | 517  | .9 6.3 |
| 596 | 524    | 532 | 539 | 546 | 554  | 561  | 568  | 575  | 583  | 590  |        |
| 597 | 597    | 604 | 612 | 619 | 626  | 634  | 641  | 648  | 655  | 663  |        |
| 598 | 670    | 677 | 684 | 692 | 699  | 706  | 713  | 721  | 728  | 735  |        |
| 599 | 742    | 750 | 757 | 764 | 771  | 779  | 786  | 793  | 800  | 808  |        |
| 600 | 815    | 822 | 829 | 837 | 844  | 851  | 858  | 866  | 873  | 880  |        |
| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6    | 7    | 8    | 9    | P. P. |
|-----|--------|-----|-----|-----|-----|-----|------|------|------|------|-------|
| 600 | 77 815 | 822 | 829 | 837 | 844 | 851 | 858  | 866  | 873  | 880  |       |
| 601 | 887    | 894 | 902 | 909 | 916 | 923 | 931  | 938  | 945  | 952  |       |
| 602 | 959    | 967 | 974 | 981 | 988 | 995 | *003 | *010 | *017 | *024 |       |
| 603 | 78 031 | 039 | 046 | 053 | 060 | 067 | 075  | 082  | 089  | 096  |       |
| 604 | 103    | 111 | 118 | 125 | 132 | 139 | 147  | 154  | 161  | 168  |       |
| 605 | 175    | 182 | 190 | 197 | 204 | 211 | 218  | 226  | 233  | 240  |       |
| 606 | 247    | 254 | 261 | 269 | 276 | 283 | 290  | 297  | 304  | 311  |       |
| 607 | 319    | 326 | 333 | 340 | 347 | 354 | 362  | 369  | 376  | 383  |       |
| 608 | 390    | 397 | 404 | 412 | 419 | 426 | 433  | 440  | 447  | 454  |       |
| 609 | 461    | 469 | 476 | 483 | 490 | 497 | 504  | 511  | 518  | 526  |       |
| 610 | 533    | 540 | 547 | 554 | 561 | 568 | 575  | 583  | 590  | 597  |       |
| 611 | 604    | 611 | 618 | 625 | 632 | 639 | 646  | 654  | 661  | 668  |       |
| 612 | 675    | 682 | 689 | 696 | 703 | 710 | 717  | 725  | 732  | 739  |       |
| 613 | 746    | 753 | 760 | 767 | 774 | 781 | 788  | 795  | 802  | 810  |       |
| 614 | 817    | 824 | 831 | 838 | 845 | 852 | 859  | 866  | 873  | 880  |       |
| 615 | 887    | 894 | 901 | 908 | 915 | 923 | 930  | 937  | 944  | 951  |       |
| 616 | 958    | 965 | 972 | 979 | 986 | 993 | *000 | *007 | *014 | *021 |       |
| 617 | 79 028 | 035 | 042 | 049 | 056 | 063 | 070  | 078  | 085  | 092  |       |
| 618 | 099    | 106 | 113 | 120 | 127 | 134 | 141  | 148  | 155  | 162  |       |
| 619 | 169    | 176 | 183 | 190 | 197 | 204 | 211  | 218  | 225  | 232  |       |
| 620 | 239    | 246 | 253 | 260 | 267 | 274 | 281  | 288  | 295  | 302  |       |
| 621 | 309    | 316 | 323 | 330 | 337 | 344 | 351  | 358  | 365  | 372  |       |
| 622 | 379    | 386 | 393 | 400 | 407 | 414 | 421  | 428  | 435  | 442  |       |
| 623 | 449    | 456 | 462 | 469 | 476 | 483 | 490  | 497  | 504  | 511  |       |
| 624 | 518    | 525 | 532 | 539 | 546 | 553 | 560  | 567  | 574  | 581  |       |
| 625 | 588    | 595 | 602 | 609 | 616 | 622 | 629  | 635  | 643  | 650  |       |
| 626 | 657    | 664 | 671 | 678 | 685 | 692 | 699  | 706  | 713  | 720  |       |
| 627 | 727    | 733 | 740 | 747 | 754 | 761 | 768  | 775  | 782  | 789  |       |
| 628 | 796    | 803 | 810 | 816 | 823 | 830 | 837  | 844  | 851  | 858  |       |
| 629 | 865    | 872 | 879 | 886 | 892 | 899 | 906  | 913  | 920  | 927  |       |
| 630 | 934    | 941 | 948 | 954 | 961 | 968 | 975  | 982  | 989  | 996  |       |
| 631 | 80 003 | 010 | 016 | 023 | 030 | 037 | 044  | 051  | 058  | 065  |       |
| 632 | 071    | 078 | 085 | 092 | 099 | 106 | 113  | 120  | 126  | 133  |       |
| 633 | 140    | 147 | 154 | 161 | 168 | 174 | 181  | 188  | 195  | 202  |       |
| 634 | 209    | 216 | 222 | 229 | 236 | 243 | 250  | 257  | 263  | 270  |       |
| 635 | 277    | 284 | 291 | 298 | 304 | 311 | 318  | 325  | 332  | 339  |       |
| 636 | 345    | 352 | 359 | 366 | 373 | 380 | 386  | 393  | 400  | 407  |       |
| 637 | 414    | 421 | 427 | 434 | 441 | 448 | 455  | 461  | 468  | 475  |       |
| 638 | 482    | 489 | 495 | 502 | 509 | 516 | 523  | 529  | 536  | 543  |       |
| 639 | 550    | 557 | 563 | 570 | 577 | 584 | 591  | 597  | 604  | 611  |       |
| 640 | 618    | 625 | 631 | 638 | 645 | 652 | 658  | 665  | 672  | 679  |       |
| 641 | 686    | 692 | 699 | 706 | 713 | 719 | 726  | 733  | 740  | 747  |       |
| 642 | 753    | 760 | 767 | 774 | 780 | 787 | 794  | 801  | 807  | 814  |       |
| 643 | 821    | 828 | 834 | 841 | 848 | 855 | 861  | 868  | 875  | 882  |       |
| 644 | 888    | 895 | 902 | 909 | 915 | 922 | 929  | 936  | 942  | 949  |       |
| 645 | 956    | 962 | 969 | 976 | 983 | 989 | 996  | *003 | *010 | *016 |       |
| 646 | 81 023 | 030 | 036 | 043 | 050 | 057 | 063  | 070  | 077  | 083  |       |
| 647 | 090    | 097 | 104 | 110 | 117 | 124 | 130  | 137  | 144  | 151  |       |
| 648 | 157    | 164 | 171 | 177 | 184 | 191 | 197  | 204  | 211  | 218  |       |
| 649 | 224    | 231 | 238 | 244 | 251 | 258 | 264  | 271  | 278  | 284  |       |
| 650 | 291    | 298 | 304 | 311 | 318 | 324 | 331  | 338  | 345  | 351  |       |
| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6    | 7    | 8    | 9    | P. P. |



TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|------|------|------|------|------|------|------|------|------|--------|
| 650 | 81 291 | 298  | 304  | 311  | 318  | 324  | 331  | 338  | 345  | 351  |        |
| 651 | 358    | 365  | 371  | 378  | 385  | 391  | 398  | 405  | 411  | 418  |        |
| 652 | 425    | 431  | 438  | 444  | 451  | 458  | 464  | 471  | 478  | 484  |        |
| 653 | 491    | 498  | 504  | 511  | 518  | 524  | 531  | 538  | 544  | 551  |        |
| 654 | 558    | 564  | 571  | 577  | 584  | 591  | 597  | 604  | 611  | 617  |        |
| 655 | 624    | 631  | 637  | 644  | 650  | 657  | 664  | 670  | 677  | 684  |        |
| 656 | 690    | 697  | 703  | 710  | 717  | 723  | 730  | 736  | 743  | 750  |        |
| 657 | 756    | 763  | 770  | 776  | 783  | 789  | 796  | 803  | 809  | 816  |        |
| 658 | 822    | 829  | 836  | 842  | 849  | 855  | 862  | 869  | 875  | 882  | .1 0.7 |
| 659 | 888    | 895  | 901  | 908  | 915  | 921  | 928  | 934  | 941  | 948  | .2 1.4 |
| 660 | 954    | 961  | 967  | 974  | 980  | 987  | 994  | *000 | *007 | *013 | .3 2.1 |
| 661 | 82 020 | 026  | 033  | 040  | 046  | 053  | 059  | 066  | 072  | 079  | .4 2.8 |
| 662 | 086    | 092  | 099  | 105  | 112  | 118  | 125  | 131  | 138  | 145  | .5 3.5 |
| 663 | 151    | 158  | 164  | 171  | 177  | 184  | 190  | 197  | 203  | 210  | .6 4.2 |
| 664 | 217    | 223  | 230  | 236  | 243  | 249  | 256  | 262  | 269  | 275  | .7 4.9 |
| 665 | 282    | 288  | 295  | 302  | 308  | 315  | 321  | 328  | 334  | 341  | .8 5.6 |
| 666 | 347    | 354  | 360  | 367  | 373  | 380  | 386  | 393  | 399  | 406  | .9 6.3 |
| 667 | 412    | 419  | 425  | 432  | 438  | 445  | 451  | 458  | 464  | 471  |        |
| 668 | 477    | 484  | 490  | 497  | 503  | 510  | 516  | 523  | 529  | 536  |        |
| 669 | 542    | 549  | 555  | 562  | 568  | 575  | 581  | 588  | 594  | 601  |        |
| 670 | 607    | 614  | 620  | 627  | 633  | 640  | 646  | 653  | 659  | 666  |        |
| 671 | 672    | 678  | 685  | 691  | 698  | 704  | 711  | 717  | 724  | 730  | .1 0.6 |
| 672 | 737    | 743  | 750  | 756  | 763  | 769  | 775  | 782  | 788  | 795  | .2 1.3 |
| 673 | 801    | 808  | 814  | 821  | 827  | 834  | 840  | 846  | 853  | 859  | .3 1.9 |
| 674 | 866    | 872  | 879  | 885  | 892  | 898  | 904  | 911  | 917  | 924  | .4 2.6 |
| 675 | 930    | 937  | 943  | 949  | 956  | 962  | 969  | 975  | 982  | 988  | .5 3.2 |
| 676 | 994    | *001 | *007 | *014 | *020 | *027 | *033 | *039 | *046 | *052 | .6 3.9 |
| 677 | 83 059 | 065  | 071  | 078  | 084  | 091  | 097  | 103  | 110  | 116  | .7 4.5 |
| 678 | 123    | 129  | 136  | 142  | 148  | 155  | 161  | 168  | 174  | 180  | .8 5.2 |
| 679 | 187    | 193  | 200  | 206  | 212  | 219  | 225  | 231  | 238  | 244  | .9 5.8 |
| 680 | 251    | 257  | 263  | 270  | 276  | 283  | 289  | 295  | 302  | 308  |        |
| 681 | 314    | 321  | 327  | 334  | 340  | 346  | 353  | 359  | 365  | 372  |        |
| 682 | 378    | 385  | 391  | 397  | 404  | 410  | 416  | 423  | 429  | 435  |        |
| 683 | 442    | 448  | 455  | 461  | 467  | 474  | 480  | 486  | 493  | 499  |        |
| 684 | 505    | 512  | 518  | 524  | 531  | 537  | 543  | 550  | 556  | 562  |        |
| 685 | 569    | 575  | 581  | 588  | 594  | 600  | 607  | 613  | 619  | 626  |        |
| 686 | 632    | 638  | 645  | 651  | 657  | 664  | 670  | 676  | 683  | 689  |        |
| 687 | 695    | 702  | 708  | 714  | 721  | 727  | 733  | 740  | 746  | 752  |        |
| 688 | 759    | 765  | 771  | 778  | 784  | 790  | 796  | 803  | 809  | 815  | .1 0.6 |
| 689 | 822    | 828  | 834  | 841  | 847  | 853  | 859  | 866  | 872  | 878  | .2 1.2 |
| 690 | 885    | 891  | 897  | 904  | 910  | 916  | 922  | 929  | 935  | 941  | .3 1.8 |
| 691 | 948    | 954  | 960  | 966  | 973  | 979  | 985  | 992  | 998  | *004 | .4 2.4 |
| 692 | 84 010 | 017  | 023  | 029  | 035  | 042  | 048  | 054  | 061  | 067  | .5 3.0 |
| 693 | 073    | 079  | 086  | 092  | 098  | 104  | 111  | 117  | 123  | 129  | .6 3.6 |
| 694 | 136    | 142  | 148  | 154  | 161  | 167  | 173  | 179  | 186  | 192  | .7 4.2 |
| 695 | 198    | 204  | 211  | 217  | 223  | 229  | 236  | 242  | 248  | 254  | .8 4.8 |
| 696 | 261    | 267  | 273  | 279  | 286  | 292  | 298  | 304  | 311  | 317  | .9 5.4 |
| 697 | 323    | 329  | 335  | 342  | 348  | 354  | 360  | 367  | 373  | 379  |        |
| 698 | 385    | 392  | 398  | 404  | 410  | 416  | 423  | 429  | 435  | 441  |        |
| 699 | 447    | 454  | 460  | 466  | 472  | 479  | 485  | 491  | 497  | 503  |        |
| 700 | 510    | 516  | 522  | 528  | 534  | 541  | 547  | 553  | 559  | 565  |        |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|-----|-----|------|------|------|------|------|------|--------|
| 700 | 84 510 | 516 | 522 | 528 | 534  | 541  | 547  | 553  | 559  | 565  |        |
| 701 | 572    | 578 | 584 | 590 | 596  | 603  | 609  | 616  | 621  | 627  |        |
| 702 | 633    | 640 | 646 | 652 | 658  | 664  | 671  | 677  | 683  | 689  |        |
| 703 | 695    | 701 | 708 | 714 | 720  | 726  | 732  | 739  | 745  | 751  |        |
| 704 | 757    | 763 | 769 | 775 | 782  | 788  | 794  | 800  | 806  | 813  |        |
| 705 | 819    | 825 | 831 | 837 | 843  | 849  | 856  | 862  | 868  | 874  |        |
| 706 | 880    | 886 | 893 | 899 | 905  | 911  | 917  | 923  | 929  | 936  |        |
| 707 | 942    | 948 | 954 | 960 | 966  | 972  | 979  | 985  | 991  | 997  |        |
| 708 | 35 003 | 009 | 015 | 021 | 028  | 034  | 040  | 046  | 052  | 058  | .1 0.6 |
| 709 | 064    | 070 | 077 | 083 | 089  | 095  | 101  | 107  | 113  | 119  | .2 1.3 |
|     |        |     |     |     |      |      |      |      |      |      | .3 1.9 |
| 710 | 126    | 132 | 138 | 144 | 150  | 156  | 162  | 168  | 174  | 181  | .4 2.6 |
|     |        |     |     |     |      |      |      |      |      |      | .5 3.2 |
| 711 | 187    | 193 | 199 | 205 | 211  | 217  | 223  | 229  | 236  | 242  | .6 3.9 |
| 712 | 248    | 254 | 260 | 266 | 272  | 278  | 284  | 290  | 297  | 303  | .7 4.5 |
| 713 | 309    | 315 | 321 | 327 | 333  | 339  | 345  | 351  | 357  | 363  | .8 5.2 |
| 714 | 370    | 376 | 382 | 388 | 394  | 400  | 406  | 412  | 418  | 424  | .9 5.8 |
| 715 | 430    | 436 | 443 | 449 | 455  | 461  | 467  | 473  | 479  | 485  |        |
| 716 | 491    | 497 | 503 | 509 | 515  | 521  | 527  | 533  | 540  | 546  |        |
| 717 | 552    | 558 | 564 | 570 | 576  | 582  | 588  | 594  | 600  | 606  |        |
| 718 | 612    | 618 | 624 | 630 | 636  | 642  | 648  | 655  | 661  | 667  |        |
| 719 | 673    | 679 | 685 | 691 | 697  | 703  | 709  | 715  | 721  | 727  |        |
| 720 | 733    | 739 | 745 | 751 | 757  | 763  | 769  | 775  | 781  | 787  |        |
|     |        |     |     |     |      |      |      |      |      |      | 6      |
| 721 | 793    | 799 | 805 | 811 | 817  | 823  | 829  | 835  | 841  | 847  | .1 0.6 |
| 722 | 853    | 859 | 865 | 872 | 878  | 884  | 890  | 896  | 902  | 908  | .2 1.2 |
| 723 | 914    | 920 | 926 | 932 | 938  | 944  | 950  | 956  | 962  | 968  | .3 1.8 |
| 724 | 974    | 980 | 986 | 992 | 998  | *004 | *010 | *016 | *022 | *028 | .4 2.4 |
| 725 | 36 034 | 040 | 046 | 052 | 058  | 063  | 069  | 075  | 081  | 087  | .5 3.0 |
| 726 | 093    | 099 | 105 | 111 | 117  | 123  | 129  | 135  | 141  | 147  | .6 3.6 |
| 727 | 153    | 159 | 165 | 171 | 177  | 183  | 189  | 195  | 201  | 207  | .7 4.2 |
| 728 | 213    | 219 | 225 | 231 | 237  | 243  | 249  | 255  | 261  | 267  | .8 4.8 |
| 729 | 273    | 278 | 284 | 290 | 296  | 302  | 308  | 314  | 320  | 326  | .9 5.4 |
| 730 | 332    | 338 | 344 | 350 | 356  | 362  | 368  | 374  | 380  | 386  |        |
| 731 | 391    | 397 | 403 | 409 | 415  | 421  | 427  | 433  | 439  | 445  |        |
| 732 | 451    | 457 | 463 | 469 | 475  | 481  | 486  | 492  | 498  | 504  |        |
| 733 | 510    | 516 | 522 | 528 | 534  | 540  | 546  | 552  | 558  | 563  |        |
| 734 | 569    | 575 | 581 | 587 | 593  | 599  | 605  | 611  | 617  | 623  |        |
| 735 | 628    | 634 | 640 | 646 | 652  | 658  | 664  | 670  | 676  | 682  |        |
| 736 | 688    | 693 | 699 | 705 | 711  | 717  | 723  | 729  | 735  | 741  |        |
| 737 | 746    | 752 | 758 | 764 | 770  | 776  | 782  | 788  | 794  | 800  |        |
| 738 | 805    | 811 | 817 | 823 | 829  | 835  | 841  | 847  | 852  | 858  | .1 0.5 |
| 739 | 864    | 870 | 876 | 882 | 888  | 894  | 899  | 905  | 911  | 917  | .2 1.1 |
|     |        |     |     |     |      |      |      |      |      |      | .3 1.6 |
| 740 | 923    | 929 | 935 | 941 | 946  | 952  | 958  | 964  | 970  | 976  | .4 2.2 |
|     |        |     |     |     |      |      |      |      |      |      | .5 2.7 |
| 741 | 982    | 987 | 993 | 999 | *005 | *011 | *017 | *023 | *028 | *034 | .6 3.3 |
| 742 | 37 040 | 046 | 052 | 058 | 064  | 069  | 075  | 081  | 087  | 093  | .7 3.8 |
| 743 | 099    | 104 | 110 | 116 | 122  | 128  | 134  | 140  | 145  | 151  | .8 4.4 |
| 744 | 157    | 163 | 169 | 175 | 180  | 186  | 192  | 198  | 204  | 210  | .9 4.9 |
| 745 | 215    | 221 | 227 | 233 | 239  | 245  | 250  | 256  | 262  | 268  |        |
| 746 | 274    | 279 | 285 | 291 | 297  | 303  | 309  | 314  | 320  | 326  |        |
| 747 | 332    | 338 | 343 | 349 | 355  | 361  | 367  | 372  | 378  | 384  |        |
| 748 | 390    | 396 | 402 | 407 | 413  | 419  | 425  | 431  | 436  | 442  |        |
| 749 | 448    | 454 | 460 | 465 | 471  | 477  | 483  | 489  | 494  | 500  |        |
| 750 | 506    | 512 | 517 | 523 | 529  | 535  | 541  | 546  | 552  | 558  |        |
| N.  | 0      | 1   | 2   | 3   | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |
|-----|--------|-----|-----|------|------|------|------|------|------|------|-------|
| 750 | 87 506 | 512 | 517 | 523  | 529  | 535  | 541  | 546  | 552  | 558  |       |
| 751 | 564    | 570 | 575 | 581  | 587  | 593  | 598  | 604  | 610  | 616  |       |
| 752 | 622    | 627 | 633 | 639  | 645  | 650  | 656  | 662  | 668  | 673  |       |
| 753 | 679    | 685 | 691 | 697  | 702  | 708  | 714  | 720  | 725  | 731  |       |
| 754 | 737    | 743 | 748 | 754  | 760  | 766  | 771  | 777  | 783  | 789  |       |
| 755 | 794    | 800 | 806 | 812  | 817  | 823  | 829  | 835  | 840  | 846  |       |
| 756 | 852    | 858 | 863 | 869  | 875  | 881  | 886  | 892  | 898  | 904  |       |
| 757 | 909    | 915 | 921 | 927  | 932  | 938  | 944  | 949  | 955  | 961  |       |
| 758 | 967    | 972 | 978 | 984  | 990  | 995  | *001 | *007 | *012 | *018 |       |
| 759 | 88 024 | 030 | 035 | 041  | 047  | 053  | 058  | 064  | 070  | 075  |       |
| 760 | 081    | 087 | 093 | 098  | 104  | 110  | 115  | 121  | 127  | 133  |       |
| 761 | 138    | 144 | 150 | 155  | 161  | 167  | 172  | 178  | 184  | 190  |       |
| 762 | 195    | 201 | 207 | 212  | 218  | 224  | 229  | 235  | 241  | 247  |       |
| 763 | 252    | 258 | 264 | 269  | 275  | 281  | 286  | 292  | 298  | 303  |       |
| 764 | 309    | 315 | 320 | 326  | 332  | 337  | 343  | 349  | 355  | 360  |       |
| 765 | 366    | 372 | 377 | 383  | 389  | 394  | 400  | 406  | 411  | 417  |       |
| 766 | 423    | 428 | 434 | 440  | 445  | 451  | 457  | 462  | 468  | 474  |       |
| 767 | 479    | 485 | 491 | 496  | 502  | 508  | 513  | 519  | 525  | 530  |       |
| 768 | 536    | 542 | 547 | 553  | 558  | 564  | 570  | 575  | 581  | 587  |       |
| 769 | 592    | 598 | 604 | 609  | 615  | 621  | 626  | 632  | 638  | 643  |       |
| 770 | 649    | 654 | 660 | 666  | 671  | 677  | 683  | 688  | 694  | 700  |       |
| 771 | 705    | 711 | 716 | 722  | 728  | 733  | 739  | 745  | 750  | 756  |       |
| 772 | 761    | 767 | 773 | 778  | 784  | 790  | 795  | 801  | 806  | 812  |       |
| 773 | 818    | 823 | 829 | 835  | 840  | 846  | 851  | 857  | 863  | 868  |       |
| 774 | 874    | 879 | 885 | 891  | 896  | 902  | 907  | 913  | 919  | 924  |       |
| 775 | 930    | 936 | 941 | 947  | 952  | 958  | 964  | 969  | 975  | 980  |       |
| 776 | 986    | 992 | 997 | *003 | *008 | *014 | *019 | *025 | *031 | *036 |       |
| 777 | 89 042 | 047 | 053 | 059  | 064  | 070  | 075  | 081  | 087  | 092  |       |
| 778 | 098    | 103 | 109 | 114  | 120  | 126  | 131  | 137  | 142  | 148  |       |
| 779 | 153    | 159 | 165 | 170  | 176  | 181  | 187  | 193  | 198  | 204  |       |
| 780 | 209    | 215 | 220 | 226  | 231  | 237  | 243  | 248  | 254  | 259  |       |
| 781 | 265    | 270 | 276 | 282  | 287  | 293  | 298  | 304  | 309  | 315  |       |
| 782 | 320    | 326 | 332 | 337  | 343  | 348  | 354  | 359  | 365  | 370  |       |
| 783 | 376    | 381 | 387 | 393  | 398  | 404  | 409  | 415  | 420  | 426  |       |
| 784 | 431    | 437 | 442 | 448  | 454  | 459  | 465  | 470  | 476  | 481  |       |
| 785 | 487    | 492 | 498 | 503  | 509  | 514  | 520  | 525  | 531  | 536  |       |
| 786 | 542    | 548 | 553 | 559  | 564  | 570  | 575  | 581  | 586  | 592  |       |
| 787 | 597    | 603 | 608 | 614  | 619  | 625  | 630  | 636  | 641  | 647  |       |
| 788 | 652    | 658 | 663 | 669  | 674  | 680  | 685  | 691  | 696  | 702  |       |
| 789 | 707    | 713 | 718 | 724  | 729  | 735  | 740  | 746  | 751  | 757  |       |
| 790 | 762    | 768 | 773 | 779  | 784  | 790  | 795  | 801  | 806  | 812  |       |
| 791 | 817    | 823 | 828 | 834  | 839  | 845  | 850  | 856  | 861  | 867  |       |
| 792 | 872    | 878 | 883 | 889  | 894  | 900  | 905  | 911  | 916  | 922  |       |
| 793 | 927    | 933 | 938 | 943  | 949  | 954  | 960  | 965  | 971  | 976  |       |
| 794 | 982    | 987 | 993 | 998  | *004 | *009 | *015 | *020 | *026 | *031 |       |
| 795 | 90 036 | 042 | 047 | 053  | 058  | 064  | 069  | 075  | 080  | 086  |       |
| 796 | 091    | 097 | 102 | 107  | 113  | 118  | 124  | 129  | 135  | 140  |       |
| 797 | 146    | 151 | 156 | 162  | 167  | 173  | 178  | 184  | 189  | 195  |       |
| 798 | 200    | 205 | 211 | 216  | 222  | 227  | 233  | 238  | 244  | 249  |       |
| 799 | 254    | 260 | 265 | 271  | 276  | 282  | 287  | 292  | 298  | 303  |       |
| 800 | 309    | 314 | 320 | 325  | 330  | 336  | 341  | 347  | 352  | 358  |       |
| N.  | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |

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TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8    | 9    | P. P. |
|-----|--------|-----|-----|-----|-----|-----|-----|-----|------|------|-------|
| 800 | 90 309 | 314 | 320 | 325 | 330 | 336 | 341 | 347 | 352  | 358  |       |
| 801 | 363    | 368 | 374 | 379 | 385 | 390 | 396 | 401 | 406  | 412  |       |
| 802 | 417    | 423 | 428 | 433 | 439 | 444 | 450 | 455 | 460  | 466  |       |
| 803 | 471    | 477 | 482 | 488 | 493 | 498 | 504 | 509 | 515  | 520  |       |
| 804 | 525    | 531 | 536 | 542 | 547 | 552 | 558 | 563 | 569  | 574  |       |
| 805 | 579    | 585 | 590 | 596 | 601 | 606 | 612 | 617 | 622  | 628  |       |
| 806 | 633    | 639 | 644 | 649 | 655 | 660 | 666 | 671 | 676  | 682  |       |
| 807 | 687    | 693 | 698 | 703 | 709 | 714 | 719 | 725 | 730  | 736  |       |
| 808 | 741    | 746 | 752 | 757 | 762 | 768 | 773 | 778 | 784  | 789  |       |
| 809 | 795    | 800 | 805 | 811 | 816 | 821 | 827 | 832 | 838  | 843  |       |
| 810 | 848    | 854 | 859 | 864 | 870 | 875 | 880 | 886 | 891  | 896  |       |
| 811 | 902    | 907 | 913 | 918 | 923 | 929 | 934 | 939 | 945  | 950  |       |
| 812 | 955    | 961 | 966 | 971 | 977 | 982 | 987 | 993 | 998  | 003  |       |
| 813 | 91 009 | 014 | 019 | 025 | 030 | 036 | 041 | 046 | 052  | 057  |       |
| 814 | 062    | 068 | 073 | 078 | 084 | 089 | 094 | 100 | 105  | 110  |       |
| 815 | 116    | 121 | 126 | 131 | 137 | 142 | 147 | 153 | 158  | 163  |       |
| 816 | 169    | 174 | 179 | 185 | 190 | 195 | 201 | 206 | 211  | 217  |       |
| 817 | 222    | 227 | 233 | 238 | 243 | 249 | 254 | 259 | 264  | 270  |       |
| 818 | 275    | 280 | 286 | 291 | 296 | 302 | 307 | 312 | 318  | 323  |       |
| 819 | 328    | 333 | 339 | 344 | 349 | 355 | 360 | 365 | 371  | 376  |       |
| 820 | 381    | 386 | 392 | 397 | 402 | 408 | 413 | 418 | 423  | 429  |       |
| 821 | 434    | 439 | 445 | 450 | 455 | 461 | 466 | 471 | 476  | 482  |       |
| 822 | 487    | 492 | 497 | 503 | 508 | 513 | 519 | 524 | 529  | 534  |       |
| 823 | 540    | 545 | 550 | 556 | 561 | 566 | 571 | 577 | 582  | 587  |       |
| 824 | 592    | 598 | 603 | 608 | 614 | 619 | 624 | 629 | 635  | 640  |       |
| 825 | 645    | 650 | 656 | 661 | 666 | 671 | 677 | 682 | 687  | 692  |       |
| 826 | 698    | 703 | 708 | 714 | 719 | 724 | 729 | 735 | 740  | 745  |       |
| 827 | 750    | 756 | 761 | 766 | 771 | 777 | 782 | 787 | 792  | 798  |       |
| 828 | 803    | 808 | 813 | 819 | 824 | 829 | 834 | 839 | 845  | 850  |       |
| 829 | 855    | 860 | 866 | 871 | 876 | 881 | 887 | 892 | 897  | 902  |       |
| 830 | 908    | 913 | 918 | 923 | 928 | 934 | 939 | 944 | 949  | 955  |       |
| 831 | 960    | 965 | 970 | 976 | 981 | 986 | 991 | 996 | *002 | *007 |       |
| 832 | 91 012 | 017 | 023 | 028 | 033 | 038 | 043 | 049 | 054  | 059  |       |
| 833 | 064    | 069 | 075 | 080 | 085 | 090 | 096 | 101 | 106  | 111  |       |
| 834 | 116    | 122 | 127 | 132 | 137 | 142 | 148 | 153 | 158  | 163  |       |
| 835 | 168    | 174 | 179 | 184 | 189 | 194 | 200 | 205 | 210  | 215  |       |
| 836 | 220    | 226 | 231 | 236 | 241 | 246 | 252 | 257 | 262  | 267  |       |
| 837 | 272    | 277 | 283 | 288 | 293 | 298 | 303 | 309 | 314  | 319  |       |
| 838 | 324    | 329 | 335 | 340 | 345 | 350 | 355 | 360 | 366  | 371  |       |
| 839 | 376    | 381 | 386 | 391 | 397 | 402 | 407 | 412 | 417  | 423  |       |
| 840 | 428    | 433 | 438 | 443 | 448 | 454 | 459 | 464 | 469  | 474  |       |
| 841 | 479    | 485 | 490 | 495 | 500 | 505 | 510 | 515 | 521  | 526  |       |
| 842 | 531    | 536 | 541 | 546 | 552 | 557 | 562 | 567 | 572  | 577  |       |
| 843 | 583    | 588 | 593 | 598 | 603 | 608 | 613 | 619 | 624  | 629  |       |
| 844 | 634    | 639 | 644 | 649 | 655 | 660 | 665 | 670 | 675  | 680  |       |
| 845 | 685    | 691 | 696 | 701 | 706 | 711 | 716 | 721 | 727  | 732  |       |
| 846 | 737    | 742 | 747 | 752 | 757 | 762 | 768 | 773 | 778  | 783  |       |
| 847 | 788    | 793 | 798 | 803 | 809 | 814 | 819 | 824 | 829  | 834  |       |
| 848 | 839    | 844 | 850 | 855 | 860 | 865 | 870 | 875 | 880  | 885  |       |
| 849 | 891    | 896 | 901 | 906 | 911 | 916 | 921 | 926 | 931  | 937  |       |
| 850 | 942    | 947 | 952 | 957 | 962 | 967 | 972 | 977 | 982  | 988  |       |
| N.  | 0      | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8    | 9    | P. P. |

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TABLE V.—LOGARITHMS OF NUMBERS.

| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|-----|------|------|------|------|------|------|------|------|--------|
| 850 | 92 942 | 947 | 952  | 957  | 962  | 967  | 972  | 977  | 982  | 988  |        |
| 851 | 993    | 998 | *003 | *008 | *013 | *018 | *023 | *028 | *034 | *039 |        |
| 852 | 93 044 | 049 | 054  | 059  | 064  | 069  | 074  | 079  | 084  | 090  |        |
| 853 | 095    | 100 | 105  | 110  | 115  | 120  | 125  | 130  | 135  | 140  |        |
| 854 | 146    | 151 | 156  | 161  | 166  | 171  | 176  | 181  | 186  | 191  |        |
| 855 | 196    | 201 | 207  | 212  | 217  | 222  | 227  | 232  | 237  | 242  |        |
| 856 | 247    | 252 | 257  | 262  | 267  | 272  | 278  | 283  | 288  | 293  |        |
| 857 | 298    | 303 | 308  | 313  | 318  | 323  | 328  | 333  | 338  | 343  |        |
| 858 | 348    | 354 | 359  | 364  | 369  | 374  | 379  | 384  | 389  | 394  | .1 0.5 |
| 859 | 399    | 404 | 409  | 414  | 419  | 424  | 429  | 434  | 439  | 445  | .2 1.1 |
|     |        |     |      |      |      |      |      |      |      |      | .3 1.6 |
| 860 | 450    | 455 | 460  | 465  | 470  | 475  | 480  | 485  | 490  | 495  | .4 2.2 |
|     |        |     |      |      |      |      |      |      |      |      | .5 2.7 |
| 861 | 500    | 505 | 510  | 515  | 520  | 525  | 530  | 535  | 540  | 545  | .6 3.3 |
| 862 | 550    | 556 | 561  | 566  | 571  | 576  | 581  | 586  | 591  | 596  | .7 3.8 |
| 863 | 601    | 606 | 611  | 616  | 621  | 626  | 631  | 636  | 641  | 646  | .8 4.4 |
| 864 | 651    | 656 | 661  | 666  | 671  | 676  | 681  | 686  | 691  | 696  | .9 4.9 |
| 865 | 701    | 706 | 711  | 716  | 721  | 726  | 731  | 736  | 742  | 747  |        |
| 866 | 752    | 757 | 762  | 767  | 772  | 777  | 782  | 787  | 792  | 797  |        |
| 867 | 802    | 807 | 812  | 817  | 822  | 827  | 832  | 837  | 842  | 847  |        |
| 868 | 852    | 857 | 862  | 867  | 872  | 877  | 882  | 887  | 892  | 897  |        |
| 869 | 902    | 907 | 912  | 917  | 922  | 927  | 932  | 937  | 942  | 947  |        |
| 870 | 952    | 957 | 962  | 967  | 972  | 977  | 982  | 987  | 992  | 997  |        |
| 871 | 94 002 | 007 | 012  | 017  | 022  | 026  | 031  | 036  | 041  | 046  | .1 5   |
| 872 | 051    | 056 | 061  | 066  | 071  | 076  | 081  | 086  | 091  | 096  | .2 0.5 |
| 873 | 101    | 106 | 111  | 116  | 121  | 126  | 131  | 136  | 141  | 146  | .3 1.0 |
| 874 | 151    | 156 | 161  | 166  | 171  | 176  | 181  | 186  | 191  | 196  | .4 1.5 |
| 875 | 201    | 206 | 210  | 215  | 220  | 225  | 230  | 235  | 240  | 245  | .5 2.0 |
| 876 | 250    | 255 | 260  | 265  | 270  | 275  | 280  | 285  | 290  | 295  | .6 2.5 |
| 877 | 300    | 305 | 310  | 315  | 320  | 324  | 329  | 334  | 339  | 344  | .7 3.0 |
| 878 | 349    | 354 | 359  | 364  | 369  | 374  | 379  | 384  | 389  | 394  | .8 3.5 |
| 879 | 399    | 404 | 409  | 413  | 418  | 423  | 428  | 433  | 438  | 443  | .9 4.0 |
|     |        |     |      |      |      |      |      |      |      |      | .5     |
| 880 | 448    | 453 | 458  | 463  | 468  | 473  | 478  | 483  | 487  | 492  |        |
| 881 | 497    | 502 | 507  | 512  | 517  | 522  | 527  | 532  | 537  | 542  |        |
| 882 | 547    | 552 | 556  | 561  | 566  | 571  | 576  | 581  | 586  | 591  |        |
| 883 | 596    | 601 | 606  | 611  | 615  | 620  | 625  | 630  | 635  | 640  |        |
| 884 | 645    | 650 | 655  | 660  | 665  | 670  | 674  | 679  | 684  | 689  |        |
| 885 | 694    | 699 | 704  | 709  | 714  | 719  | 724  | 728  | 733  | 738  |        |
| 886 | 743    | 748 | 753  | 758  | 763  | 768  | 773  | 777  | 782  | 787  |        |
| 887 | 792    | 797 | 802  | 807  | 812  | 817  | 821  | 826  | 831  | 836  |        |
| 888 | 841    | 846 | 851  | 856  | 861  | 865  | 870  | 875  | 880  | 885  | .1 4   |
| 889 | 890    | 895 | 900  | 905  | 909  | 914  | 919  | 924  | 929  | 934  | .2 0.4 |
|     |        |     |      |      |      |      |      |      |      |      | .3 0.9 |
| 890 | 939    | 944 | 949  | 953  | 958  | 963  | 968  | 973  | 978  | 983  | .4 1.3 |
|     |        |     |      |      |      |      |      |      |      |      | .5 1.8 |
| 891 | 988    | 992 | 997  | *002 | *007 | *012 | *017 | *022 | *026 | 031  | .6 2.2 |
| 892 | 95 036 | 041 | 046  | 051  | 056  | 061  | 065  | 070  | 075  | 080  | .7 2.7 |
| 893 | 085    | 090 | 095  | 099  | 104  | 109  | 114  | 119  | 124  | 129  | .8 3.1 |
| 894 | 134    | 138 | 143  | 148  | 153  | 158  | 163  | 167  | 172  | 177  | .9 3.6 |
| 895 | 182    | 187 | 192  | 197  | 201  | 206  | 211  | 216  | 221  | 226  |        |
| 896 | 231    | 235 | 240  | 245  | 250  | 255  | 260  | 264  | 269  | 274  |        |
| 897 | 279    | 284 | 289  | 294  | 298  | 303  | 308  | 313  | 318  | 323  |        |
| 898 | 327    | 332 | 337  | 342  | 347  | 352  | 356  | 361  | 366  | 371  |        |
| 899 | 376    | 381 | 385  | 390  | 395  | 400  | 405  | 410  | 414  | 419  |        |
| 900 | 424    | 429 | 434  | 438  | 443  | 448  | 453  | 458  | 463  | 467  |        |
| N.  | 0      | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |



TABLE V.—LOGARITMS OF NUMBERS.

| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |
|-----|--------|------|------|------|------|------|------|------|------|------|--------|
| 900 | 95 424 | 429  | 434  | 438  | 443  | 448  | 453  | 458  | 463  | 467  |        |
| 901 | 472    | 477  | 482  | 487  | 492  | 496  | 501  | 506  | 511  | 516  |        |
| 902 | 520    | 525  | 530  | 535  | 540  | 544  | 549  | 554  | 559  | 564  |        |
| 903 | 569    | 573  | 578  | 583  | 588  | 593  | 597  | 602  | 607  | 612  |        |
| 904 | 617    | 621  | 626  | 631  | 636  | 641  | 645  | 650  | 655  | 660  |        |
| 905 | 665    | 669  | 674  | 679  | 684  | 689  | 693  | 698  | 703  | 708  |        |
| 906 | 713    | 717  | 722  | 727  | 732  | 737  | 741  | 746  | 751  | 756  |        |
| 907 | 760    | 765  | 770  | 775  | 780  | 784  | 789  | 794  | 799  | 804  |        |
| 908 | 808    | 813  | 818  | 823  | 827  | 832  | 837  | 842  | 847  | 851  |        |
| 909 | 856    | 861  | 866  | 870  | 875  | 880  | 885  | 890  | 894  | 899  |        |
| 910 | 904    | 909  | 913  | 918  | 923  | 928  | 933  | 937  | 942  | 947  |        |
| 911 | 952    | 956  | 961  | 966  | 971  | 975  | 980  | 985  | 990  | 994  | .5     |
| 912 | 999    | *004 | *009 | *014 | *018 | *023 | *028 | *033 | *037 | *042 | .1 0.5 |
| 913 | 96     | 047  | 052  | 056  | 061  | 066  | 071  | 075  | 080  | 085  | .2 1.0 |
| 914 | 094    | 099  | 104  | 109  | 113  | 118  | 123  | 128  | 132  | 137  | .3 1.5 |
| 915 | 142    | 147  | 151  | 156  | 161  | 166  | 170  | 175  | 180  | 185  | .4 2.0 |
| 916 | 189    | 194  | 199  | 204  | 208  | 213  | 218  | 222  | 227  | 232  | .5 2.5 |
| 917 | 237    | 241  | 246  | 251  | 256  | 260  | 265  | 270  | 275  | 279  | .6 3.0 |
| 918 | 284    | 289  | 293  | 298  | 303  | 308  | 312  | 317  | 322  | 327  | .7 3.5 |
| 919 | 331    | 336  | 341  | 345  | 350  | 355  | 360  | 364  | 369  | 374  | .8 4.0 |
| 920 | 379    | 383  | 388  | 393  | 397  | 402  | 407  | 412  | 416  | 421  | .9 4.5 |
| 921 | 426    | 430  | 435  | 440  | 445  | 449  | 454  | 459  | 463  | 468  |        |
| 922 | 473    | 478  | 482  | 487  | 492  | 496  | 501  | 506  | 511  | 515  |        |
| 923 | 520    | 525  | 529  | 534  | 539  | 543  | 548  | 553  | 558  | 562  |        |
| 924 | 567    | 572  | 576  | 581  | 586  | 590  | 595  | 600  | 605  | 609  |        |
| 925 | 614    | 619  | 623  | 628  | 633  | 637  | 642  | 647  | 651  | 656  |        |
| 926 | 661    | 666  | 670  | 675  | 680  | 684  | 689  | 694  | 698  | 703  |        |
| 927 | 708    | 712  | 717  | 722  | 726  | 731  | 736  | 741  | 745  | 750  |        |
| 928 | 755    | 759  | 764  | 769  | 773  | 778  | 783  | 787  | 792  | 797  |        |
| 929 | 801    | 806  | 811  | 815  | 820  | 825  | 829  | 834  | 839  | 843  |        |
| 930 | 848    | 853  | 857  | 862  | 867  | 871  | 876  | 881  | 885  | 890  |        |
| 931 | 895    | 899  | 904  | 909  | 913  | 918  | 923  | 927  | 932  | 937  | .4     |
| 932 | 941    | 946  | 951  | 955  | 960  | 965  | 969  | 974  | 979  | 983  | .1 0.4 |
| 933 | 988    | 993  | 997  | *002 | *007 | *011 | *016 | *020 | *025 | *030 | .2 0.9 |
| 934 | 97     | 034  | 039  | 044  | 048  | 053  | 058  | 062  | 067  | 072  | .3 1.3 |
| 935 | 081    | 086  | 090  | 095  | 099  | 104  | 109  | 113  | 118  | 123  | .4 1.8 |
| 936 | 127    | 132  | 137  | 141  | 146  | 151  | 155  | 160  | 164  | 169  | .5 2.2 |
| 937 | 174    | 178  | 183  | 188  | 192  | 197  | 202  | 206  | 211  | 215  | .6 2.7 |
| 938 | 220    | 225  | 229  | 234  | 239  | 243  | 248  | 252  | 257  | 262  | .7 3.1 |
| 939 | 266    | 271  | 276  | 280  | 285  | 289  | 294  | 299  | 303  | 308  | .8 3.6 |
| 940 | 313    | 317  | 322  | 326  | 331  | 336  | 340  | 345  | 349  | 354  | .9 4.0 |
| 941 | 359    | 363  | 368  | 373  | 377  | 382  | 386  | 391  | 396  | 400  |        |
| 942 | 405    | 409  | 414  | 419  | 423  | 428  | 432  | 437  | 442  | 446  |        |
| 943 | 451    | 456  | 460  | 465  | 469  | 474  | 479  | 483  | 488  | 492  |        |
| 944 | 497    | 502  | 506  | 511  | 515  | 520  | 525  | 529  | 534  | 538  |        |
| 945 | 543    | 548  | 552  | 557  | 561  | 566  | 570  | 575  | 580  | 584  |        |
| 946 | 589    | 593  | 598  | 603  | 607  | 612  | 616  | 621  | 626  | 630  |        |
| 947 | 635    | 639  | 644  | 649  | 653  | 658  | 662  | 667  | 671  | 676  |        |
| 948 | 681    | 685  | 690  | 694  | 699  | 703  | 708  | 713  | 717  | 722  |        |
| 949 | 726    | 731  | 736  | 740  | 745  | 749  | 754  | 758  | 763  | 768  |        |
| 950 | 772    | 777  | 781  | 786  | 790  | 795  | 800  | 804  | 809  | 813  |        |
| N.  | 0      | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.   | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |
|------|--------|-----|-----|------|------|------|------|------|------|------|-------|
| 950  | 97 772 | 777 | 781 | 786  | 790  | 795  | 800  | 804  | 809  | 813  |       |
| 951  | 818    | 822 | 827 | 831  | 836  | 841  | 845  | 850  | 854  | 859  |       |
| 952  | 863    | 868 | 873 | 877  | 882  | 886  | 891  | 895  | 900  | 904  |       |
| 953  | 909    | 914 | 918 | 923  | 927  | 932  | 936  | 941  | 945  | 950  |       |
| 954  | 955    | 959 | 964 | 968  | 973  | 977  | 982  | 986  | 991  | 996  |       |
| 955  | 98 000 | 005 | 009 | 014  | 018  | 023  | 027  | 032  | 036  | 041  |       |
| 956  | 046    | 050 | 055 | 059  | 064  | 068  | 073  | 077  | 082  | 086  |       |
| 957  | 091    | 095 | 100 | 105  | 109  | 114  | 118  | 123  | 127  | 132  |       |
| 958  | 136    | 141 | 145 | 150  | 154  | 159  | 163  | 168  | 173  | 177  |       |
| 959  | 182    | 186 | 191 | 195  | 200  | 204  | 209  | 213  | 218  | 222  |       |
| 960  | 227    | 231 | 236 | 240  | 245  | 249  | 254  | 259  | 263  | 268  |       |
| 961  | 272    | 277 | 281 | 286  | 290  | 295  | 299  | 304  | 308  | 313  |       |
| 962  | 317    | 322 | 326 | 331  | 335  | 340  | 344  | 349  | 353  | 358  |       |
| 963  | 362    | 367 | 371 | 376  | 380  | 385  | 389  | 394  | 398  | 403  |       |
| 964  | 407    | 412 | 416 | 421  | 425  | 430  | 434  | 439  | 443  | 448  |       |
| 965  | 452    | 457 | 461 | 466  | 470  | 475  | 479  | 484  | 488  | 493  |       |
| 966  | 497    | 502 | 506 | 511  | 515  | 520  | 524  | 529  | 533  | 538  |       |
| 967  | 542    | 547 | 551 | 556  | 560  | 565  | 569  | 574  | 578  | 583  |       |
| 968  | 587    | 592 | 596 | 601  | 605  | 610  | 614  | 619  | 623  | 628  |       |
| 969  | 632    | 637 | 641 | 646  | 650  | 655  | 659  | 663  | 668  | 672  |       |
| 970  | 677    | 681 | 686 | 690  | 695  | 699  | 704  | 708  | 713  | 717  |       |
| 971  | 722    | 726 | 731 | 735  | 740  | 744  | 749  | 753  | 757  | 762  |       |
| 972  | 766    | 771 | 775 | 780  | 784  | 789  | 793  | 798  | 802  | 807  |       |
| 973  | 811    | 815 | 820 | 824  | 829  | 833  | 838  | 842  | 847  | 851  |       |
| 974  | 856    | 860 | 865 | 869  | 873  | 878  | 882  | 887  | 891  | 896  |       |
| 975  | 900    | 905 | 909 | 914  | 918  | 922  | 927  | 931  | 936  | 940  |       |
| 976  | 945    | 949 | 954 | 958  | 963  | 967  | 971  | 976  | 980  | 985  |       |
| 977  | 989    | 994 | 998 | *003 | *007 | *011 | *016 | *020 | *025 | *029 |       |
| 978  | 99 034 | 038 | 043 | 047  | 051  | 056  | 060  | 065  | 069  | 074  |       |
| 979  | 078    | 082 | 087 | 091  | 096  | 100  | 105  | 109  | 113  | 118  |       |
| 980  | 122    | 127 | 131 | 136  | 140  | 145  | 149  | 153  | 158  | 162  |       |
| 981  | 167    | 171 | 176 | 180  | 184  | 189  | 193  | 198  | 202  | 206  |       |
| 982  | 211    | 215 | 220 | 224  | 229  | 233  | 237  | 242  | 246  | 251  |       |
| 983  | 255    | 260 | 264 | 268  | 273  | 277  | 282  | 286  | 290  | 295  |       |
| 984  | 299    | 304 | 308 | 312  | 317  | 321  | 326  | 330  | 335  | 339  |       |
| 985  | 343    | 348 | 352 | 357  | 361  | 365  | 370  | 374  | 379  | 383  |       |
| 986  | 387    | 392 | 396 | 401  | 405  | 409  | 414  | 418  | 423  | 427  |       |
| 987  | 431    | 436 | 440 | 445  | 449  | 453  | 458  | 462  | 467  | 471  |       |
| 988  | 475    | 480 | 484 | 489  | 493  | 497  | 502  | 506  | 511  | 515  |       |
| 989  | 519    | 524 | 528 | 533  | 537  | 541  | 546  | 550  | 554  | 559  |       |
| 990  | 563    | 568 | 572 | 576  | 581  | 585  | 590  | 594  | 598  | 603  |       |
| 991  | 607    | 611 | 616 | 620  | 625  | 629  | 633  | 638  | 642  | 647  |       |
| 992  | 651    | 655 | 660 | 664  | 668  | 673  | 677  | 682  | 686  | 690  |       |
| 993  | 695    | 699 | 703 | 708  | 712  | 717  | 721  | 725  | 730  | 734  |       |
| 994  | 738    | 743 | 747 | 751  | 756  | 760  | 765  | 769  | 773  | 778  |       |
| 995  | 782    | 786 | 791 | 795  | 800  | 804  | 808  | 813  | 817  | 821  |       |
| 996  | 826    | 830 | 834 | 839  | 843  | 847  | 852  | 856  | 861  | 865  |       |
| 997  | 869    | 874 | 878 | 882  | 887  | 891  | 895  | 900  | 904  | 908  |       |
| 998  | 913    | 917 | 922 | 926  | 930  | 935  | 939  | 943  | 948  | 952  |       |
| 999  | 956    | 961 | 965 | 969  | 974  | 978  | 982  | 987  | 991  | 995  |       |
| 1000 | 00 000 | 004 | 008 | 013  | 017  | 021  | 026  | 030  | 034  | 039  |       |
| N.   | 0      | 1   | 2   | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |

5  
.1 0.5  
.2 1.0  
.3 1.5  
.4 2.0  
.5 2.5  
.6 3.0  
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.8 4.0  
.9 4.5

4  
.1 0.4  
.2 0.9  
.3 1.3  
.4 1.8  
.5 2.2  
.6 2.7  
.7 3.1  
.8 3.6  
.9 4.0

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.1 0.4  
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.3 1.2  
.4 1.6  
.5 2.0  
.6 2.4  
.7 2.8  
.8 3.2  
.9 3.6



TABLE V.—LOGARITHMS OF NUMBERS.

| N.   | 0   | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |  |
|------|-----|-----|------|------|------|------|------|------|------|------|-------|--|
| 1000 | 000 | 000 | 043  | 087  | 130  | 173  | 217  | 260  | 304  | 347  | 390   |  |
| 01   |     | 434 | 477  | 521  | 564  | 607  | 651  | 694  | 737  | 781  | 824   |  |
| 02   |     | 867 | 911  | 954  | 997  | *041 | *084 | *127 | *171 | *214 | *257  |  |
| 03   | 001 | 301 | 344  | 387  | 431  | 474  | 517  | 560  | 604  | 647  | 690   |  |
| 04   |     | 733 | 777  | 820  | 863  | 906  | 950  | 993  | *036 | *079 | *123  |  |
| 05   | 002 | 166 | 209  | 252  | 295  | 339  | 382  | 425  | 468  | 511  | 555   |  |
| 06   |     | 598 | 641  | 684  | 727  | 770  | 814  | 857  | 900  | 943  | 986   |  |
| 07   | 003 | 029 | 072  | 115  | 159  | 202  | 245  | 288  | 331  | 374  | 417   |  |
| 08   |     | 460 | 503  | 546  | 590  | 633  | 676  | 719  | 762  | 805  | 848   |  |
| 09   |     | 891 | 934  | 977  | *020 | *063 | *106 | *149 | *192 | *235 | *278  |  |
| 1010 | 004 | 321 | 364  | 407  | 450  | 493  | 536  | 579  | 622  | 665  | 708   |  |
| 11   |     | 751 | 794  | 837  | 880  | 923  | 966  | *009 | *051 | *094 | *137  |  |
| 12   | 005 | 180 | 223  | 266  | 309  | 352  | 395  | 438  | 481  | 523  | 566   |  |
| 13   |     | 609 | 652  | 695  | 738  | 781  | 824  | 866  | 909  | 952  | 995   |  |
| 14   | 006 | 038 | 081  | 123  | 166  | 209  | 252  | 295  | 337  | 380  | 423   |  |
| 15   |     | 466 | 509  | 551  | 594  | 637  | 680  | 722  | 765  | 808  | 851   |  |
| 16   |     | 893 | 936  | 979  | *022 | *064 | *107 | *150 | *193 | *235 | *278  |  |
| 17   | 007 | 321 | 363  | 406  | 449  | 491  | 534  | 577  | 620  | 662  | 705   |  |
| 18   |     | 748 | 790  | 833  | 875  | 918  | 961  | *003 | *046 | *089 | 131   |  |
| 19   | 008 | 174 | 217  | 259  | 302  | 344  | 387  | 430  | 472  | 515  | 557   |  |
| 1020 |     | 600 | 642  | 685  | 728  | 770  | 813  | 855  | 898  | 940  | 983   |  |
| 21   | 009 | 025 | 068  | 111  | 153  | 196  | 238  | 281  | 323  | 366  | 408   |  |
| 22   |     | 451 | 493  | 536  | 578  | 621  | 663  | 706  | 748  | 790  | 833   |  |
| 23   |     | 875 | 918  | 960  | *003 | *045 | *088 | *130 | *172 | *215 | *257  |  |
| 24   | 010 | 300 | 342  | 385  | 427  | 469  | 512  | 554  | 596  | 639  | 681   |  |
| 25   |     | 724 | 766  | 808  | 851  | 893  | 935  | 978  | *020 | *062 | *105  |  |
| 26   | 011 | 147 | 189  | 232  | 274  | 316  | 359  | 401  | 443  | 486  | 528   |  |
| 27   |     | 570 | 612  | 655  | 697  | 739  | 782  | 824  | 866  | 908  | 951   |  |
| 28   |     | 993 | *035 | *077 | *120 | *162 | *204 | *246 | *288 | *331 | *373  |  |
| 29   | 012 | 415 | 457  | 500  | 542  | 584  | 626  | 668  | 710  | 753  | 795   |  |
| 1030 |     | 837 | 879  | 921  | 963  | *006 | *048 | *090 | *132 | 174  | 216   |  |
| 31   | 013 | 258 | 301  | 343  | 385  | 427  | 469  | 511  | 553  | 595  | 637   |  |
| 32   |     | 679 | 722  | 764  | 806  | 848  | 890  | 932  | 974  | *016 | *058  |  |
| 33   | 014 | 100 | 142  | 184  | 226  | 268  | 310  | 352  | 394  | 436  | 478   |  |
| 34   |     | 520 | 562  | 604  | 646  | 688  | 730  | 772  | 814  | 856  | 898   |  |
| 35   |     | 940 | 982  | *024 | *066 | *108 | *150 | *192 | *234 | *276 | *318  |  |
| 36   | 015 | 360 | 401  | 443  | 485  | 527  | 569  | 611  | 653  | 695  | 737   |  |
| 37   |     | 779 | 820  | 862  | 904  | 946  | 988  | *030 | *072 | *113 | 155   |  |
| 38   | 016 | 197 | 239  | 281  | 323  | 364  | 406  | 448  | 490  | 532  | 573   |  |
| 39   |     | 615 | 657  | 699  | 741  | 782  | 824  | 866  | 908  | 950  | 991   |  |
| 1040 | 017 | 033 | 075  | 117  | 158  | 200  | 242  | 284  | 325  | 367  | 409   |  |
| 41   |     | 450 | 492  | 534  | 576  | 617  | 659  | 701  | 742  | 784  | 826   |  |
| 42   |     | 867 | 909  | 951  | 992  | *034 | *076 | *117 | *159 | *201 | *242  |  |
| 43   | 018 | 284 | 326  | 367  | 409  | 451  | 492  | 534  | 575  | 617  | 659   |  |
| 44   |     | 700 | 742  | 783  | 825  | 867  | 908  | 950  | 991  | *033 | *074  |  |
| 45   | 019 | 116 | 158  | 199  | 241  | 282  | 324  | 365  | 407  | 448  | 490   |  |
| 46   |     | 531 | 573  | 614  | 656  | 697  | 739  | 780  | 822  | 863  | 905   |  |
| 47   |     | 946 | 988  | *029 | *071 | *112 | *154 | *195 | *237 | *278 | *320  |  |
| 48   | 020 | 361 | 402  | 444  | 485  | 527  | 568  | 610  | 651  | 692  | 734   |  |
| 49   |     | 775 | 817  | 858  | 899  | 941  | 982  | *024 | *065 | *106 | *148  |  |
| 1050 | 021 | 189 | 230  | 272  | 313  | 354  | 396  | 437  | 478  | 520  | 561   |  |
| N.   | 0   | 1   | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P. |  |

TABLE V.—LOGARITHMS OF NUMBERS.

| N.   | 0       | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.   |
|------|---------|------|------|------|------|------|------|------|------|------|---------|
| 1050 | 021 189 | 230  | 272  | 313  | 354  | 396  | 437  | 478  | 520  | 561  | 41      |
| 51   | 602     | 644  | 685  | 726  | 768  | 809  | 850  | 892  | 933  | 974  | .1 4.1  |
| 52   | 022 015 | 057  | 098  | 139  | 181  | 222  | 263  | 304  | 346  | 387  | .2 8.3  |
| 53   | 428     | 469  | 511  | 552  | 593  | 634  | 676  | 717  | 758  | 799  | .3 12.4 |
| 54   | 840     | 882  | 923  | 964  | *005 | *046 | *088 | *129 | *170 | *211 | .4 16.6 |
| 55   | 023 252 | 293  | 335  | 376  | 417  | 458  | 499  | 540  | 581  | 623  | .5 20.7 |
| 56   | 664     | 705  | 746  | 787  | 828  | 869  | 910  | 951  | 993  | *034 | .6 24.9 |
| 57   | 024 075 | 116  | 157  | 198  | 239  | 280  | 321  | 362  | 403  | 444  | .7 29.0 |
| 58   | 485     | 526  | 568  | 609  | 650  | 691  | 732  | 773  | 814  | 855  | .8 33.2 |
| 59   | 896     | 937  | 978  | *019 | *060 | *101 | *142 | *183 | *224 | *265 | .9 37.3 |
| 1060 | 025 306 | 347  | 388  | 429  | 469  | 510  | 551  | 592  | 633  | 674  | 41      |
| 61   | 715     | 756  | 797  | 838  | 879  | 920  | 961  | *002 | *042 | *083 | .1 4.1  |
| 62   | 026 124 | 165  | 206  | 247  | 288  | 329  | 370  | 410  | 451  | 492  | .2 8.2  |
| 63   | 533     | 574  | 615  | 656  | 696  | 737  | 778  | 819  | 860  | 901  | .3 12.3 |
| 64   | 941     | 982  | *023 | *064 | *105 | *145 | *186 | *227 | *268 | *309 | .4 16.4 |
| 65   | 027 349 | 390  | 431  | 472  | 512  | 553  | 594  | 635  | 675  | 716  | .5 20.5 |
| 66   | 757     | 798  | 838  | 879  | 920  | 961  | *001 | *042 | *083 | *123 | .6 24.6 |
| 67   | 028 164 | 205  | 246  | 286  | 327  | 368  | 408  | 449  | 490  | 530  | .7 28.7 |
| 68   | 571     | 612  | 652  | 693  | 734  | 774  | 815  | 856  | 896  | 937  | .8 32.8 |
| 69   | 977     | *018 | *059 | *099 | *140 | *181 | *221 | *262 | *302 | *343 | .9 36.9 |
| 1070 | 029 384 | 424  | 465  | 505  | 546  | 586  | 627  | 668  | 708  | 749  | 40      |
| 71   | 789     | 830  | 870  | 911  | 951  | 992  | *032 | *073 | *114 | *154 | .1 4.0  |
| 72   | 030 195 | 235  | 276  | 316  | 357  | 397  | 438  | 478  | 519  | 559  | .2 8.1  |
| 73   | 599     | 640  | 680  | 721  | 761  | 802  | 842  | 883  | 923  | 964  | .3 12.1 |
| 74   | 031 004 | 044  | 085  | 125  | 166  | 206  | 247  | 287  | 327  | 368  | .4 16.2 |
| 75   | 408     | 449  | 489  | 529  | 570  | 610  | 651  | 691  | 731  | 772  | .5 20.2 |
| 76   | 812     | 852  | 893  | 933  | 973  | *014 | *054 | *094 | *135 | *175 | .6 24.3 |
| 77   | 032 215 | 256  | 296  | 336  | 377  | 417  | 457  | 498  | 538  | 578  | .7 28.3 |
| 78   | 619     | 659  | 699  | 739  | 780  | 820  | 860  | 900  | 941  | 981  | .8 32.4 |
| 79   | 033 021 | 061  | 102  | 142  | 182  | 222  | 263  | 303  | 343  | 383  | .9 36.4 |
| 1080 | 424     | 464  | 504  | 544  | 584  | 625  | 665  | 705  | 745  | 785  | 40      |
| 81   | 825     | 866  | 906  | 946  | 986  | *026 | *066 | *107 | 147  | 187  | .1 4.0  |
| 82   | 034 227 | 267  | 307  | 347  | 388  | 428  | 468  | 508  | 548  | 588  | .2 8.0  |
| 83   | 628     | 668  | 708  | 748  | 789  | 829  | 869  | 909  | 949  | 989  | .3 12.0 |
| 84   | 035 029 | 069  | 109  | 149  | 189  | 229  | 269  | 309  | 349  | 389  | .4 16.0 |
| 85   | 429     | 470  | 510  | 550  | 590  | 630  | 670  | 710  | 750  | 790  | .5 20.0 |
| 86   | 830     | 870  | 910  | 950  | 990  | *029 | *069 | *109 | *149 | *189 | .6 24.0 |
| 87   | 036 229 | 269  | 309  | 349  | 389  | 429  | 469  | 509  | 549  | 589  | .7 28.0 |
| 88   | 629     | 669  | 708  | 748  | 788  | 828  | 868  | 908  | 948  | 988  | .8 32.0 |
| 89   | 037 028 | 068  | 107  | 147  | 187  | 227  | 267  | 307  | 347  | 386  | .9 36.0 |
| 1090 | 426     | 466  | 506  | 546  | 586  | 625  | 665  | 705  | 745  | 785  | 39      |
| 91   | 825     | 864  | 904  | 944  | 984  | *023 | *063 | *103 | 143  | 183  | .1 3.9  |
| 92   | 038 222 | 262  | 302  | 342  | 381  | 421  | 461  | 501  | 540  | 580  | .2 7.9  |
| 93   | 620     | 660  | 699  | 739  | 779  | 819  | 858  | 898  | 938  | 977  | .3 11.8 |
| 94   | 039 017 | 057  | 096  | 136  | 176  | 216  | 255  | 295  | 335  | 374  | .4 15.8 |
| 95   | 414     | 454  | 493  | 533  | 572  | 612  | 652  | 691  | 731  | 771  | .5 19.7 |
| 96   | 810     | 850  | 890  | 929  | 969  | *008 | *048 | *088 | *127 | *167 | .6 23.7 |
| 97   | 040 206 | 246  | 286  | 325  | 365  | 404  | 444  | 483  | 523  | 563  | .7 27.6 |
| 98   | 602     | 642  | 681  | 721  | 760  | 800  | 839  | 879  | 918  | 958  | .8 31.6 |
| 99   | 997     | *037 | *076 | *116 | *155 | *195 | *234 | *274 | *313 | *353 | .9 35.5 |
| 1100 | 041 392 | 432  | 471  | 511  | 550  | 590  | 629  | 669  | 708  | 748  |         |
| N.   | 0       | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | P. P.   |

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES.

$$\begin{aligned}\log \sin \phi &= \log \phi'' + S. \\ \log \tan \phi &= \log \phi'' + T.\end{aligned}$$

 $0^\circ$ 

$$\begin{aligned}\log \phi'' &= \log \sin \phi + S'. \\ \log \phi'' &= \log \tan \phi + T'.\end{aligned}$$

| "    | '  | S        | T  | Log. Sin.  | S'       | T' | Log. Tan.  |
|------|----|----------|----|------------|----------|----|------------|
| 0    | 0  | 4.685 57 | 57 | — $\infty$ | 5.314 42 | 42 | — $\infty$ |
| 60   | 1  | 57       | 57 | 6.46 372   | 42       | 42 | 6.46 372   |
| 120  | 2  | 57       | 57 | .76 475    | 42       | 42 | .76 475    |
| 180  | 3  | 57       | 57 | .94 084    | 42       | 42 | .94 084    |
| 240  | 4  | 57       | 57 | 7.06 578   | 42       | 42 | 7.06 578   |
| 300  | 5  | 4.685 57 | 57 | 7.16 269   | 5.314 42 | 42 | 7.16 269   |
| 360  | 6  | 57       | 57 | .24 187    | 42       | 42 | .24 188    |
| 420  | 7  | 57       | 57 | .30 882    | 42       | 42 | .30 882    |
| 480  | 8  | 57       | 57 | .36 681    | 42       | 42 | .36 681    |
| 540  | 9  | 57       | 57 | .41 797    | 42       | 42 | .41 797    |
| 600  | 10 | 4.685 57 | 57 | 7.46 372   | 5.314 42 | 42 | 7.46 372   |
| 660  | 11 | 57       | 57 | .50 512    | 42       | 42 | .50 512    |
| 720  | 12 | 57       | 57 | .54 290    | 42       | 42 | .54 291    |
| 780  | 13 | 57       | 57 | .57 767    | 42       | 42 | .57 767    |
| 840  | 14 | 57       | 57 | .60 995    | 42       | 42 | .60 985    |
| 900  | 15 | 4.685 57 | 58 | 7.63 981   | 5.314 42 | 42 | 7.63 982   |
| 960  | 16 | 57       | 58 | .66 784    | 42       | 42 | .66 785    |
| 1020 | 17 | 57       | 58 | .69 417    | 42       | 42 | .69 418    |
| 1080 | 18 | 57       | 58 | .71 899    | 42       | 42 | .71 900    |
| 1140 | 19 | 57       | 58 | .74 248    | 42       | 42 | .74 248    |
| 1200 | 20 | 4.685 57 | 58 | 7.76 475   | 5.314 43 | 42 | 7.76 476   |
| 1260 | 21 | 57       | 58 | .78 594    | 43       | 42 | .78 595    |
| 1320 | 22 | 57       | 58 | .80 614    | 43       | 42 | .80 615    |
| 1380 | 23 | 57       | 58 | .82 545    | 43       | 42 | .82 546    |
| 1440 | 24 | 57       | 58 | .84 393    | 43       | 42 | .84 394    |
| 1500 | 25 | 4.685 57 | 58 | 7.86 166   | 5.314 43 | 41 | 7.86 167   |
| 1560 | 26 | 57       | 58 | .87 869    | 43       | 41 | .87 871    |
| 1620 | 27 | 57       | 58 | .89 508    | 43       | 41 | .89 510    |
| 1680 | 28 | 57       | 58 | .91 088    | 43       | 41 | .91 089    |
| 1740 | 29 | 57       | 58 | .92 612    | 43       | 41 | .92 613    |
| 1800 | 30 | 4.685 57 | 58 | 7.94 084   | 5.314 43 | 41 | 7.94 086   |
| 1860 | 31 | 57       | 58 | .95 508    | 43       | 41 | .95 510    |
| 1920 | 32 | 57       | 58 | .96 887    | 43       | 41 | .96 889    |
| 1980 | 33 | 57       | 59 | .98 223    | 43       | 41 | .98 225    |
| 2040 | 34 | 57       | 59 | .99 520    | 43       | 41 | .99 522    |
| 2100 | 35 | 4.685 56 | 59 | 8.00 778   | 5.314 43 | 41 | 8.00 781   |
| 2160 | 36 | 56       | 59 | .02 002    | 43       | 41 | .02 004    |
| 2220 | 37 | 56       | 59 | .03 192    | 43       | 41 | .03 194    |
| 2280 | 38 | 56       | 59 | .04 350    | 43       | 40 | .04 352    |
| 2340 | 39 | 56       | 59 | .05 478    | 43       | 40 | .05 481    |
| 2400 | 40 | 4.685 56 | 59 | 8.08 577   | 5.314 43 | 40 | 8.08 580   |
| 2460 | 41 | 56       | 59 | .07 650    | 43       | 40 | .07 653    |
| 2520 | 42 | 56       | 59 | .08 696    | 43       | 40 | .08 699    |
| 2580 | 43 | 56       | 60 | .09 718    | 43       | 40 | .09 721    |
| 2640 | 44 | 56       | 60 | .10 716    | 43       | 40 | .10 720    |
| 2700 | 45 | 4.685 56 | 60 | 8.11 692   | 5.314 44 | 40 | 8.11 696   |
| 2760 | 46 | 56       | 60 | .12 647    | 44       | 40 | .12 651    |
| 2820 | 47 | 56       | 60 | .13 581    | 44       | 40 | .13 585    |
| 2880 | 48 | 56       | 60 | .14 495    | 44       | 39 | .14 499    |
| 2940 | 49 | 56       | 60 | .15 390    | 44       | 39 | .15 395    |
| 3000 | 50 | 4.685 56 | 60 | 8.16 268   | 5.314 44 | 39 | 8.16 272   |
| 3060 | 51 | 56       | 60 | .17 128    | 44       | 39 | .17 133    |
| 3120 | 52 | 56       | 61 | .17 971    | 44       | 39 | .17 976    |
| 3180 | 53 | 56       | 61 | .18 798    | 44       | 39 | .18 803    |
| 3240 | 54 | 55       | 61 | .19 610    | 44       | 39 | .19 615    |
| 3300 | 55 | 4.685 55 | 61 | 8.20 407   | 5.314 44 | 39 | 8.20 412   |
| 3360 | 56 | 55       | 61 | .21 189    | 44       | 38 | .21 195    |
| 3420 | 57 | 55       | 61 | .21 958    | 44       | 38 | .21 964    |
| 3480 | 58 | 55       | 61 | .22 713    | 44       | 38 | .22 719    |
| 3540 | 59 | 55       | 62 | .23 455    | 44       | 38 | .23 462    |

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES

$$\begin{aligned}\log \sin \phi &= \log \phi'' + S. \\ \log \tan \phi &= \log \phi'' + T.\end{aligned}$$

1°

$$\begin{aligned}\log \phi'' &= \log \sin \phi + S' \\ \log \phi'' &= \log \tan \phi + T'\end{aligned}$$

| "    | '  | S        | T  | Log. Sin. | S'       | T' | Log. Tan |
|------|----|----------|----|-----------|----------|----|----------|
| 3600 | 0  | 4.685 55 | 62 | 8.24 185  | 5.314 44 | 38 | 8.24 19  |
| 3660 | 1  | 55       | 62 | .24 903   | 45       | 38 | .24 91   |
| 3720 | 2  | 55       | 62 | .25 609   | 45       | 38 | .25 61   |
| 3780 | 3  | 55       | 62 | .26 304   | 45       | 37 | .26 31   |
| 3840 | 4  | 55       | 62 | .26 988   | 45       | 37 | .26 99   |
| 3900 | 5  | 4.685 55 | 62 | 8.27 661  | 5.314 45 | 37 | 8.27 66  |
| 3960 | 6  | 55       | 63 | .28 324   | 45       | 37 | .28 33   |
| 4020 | 7  | 54       | 63 | .28 977   | 45       | 37 | .28 98   |
| 4080 | 8  | 54       | 63 | .29 620   | 45       | 37 | .29 62   |
| 4140 | 9  | 54       | 63 | .30 254   | 45       | 36 | .30 26   |
| 4200 | 10 | 4.685 54 | 63 | 8.30 879  | 5.314 45 | 36 | 8.30 88  |
| 4260 | 11 | 54       | 63 | .31 495   | 45       | 36 | .31 50   |
| 4320 | 12 | 54       | 64 | .32 102   | 45       | 36 | .32 11   |
| 4380 | 13 | 54       | 64 | .32 701   | 46       | 36 | .32 71   |
| 4440 | 14 | 54       | 64 | .33 292   | 46       | 36 | .33 30   |
| 4500 | 15 | 4.685 54 | 64 | 8.33 875  | 5.314 46 | 35 | 8.33 88  |
| 4560 | 16 | 54       | 64 | .34 450   | 46       | 35 | .34 46   |
| 4620 | 17 | 54       | 65 | .35 018   | 46       | 35 | .35 02   |
| 4680 | 18 | 54       | 65 | .35 578   | 46       | 35 | .35 58   |
| 4740 | 19 | 53       | 65 | .36 131   | 46       | 35 | .36 14   |
| 4800 | 20 | 4.685 53 | 65 | 8.36 677  | 5.314 46 | 34 | 8.36 68  |
| 4860 | 21 | 53       | 65 | .37 217   | 46       | 34 | .37 22   |
| 4920 | 22 | 53       | 65 | .37 750   | 46       | 34 | .37 76   |
| 4980 | 23 | 53       | 66 | .38 276   | 46       | 34 | .38 28   |
| 5040 | 24 | 53       | 66 | .38 796   | 47       | 34 | .38 80   |
| 5100 | 25 | 4.685 53 | 66 | 8.39 310  | 5.314 47 | 33 | 8.39 32  |
| 5160 | 26 | 53       | 66 | .39 818   | 47       | 33 | .39 83   |
| 5220 | 27 | 53       | 67 | .40 320   | 47       | 33 | .40 33   |
| 5280 | 28 | 52       | 67 | .40 816   | 47       | 33 | .40 83   |
| 5340 | 29 | 52       | 67 | .41 307   | 47       | 33 | .41 32   |
| 5400 | 30 | 4.685 52 | 67 | 8.41 792  | 5.314 47 | 32 | 8.41 80  |
| 5460 | 31 | 52       | 67 | .42 271   | 47       | 32 | .42 28   |
| 5520 | 32 | 52       | 68 | .42 746   | 47       | 32 | .42 76   |
| 5580 | 33 | 52       | 68 | .43 215   | 48       | 32 | .43 23   |
| 5640 | 34 | 52       | 68 | .43 680   | 48       | 31 | .43 69   |
| 5700 | 35 | 4.685 52 | 68 | 8.44 139  | 5.314 48 | 31 | 8.44 15  |
| 5760 | 36 | 52       | 69 | .44 594   | 48       | 31 | .44 61   |
| 5820 | 37 | 51       | 69 | .45 044   | 48       | 31 | .45 06   |
| 5880 | 38 | 51       | 69 | .45 489   | 48       | 30 | .45 50   |
| 5940 | 39 | 51       | 69 | .45 930   | 48       | 30 | .45 94   |
| 6000 | 40 | 4.685 51 | 69 | 8.46 366  | 5.314 48 | 30 | 8.46 38  |
| 6060 | 41 | 51       | 70 | .46 798   | 49       | 30 | .46 81   |
| 6120 | 42 | 51       | 70 | .47 226   | 49       | 30 | .47 24   |
| 6180 | 43 | 51       | 70 | .47 650   | 49       | 29 | .47 66   |
| 6240 | 44 | 51       | 70 | .48 069   | 49       | 29 | .48 08   |
| 6300 | 45 | 4.685 50 | 71 | 8.48 485  | 5.314 49 | 29 | 8.48 50  |
| 6360 | 46 | 50       | 71 | .48 896   | 49       | 28 | .48 91   |
| 6420 | 47 | 50       | 71 | .49 304   | 49       | 28 | .49 32   |
| 6480 | 48 | 50       | 72 | .49 708   | 49       | 28 | .49 72   |
| 6540 | 49 | 50       | 72 | .50 108   | 50       | 28 | .50 13   |
| 6600 | 50 | 4.685 50 | 72 | 8.50 504  | 5.314 50 | 27 | 8.50 52  |
| 6660 | 51 | 50       | 72 | .50 897   | 50       | 27 | .50 92   |
| 6720 | 52 | 50       | 73 | .51 286   | 50       | 27 | .51 31   |
| 6780 | 53 | 49       | 73 | .51 672   | 50       | 27 | .51 69   |
| 6840 | 54 | 49       | 73 | .52 055   | 50       | 26 | .52 07   |
| 6900 | 55 | 4.685 49 | 73 | 8.52 434  | 5.314 50 | 26 | 8.52 45  |
| 6960 | 56 | 49       | 74 | .52 810   | 51       | 26 | .52 83   |
| 7020 | 57 | 49       | 74 | .53 183   | 51       | 25 | .53 20   |
| 7080 | 58 | 49       | 74 | .53 552   | 51       | 25 | .53 57   |
| 7140 | 59 | 49       | 75 | .53 918   | 51       | 25 | .53 94   |

TABLE VI.—LOGARITHMIC SINES AND TANGENTS OF SMALL ANGLES

$$\begin{aligned}\log \sin \phi &= \log \phi'' + S. \\ \log \tan \phi &= \log \phi'' + T.\end{aligned}$$

2°

$$\begin{aligned}\log \phi'' &= \log \sin \phi + S'. \\ \log \phi'' &= \log \tan \phi + T'.\end{aligned}$$

| "     | '  | S        | T  | Log. Sin. | S'       | T' | Log. Tan. |
|-------|----|----------|----|-----------|----------|----|-----------|
| 7200  | 0  | 4.635 48 | 75 | 8.54 282  | 5.314 51 | 25 | 8.54 308  |
| 7260  | 1  | 48       | 75 | .54 642   | 51       | 24 | .54 669   |
| 7320  | 2  | 49       | 75 | .54 999   | 51       | 24 | .55 027   |
| 7380  | 3  | 48       | 76 | .55 354   | 52       | 24 | .55 381   |
| 7440  | 4  | 48       | 76 | .55 705   | 52       | 23 | .55 733   |
| 7500  | 5  | 4.685 48 | 76 | 8.56 054  | 5.314 52 | 23 | 8.56 083  |
| 7560  | 6  | 48       | 77 | .56 400   | 52       | 23 | .56 429   |
| 7620  | 7  | 47       | 77 | .56 743   | 52       | 22 | .56 772   |
| 7680  | 8  | 47       | 77 | .57 083   | 52       | 22 | .57 113   |
| 7740  | 9  | 47       | 78 | .57 421   | 52       | 22 | .57 452   |
| 7800  | 10 | 4.685 47 | 78 | 8.57 756  | 5.314 53 | 22 | 8.57 787  |
| 7860  | 11 | 47       | 78 | .58 089   | 53       | 21 | .58 121   |
| 7920  | 12 | 47       | 79 | .58 419   | 53       | 21 | .58 451   |
| 7980  | 13 | 46       | 79 | .58 747   | 53       | 21 | .58 779   |
| 8040  | 14 | 46       | 79 | .59 072   | 53       | 20 | .59 105   |
| 8100  | 15 | 4.685 46 | 80 | 8.59 395  | 5.314 53 | 20 | 8.59 428  |
| 8160  | 16 | 46       | 80 | .59 715   | 54       | 20 | .59 749   |
| 8220  | 17 | 46       | 80 | .60 033   | 54       | 19 | .60 067   |
| 8280  | 18 | 46       | 81 | .60 349   | 54       | 19 | .60 384   |
| 8340  | 19 | 45       | 81 | .60 662   | 54       | 19 | .60 698   |
| 8400  | 20 | 4.685 45 | 81 | 8.60 973  | 5.314 54 | 18 | 8.61 009  |
| 8460  | 21 | 45       | 82 | .61 282   | 54       | 18 | .61 319   |
| 8520  | 22 | 45       | 82 | .61 589   | 55       | 18 | .61 626   |
| 8580  | 23 | 45       | 82 | .61 893   | 55       | 17 | .61 931   |
| 8640  | 24 | 45       | 83 | .62 196   | 55       | 17 | .62 224   |
| 8700  | 25 | 4.685 44 | 83 | 8.62 496  | 5.314 55 | 16 | 8.62 535  |
| 8760  | 26 | 44       | 83 | .62 795   | 55       | 16 | .62 834   |
| 8820  | 27 | 44       | 84 | .63 091   | 55       | 16 | .63 131   |
| 8880  | 28 | 44       | 84 | .63 385   | 56       | 15 | .63 425   |
| 8940  | 29 | 44       | 84 | .63 677   | 56       | 15 | .63 718   |
| 9000  | 30 | 4.685 43 | 85 | 8.63 968  | 5.314 56 | 15 | 8.64 009  |
| 9060  | 31 | 43       | 85 | .64 256   | 56       | 14 | .64 298   |
| 9120  | 32 | 43       | 86 | .64 543   | 56       | 14 | .64 585   |
| 9180  | 33 | 43       | 86 | .64 827   | 57       | 14 | .64 870   |
| 9240  | 34 | 42       | 86 | .65 110   | 57       | 13 | .65 153   |
| 9300  | 35 | 4.685 43 | 87 | 8.65 391  | 5.314 57 | 13 | 8.65 435  |
| 9360  | 36 | 42       | 87 | .65 670   | 57       | 12 | .65 715   |
| 9420  | 37 | 42       | 87 | .65 947   | 57       | 12 | .65 993   |
| 9480  | 38 | 42       | 88 | .66 223   | 58       | 12 | .66 269   |
| 9540  | 39 | 42       | 88 | .66 497   | 58       | 11 | .66 543   |
| 9600  | 40 | 4.685 42 | 89 | 8.66 769  | 5.314 58 | 11 | 8.66 816  |
| 9660  | 41 | 41       | 89 | .67 039   | 58       | 10 | .67 087   |
| 9720  | 42 | 41       | 89 | .67 308   | 58       | 10 | .67 356   |
| 9780  | 43 | 41       | 90 | .67 575   | 59       | 10 | .67 624   |
| 9840  | 44 | 41       | 90 | .67 840   | 59       | 09 | .67 890   |
| 9900  | 45 | 4.685 41 | 91 | 8.68 104  | 5.314 59 | 09 | 8.68 154  |
| 9960  | 46 | 40       | 91 | .68 366   | 59       | 08 | .68 417   |
| 10020 | 47 | 40       | 91 | .68 627   | 59       | 08 | .68 678   |
| 10080 | 48 | 40       | 92 | .68 886   | 60       | 08 | .68 938   |
| 10140 | 49 | 40       | 92 | .69 144   | 60       | 07 | .69 196   |
| 10200 | 50 | 4.685 40 | 93 | 8.69 400  | 5.314 60 | 07 | 8.69 453  |
| 10260 | 51 | 39       | 93 | .69 654   | 60       | 06 | .69 708   |
| 10320 | 52 | 39       | 93 | .69 907   | 60       | 06 | .69 961   |
| 10380 | 53 | 39       | 94 | .70 159   | 61       | 06 | .70 214   |
| 10440 | 54 | 39       | 94 | .70 409   | 61       | 05 | .70 464   |
| 10500 | 55 | 4.685 38 | 95 | 8.70 657  | 5.314 61 | 05 | 8.70 714  |
| 10560 | 56 | 38       | 95 | .70 905   | 61       | 04 | .70 962   |
| 10620 | 57 | 38       | 96 | .71 150   | 61       | 04 | .71 208   |
| 10680 | 58 | 38       | 96 | .71 395   | 62       | 03 | .71 453   |
| 10740 | 59 | 38       | 97 | .71 638   | 62       | 03 | .71 697   |



TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

0°

|    | Log. Sin. | D     | Log. Tan. | Com. D. | Log. Cot. | Log. Cos. |
|----|-----------|-------|-----------|---------|-----------|-----------|
| 0  | —∞        |       | —∞        |         | +∞        | 0.00 000  |
| 1  | 6.46 372  | 30103 | 6.46 372  | 30103   | 3 53 627  | 0.00 000  |
| 2  | 6.76 475  | 17609 | 6.76 475  | 17609   | 3 23 524  | 0.00 000  |
| 3  | 6.94 084  | 12494 | 6.94 084  | 12494   | 3 05 915  | 0.00 000  |
| 4  | 7.06 578  |       | 7.06 578  |         | 2 53 421  | 0.00 000  |
| 5  | 7.16 269  | 9691  | 7.16 269  | 9691    | 2 43 730  | 0.00 000  |
| 6  | 7.24 187  | 7918  | 7.24 188  | 7918    | 2 35 812  | 0.00 000  |
| 7  | 7.30 882  | 6895  | 7.30 882  | 6894    | 2 29 117  | 0.00 000  |
| 8  | 7.36 681  | 5799  | 7.36 681  | 5799    | 2 23 318  | 0.00 000  |
| 9  | 7.41 797  | 5115  | 7.41 797  | 5115    | 2 18 203  | 0.00 000  |
| 10 | 7.46 372  | 4575  | 7.46 372  | 4575    | 2 13 627  | 0.00 000  |
| 11 | 7.50 512  | 4139  | 7.50 512  | 4139    | 2 09 488  | 0.00 000  |
| 12 | 7.54 290  | 3778  | 7.54 291  | 3779    | 2 05 709  | 9.99 999  |
| 13 | 7.57 767  | 3476  | 7.57 767  | 3476    | 2 02 233  | 9.99 999  |
| 14 | 7.60 985  | 3218  | 7.60 985  | 3218    | 2 00 014  | 9.99 999  |
| 15 | 7.63 981  | 2996  | 7.63 982  | 2996    | 2 36 018  | 9.99 999  |
| 16 | 7.66 784  | 2803  | 7.66 785  | 2803    | 2 33 215  | 9.99 999  |
| 17 | 7.69 417  | 2633  | 7.69 418  | 2633    | 2 30 582  | 9.99 999  |
| 18 | 7.71 899  | 2482  | 7.71 900  | 2482    | 2 28 099  | 9.99 999  |
| 19 | 7.74 248  | 2348  | 7.74 248  | 2348    | 2 25 751  | 9.99 999  |
| 20 | 7.76 475  | 2227  | 7.76 476  | 2227    | 2 23 524  | 9.99 999  |
| 21 | 7.78 594  | 2119  | 7.78 595  | 2119    | 2 21 405  | 9.99 999  |
| 22 | 7.80 614  | 2020  | 7.80 615  | 2020    | 2 19 384  | 9.99 999  |
| 23 | 7.82 545  | 1930  | 7.82 546  | 1930    | 2 17 454  | 9.99 999  |
| 24 | 7.84 393  | 1848  | 7.84 394  | 1848    | 2 15 605  | 9.99 999  |
| 25 | 7.86 166  | 1772  | 7.86 167  | 1773    | 2 13 832  | 9.99 999  |
| 26 | 7.87 869  | 1703  | 7.87 871  | 1703    | 2 12 129  | 9.99 999  |
| 27 | 7.89 508  | 1639  | 7.89 510  | 1639    | 2 10 490  | 9.99 998  |
| 28 | 7.91 088  | 1579  | 7.91 089  | 1579    | 2 08 910  | 9.99 998  |
| 29 | 7.92 612  | 1524  | 7.92 613  | 1524    | 2 07 386  | 9.99 998  |
| 30 | 7.94 084  | 1472  | 7.94 086  | 1472    | 2 05 914  | 9.99 998  |
| 31 | 7.95 508  | 1424  | 7.95 510  | 1424    | 2 04 490  | 9.99 998  |
| 32 | 7.96 887  | 1379  | 7.96 889  | 1379    | 2 03 111  | 9.99 998  |
| 33 | 7.98 223  | 1336  | 7.98 225  | 1336    | 2 01 774  | 9.99 998  |
| 34 | 7.99 520  | 1296  | 7.99 522  | 1296    | 2 00 478  | 9.99 998  |
| 35 | 8 00 778  | 1258  | 8 00 781  | 1259    | 1 99 219  | 9.99 997  |
| 36 | 8 02 002  | 1223  | 8 02 004  | 1223    | 1 97 995  | 9.99 997  |
| 37 | 8 03 192  | 1190  | 8 03 194  | 1190    | 1 96 805  | 9.99 997  |
| 38 | 8 04 350  | 1158  | 8 04 352  | 1158    | 1 95 647  | 9.99 997  |
| 39 | 8 05 478  | 1128  | 8 05 481  | 1128    | 1 94 519  | 9.99 997  |
| 40 | 8 06 577  | 1099  | 8 06 580  | 1099    | 1 93 419  | 9.99 997  |
| 41 | 8 07 650  | 1072  | 8 07 653  | 1072    | 1 92 347  | 9.99 997  |
| 42 | 8 08 696  | 1046  | 8 08 699  | 1046    | 1 91 300  | 9.99 997  |
| 43 | 8 09 718  | 1022  | 8 09 721  | 1022    | 1 90 278  | 9.99 996  |
| 44 | 8 10 716  | 998   | 8 10 720  | 999     | 1 89 279  | 9.99 996  |
| 45 | 8 11 692  | 976   | 8 11 696  | 976     | 1 88 303  | 9.99 996  |
| 46 | 8 12 647  | 954   | 8 12 651  | 954     | 1 87 349  | 9.99 996  |
| 47 | 8 13 581  | 934   | 8 13 585  | 934     | 1 86 415  | 9.99 996  |
| 48 | 8 14 495  | 914   | 8 14 499  | 914     | 1 85 500  | 9.99 996  |
| 49 | 8 15 390  | 895   | 8 15 395  | 895     | 1 84 605  | 9.99 995  |
| 50 | 8 16 268  | 877   | 8 16 272  | 877     | 1 83 727  | 9.99 995  |
| 51 | 8 17 128  | 860   | 8 17 133  | 860     | 1 82 867  | 9.99 995  |
| 52 | 8 17 971  | 843   | 8 17 976  | 843     | 1 82 023  | 9.99 995  |
| 53 | 8 18 798  | 827   | 8 18 803  | 827     | 1 81 196  | 9.99 995  |
| 54 | 8 19 610  | 811   | 8 19 615  | 812     | 1 80 384  | 9.99 994  |
| 55 | 8 20 407  | 797   | 8 20 412  | 797     | 1 79 587  | 9.99 994  |
| 56 | 8 21 189  | 782   | 8 21 195  | 783     | 1 78 804  | 9.99 994  |
| 57 | 8 21 958  | 768   | 8 21 964  | 768     | 1 78 036  | 9.99 994  |
| 58 | 8 22 713  | 755   | 8 22 719  | 755     | 1 77 280  | 9.99 994  |
| 59 | 8 22 455  | 742   | 8 22 462  | 742     | 1 76 538  | 9.99 993  |
| 60 | 8 24 185  | 730   | 8 24 192  | 730     | 1 75 808  | 9 99 993  |
|    | Log. Cos. | D     | Log. Cot. | Com. D. | Log. Tan. | Log. Sin. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | D   | Log. Tan. | Com. D. | Log. Cot. | Log. Cos. |    |
|----|-----------|-----|-----------|---------|-----------|-----------|----|
| 0  | 8.24 185  |     | 8.24 192  |         | 1.75 808  | 9.99 993  | 60 |
| 1  | 8.24 903  | 712 | 8.24 910  | 718     | 1.75 090  | 9.99 993  | 59 |
| 2  | 8.25 609  | 706 | 8.25 616  | 706     | 1.74 383  | 9.99 993  | 58 |
| 3  | 8.26 304  | 694 | 8.26 311  | 695     | 1.73 688  | 9.99 992  | 57 |
| 4  | 8.26 988  | 684 | 8.26 995  | 684     | 1.73 004  | 9.99 992  | 56 |
| 5  | 8.27 661  | 673 | 8.27 669  | 673     | 1.72 331  | 9.99 992  | 55 |
| 6  | 8.28 324  | 663 | 8.28 332  | 663     | 1.71 667  | 9.99 992  | 54 |
| 7  | 8.28 977  | 653 | 8.28 985  | 653     | 1.71 014  | 9.99 992  | 53 |
| 8  | 8.29 620  | 643 | 8.29 629  | 643     | 1.70 371  | 9.99 991  | 52 |
| 9  | 8.30 254  | 634 | 8.30 263  | 634     | 1.69 736  | 9.99 991  | 51 |
| 10 | 8.30 879  | 625 | 8.30 888  | 625     | 1.69 111  | 9.99 991  | 50 |
| 11 | 8.31 495  | 616 | 8.31 504  | 616     | 1.68 495  | 9.99 990  | 49 |
| 12 | 8.32 102  | 607 | 8.32 112  | 607     | 1.67 888  | 9.99 990  | 48 |
| 13 | 8.32 701  | 599 | 8.32 711  | 599     | 1.67 288  | 9.99 990  | 47 |
| 14 | 8.33 292  | 591 | 8.33 302  | 591     | 1.66 697  | 9.99 990  | 46 |
| 15 | 8.33 875  | 583 | 8.33 885  | 583     | 1.66 114  | 9.99 989  | 45 |
| 16 | 8.34 450  | 575 | 8.34 461  | 575     | 1.65 539  | 9.99 989  | 44 |
| 17 | 8.35 018  | 567 | 8.35 029  | 568     | 1.64 971  | 9.99 989  | 43 |
| 18 | 8.35 578  | 560 | 8.35 589  | 560     | 1.64 410  | 9.99 989  | 42 |
| 19 | 8.36 131  | 553 | 8.36 143  | 553     | 1.63 857  | 9.99 988  | 41 |
| 20 | 8.36 677  | 546 | 8.36 689  | 546     | 1.63 310  | 9.99 988  | 40 |
| 21 | 8.37 217  | 539 | 8.37 229  | 539     | 1.62 771  | 9.99 988  | 39 |
| 22 | 8.37 750  | 533 | 8.37 762  | 533     | 1.62 238  | 9.99 987  | 38 |
| 23 | 8.38 276  | 526 | 8.38 289  | 527     | 1.61 711  | 9.99 987  | 37 |
| 24 | 8.38 796  | 520 | 8.38 809  | 520     | 1.61 191  | 9.99 987  | 36 |
| 25 | 8.39 310  | 514 | 8.39 323  | 514     | 1.60 676  | 9.99 986  | 35 |
| 26 | 8.39 818  | 508 | 8.39 831  | 508     | 1.60 168  | 9.99 986  | 34 |
| 27 | 8.40 320  | 502 | 8.40 334  | 502     | 1.59 666  | 9.99 986  | 33 |
| 28 | 8.40 816  | 496 | 8.40 830  | 496     | 1.59 169  | 9.99 986  | 32 |
| 29 | 8.41 307  | 491 | 8.41 321  | 491     | 1.58 678  | 9.99 985  | 31 |
| 30 | 8.41 792  | 485 | 8.41 807  | 485     | 1.58 193  | 9.99 985  | 30 |
| 31 | 8.42 271  | 479 | 8.42 287  | 480     | 1.57 713  | 9.99 985  | 29 |
| 32 | 8.42 746  | 474 | 8.42 762  | 475     | 1.57 238  | 9.99 984  | 28 |
| 33 | 8.43 215  | 469 | 8.43 231  | 469     | 1.56 768  | 9.99 984  | 27 |
| 34 | 8.43 680  | 464 | 8.43 696  | 464     | 1.56 304  | 9.99 984  | 26 |
| 35 | 8.44 139  | 459 | 8.44 156  | 460     | 1.55 844  | 9.99 983  | 25 |
| 36 | 8.44 594  | 454 | 8.44 611  | 455     | 1.55 389  | 9.99 983  | 24 |
| 37 | 8.45 044  | 450 | 8.45 061  | 450     | 1.54 938  | 9.99 982  | 23 |
| 38 | 8.45 489  | 445 | 8.45 507  | 445     | 1.54 493  | 9.99 982  | 22 |
| 39 | 8.45 930  | 440 | 8.45 948  | 441     | 1.54 052  | 9.99 982  | 21 |
| 40 | 8.46 366  | 436 | 8.46 385  | 437     | 1.53 615  | 9.99 981  | 20 |
| 41 | 8.46 798  | 432 | 8.46 817  | 432     | 1.53 183  | 9.99 981  | 19 |
| 42 | 8.47 226  | 428 | 8.47 245  | 428     | 1.52 754  | 9.99 981  | 18 |
| 43 | 8.47 650  | 423 | 8.47 669  | 424     | 1.52 330  | 9.99 980  | 17 |
| 44 | 8.48 069  | 419 | 8.48 089  | 419     | 1.51 911  | 9.99 980  | 16 |
| 45 | 8.48 485  | 415 | 8.48 505  | 416     | 1.51 495  | 9.99 979  | 15 |
| 46 | 8.48 896  | 411 | 8.48 917  | 412     | 1.51 083  | 9.99 979  | 14 |
| 47 | 8.49 304  | 407 | 8.49 325  | 408     | 1.50 675  | 9.99 979  | 13 |
| 48 | 8.49 708  | 404 | 8.49 729  | 404     | 1.50 270  | 9.99 978  | 12 |
| 49 | 8.50 108  | 400 | 8.50 130  | 400     | 1.49 870  | 9.99 978  | 11 |
| 50 | 8.50 504  | 396 | 8.50 526  | 396     | 1.49 473  | 9.99 978  | 10 |
| 51 | 8.50 897  | 393 | 8.50 920  | 393     | 1.49 080  | 9.99 977  | 9  |
| 52 | 8.51 286  | 389 | 8.51 310  | 390     | 1.48 690  | 9.99 977  | 8  |
| 53 | 8.51 672  | 386 | 8.51 696  | 386     | 1.48 304  | 9.99 976  | 7  |
| 54 | 8.52 055  | 382 | 8.52 079  | 383     | 1.47 921  | 9.99 976  | 6  |
| 55 | 8.52 434  | 379 | 8.52 458  | 379     | 1.47 541  | 9.99 975  | 5  |
| 56 | 8.52 810  | 375 | 8.52 835  | 376     | 1.47 165  | 9.99 975  | 4  |
| 57 | 8.53 183  | 373 | 8.53 208  | 373     | 1.46 792  | 9.99 975  | 3  |
| 58 | 8.53 552  | 369 | 8.53 578  | 370     | 1.46 422  | 9.99 974  | 2  |
| 59 | 8.53 918  | 366 | 8.53 944  | 366     | 1.46 055  | 9.99 974  | 1  |
| 60 | 8.54 282  | 363 | 8.54 308  | 364     | 1.45 691  | 9.99 973  | 0  |
|    | Log. Cos. | D   | Log. Cot. | Com. D. | Log. Tan. | Log. Sin. |    |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

|    | Log. Sin. | D   | Log. Tan. | Com. D. | Log. Cot. | Log. Cos. |
|----|-----------|-----|-----------|---------|-----------|-----------|
| 0  | 8.54 282  | 360 | 8.54 308  | 360     | 1.45 691  | 9.99 973  |
| 1  | 8.54 642  | 357 | 8.54 669  | 358     | 1.45 531  | 9.99 973  |
| 2  | 8.54 999  | 354 | 8.55 027  | 354     | 1.44 973  | 9.99 972  |
| 3  | 8.55 354  | 351 | 8.55 381  | 352     | 1.44 618  | 9.99 972  |
| 4  | 8.55 705  | 348 | 8.55 733  | 349     | 1.44 265  | 9.99 971  |
| 5  | 8.56 054  | 346 | 8.56 083  | 346     | 1.43 917  | 9.99 971  |
| 6  | 8.56 400  | 343 | 8.56 429  | 343     | 1.43 571  | 9.99 971  |
| 7  | 8.56 743  | 340 | 8.56 772  | 341     | 1.43 227  | 9.99 970  |
| 8  | 8.57 083  | 338 | 8.57 113  | 338     | 1.42 886  | 9.99 970  |
| 9  | 8.57 421  | 335 | 8.57 452  | 335     | 1.42 548  | 9.99 969  |
| 10 | 8.57 756  | 332 | 8.57 787  | 333     | 1.42 212  | 9.99 969  |
| 11 | 8.58 039  | 330 | 8.58 121  | 330     | 1.41 879  | 9.99 968  |
| 12 | 8.58 419  | 327 | 8.58 451  | 328     | 1.41 548  | 9.99 968  |
| 13 | 8.58 747  | 325 | 8.58 779  | 325     | 1.41 220  | 9.99 967  |
| 14 | 8.59 072  | 323 | 8.59 105  | 323     | 1.40 895  | 9.99 967  |
| 15 | 8.59 395  | 320 | 8.59 428  | 320     | 1.40 571  | 9.99 966  |
| 16 | 8.59 715  | 318 | 8.59 749  | 318     | 1.40 251  | 9.99 966  |
| 17 | 8.60 033  | 316 | 8.60 067  | 316     | 1.39 932  | 9.99 965  |
| 18 | 8.60 349  | 313 | 8.60 384  | 314     | 1.39 616  | 9.99 965  |
| 19 | 8.60 662  | 311 | 8.60 698  | 311     | 1.39 302  | 9.99 964  |
| 20 | 8.60 973  | 309 | 8.61 009  | 309     | 1.38 990  | 9.99 964  |
| 21 | 8.61 282  | 306 | 8.61 319  | 307     | 1.38 681  | 9.99 963  |
| 22 | 8.61 589  | 304 | 8.61 626  | 305     | 1.38 374  | 9.99 963  |
| 23 | 8.61 893  | 302 | 8.61 931  | 303     | 1.38 068  | 9.99 962  |
| 24 | 8.62 196  | 300 | 8.62 234  | 300     | 1.37 765  | 9.99 962  |
| 25 | 8.62 496  | 298 | 8.62 535  | 299     | 1.37 465  | 9.99 961  |
| 26 | 8.62 795  | 296 | 8.62 834  | 297     | 1.37 166  | 9.99 961  |
| 27 | 8.63 091  | 294 | 8.63 131  | 294     | 1.36 869  | 9.99 960  |
| 28 | 8.63 385  | 292 | 8.63 425  | 293     | 1.36 574  | 9.99 959  |
| 29 | 8.63 677  | 290 | 8.63 718  | 291     | 1.36 281  | 9.99 959  |
| 30 | 8.63 968  | 288 | 8.64 009  | 288     | 1.35 990  | 9.99 958  |
| 31 | 8.64 256  | 286 | 8.64 298  | 287     | 1.35 702  | 9.99 958  |
| 32 | 8.64 543  | 284 | 8.64 585  | 285     | 1.35 414  | 9.99 957  |
| 33 | 8.64 827  | 282 | 8.64 870  | 283     | 1.35 129  | 9.99 957  |
| 34 | 8.65 110  | 281 | 8.65 153  | 281     | 1.34 846  | 9.99 956  |
| 35 | 8.65 391  | 279 | 8.65 435  | 280     | 1.34 565  | 9.99 956  |
| 36 | 8.65 670  | 277 | 8.65 715  | 278     | 1.34 285  | 9.99 955  |
| 37 | 8.65 947  | 275 | 8.65 993  | 276     | 1.34 007  | 9.99 954  |
| 38 | 8.66 223  | 274 | 8.66 269  | 274     | 1.33 731  | 9.99 954  |
| 39 | 8.66 497  | 272 | 8.66 543  | 272     | 1.33 456  | 9.99 953  |
| 40 | 8.66 769  | 270 | 8.66 816  | 271     | 1.33 184  | 9.99 953  |
| 41 | 8.67 039  | 268 | 8.67 087  | 269     | 1.32 913  | 9.99 952  |
| 42 | 8.67 308  | 267 | 8.67 356  | 267     | 1.32 643  | 9.99 952  |
| 43 | 8.67 575  | 265 | 8.67 624  | 266     | 1.32 376  | 9.99 951  |
| 44 | 8.67 840  | 264 | 8.67 890  | 264     | 1.32 110  | 9.99 950  |
| 45 | 8.68 104  | 262 | 8.68 154  | 262     | 1.31 845  | 9.99 950  |
| 46 | 8.68 366  | 260 | 8.68 417  | 261     | 1.31 583  | 9.99 949  |
| 47 | 8.68 627  | 259 | 8.68 678  | 259     | 1.31 321  | 9.99 948  |
| 48 | 8.68 886  | 257 | 8.68 938  | 258     | 1.31 062  | 9.99 948  |
| 49 | 8.69 144  | 256 | 8.69 196  | 256     | 1.30 803  | 9.99 947  |
| 50 | 8.69 400  | 254 | 8.69 453  | 255     | 1.30 547  | 9.99 947  |
| 51 | 8.69 654  | 253 | 8.69 708  | 253     | 1.30 292  | 9.99 946  |
| 52 | 8.69 907  | 251 | 8.69 961  | 252     | 1.30 038  | 9.99 945  |
| 53 | 8.70 159  | 250 | 8.70 214  | 250     | 1.29 786  | 9.99 945  |
| 54 | 8.70 409  | 248 | 8.70 464  | 249     | 1.29 535  | 9.99 944  |
| 55 | 8.70 657  | 247 | 8.70 714  | 248     | 1.29 286  | 9.99 943  |
| 56 | 8.70 905  | 245 | 8.70 962  | 246     | 1.29 038  | 9.99 943  |
| 57 | 8.71 150  | 244 | 8.71 208  | 245     | 1.28 791  | 9.99 942  |
| 58 | 8.71 395  | 243 | 8.71 453  | 243     | 1.28 546  | 9.99 942  |
| 59 | 8.71 638  | 241 | 8.71 697  | 242     | 1.28 303  | 9.99 941  |
| 60 | 8.71 880  | 241 | 8.71 939  | 242     | 1.28 060  | 9.99 940  |
|    | Log. Cos. | D   | Log. Cot. | Com. D. | Log. Tan. | Log. Sin. |

# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d.  | Log. Tan. | c.d. | Log. Cot. | Log. Cos. |    | P. P.           |       |       |       |
|----|-----------|-----|-----------|------|-----------|-----------|----|-----------------|-------|-------|-------|
| 0  | 8.71 880  |     | 8.71 939  |      | 1.28 060  | 9.99 940  | 60 | 330             | 320   | 310   | 300   |
| 1  | 8.72 120  | 240 | 8.72 180  | 241  | 1.27 819  | 9.99 940  | 59 | 6 33.0          | 32.0  | 31.0  | 30.0  |
| 2  | 8.72 359  | 239 | 8.72 420  | 240  | 1.27 579  | 9.99 939  | 58 | 7 38.5          | 37.3  | 36.1  | 35.0  |
| 3  | 8.72 597  | 237 | 8.72 659  | 238  | 1.27 341  | 9.99 938  | 57 | 8 44.0          | 42.6  | 41.3  | 40.0  |
| 4  | 8.72 833  | 236 | 8.72 896  | 237  | 1.27 104  | 9.99 938  | 56 | 9 49.5          | 48.0  | 46.5  | 45.0  |
| 5  | 8.73 069  | 235 | 8.73 131  | 235  | 1.26 868  | 9.99 937  | 55 | 10 55.0         | 53.3  | 51.6  | 50.0  |
| 6  | 8.73 302  | 233 | 8.73 366  | 233  | 1.26 633  | 9.99 936  | 54 | 20 110.0        | 106.6 | 103.3 | 100.0 |
| 7  | 8.73 535  | 233 | 8.73 599  | 233  | 1.26 400  | 9.99 935  | 53 | 30 165.0        | 160.0 | 155.0 | 150.0 |
| 8  | 8.73 766  | 231 | 8.73 831  | 232  | 1.26 168  | 9.99 935  | 52 | 40 220.0        | 213.3 | 206.6 | 200.0 |
| 9  | 8.73 997  | 230 | 8.74 062  | 231  | 1.25 937  | 9.99 934  | 51 | 50 275.0        | 266.6 | 258.3 | 250.0 |
| 10 | 8.74 226  | 229 | 8.74 292  | 229  | 1.25 708  | 9.99 933  | 50 | 290 280 270 260 |       |       |       |
| 11 | 8.74 453  | 227 | 8.74 520  | 228  | 1.25 479  | 9.99 933  | 49 | 6 29.0          | 28.0  | 27.0  | 26.0  |
| 12 | 8.74 680  | 226 | 8.74 748  | 227  | 1.25 252  | 9.99 932  | 48 | 7 33.8          | 32.6  | 31.5  | 30.3  |
| 13 | 8.74 905  | 225 | 8.74 974  | 226  | 1.25 026  | 9.99 931  | 47 | 8 38.6          | 37.3  | 36.0  | 34.6  |
| 14 | 8.75 129  | 224 | 8.75 199  | 225  | 1.24 801  | 9.99 931  | 46 | 9 43.5          | 42.0  | 40.5  | 39.0  |
| 15 | 8.75 353  | 223 | 8.75 422  | 223  | 1.24 577  | 9.99 930  | 45 | 10 48.3         | 46.6  | 45.0  | 43.3  |
| 16 | 8.75 574  | 221 | 8.75 645  | 221  | 1.24 354  | 9.99 929  | 44 | 20 96.6         | 93.3  | 90.0  | 86.6  |
| 17 | 8.75 795  | 219 | 8.75 867  | 220  | 1.24 133  | 9.99 928  | 43 | 30 145.0        | 140.0 | 135.0 | 130.0 |
| 18 | 8.76 015  | 218 | 8.76 087  | 219  | 1.23 913  | 9.99 928  | 42 | 40 193.3        | 186.6 | 180.0 | 173.3 |
| 19 | 8.76 233  | 217 | 8.76 306  | 218  | 1.23 693  | 9.99 927  | 41 | 50 241.6        | 233.3 | 225.0 | 216.6 |
| 20 | 8.76 451  | 216 | 8.76 524  | 217  | 1.23 475  | 9.99 926  | 40 | 250 240 230 220 |       |       |       |
| 21 | 8.76 667  | 215 | 8.76 741  | 216  | 1.23 258  | 9.99 925  | 39 | 6 25.0          | 24.0  | 23.0  | 22.0  |
| 22 | 8.76 883  | 214 | 8.76 958  | 215  | 1.23 041  | 9.99 925  | 38 | 7 29.1          | 28.0  | 26.8  | 25.6  |
| 23 | 8.77 097  | 213 | 8.77 172  | 214  | 1.22 827  | 9.99 924  | 37 | 8 33.3          | 32.0  | 30.6  | 29.3  |
| 24 | 8.77 310  | 212 | 8.77 386  | 213  | 1.22 613  | 9.99 923  | 36 | 9 37.5          | 36.0  | 34.5  | 33.0  |
| 25 | 8.77 522  | 211 | 8.77 599  | 212  | 1.22 400  | 9.99 922  | 35 | 10 41.6         | 40.0  | 38.3  | 36.6  |
| 26 | 8.77 733  | 210 | 8.77 811  | 211  | 1.22 188  | 9.99 922  | 34 | 20 85.0         | 80.0  | 76.6  | 73.3  |
| 27 | 8.77 943  | 209 | 8.78 022  | 210  | 1.21 978  | 9.99 921  | 33 | 30 123.0        | 120.0 | 115.0 | 110.0 |
| 28 | 8.78 152  | 208 | 8.78 232  | 209  | 1.21 768  | 9.99 920  | 32 | 40 166.6        | 160.0 | 153.3 | 146.6 |
| 29 | 8.78 360  | 207 | 8.78 441  | 208  | 1.21 559  | 9.99 919  | 31 | 50 208.3        | 200.0 | 191.6 | 183.3 |
| 30 | 8.78 567  | 206 | 8.78 648  | 207  | 1.21 351  | 9.99 918  | 30 | 210 200 190 180 |       |       |       |
| 31 | 8.78 773  | 205 | 8.78 855  | 206  | 1.21 144  | 9.99 918  | 29 | 6 21.0          | 20.0  | 19.0  | 18.0  |
| 32 | 8.78 978  | 204 | 8.79 061  | 205  | 1.20 938  | 9.99 917  | 28 | 7 24.5          | 23.3  | 22.2  | 21.0  |
| 33 | 8.79 183  | 203 | 8.79 266  | 204  | 1.20 734  | 9.99 916  | 27 | 8 28.0          | 26.6  | 25.3  | 24.0  |
| 34 | 8.79 386  | 202 | 8.79 470  | 203  | 1.20 530  | 9.99 916  | 26 | 9 31.5          | 30.0  | 28.7  | 27.0  |
| 35 | 8.79 588  | 201 | 8.79 673  | 202  | 1.20 327  | 9.99 915  | 25 | 10 35.0         | 33.3  | 31.6  | 30.0  |
| 36 | 8.79 789  | 200 | 8.79 875  | 201  | 1.20 125  | 9.99 914  | 24 | 20 70.0         | 66.6  | 63.3  | 60.0  |
| 37 | 8.79 989  | 199 | 8.80 076  | 200  | 1.19 923  | 9.99 913  | 23 | 30 105.0        | 100.0 | 95.0  | 90.0  |
| 38 | 8.80 189  | 198 | 8.80 276  | 199  | 1.19 723  | 9.99 912  | 22 | 40 140.0        | 133.3 | 126.6 | 120.0 |
| 39 | 8.80 387  | 197 | 8.80 476  | 198  | 1.19 524  | 9.99 912  | 21 | 50 175.0        | 166.6 | 158.3 | 150.0 |
| 40 | 8.80 585  | 197 | 8.80 674  | 197  | 1.19 326  | 9.99 911  | 20 | 9 9 8 7 6 5     |       |       |       |
| 41 | 8.80 782  | 195 | 8.80 871  | 197  | 1.19 128  | 9.99 910  | 19 | 6 0.0           | 0.0   | 0.0   | 0.5   |
| 42 | 8.80 977  | 195 | 8.81 068  | 195  | 1.18 931  | 9.99 909  | 18 | 7 1.1           | 1.0   | 0.0   | 0.6   |
| 43 | 8.81 172  | 194 | 8.81 264  | 195  | 1.18 736  | 9.99 908  | 17 | 8 1.2           | 1.1   | 0.0   | 0.7   |
| 44 | 8.81 366  | 193 | 8.81 459  | 194  | 1.18 541  | 9.99 907  | 16 | 9 1.4           | 1.3   | 1.2   | 1.0   |
| 45 | 8.81 560  | 192 | 8.81 653  | 193  | 1.18 347  | 9.99 907  | 15 | 10 1.6          | 1.5   | 1.3   | 1.1   |
| 46 | 8.81 752  | 191 | 8.81 846  | 192  | 1.18 154  | 9.99 906  | 14 | 20 3.1          | 3.0   | 2.6   | 2.3   |
| 47 | 8.81 943  | 191 | 8.82 038  | 191  | 1.17 961  | 9.99 905  | 13 | 30 4.7          | 4.4   | 3.5   | 3.0   |
| 48 | 8.82 134  | 189 | 8.82 230  | 190  | 1.17 770  | 9.99 904  | 12 | 40 6.3          | 6.0   | 5.3   | 4.6   |
| 49 | 8.82 324  | 188 | 8.82 420  | 189  | 1.17 579  | 9.99 903  | 11 | 50 7.9          | 7.5   | 6.6   | 5.8   |
| 50 | 8.82 513  | 188 | 8.82 610  | 188  | 1.17 389  | 9.99 902  | 10 | 4 4 3 2 1 0     |       |       |       |
| 51 | 8.82 701  | 187 | 8.82 799  | 188  | 1.17 201  | 9.99 902  | 9  | 6 0.4           | 0.4   | 0.3   | 0.2   |
| 52 | 8.82 888  | 186 | 8.82 987  | 187  | 1.17 012  | 9.99 901  | 8  | 7 0.5           | 0.5   | 0.4   | 0.3   |
| 53 | 8.83 075  | 185 | 8.83 175  | 186  | 1.16 825  | 9.99 900  | 7  | 8 0.6           | 0.6   | 0.5   | 0.4   |
| 54 | 8.83 260  | 185 | 8.83 361  | 185  | 1.16 638  | 9.99 899  | 6  | 9 0.7           | 0.6   | 0.5   | 0.4   |
| 55 | 8.83 445  | 184 | 8.83 547  | 185  | 1.16 453  | 9.99 898  | 5  | 10 0.7          | 0.6   | 0.5   | 0.4   |
| 56 | 8.83 629  | 183 | 8.83 732  | 184  | 1.16 268  | 9.99 897  | 4  | 20 1.1          | 1.1   | 1.0   | 0.9   |
| 57 | 8.83 813  | 182 | 8.83 916  | 183  | 1.16 083  | 9.99 896  | 3  | 30 2.2          | 2.2   | 2.0   | 1.5   |
| 58 | 8.83 995  | 182 | 8.84 100  | 182  | 1.15 900  | 9.99 896  | 2  | 40 3.0          | 2.5   | 2.0   | 1.3   |
| 59 | 8.84 177  | 181 | 8.84 282  | 182  | 1.15 717  | 9.99 895  | 1  | 50 3.7          | 3.2   | 2.5   | 1.6   |
| 60 | 8.84 358  |     | 8.84 464  |      | 1.15 535  | 9.99 894  | 0  | P. P.           |       |       |       |
|    | Log. Cos. | d.  | Log. Cot. | c.d. | Log. Tan. | Log. Sin. |    |                 |       |       |       |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d.  | Log. Tan. | c.d. | Log. Cot. | Log. Cos. |    | P. P. |       |       |       |
|----|-----------|-----|-----------|------|-----------|-----------|----|-------|-------|-------|-------|
| 0  | 8.84 358  |     | 8.84 464  |      | 1.15 535  | 9.99 894  | 60 | 181   | 180   | 178   | 176   |
| 1  | 8.84 538  | 180 | 8.84 645  | 181  | 1.15 354  | 9.99 893  | 59 | 6     | 18.1  | 18.0  | 17.8  |
| 2  | 8.84 718  | 180 | 8.84 826  | 180  | 1.15 174  | 9.99 892  | 58 | 7     | 21.1  | 21.0  | 20.7  |
| 3  | 8.84 897  | 178 | 8.85 005  | 179  | 1.14 994  | 9.99 891  | 57 | 8     | 24.1  | 24.0  | 23.7  |
| 4  | 8.85 075  | 178 | 8.85 184  | 179  | 1.14 815  | 9.99 890  | 56 | 9     | 27.1  | 27.0  | 26.7  |
| 5  | 8.85 252  | 177 | 8.85 363  | 178  | 1.14 637  | 9.99 889  | 55 | 10    | 30.1  | 30.0  | 29.6  |
| 6  | 8.85 429  | 176 | 8.85 540  | 177  | 1.14 459  | 9.99 888  | 54 | 20    | 60.3  | 60.0  | 59.3  |
| 7  | 8.85 605  | 176 | 8.85 717  | 176  | 1.14 283  | 9.99 888  | 53 | 30    | 90.5  | 90.0  | 89.0  |
| 8  | 8.85 780  | 175 | 8.85 893  | 175  | 1.14 107  | 9.99 887  | 52 | 40    | 120.6 | 120.0 | 118.6 |
| 9  | 8.85 954  | 174 | 8.86 068  | 175  | 1.13 931  | 9.99 886  | 51 | 50    | 150.8 | 150.0 | 148.3 |
| 10 | 8.86 128  | 174 | 8.86 243  | 175  | 1.13 756  | 9.99 885  | 50 |       |       |       |       |
| 11 | 8.86 301  | 173 | 8.86 417  | 174  | 1.13 582  | 9.99 884  | 49 | 6     | 17.4  | 17.2  | 17.0  |
| 12 | 8.86 474  | 172 | 8.86 590  | 173  | 1.13 409  | 9.99 883  | 48 | 7     | 20.3  | 20.0  | 19.8  |
| 13 | 8.86 645  | 171 | 8.86 763  | 172  | 1.13 237  | 9.99 882  | 47 | 8     | 23.2  | 22.9  | 22.6  |
| 14 | 8.86 816  | 171 | 8.86 935  | 172  | 1.13 065  | 9.99 881  | 46 | 9     | 26.1  | 25.8  | 25.5  |
| 15 | 8.86 987  | 170 | 8.87 106  | 171  | 1.12 893  | 9.99 880  | 45 | 10    | 29.0  | 28.6  | 28.3  |
| 16 | 8.87 156  | 169 | 8.87 277  | 170  | 1.12 723  | 9.99 879  | 44 | 20    | 58.0  | 57.3  | 56.6  |
| 17 | 8.87 325  | 169 | 8.87 447  | 169  | 1.12 553  | 9.99 878  | 43 | 30    | 87.0  | 86.0  | 85.0  |
| 18 | 8.87 494  | 167 | 8.87 616  | 169  | 1.12 384  | 9.99 877  | 42 | 40    | 116.0 | 114.6 | 113.3 |
| 19 | 8.87 661  | 167 | 8.87 785  | 169  | 1.12 215  | 9.99 876  | 41 | 50    | 145.0 | 143.3 | 141.6 |
| 20 | 8.87 828  | 166 | 8.87 953  | 168  | 1.12 047  | 9.99 875  | 40 |       |       |       |       |
| 21 | 8.87 995  | 165 | 8.88 120  | 167  | 1.11 880  | 9.99 874  | 39 | 6     | 16.6  | 16.4  | 16.2  |
| 22 | 8.88 160  | 165 | 8.88 287  | 167  | 1.11 713  | 9.99 873  | 38 | 7     | 19.3  | 19.1  | 18.9  |
| 23 | 8.88 326  | 164 | 8.88 453  | 166  | 1.11 547  | 9.99 873  | 37 | 8     | 22.1  | 21.8  | 21.6  |
| 24 | 8.88 490  | 164 | 8.88 618  | 165  | 1.11 381  | 9.99 872  | 36 | 9     | 24.9  | 24.6  | 24.3  |
| 25 | 8.88 654  | 163 | 8.88 783  | 165  | 1.11 216  | 9.99 871  | 35 | 10    | 27.6  | 27.3  | 27.0  |
| 26 | 8.88 817  | 162 | 8.88 947  | 164  | 1.11 052  | 9.99 870  | 34 | 20    | 55.3  | 54.6  | 54.0  |
| 27 | 8.88 980  | 162 | 8.89 111  | 163  | 1.10 889  | 9.99 869  | 33 | 30    | 83.0  | 82.0  | 81.0  |
| 28 | 8.89 142  | 161 | 8.89 274  | 162  | 1.10 726  | 9.99 868  | 32 | 40    | 110.6 | 109.3 | 108.0 |
| 29 | 8.89 303  | 161 | 8.89 436  | 162  | 1.10 563  | 9.99 867  | 31 | 50    | 138.3 | 136.6 | 135.0 |
| 30 | 8.89 464  | 160 | 8.89 598  | 161  | 1.10 401  | 9.99 866  | 30 |       |       |       |       |
| 31 | 8.89 624  | 159 | 8.89 759  | 161  | 1.10 240  | 9.99 865  | 29 | 6     | 15.8  | 15.6  | 15.4  |
| 32 | 8.89 784  | 159 | 8.89 920  | 160  | 1.10 079  | 9.99 864  | 28 | 7     | 18.4  | 18.2  | 17.9  |
| 33 | 8.89 943  | 158 | 8.90 080  | 159  | 1.09 919  | 9.99 863  | 27 | 8     | 21.0  | 20.8  | 20.5  |
| 34 | 8.90 101  | 158 | 8.90 240  | 159  | 1.09 760  | 9.99 862  | 26 | 9     | 23.7  | 23.4  | 23.1  |
| 35 | 8.90 259  | 157 | 8.90 398  | 158  | 1.09 601  | 9.99 861  | 25 | 10    | 26.3  | 26.0  | 25.6  |
| 36 | 8.90 417  | 156 | 8.90 557  | 157  | 1.09 442  | 9.99 860  | 24 | 20    | 52.6  | 52.0  | 51.3  |
| 37 | 8.90 573  | 156 | 8.90 714  | 157  | 1.09 285  | 9.99 859  | 23 | 30    | 79.0  | 78.0  | 77.0  |
| 38 | 8.90 729  | 156 | 8.90 872  | 156  | 1.09 128  | 9.99 858  | 22 | 40    | 105.3 | 104.0 | 102.6 |
| 39 | 8.90 885  | 155 | 8.91 028  | 156  | 1.08 971  | 9.99 857  | 21 | 50    | 131.6 | 130.0 | 128.3 |
| 40 | 8.91 040  | 154 | 8.91 184  | 155  | 1.08 815  | 9.99 856  | 20 |       |       |       |       |
| 41 | 8.91 195  | 154 | 8.91 340  | 155  | 1.08 660  | 9.99 855  | 19 | 6     | 15.0  | 14.9  | 14.8  |
| 42 | 8.91 349  | 153 | 8.91 495  | 154  | 1.08 505  | 9.99 854  | 18 | 7     | 17.5  | 17.4  | 17.2  |
| 43 | 8.91 502  | 153 | 8.91 649  | 154  | 1.08 350  | 9.99 853  | 17 | 8     | 20.0  | 19.8  | 19.7  |
| 44 | 8.91 655  | 152 | 8.91 803  | 154  | 1.08 196  | 9.99 852  | 16 | 9     | 22.5  | 22.3  | 22.2  |
| 45 | 8.91 807  | 151 | 8.91 957  | 153  | 1.08 043  | 9.99 851  | 15 | 10    | 25.0  | 24.8  | 24.6  |
| 46 | 8.91 959  | 151 | 8.92 109  | 152  | 1.07 890  | 9.99 849  | 14 | 20    | 50.0  | 49.6  | 49.3  |
| 47 | 8.92 110  | 150 | 8.92 262  | 151  | 1.07 738  | 9.99 848  | 13 | 30    | 75.0  | 74.5  | 74.0  |
| 48 | 8.92 261  | 150 | 8.92 413  | 151  | 1.07 586  | 9.99 847  | 12 | 40    | 100.0 | 99.3  | 98.6  |
| 49 | 8.92 411  | 150 | 8.92 565  | 151  | 1.07 435  | 9.99 846  | 11 | 50    | 125.0 | 124.1 | 123.3 |
| 50 | 8.92 561  | 149 | 8.92 715  | 150  | 1.07 284  | 9.99 845  | 10 |       |       |       |       |
| 51 | 8.92 710  | 148 | 8.92 866  | 149  | 1.07 134  | 9.99 844  | 9  | 6     | 14.6  | 14.5  | 14.4  |
| 52 | 8.92 858  | 148 | 8.93 016  | 149  | 1.06 984  | 9.99 843  | 8  | 7     | 17.0  | 16.9  | 16.8  |
| 53 | 8.93 007  | 147 | 8.93 164  | 149  | 1.06 835  | 9.99 842  | 7  | 8     | 19.4  | 19.3  | 19.2  |
| 54 | 8.93 154  | 147 | 8.93 313  | 149  | 1.06 686  | 9.99 841  | 6  | 9     | 21.9  | 21.7  | 21.6  |
| 55 | 8.93 301  | 146 | 8.93 461  | 148  | 1.06 538  | 9.99 840  | 5  | 10    | 24.3  | 24.1  | 24.0  |
| 56 | 8.93 448  | 146 | 8.93 609  | 147  | 1.06 390  | 9.99 839  | 4  | 20    | 48.6  | 48.3  | 48.0  |
| 57 | 8.93 594  | 146 | 8.93 756  | 146  | 1.06 243  | 9.99 837  | 3  | 30    | 73.0  | 72.5  | 72.0  |
| 58 | 8.93 740  | 145 | 8.93 903  | 146  | 1.06 097  | 9.99 836  | 2  | 40    | 97.3  | 96.6  | 96.0  |
| 59 | 8.93 885  | 144 | 8.94 049  | 145  | 1.05 950  | 9.99 835  | 1  | 50    | 121.6 | 120.8 | 120.0 |
| 60 | 8.94 029  |     | 8.94 195  |      | 1.05 805  | 9.99 834  | 0  |       |       |       |       |
|    | Log. Cos. | d.  | Log. Cot. | c.d. | Log. Tan. | Log. Sin. |    | P. P. |       |       |       |

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# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,

AND COTANGENTS.

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|    | Log. Sin. | d.  | Log. Tan. | c.d. | Log. Cot. | Log. Cos. |    | P. P.    |       |       |       |       |
|----|-----------|-----|-----------|------|-----------|-----------|----|----------|-------|-------|-------|-------|
| 0  | 8.94 029  | 141 | 8.94 195  | 145  | 1.05 805  | 9.99 834  | 60 | 145      | 144   | 143   | 142   | 141   |
| 1  | 8.94 174  | 143 | 8.94 340  | 144  | 1.05 659  | 9.99 835  | 59 | 6 14.5   | 14.4  | 14.3  | 14.2  | 14.1  |
| 2  | 8.94 317  | 143 | 8.94 485  | 144  | 1.05 513  | 9.99 832  | 58 | 7 16.5   | 16.8  | 16.7  | 16.5  | 16.4  |
| 3  | 8.94 430  | 143 | 8.94 629  | 144  | 1.05 370  | 9.99 831  | 57 | 8 19.3   | 19.2  | 19.0  | 18.9  | 18.8  |
| 4  | 8.94 603  | 143 | 8.94 773  | 143  | 1.05 226  | 9.99 830  | 56 | 9 21.7   | 21.6  | 21.4  | 21.3  | 21.1  |
| 5  | 8.94 745  | 142 | 8.94 917  | 142  | 1.05 083  | 9.99 829  | 55 | 10 24.1  | 24.0  | 23.8  | 23.6  | 23.5  |
| 6  | 8.94 887  | 141 | 8.95 059  | 142  | 1.04 940  | 9.99 827  | 54 | 20 48.3  | 48.0  | 47.6  | 47.3  | 47.0  |
| 7  | 8.95 028  | 141 | 8.95 202  | 142  | 1.04 796  | 9.99 826  | 53 | 30 72.5  | 72.0  | 71.5  | 71.0  | 70.5  |
| 8  | 8.95 169  | 140 | 8.95 344  | 141  | 1.04 656  | 9.99 825  | 52 | 40 96.6  | 96.0  | 95.3  | 94.6  | 94.0  |
| 9  | 8.95 310  | 140 | 8.95 485  | 141  | 1.04 514  | 9.99 824  | 51 | 50 123.8 | 120.6 | 119.1 | 118.3 | 117.5 |
| 10 | 8.95 450  | 139 | 8.95 626  | 141  | 1.04 373  | 9.99 823  | 50 |          | 140   | 139   | 138   | 137   |
| 11 | 8.95 589  | 139 | 8.95 767  | 140  | 1.04 232  | 9.99 822  | 49 | 6 14.0   | 13.9  | 13.8  | 13.7  | 13.6  |
| 12 | 8.95 728  | 138 | 8.95 907  | 140  | 1.04 092  | 9.99 821  | 48 | 7 16.3   | 16.2  | 16.1  | 16.0  | 15.8  |
| 13 | 8.95 867  | 138 | 8.96 047  | 139  | 1.03 952  | 9.99 819  | 47 | 8 18.6   | 18.5  | 18.4  | 18.2  | 18.1  |
| 14 | 8.96 005  | 138 | 8.96 186  | 139  | 1.03 813  | 9.99 818  | 46 | 9 21.0   | 20.8  | 20.7  | 20.5  | 20.4  |
| 15 | 8.96 143  | 137 | 8.96 325  | 138  | 1.03 674  | 9.99 817  | 45 | 10 23.3  | 23.1  | 23.0  | 22.8  | 22.6  |
| 16 | 8.96 280  | 137 | 8.96 464  | 138  | 1.03 536  | 9.99 815  | 44 | 20 46.6  | 46.3  | 46.0  | 45.6  | 45.3  |
| 17 | 8.96 417  | 137 | 8.96 602  | 137  | 1.03 398  | 9.99 815  | 43 | 30 70.0  | 69.5  | 69.0  | 68.5  | 68.0  |
| 18 | 8.96 553  | 136 | 8.96 739  | 137  | 1.03 260  | 9.99 814  | 42 | 40 93.3  | 92.6  | 92.0  | 91.3  | 90.6  |
| 19 | 8.96 689  | 136 | 8.96 876  | 137  | 1.03 123  | 9.99 813  | 41 | 50 116.6 | 115.2 | 115.0 | 114.1 | 113.3 |
| 20 | 8.96 825  | 135 | 8.97 013  | 137  | 1.02 986  | 9.99 811  | 40 |          | 135   | 134   | 133   | 132   |
| 21 | 8.96 960  | 135 | 8.97 149  | 136  | 1.02 850  | 9.99 810  | 39 | 6 13.5   | 13.4  | 13.3  | 13.2  |       |
| 22 | 8.97 094  | 134 | 8.97 285  | 136  | 1.02 714  | 9.99 809  | 38 | 7 15.7   | 15.6  | 15.5  | 15.4  |       |
| 23 | 8.97 229  | 134 | 8.97 421  | 135  | 1.02 579  | 9.99 808  | 37 | 8 18.0   | 17.8  | 17.7  | 17.6  |       |
| 24 | 8.97 363  | 134 | 8.97 556  | 135  | 1.02 444  | 9.99 807  | 36 | 9 20.2   | 20.1  | 19.9  | 19.8  |       |
| 25 | 8.97 493  | 133 | 8.97 690  | 134  | 1.02 309  | 9.99 805  | 35 | 10 22.5  | 22.3  | 22.1  | 22.0  |       |
| 26 | 8.97 629  | 133 | 8.97 825  | 134  | 1.02 175  | 9.99 804  | 34 | 20 45.0  | 44.6  | 44.3  | 44.0  |       |
| 27 | 8.97 762  | 132 | 8.97 953  | 133  | 1.02 041  | 9.99 803  | 33 | 30 67.5  | 67.0  | 66.5  | 66.0  |       |
| 28 | 8.97 894  | 132 | 8.98 092  | 133  | 1.01 908  | 9.99 802  | 32 | 40 90.0  | 89.3  | 88.6  | 88.0  |       |
| 29 | 8.98 026  | 132 | 8.98 225  | 133  | 1.01 775  | 9.99 801  | 31 | 50 112.5 | 111.6 | 110.8 | 110.0 |       |
| 30 | 8.98 157  | 131 | 8.98 357  | 132  | 1.01 642  | 9.99 799  | 30 |          | 131   | 130   | 129   | 128   |
| 31 | 8.98 288  | 131 | 8.98 490  | 132  | 1.01 510  | 9.99 798  | 29 | 6 13.1   | 13.0  | 12.9  | 12.8  |       |
| 32 | 8.98 419  | 130 | 8.98 621  | 131  | 1.01 378  | 9.99 797  | 28 | 7 15.3   | 15.1  | 15.0  | 14.9  |       |
| 33 | 8.98 549  | 130 | 8.98 753  | 131  | 1.01 247  | 9.99 796  | 27 | 8 17.4   | 17.3  | 17.2  | 17.0  |       |
| 34 | 8.98 679  | 130 | 8.98 884  | 131  | 1.01 116  | 9.99 794  | 26 | 9 19.6   | 19.5  | 19.3  | 19.2  |       |
| 35 | 8.98 803  | 129 | 8.99 015  | 131  | 1.00 985  | 9.99 793  | 25 | 10 21.8  | 21.6  | 21.5  | 21.3  |       |
| 36 | 8.98 937  | 129 | 8.99 145  | 130  | 1.00 855  | 9.99 792  | 24 | 20 43.6  | 43.3  | 43.0  | 42.6  |       |
| 37 | 8.99 066  | 128 | 8.99 275  | 129  | 1.00 725  | 9.99 791  | 23 | 30 65.5  | 65.0  | 64.5  | 64.0  |       |
| 38 | 8.99 194  | 127 | 8.99 404  | 129  | 1.00 595  | 9.99 789  | 22 | 40 87.3  | 86.6  | 86.0  | 85.3  |       |
| 39 | 8.99 322  | 127 | 8.99 533  | 129  | 1.00 466  | 9.99 788  | 21 | 50 109.1 | 108.3 | 107.5 | 106.6 |       |
| 40 | 8.99 449  | 127 | 8.99 662  | 128  | 1.00 337  | 9.99 787  | 20 |          | 127   | 126   | 125   | 124   |
| 41 | 8.99 577  | 126 | 8.99 791  | 128  | 1.00 209  | 9.99 786  | 19 | 6 12.7   | 12.6  | 12.5  | 12.4  | 12.3  |
| 42 | 8.99 703  | 126 | 8.99 919  | 127  | 1.00 081  | 9.99 784  | 18 | 7 14.8   | 14.7  | 14.6  | 14.4  | 14.3  |
| 43 | 8.99 830  | 126 | 9.00 046  | 127  | 0.99 953  | 9.99 783  | 17 | 8 16.9   | 16.8  | 16.6  | 16.5  | 16.4  |
| 44 | 8.99 958  | 125 | 9.00 174  | 126  | 0.99 826  | 9.99 782  | 16 | 9 19.0   | 18.9  | 18.7  | 18.5  | 18.4  |
| 45 | 9.00 081  | 125 | 9.00 300  | 126  | 0.99 699  | 9.99 781  | 15 | 10 21.1  | 21.0  | 20.8  | 20.6  | 20.5  |
| 46 | 9.00 207  | 125 | 9.00 427  | 126  | 0.99 573  | 9.99 779  | 14 | 20 42.5  | 42.0  | 41.6  | 41.3  | 41.0  |
| 47 | 9.00 332  | 124 | 9.00 553  | 125  | 0.99 446  | 9.99 778  | 13 | 30 63.5  | 63.0  | 62.5  | 62.0  | 61.5  |
| 48 | 9.00 456  | 124 | 9.00 679  | 125  | 0.99 320  | 9.99 777  | 12 | 40 84.6  | 84.0  | 83.3  | 82.6  | 82.0  |
| 49 | 9.00 580  | 124 | 9.00 804  | 125  | 0.99 195  | 9.99 776  | 11 | 50 105.8 | 105.0 | 104.1 | 103.3 | 102.5 |
| 50 | 9.00 704  | 123 | 9.00 930  | 125  | 0.99 070  | 9.99 774  | 10 |          | 122   | 121   | 120   | 119   |
| 51 | 9.00 828  | 123 | 9.01 054  | 124  | 0.98 945  | 9.99 773  | 9  | 6 12.2   | 12.1  | 12.0  | 11.9  | 11.8  |
| 52 | 9.00 951  | 122 | 9.01 179  | 124  | 0.98 821  | 9.99 772  | 8  | 7 14.2   | 14.1  | 14.0  | 13.9  | 13.8  |
| 53 | 9.01 073  | 122 | 9.01 303  | 124  | 0.98 697  | 9.99 770  | 7  | 8 16.2   | 16.1  | 16.0  | 15.9  | 15.8  |
| 54 | 9.01 198  | 122 | 9.01 427  | 124  | 0.98 573  | 9.99 769  | 6  | 9 18.3   | 18.1  | 18.0  | 17.9  | 17.8  |
| 55 | 9.01 318  | 122 | 9.01 550  | 123  | 0.98 450  | 9.99 768  | 5  | 10 20.3  | 20.1  | 20.0  | 19.9  | 19.8  |
| 56 | 9.01 440  | 121 | 9.01 673  | 123  | 0.98 327  | 9.99 766  | 4  | 20 40.6  | 40.3  | 40.0  | 39.7  | 39.5  |
| 57 | 9.01 561  | 121 | 9.01 796  | 122  | 0.98 204  | 9.99 765  | 3  | 30 61.0  | 60.5  | 60.0  | 59.5  | 59.2  |
| 58 | 9.01 682  | 120 | 9.01 918  | 122  | 0.98 081  | 9.99 764  | 2  | 40 81.3  | 80.6  | 80.0  | 79.3  | 78.8  |
| 59 | 9.01 803  | 120 | 9.02 040  | 121  | 0.97 959  | 9.99 763  | 1  | 50 101.6 | 100.8 | 100.0 | 99.2  | 98.4  |
| 60 | 9.01 923  | 120 | 9.02 162  | 121  | 0.97 838  | 9.99 761  | 0  |          | 122   | 121   | 120   | 119   |
|    | Log. Cos. | d.  | Log. Cot. | c.d. | Log. Tan. | Log. Sin. |    | P. P.    |       |       |       |       |



**TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.**

**6°**

|     | Log. Sin. | d.    | Log. Tan. c.d. | Log. Cot. | Log. Cos. |           | P. P. |
|-----|-----------|-------|----------------|-----------|-----------|-----------|-------|
| 0   | 9.01 923  | 120   | 9.02 162       | 121       | 0.97 838  | 9.99 761  | 60    |
| 1   | 9.02 043  | 119   | 9.02 283       | 121       | 0.97 716  | 9.99 760  | 59    |
| 2   | 9.02 163  | 119   | 9.02 404       | 120       | 0.97 595  | 9.99 759  | 58    |
| 3   | 9.02 282  | 119   | 9.02 525       | 120       | 0.97 475  | 9.99 757  | 57    |
| 4   | 9.02 401  | 119   | 9.02 645       | 120       | 0.97 354  | 9.99 756  | 56    |
| 5   | 9.02 520  | 119   | 9.02 765       | 120       | 0.97 234  | 9.99 754  | 55    |
| 6   | 9.02 638  | 118   | 9.02 885       | 119       | 0.97 115  | 9.99 753  | 54    |
| 7   | 9.02 756  | 118   | 9.03 004       | 119       | 0.96 995  | 9.99 752  | 53    |
| 8   | 9.02 874  | 118   | 9.03 123       | 119       | 0.96 876  | 9.99 750  | 52    |
| 9   | 9.02 992  | 117   | 9.03 242       | 119       | 0.96 757  | 9.99 748  | 51    |
| 10  | 9.03 109  | 117   | 9.03 361       | 118       | 0.96 639  | 9.99 748  | 50    |
| 11  | 9.03 225  | 116   | 9.03 479       | 118       | 0.96 521  | 9.99 746  | 49    |
| 12  | 9.03 342  | 116   | 9.03 597       | 118       | 0.96 403  | 9.99 745  | 48    |
| 13  | 9.03 458  | 116   | 9.03 714       | 117       | 0.96 285  | 9.99 744  | 47    |
| 14  | 9.03 574  | 116   | 9.03 831       | 117       | 0.96 168  | 9.99 742  | 46    |
| 15  | 9.03 689  | 115   | 9.03 948       | 117       | 0.96 051  | 9.99 741  | 45    |
| 16  | 9.03 805  | 115   | 9.04 065       | 116       | 0.95 935  | 9.99 739  | 44    |
| 17  | 9.03 919  | 114   | 9.04 181       | 116       | 0.95 818  | 9.99 738  | 43    |
| 18  | 9.04 034  | 114   | 9.04 297       | 115       | 0.95 702  | 9.99 737  | 42    |
| 19  | 9.04 148  | 114   | 9.04 413       | 115       | 0.95 587  | 9.99 735  | 41    |
| 20  | 9.04 262  | 114   | 9.04 528       | 115       | 0.95 471  | 9.99 734  | 40    |
| 21  | 9.04 376  | 113   | 9.04 643       | 115       | 0.95 356  | 9.99 732  | 39    |
| 22  | 9.04 489  | 113   | 9.04 758       | 114       | 0.95 240  | 9.99 731  | 38    |
| 23  | 9.04 602  | 113   | 9.04 872       | 114       | 0.95 127  | 9.99 730  | 37    |
| 24  | 9.04 715  | 113   | 9.04 987       | 114       | 0.95 013  | 9.99 728  | 36    |
| 25  | 9.04 828  | 112   | 9.05 101       | 114       | 0.94 899  | 9.99 727  | 35    |
| 26  | 9.04 940  | 112   | 9.05 214       | 113       | 0.94 785  | 9.99 725  | 34    |
| 27  | 9.05 052  | 111   | 9.05 327       | 113       | 0.94 672  | 9.99 724  | 33    |
| 28  | 9.05 163  | 111   | 9.05 440       | 113       | 0.94 559  | 9.99 723  | 32    |
| 29  | 9.05 275  | 111   | 9.05 553       | 113       | 0.94 446  | 9.99 721  | 31    |
| 30  | 9.05 388  | 111   | 9.05 666       | 112       | 0.94 334  | 9.99 720  | 30    |
| 31  | 9.05 496  | 110   | 9.05 778       | 112       | 0.94 222  | 9.99 718  | 29    |
| 32  | 9.05 607  | 110   | 9.05 890       | 112       | 0.94 110  | 9.99 717  | 28    |
| 33  | 9.05 717  | 110   | 9.06 001       | 111       | 0.93 998  | 9.99 715  | 27    |
| 34  | 9.05 827  | 109   | 9.06 113       | 111       | 0.93 887  | 9.99 714  | 26    |
| 35  | 9.05 936  | 109   | 9.06 224       | 111       | 0.93 776  | 9.99 712  | 25    |
| 36  | 9.06 046  | 109   | 9.06 335       | 110       | 0.93 665  | 9.99 711  | 24    |
| 37  | 9.06 155  | 108   | 9.06 445       | 110       | 0.93 554  | 9.99 710  | 23    |
| 38  | 9.06 264  | 108   | 9.06 555       | 110       | 0.93 444  | 9.99 708  | 22    |
| 39  | 9.06 373  | 108   | 9.06 665       | 110       | 0.93 334  | 9.99 707  | 21    |
| 40  | 9.06 480  | 108   | 9.06 775       | 109       | 0.93 223  | 9.99 705  | 20    |
| 41  | 9.06 588  | 107   | 9.06 884       | 109       | 0.93 115  | 9.99 704  | 19    |
| 42  | 9.06 696  | 107   | 9.06 994       | 109       | 0.93 006  | 9.99 702  | 18    |
| 43  | 9.06 803  | 107   | 9.07 102       | 109       | 0.92 897  | 9.99 701  | 17    |
| 44  | 9.06 910  | 107   | 9.07 211       | 109       | 0.92 788  | 9.99 699  | 16    |
| 45  | 9.07 017  | 107   | 9.07 319       | 108       | 0.92 680  | 9.99 698  | 15    |
| 46  | 9.07 124  | 106   | 9.07 428       | 107       | 0.92 572  | 9.99 696  | 14    |
| 47  | 9.07 230  | 106   | 9.07 535       | 107       | 0.92 464  | 9.99 695  | 13    |
| 48  | 9.07 336  | 106   | 9.07 643       | 107       | 0.92 357  | 9.99 693  | 12    |
| 49  | 9.07 442  | 105   | 9.07 750       | 107       | 0.92 249  | 9.99 692  | 11    |
| 50  | 9.07 548  | 105   | 9.07 857       | 107       | 0.92 142  | 9.99 690  | 10    |
| 51  | 9.07 653  | 105   | 9.07 964       | 106       | 0.92 035  | 9.99 689  | 9     |
| 52  | 9.07 758  | 104   | 9.08 071       | 106       | 0.91 929  | 9.99 687  | 8     |
| 53  | 9.07 863  | 104   | 9.08 177       | 106       | 0.91 822  | 9.99 686  | 7     |
| 54  | 9.07 967  | 104   | 9.08 283       | 106       | 0.91 716  | 9.99 684  | 6     |
| 55  | 9.08 072  | 104   | 9.08 389       | 105       | 0.91 611  | 9.99 683  | 5     |
| 56  | 9.08 176  | 103   | 9.08 494       | 105       | 0.91 505  | 9.99 681  | 4     |
| 57  | 9.08 279  | 103   | 9.08 600       | 105       | 0.91 400  | 9.99 679  | 3     |
| 58  | 9.08 383  | 103   | 9.08 705       | 105       | 0.91 295  | 9.99 678  | 2     |
| 59  | 9.08 486  | 103   | 9.08 810       | 104       | 0.91 190  | 9.99 676  | 1     |
| 60  | 9.08 589  | 103   | 9.08 914       | 104       | 0.91 085  | 9.99 675  | 0     |
|     | Log. Cos. | d.    | Log. Cot.      | c.d.      | Log. Tan. | Log. Sin. |       |
| 61  | 121       | 121   | 120            | 119       | 121       | 121       | 119   |
| 62  | 12.1      | 12.1  | 12.0           | 11.9      | 12.1      | 12.1      | 11.9  |
| 63  | 14.2      | 14.1  | 14.0           | 13.9      | 14.2      | 14.1      | 14.0  |
| 64  | 16.2      | 16.1  | 16.0           | 15.9      | 16.2      | 16.1      | 16.0  |
| 65  | 18.2      | 18.1  | 18.0           | 17.8      | 18.2      | 18.1      | 18.0  |
| 66  | 20.2      | 20.1  | 20.0           | 19.8      | 20.2      | 20.1      | 20.0  |
| 67  | 40.5      | 40.3  | 40.0           | 39.6      | 40.5      | 40.3      | 40.0  |
| 68  | 60.7      | 60.5  | 60.0           | 59.5      | 60.7      | 60.5      | 60.0  |
| 69  | 81.0      | 80.6  | 80.0           | 79.3      | 81.0      | 80.6      | 80.0  |
| 70  | 101.2     | 100.8 | 100.0          | 99.1      | 101.2     | 100.8     | 100.0 |
| 71  | 117       | 117   | 116            | 115       | 117       | 117       | 116   |
| 72  | 6 11.7    | 11.7  | 11.6           | 11.5      | 6 11.7    | 11.7      | 11.6  |
| 73  | 7 13.7    | 13.6  | 13.5           | 13.4      | 7 13.7    | 13.6      | 13.5  |
| 74  | 8 15.6    | 15.6  | 15.4           | 15.3      | 8 15.6    | 15.6      | 15.4  |
| 75  | 9 17.6    | 17.5  | 17.4           | 17.2      | 9 17.6    | 17.5      | 17.4  |
| 76  | 10 19.6   | 19.5  | 19.3           | 19.1      | 10 19.6   | 19.5      | 19.3  |
| 77  | 20 39.1   | 39.0  | 38.6           | 38.3      | 20 39.1   | 39.0      | 38.6  |
| 78  | 30 58.7   | 58.5  | 58.0           | 57.5      | 30 58.7   | 58.5      | 58.0  |
| 79  | 40 78.3   | 78.0  | 77.3           | 76.6      | 40 78.3   | 78.0      | 77.3  |
| 80  | 50 97.9   | 97.5  | 96.6           | 95.8      | 50 97.9   | 97.5      | 96.6  |
| 81  | 114       | 114   | 113            | 112       | 114       | 114       | 113   |
| 82  | 6 11.4    | 11.4  | 11.3           | 11.2      | 6 11.4    | 11.4      | 11.3  |
| 83  | 7 13.3    | 13.3  | 13.2           | 13.0      | 7 13.3    | 13.3      | 13.2  |
| 84  | 8 15.2    | 15.2  | 15.0           | 14.9      | 8 15.2    | 15.2      | 15.0  |
| 85  | 9 17.2    | 17.1  | 16.9           | 16.8      | 9 17.2    | 17.1      | 16.9  |
| 86  | 10 19.1   | 19.0  | 18.8           | 18.6      | 10 19.1   | 19.0      | 18.8  |
| 87  | 20 38.1   | 38.0  | 37.6           | 37.3      | 20 38.1   | 38.0      | 37.6  |
| 88  | 30 57.2   | 57.0  | 56.5           | 56.0      | 30 57.2   | 57.0      | 56.5  |
| 89  | 40 76.3   | 76.0  | 75.3           | 74.6      | 40 76.3   | 76.0      | 75.3  |
| 90  | 50 95.4   | 95.0  | 94.1           | 93.3      | 50 95.4   | 95.0      | 94.1  |
| 91  | 110       | 110   | 109            | 108       | 110       | 110       | 109   |
| 92  | 6 11.0    | 11.0  | 10.9           | 10.8      | 6 11.0    | 11.0      | 10.9  |
| 93  | 7 12.9    | 12.8  | 12.7           | 12.6      | 7 12.9    | 12.8      | 12.7  |
| 94  | 8 14.7    | 14.6  | 14.5           | 14.4      | 8 14.7    | 14.6      | 14.5  |
| 95  | 9 16.6    | 16.5  | 16.3           | 16.2      | 9 16.6    | 16.5      | 16.3  |
| 96  | 10 18.4   | 18.3  | 18.1           | 18.0      | 10 18.4   | 18.3      | 18.1  |
| 97  | 20 36.8   | 36.6  | 36.3           | 36.0      | 20 36.8   | 36.6      | 36.3  |
| 98  | 30 55.2   | 55.0  | 54.5           | 54.0      | 30 55.2   | 55.0      | 54.5  |
| 99  | 40 73.6   | 73.3  | 72.6           | 72.0      | 40 73.6   | 73.3      | 72.6  |
| 100 | 50 92.1   | 91.6  | 90.8           | 90.0      | 50 92.1   | 91.6      | 90.8  |
| 101 | 107       | 107   | 106            | 105       | 107       | 107       | 106   |
| 102 | 6 10.7    | 10.7  | 10.6           | 10.5      | 6 10.7    | 10.7      | 10.6  |
| 103 | 7 12.5    | 12.5  | 12.3           | 12.2      | 7 12.5    | 12.5      | 12.3  |
| 104 | 8 14.3    | 14.2  | 14.1           | 14.0      | 8 14.3    | 14.2      | 14.1  |
| 105 | 9 16.1    | 16.0  | 15.9           | 15.7      | 9 16.1    | 16.0      | 15.9  |
| 106 | 10 17.9   | 17.8  | 17.6           | 17.5      | 10 17.9   | 17.8      | 17.6  |
| 107 | 20 35.8   | 35.6  | 35.3           | 35.0      | 20 35.8   | 35.6      | 35.3  |
| 108 | 30 53.7   | 53.5  | 53.0           | 52.5      | 30 53.7   | 53.5      | 53.0  |
| 109 | 40 71.6   | 71.3  | 70.6           | 70.0      | 40 71.6   | 71.3      | 70.6  |
| 110 | 50 89.6   | 89.1  | 88.3           | 87.5      | 50 89.6   | 89.1      | 88.3  |
| 111 | 103       | 103   | 102            | 101       | 103       | 103       | 102   |
| 112 | 6 10.3    | 10.3  | 0.2            | 0.1       | 6 10.3    | 10.3      | 0.2   |
| 113 | 7 12.1    | 12.0  | 0.2            | 0.1       | 7 12.1    | 12.0      | 0.2   |
| 114 | 8 13.8    | 13.7  | 0.2            | 0.1       | 8 13.8    | 13.7      | 0.2   |
| 115 | 9 15.5    | 15.4  | 0.3            | 0.2       | 9 15.5    | 15.4      | 0.3   |
| 116 | 10 17.2   | 17.1  | 0.3            | 0.2       | 10 17.2   | 17.1      | 0.3   |
| 117 | 20 34.5   | 34.3  | 0.6            | 0.5       | 20 34.5   | 34.3      | 0.6   |
| 118 | 30 51.7   | 51.5  | 1.0            | 0.7       | 30 51.7   | 51.5      | 1.0   |
| 119 | 40 69.0   | 68.6  | 1.3            | 1.0       | 40 69.0   | 68.6      | 1.3   |
| 120 | 50 86.2   | 85.8  | 1.6            | 1.2       | 50 86.2   | 85.8      | 1.6   |
|     | P. P.     |       |                |           |           |           |       |

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# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d.  | Log. Tan. | c.d. | Log. Cot. | Log. Cos. |    | P. P.                  |
|----|-----------|-----|-----------|------|-----------|-----------|----|------------------------|
| 0  | 9.08 589  | 102 | 9.08 914  | 104  | 0.91 085  | 9.99 675  | 60 |                        |
| 1  | 9.08 692  | 102 | 9.09 018  | 104  | 0.90 981  | 9.99 673  | 59 | 104 103 102 101        |
| 2  | 9.08 794  | 102 | 9.09 123  | 103  | 0.90 877  | 9.99 672  | 58 | 6 10.4 10.3 10.2 10.1  |
| 3  | 9.08 897  | 102 | 9.09 226  | 103  | 0.90 773  | 9.99 670  | 57 | 7 12.1 12.0 11.9 11.8  |
| 4  | 9.08 999  | 102 | 9.09 330  | 103  | 0.90 670  | 9.99 669  | 56 | 8 13.8 13.7 13.6 13.5  |
| 5  | 9.09 101  | 101 | 9.09 433  | 103  | 0.90 566  | 9.99 667  | 55 | 9 15.6 15.4 15.3 15.1  |
| 6  | 9.09 202  | 101 | 9.09 536  | 103  | 0.90 463  | 9.99 665  | 54 | 10 17.3 17.1 17.0 16.8 |
| 7  | 9.09 303  | 101 | 9.09 639  | 102  | 0.90 360  | 9.99 664  | 53 | 20 34.6 34.3 34.0 33.6 |
| 8  | 9.09 404  | 101 | 9.09 742  | 102  | 0.90 258  | 9.99 662  | 52 | 30 52.0 51.5 51.0 50.5 |
| 9  | 9.09 505  | 100 | 9.09 844  | 102  | 0.90 155  | 9.99 661  | 51 | 40 69.3 68.6 68.0 67.3 |
| 10 | 9.09 606  | 100 | 9.09 947  | 102  | 0.90 053  | 9.99 659  | 50 | 50 86.6 85.3 85.0 84.1 |
| 11 | 9.09 706  | 100 | 9.10 048  | 101  | 0.89 951  | 9.99 658  | 49 |                        |
| 12 | 9.09 806  | 100 | 9.10 150  | 101  | 0.89 849  | 9.99 656  | 48 |                        |
| 13 | 9.09 906  | 99  | 9.10 252  | 101  | 0.89 748  | 9.99 654  | 47 |                        |
| 14 | 9.10 003  | 99  | 9.10 353  | 101  | 0.89 647  | 9.99 653  | 46 | 100 100 99 98          |
| 15 | 9.10 105  | 99  | 9.10 454  | 101  | 0.89 546  | 9.99 651  | 45 | 6 11.0 11.0 9.9 9.8    |
| 16 | 9.10 205  | 98  | 9.10 555  | 100  | 0.89 445  | 9.99 650  | 44 | 7 11.7 11.6 11.5 11.4  |
| 17 | 9.10 303  | 98  | 9.10 655  | 100  | 0.89 344  | 9.99 648  | 43 | 8 13.4 13.3 13.2 13.0  |
| 18 | 9.10 402  | 98  | 9.10 756  | 100  | 0.89 244  | 9.99 646  | 42 | 9 15.1 15.0 14.8 14.7  |
| 19 | 9.10 501  | 98  | 9.10 854  | 100  | 0.89 144  | 9.99 645  | 41 | 10 16.7 16.6 16.5 16.3 |
| 20 | 9.10 599  | 98  | 9.10 956  | 99   | 0.89 044  | 9.99 643  | 40 | 20 33.5 33.3 33.0 32.6 |
| 21 | 9.10 697  | 97  | 9.11 055  | 99   | 0.88 944  | 9.99 641  | 39 | 30 50.2 50.0 49.5 49.0 |
| 22 | 9.10 795  | 97  | 9.11 155  | 99   | 0.88 845  | 9.99 640  | 38 | 40 67.0 66.6 66.0 65.3 |
| 23 | 9.10 892  | 97  | 9.11 254  | 99   | 0.88 745  | 9.99 638  | 37 | 50 83.7 83.3 82.5 81.6 |
| 24 | 9.10 990  | 97  | 9.11 353  | 98   | 0.88 646  | 9.99 637  | 36 |                        |
| 25 | 9.11 087  | 96  | 9.11 452  | 98   | 0.88 548  | 9.99 635  | 35 |                        |
| 26 | 9.11 184  | 96  | 9.11 550  | 98   | 0.88 449  | 9.99 633  | 34 | 97 97 96 95            |
| 27 | 9.11 281  | 96  | 9.11 649  | 98   | 0.88 351  | 9.99 632  | 33 | 6 9.7 9.7 9.6 9.5      |
| 28 | 9.11 377  | 96  | 9.11 747  | 98   | 0.88 253  | 9.99 630  | 32 | 7 11.4 11.3 11.2 11.1  |
| 29 | 9.11 473  | 95  | 9.11 845  | 98   | 0.88 155  | 9.99 628  | 31 | 8 13.0 12.9 12.8 12.6  |
| 30 | 9.11 570  | 95  | 9.11 943  | 98   | 0.88 057  | 9.99 627  | 30 | 9 14.6 14.5 14.4 14.1  |
| 31 | 9.11 665  | 96  | 9.12 040  | 97   | 0.87 959  | 9.99 625  | 29 | 10 16.2 16.1 16.0 15.8 |
| 32 | 9.11 761  | 95  | 9.12 137  | 97   | 0.87 862  | 9.99 623  | 28 | 20 32.5 32.3 32.0 31.6 |
| 33 | 9.11 856  | 95  | 9.12 235  | 97   | 0.87 763  | 9.99 622  | 27 | 30 48.7 48.5 48.0 47.5 |
| 34 | 9.11 952  | 95  | 9.12 331  | 96   | 0.87 668  | 9.99 620  | 26 | 40 65.0 64.6 64.0 63.3 |
| 35 | 9.12 047  | 94  | 9.12 428  | 97   | 0.87 571  | 9.99 618  | 25 | 50 81.2 80.8 80.0 79.1 |
| 36 | 9.12 141  | 94  | 9.12 525  | 96   | 0.87 475  | 9.99 617  | 24 |                        |
| 37 | 9.12 236  | 94  | 9.12 621  | 96   | 0.87 379  | 9.99 615  | 23 |                        |
| 38 | 9.12 330  | 94  | 9.12 717  | 96   | 0.87 283  | 9.99 613  | 22 | 94 94 93 92            |
| 39 | 9.12 425  | 93  | 9.12 813  | 95   | 0.87 187  | 9.99 611  | 21 | 6 9.4 9.4 9.3 9.2      |
| 40 | 9.12 518  | 94  | 9.12 908  | 95   | 0.87 091  | 9.99 610  | 20 | 7 11.0 10.9 10.8 10.7  |
| 41 | 9.12 612  | 93  | 9.13 004  | 95   | 0.86 996  | 9.99 608  | 19 | 8 12.6 12.5 12.4 12.2  |
| 42 | 9.12 706  | 93  | 9.13 099  | 95   | 0.86 900  | 9.99 606  | 18 | 9 14.2 14.1 13.9 13.8  |
| 43 | 9.12 799  | 93  | 9.13 194  | 95   | 0.86 805  | 9.99 605  | 17 | 10 15.7 15.6 15.5 15.3 |
| 44 | 9.12 892  | 93  | 9.13 289  | 94   | 0.86 710  | 9.99 603  | 16 | 20 31.5 31.3 31.0 30.6 |
| 45 | 9.12 985  | 92  | 9.13 384  | 94   | 0.86 616  | 9.99 601  | 15 | 30 47.2 47.0 46.5 46.0 |
| 46 | 9.13 078  | 92  | 9.13 478  | 94   | 0.86 521  | 9.99 600  | 14 | 40 63.0 62.6 62.0 61.3 |
| 47 | 9.13 170  | 92  | 9.13 572  | 94   | 0.86 427  | 9.99 598  | 13 | 50 78.7 78.3 77.5 76.6 |
| 48 | 9.13 263  | 92  | 9.13 666  | 94   | 0.86 333  | 9.99 596  | 12 |                        |
| 49 | 9.13 355  | 92  | 9.13 760  | 94   | 0.86 239  | 9.99 594  | 11 |                        |
| 50 | 9.13 447  | 91  | 9.13 854  | 93   | 0.86 146  | 9.99 593  | 10 | 91 91 90 2             |
| 51 | 9.13 538  | 92  | 9.13 947  | 93   | 0.86 052  | 9.99 591  | 9  | 6 9.1 9.1 9.0 0.2      |
| 52 | 9.13 630  | 91  | 9.14 041  | 93   | 0.85 959  | 9.99 589  | 8  | 7 10.7 10.6 10.5 0.2   |
| 53 | 9.13 721  | 91  | 9.14 134  | 93   | 0.85 866  | 9.99 587  | 7  | 8 12.2 12.1 12.0 0.2   |
| 54 | 9.13 813  | 90  | 9.14 227  | 93   | 0.85 773  | 9.99 586  | 6  | 9 13.7 13.6 13.5 0.3   |
| 55 | 9.13 903  | 91  | 9.14 319  | 92   | 0.85 680  | 9.99 584  | 5  | 10 15.2 15.1 15.0 0.3  |
| 56 | 9.13 994  | 90  | 9.14 412  | 92   | 0.85 588  | 9.99 582  | 4  | 20 30.5 30.3 30.0 0.6  |
| 57 | 9.14 085  | 90  | 9.14 504  | 92   | 0.85 495  | 9.99 580  | 3  | 30 45.7 45.5 45.0 1.0  |
| 58 | 9.14 175  | 90  | 9.14 596  | 92   | 0.85 403  | 9.99 579  | 2  | 40 61.0 60.6 60.0 1.3  |
| 59 | 9.14 265  | 90  | 9.14 688  | 92   | 0.85 311  | 9.99 577  | 1  | 50 76.2 75.8 75.0 1.6  |
| 60 | 9.14 355  | 90  | 9.14 780  | 92   | 0.85 219  | 9.99 575  | 0  |                        |
|    | Log. Cos. | d.  | Log. Cot. | c.d. | Log. Tan. | Log. Sin. |    | P. P.                  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c.d. | Log. Cot. | Log. Cos. |    | P. P. |
|----|-----------|----|-----------|------|-----------|-----------|----|-------|
| 0  | 9.19 433  |    | 9.19 971  | 81   | 0.80 029  | 9.99 462  | 60 |       |
| 1  | 9.19 513  | 80 | 9.20 033  | 81   | 0.79 947  | 9.99 460  | 59 |       |
| 2  | 9.19 532  | 79 | 9.20 134  | 81   | 0.79 835  | 9.99 458  | 58 |       |
| 3  | 9.19 672  | 79 | 9.20 216  | 81   | 0.79 784  | 9.99 456  | 57 |       |
| 4  | 9.19 751  | 79 | 9.20 297  | 81   | 0.79 703  | 9.99 454  | 56 |       |
| 5  | 9.19 830  | 79 | 9.20 378  | 81   | 0.79 622  | 9.99 452  | 55 |       |
| 6  | 9.19 909  | 79 | 9.20 459  | 81   | 0.79 541  | 9.99 450  | 54 |       |
| 7  | 9.19 986  | 78 | 9.20 540  | 80   | 0.79 460  | 9.99 448  | 53 |       |
| 8  | 9.20 066  | 78 | 9.20 620  | 81   | 0.79 379  | 9.99 446  | 52 |       |
| 9  | 9.20 145  | 78 | 9.20 701  | 81   | 0.79 298  | 9.99 444  | 51 |       |
| 10 | 9.20 223  | 78 | 9.20 781  | 80   | 0.79 218  | 9.99 442  | 50 |       |
| 11 | 9.20 301  | 78 | 9.20 862  | 80   | 0.79 138  | 9.99 440  | 49 |       |
| 12 | 9.20 379  | 78 | 9.20 942  | 80   | 0.79 056  | 9.99 437  | 48 |       |
| 13 | 9.20 457  | 78 | 9.21 022  | 80   | 0.78 976  | 9.99 435  | 47 |       |
| 14 | 9.20 535  | 78 | 9.21 102  | 80   | 0.78 896  | 9.99 433  | 46 |       |
| 15 | 9.20 612  | 77 | 9.21 181  | 79   | 0.78 816  | 9.99 431  | 45 |       |
| 16 | 9.20 690  | 77 | 9.21 261  | 79   | 0.78 733  | 9.99 429  | 44 |       |
| 17 | 9.20 768  | 77 | 9.21 340  | 79   | 0.78 650  | 9.99 427  | 43 |       |
| 18 | 9.20 845  | 77 | 9.21 420  | 79   | 0.78 568  | 9.99 425  | 42 |       |
| 19 | 9.20 922  | 77 | 9.21 499  | 79   | 0.78 501  | 9.99 423  | 41 |       |
| 20 | 9.20 999  | 77 | 9.21 578  | 79   | 0.78 422  | 9.99 421  | 40 |       |
| 21 | 9.21 073  | 76 | 9.21 657  | 78   | 0.78 343  | 9.99 419  | 39 |       |
| 22 | 9.21 152  | 76 | 9.21 735  | 78   | 0.78 264  | 9.99 417  | 38 |       |
| 23 | 9.21 229  | 76 | 9.21 814  | 78   | 0.78 185  | 9.99 415  | 37 |       |
| 24 | 9.21 305  | 76 | 9.21 892  | 78   | 0.78 107  | 9.99 413  | 36 |       |
| 25 | 9.21 382  | 76 | 9.21 971  | 78   | 0.78 029  | 9.99 411  | 35 |       |
| 26 | 9.21 458  | 76 | 9.22 049  | 78   | 0.77 951  | 9.99 408  | 34 |       |
| 27 | 9.21 534  | 75 | 9.22 127  | 78   | 0.77 873  | 9.99 406  | 33 |       |
| 28 | 9.21 609  | 76 | 9.22 205  | 78   | 0.77 795  | 9.99 404  | 32 |       |
| 29 | 9.21 685  | 75 | 9.22 283  | 77   | 0.77 717  | 9.99 402  | 31 |       |
| 30 | 9.21 761  | 75 | 9.22 360  | 77   | 0.77 639  | 9.99 400  | 30 |       |
| 31 | 9.21 836  | 75 | 9.22 438  | 77   | 0.77 562  | 9.99 398  | 29 |       |
| 32 | 9.21 911  | 75 | 9.22 515  | 77   | 0.77 484  | 9.99 396  | 28 |       |
| 33 | 9.21 987  | 75 | 9.22 593  | 77   | 0.77 407  | 9.99 394  | 27 |       |
| 34 | 9.22 062  | 74 | 9.22 670  | 77   | 0.77 330  | 9.99 392  | 26 |       |
| 35 | 9.22 136  | 74 | 9.22 747  | 77   | 0.77 253  | 9.99 389  | 25 |       |
| 36 | 9.22 211  | 74 | 9.22 824  | 76   | 0.77 176  | 9.99 387  | 24 |       |
| 37 | 9.22 283  | 74 | 9.22 900  | 76   | 0.77 099  | 9.99 385  | 23 |       |
| 38 | 9.22 360  | 74 | 9.22 977  | 76   | 0.77 022  | 9.99 383  | 22 |       |
| 39 | 9.22 435  | 74 | 9.23 054  | 76   | 0.76 946  | 9.99 381  | 21 |       |
| 40 | 9.22 509  | 74 | 9.23 130  | 76   | 0.76 870  | 9.99 379  | 20 |       |
| 41 | 9.22 583  | 74 | 9.23 206  | 76   | 0.76 793  | 9.99 377  | 19 |       |
| 42 | 9.22 657  | 73 | 9.23 282  | 76   | 0.76 717  | 9.99 374  | 18 |       |
| 43 | 9.22 731  | 74 | 9.23 358  | 76   | 0.76 641  | 9.99 372  | 17 |       |
| 44 | 9.22 805  | 73 | 9.23 434  | 76   | 0.76 565  | 9.99 370  | 16 |       |
| 45 | 9.22 878  | 73 | 9.23 510  | 75   | 0.76 489  | 9.99 368  | 15 |       |
| 46 | 9.22 952  | 73 | 9.23 586  | 75   | 0.76 414  | 9.99 366  | 14 |       |
| 47 | 9.23 025  | 73 | 9.23 661  | 75   | 0.76 338  | 9.99 364  | 13 |       |
| 48 | 9.23 098  | 73 | 9.23 737  | 75   | 0.76 263  | 9.99 361  | 12 |       |
| 49 | 9.23 171  | 73 | 9.23 812  | 75   | 0.76 188  | 9.99 359  | 11 |       |
| 50 | 9.23 244  | 72 | 9.23 887  | 75   | 0.76 111  | 9.99 357  | 10 |       |
| 51 | 9.23 317  | 73 | 9.23 962  | 75   | 0.76 032  | 9.99 355  | 9  |       |
| 52 | 9.23 390  | 72 | 9.24 037  | 75   | 0.75 953  | 9.99 353  | 8  |       |
| 53 | 9.23 462  | 72 | 9.24 112  | 74   | 0.75 878  | 9.99 350  | 7  |       |
| 54 | 9.23 535  | 72 | 9.24 186  | 74   | 0.75 803  | 9.99 348  | 6  |       |
| 55 | 9.23 607  | 72 | 9.24 261  | 74   | 0.75 729  | 9.99 346  | 5  |       |
| 56 | 9.23 679  | 72 | 9.24 335  | 74   | 0.75 654  | 9.99 344  | 4  |       |
| 57 | 9.23 751  | 72 | 9.24 409  | 74   | 0.75 579  | 9.99 342  | 3  |       |
| 58 | 9.23 823  | 72 | 9.24 484  | 74   | 0.75 514  | 9.99 339  | 2  |       |
| 59 | 9.23 895  | 72 | 9.24 558  | 74   | 0.75 442  | 9.99 337  | 1  |       |
| 60 | 9.23 967  | 71 | 9.24 632  | 74   | 0.75 368  | 9.99 335  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c.d. | Log. Tan. | Log. Sin. |    | P. P. |

|    |      |      |      |      |
|----|------|------|------|------|
| 6  | 8.1  | 8.1  | 8.0  | 7.9  |
| 7  | 9.5  | 9.4  | 9.3  | 9.2  |
| 8  | 10.8 | 10.8 | 10.6 | 10.5 |
| 9  | 12.2 | 12.1 | 12.0 | 11.9 |
| 10 | 13.6 | 13.5 | 13.3 | 13.1 |
| 20 | 27.1 | 27.0 | 26.6 | 26.3 |
| 30 | 40.7 | 40.5 | 40.0 | 39.9 |
| 40 | 54.3 | 54.0 | 53.3 | 52.9 |
| 50 | 67.9 | 67.5 | 66.6 | 65.8 |

|    |      |      |      |
|----|------|------|------|
| 6  | 7.8  | 7.8  | 7.7  |
| 7  | 9.1  | 9.1  | 9.0  |
| 8  | 10.4 | 10.4 | 10.2 |
| 9  | 11.8 | 11.7 | 11.5 |
| 10 | 13.1 | 13.0 | 12.8 |
| 20 | 26.1 | 26.0 | 25.6 |
| 30 | 39.9 | 39.9 | 38.5 |
| 40 | 52.2 | 52.0 | 51.3 |
| 50 | 65.4 | 65.0 | 64.1 |

|    |      |      |      |      |
|----|------|------|------|------|
| 6  | 7.6  | 7.6  | 7.5  | 7.4  |
| 7  | 8.9  | 8.8  | 8.7  | 8.6  |
| 8  | 10.2 | 10.1 | 10.0 | 9.9  |
| 9  | 11.5 | 11.4 | 11.2 | 11.1 |
| 10 | 12.7 | 12.6 | 12.5 | 12.4 |
| 20 | 25.5 | 25.3 | 25.0 | 24.8 |
| 30 | 38.2 | 38.0 | 37.5 | 37.3 |
| 40 | 51.0 | 50.8 | 50.0 | 49.9 |
| 50 | 63.7 | 63.3 | 62.5 | 61.6 |

|    |      |      |      |
|----|------|------|------|
| 6  | 7.3  | 7.3  | 7.2  |
| 7  | 8.6  | 8.5  | 8.4  |
| 8  | 9.9  | 9.7  | 9.6  |
| 9  | 11.0 | 10.9 | 10.8 |
| 10 | 12.2 | 12.1 | 12.0 |
| 20 | 24.5 | 24.3 | 24.0 |
| 30 | 36.7 | 36.5 | 36.0 |
| 40 | 49.0 | 48.8 | 48.0 |
| 50 | 61.2 | 60.8 | 60.0 |

|    |      |      |      |      |
|----|------|------|------|------|
| 6  | 7.1  | 7.1  | 7.0  | 6.9  |
| 7  | 8.4  | 8.3  | 8.2  | 8.1  |
| 8  | 9.7  | 9.6  | 9.5  | 9.4  |
| 9  | 10.9 | 10.8 | 10.7 | 10.6 |
| 10 | 12.1 | 12.0 | 11.9 | 11.8 |
| 20 | 23.8 | 23.6 | 23.4 | 23.3 |
| 30 | 35.7 | 35.5 | 35.1 | 34.9 |
| 40 | 47.6 | 47.3 | 46.1 | 45.9 |
| 50 | 59.5 | 59.1 | 58.1 | 57.6 |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |      |      |       |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|------|------|-------|
| 0  | 9.23 967  | 71 | 9.24 632  | 73    | 0.75 368  | 9.99 335  | 60 |       | 74   | 73   | 73    |
| 1  | 9.24 038  | 71 | 9.24 705  | 74    | 0.75 294  | 9.99 333  | 59 | 6     | 7.4  | 7.3  | 7.3   |
| 2  | 9.24 110  | 71 | 9.24 779  | 75    | 0.75 220  | 9.99 330  | 58 | 7     | 8.6  | 8.6  | 8.5   |
| 3  | 9.24 181  | 71 | 9.24 853  | 76    | 0.75 147  | 9.99 328  | 57 | 8     | 9.8  | 9.8  | 9.7   |
| 4  | 9.24 252  | 71 | 9.24 926  | 77    | 0.75 073  | 9.99 326  | 56 | 9     | 11.1 | 11.0 | 10.9  |
| 5  | 9.24 323  | 71 | 9.25 000  | 78    | 0.75 000  | 9.99 324  | 55 | 10    | 12.3 | 12.2 | 12.1  |
| 6  | 9.24 394  | 71 | 9.25 073  | 79    | 0.74 927  | 9.99 321  | 54 | 20    | 24.6 | 24.5 | 24.3  |
| 7  | 9.24 465  | 71 | 9.25 146  | 80    | 0.74 854  | 9.99 319  | 53 | 30    | 37.0 | 36.7 | 36.5  |
| 8  | 9.24 536  | 70 | 9.25 219  | 81    | 0.74 781  | 9.99 317  | 52 | 40    | 49.3 | 49.0 | 48.6  |
| 9  | 9.24 607  | 70 | 9.25 292  | 82    | 0.74 708  | 9.99 315  | 51 | 50    | 61.6 | 61.2 | 60.8  |
| 10 | 9.24 677  | 70 | 9.25 365  | 83    | 0.74 635  | 9.99 312  | 50 |       | 72   | 72   | 71    |
| 11 | 9.24 748  | 70 | 9.25 437  | 84    | 0.74 562  | 9.99 310  | 49 | 6     | 7.2  | 7.2  | 7.1   |
| 12 | 9.24 818  | 70 | 9.25 510  | 85    | 0.74 490  | 9.99 308  | 48 | 7     | 8.4  | 8.4  | 8.3   |
| 13 | 9.24 888  | 70 | 9.25 582  | 86    | 0.74 417  | 9.99 306  | 47 | 8     | 9.6  | 9.6  | 9.5   |
| 14 | 9.24 958  | 69 | 9.25 654  | 87    | 0.74 345  | 9.99 303  | 46 | 9     | 10.9 | 10.8 | 10.7  |
| 15 | 9.25 028  | 70 | 9.25 727  | 88    | 0.74 273  | 9.99 301  | 45 | 10    | 12.1 | 12.0 | 11.9  |
| 16 | 9.25 098  | 69 | 9.25 799  | 89    | 0.74 201  | 9.99 299  | 44 | 20    | 24.1 | 24.0 | 23.8  |
| 17 | 9.25 167  | 70 | 9.25 871  | 90    | 0.74 129  | 9.99 296  | 43 | 30    | 36.2 | 36.0 | 35.7  |
| 18 | 9.25 237  | 69 | 9.25 943  | 91    | 0.74 057  | 9.99 294  | 42 | 40    | 48.3 | 48.0 | 47.6  |
| 19 | 9.25 306  | 69 | 9.26 014  | 92    | 0.73 985  | 9.99 292  | 41 | 50    | 60.4 | 60.0 | 59.6  |
| 20 | 9.25 376  | 69 | 9.26 086  | 93    | 0.73 913  | 9.99 290  | 40 |       | 70   | 70   | 69    |
| 21 | 9.25 445  | 69 | 9.26 158  | 94    | 0.73 842  | 9.99 287  | 39 | 6     | 7.0  | 7.0  | 6.9   |
| 22 | 9.25 514  | 69 | 9.26 229  | 95    | 0.73 771  | 9.99 285  | 38 | 7     | 8.2  | 8.1  | 8.0   |
| 23 | 9.25 583  | 69 | 9.26 300  | 96    | 0.73 699  | 9.99 283  | 37 | 8     | 9.4  | 9.3  | 9.2   |
| 24 | 9.25 652  | 68 | 9.26 371  | 97    | 0.73 628  | 9.99 280  | 36 | 9     | 10.6 | 10.5 | 10.4  |
| 25 | 9.25 721  | 69 | 9.26 443  | 98    | 0.73 557  | 9.99 278  | 35 | 10    | 11.7 | 11.6 | 11.5  |
| 26 | 9.25 790  | 68 | 9.26 514  | 99    | 0.73 486  | 9.99 276  | 34 | 20    | 23.5 | 23.3 | 23.0  |
| 27 | 9.25 858  | 68 | 9.26 584  | 100   | 0.73 415  | 9.99 273  | 33 | 30    | 35.2 | 35.0 | 34.6  |
| 28 | 9.25 927  | 68 | 9.26 655  | 101   | 0.73 344  | 9.99 271  | 32 | 40    | 47.0 | 46.6 | 46.0  |
| 29 | 9.25 995  | 68 | 9.26 726  | 102   | 0.73 274  | 9.99 268  | 31 | 50    | 58.7 | 58.3 | 57.5  |
| 30 | 9.26 063  | 68 | 9.26 796  | 103   | 0.73 203  | 9.99 266  | 30 |       | 68   | 68   | 67    |
| 31 | 9.26 131  | 68 | 9.26 867  | 104   | 0.73 133  | 9.99 264  | 29 | 6     | 6.8  | 6.8  | 6.7   |
| 32 | 9.26 199  | 68 | 9.26 937  | 105   | 0.73 062  | 9.99 262  | 28 | 7     | 8.0  | 7.9  | 7.8   |
| 33 | 9.26 267  | 67 | 9.27 007  | 106   | 0.72 992  | 9.99 259  | 27 | 8     | 9.1  | 9.0  | 8.9   |
| 34 | 9.26 335  | 67 | 9.27 078  | 107   | 0.72 922  | 9.99 257  | 26 | 9     | 10.3 | 10.2 | 10.1  |
| 35 | 9.26 402  | 67 | 9.27 148  | 108   | 0.72 852  | 9.99 255  | 25 | 10    | 11.4 | 11.3 | 11.2  |
| 36 | 9.26 470  | 67 | 9.27 218  | 109   | 0.72 782  | 9.99 252  | 24 | 20    | 22.8 | 22.6 | 22.3  |
| 37 | 9.26 537  | 67 | 9.27 287  | 110   | 0.72 712  | 9.99 250  | 23 | 30    | 34.2 | 34.0 | 33.5  |
| 38 | 9.26 605  | 67 | 9.27 357  | 111   | 0.72 642  | 9.99 248  | 22 | 40    | 45.6 | 45.3 | 44.6  |
| 39 | 9.26 672  | 67 | 9.27 427  | 112   | 0.72 573  | 9.99 245  | 21 | 50    | 57.1 | 56.6 | 55.8  |
| 40 | 9.26 739  | 67 | 9.27 496  | 113   | 0.72 503  | 9.99 243  | 20 |       | 66   | 66   | 65    |
| 41 | 9.26 806  | 67 | 9.27 566  | 114   | 0.72 434  | 9.99 240  | 19 | 6     | 6.6  | 6.6  | 6.5   |
| 42 | 9.26 873  | 67 | 9.27 635  | 115   | 0.72 365  | 9.99 238  | 18 | 7     | 7.7  | 7.7  | 7.6   |
| 43 | 9.26 940  | 66 | 9.27 704  | 116   | 0.72 295  | 9.99 236  | 17 | 8     | 8.8  | 8.8  | 8.7   |
| 44 | 9.27 007  | 66 | 9.27 773  | 117   | 0.72 226  | 9.99 233  | 16 | 9     | 10.0 | 9.9  | 9.8   |
| 45 | 9.27 073  | 66 | 9.27 842  | 118   | 0.72 157  | 9.99 231  | 15 | 10    | 11.1 | 11.0 | 10.9  |
| 46 | 9.27 140  | 66 | 9.27 911  | 119   | 0.72 088  | 9.99 228  | 14 | 20    | 22.1 | 22.0 | 21.8  |
| 47 | 9.27 206  | 66 | 9.27 980  | 120   | 0.72 020  | 9.99 226  | 13 | 30    | 33.2 | 33.0 | 32.5  |
| 48 | 9.27 272  | 66 | 9.28 049  | 121   | 0.71 951  | 9.99 224  | 12 | 40    | 44.3 | 44.0 | 43.6  |
| 49 | 9.27 339  | 66 | 9.28 117  | 122   | 0.71 882  | 9.99 221  | 11 | 50    | 55.4 | 55.0 | 54.1  |
| 50 | 9.27 405  | 66 | 9.28 186  | 123   | 0.71 814  | 9.99 219  | 10 |       | 2    | 2    |       |
| 51 | 9.27 471  | 65 | 9.28 254  | 124   | 0.71 746  | 9.99 216  | 9  | 6     | 0.2  | 0.2  |       |
| 52 | 9.27 536  | 65 | 9.28 322  | 125   | 0.71 677  | 9.99 214  | 8  | 7     | 0.3  | 0.3  |       |
| 53 | 9.27 602  | 65 | 9.28 390  | 126   | 0.71 609  | 9.99 212  | 7  | 8     | 0.3  | 0.3  |       |
| 54 | 9.27 668  | 65 | 9.28 459  | 127   | 0.71 541  | 9.99 209  | 6  | 9     | 0.4  | 0.4  |       |
| 55 | 9.27 733  | 65 | 9.28 527  | 128   | 0.71 473  | 9.99 207  | 5  | 10    | 0.4  | 0.4  |       |
| 56 | 9.27 799  | 65 | 9.28 594  | 129   | 0.71 405  | 9.99 204  | 4  | 20    | 0.8  | 0.6  |       |
| 57 | 9.27 864  | 65 | 9.28 662  | 130   | 0.71 337  | 9.99 202  | 3  | 30    | 1.2  | 1.0  |       |
| 58 | 9.27 929  | 65 | 9.28 730  | 131   | 0.71 270  | 9.99 199  | 2  | 40    | 1.6  | 1.3  |       |
| 59 | 9.27 995  | 65 | 9.28 797  | 132   | 0.71 202  | 9.99 197  | 1  | 50    | 2.1  | 1.6  |       |
| 60 | 9.28 060  | 65 | 9.28 865  | 133   | 0.71 135  | 9.99 194  | 0  |       |      |      |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |    |       |      |      | P. P. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P.                  |
|----|-----------|----|-----------|-------|-----------|-----------|----|------------------------|
| 0  | 9 28 060  | 65 | 9 28 865  | 67    | 0 71 135  | 9 99 194  | 60 | 67 67                  |
| 1  | 9 28 125  | 64 | 9 28 932  | 67    | 0 71 067  | 9 99 191  | 59 | 6 6.7 6 7              |
| 2  | 9 28 189  | 65 | 9 29 000  | 67    | 0 71 006  | 9 99 189  | 58 | 7 7.9 7 8              |
| 3  | 9 28 254  | 64 | 9 29 067  | 67    | 0 70 933  | 9 99 187  | 57 | 8 9.0 8 9              |
| 4  | 9 28 319  | 64 | 9 29 134  | 67    | 0 70 866  | 9 99 185  | 56 | 9 10.1 10 0            |
| 5  | 9 28 383  | 64 | 9 29 201  | 66    | 0 70 798  | 9 99 182  | 55 | 10 11.2 11.1           |
| 6  | 9 28 448  | 64 | 9 29 268  | 67    | 0 70 732  | 9 99 180  | 54 | 20 22.5 22.3           |
| 7  | 9 28 512  | 64 | 9 29 335  | 66    | 0 70 665  | 9 99 177  | 53 | 30 33.7 33.5           |
| 8  | 9 28 576  | 64 | 9 29 401  | 66    | 0 70 598  | 9 99 175  | 52 | 40 45.0 44.6           |
| 9  | 9 28 641  | 64 | 9 29 468  | 67    | 0 70 531  | 9 99 172  | 51 | 50 56.2 55.8           |
| 10 | 9 28 705  | 64 | 9 29 535  | 66    | 0 70 465  | 9 99 170  | 50 | 66 66 65 65            |
| 11 | 9 28 769  | 63 | 9 29 601  | 66    | 0 70 398  | 9 99 167  | 49 | 6 6.0 6 6 6 5          |
| 12 | 9 28 832  | 64 | 9 29 667  | 66    | 0 70 332  | 9 99 165  | 48 | 7 7.7 7 7 7 6          |
| 13 | 9 28 896  | 63 | 9 29 734  | 66    | 0 70 266  | 9 99 162  | 47 | 8 8.8 8 8 8 7          |
| 14 | 9 28 960  | 63 | 9 29 800  | 66    | 0 70 200  | 9 99 160  | 46 | 8 10.0 9 9 9 8         |
| 15 | 9 29 023  | 63 | 9 29 866  | 66    | 0 70 134  | 9 99 157  | 45 | 10 11.1 11.0 10 9 10 8 |
| 16 | 9 29 087  | 63 | 9 29 932  | 66    | 0 70 068  | 9 99 155  | 44 | 20 22.1 22.0 21 8 21 6 |
| 17 | 9 29 150  | 63 | 9 29 998  | 66    | 0 70 002  | 9 99 152  | 43 | 30 33.2 33.0 32 7 32 5 |
| 18 | 9 29 213  | 63 | 9 30 064  | 65    | 0 69 936  | 9 99 150  | 42 | 40 44.3 44.0 43 6 43 3 |
| 19 | 9 29 277  | 63 | 9 30 129  | 65    | 0 69 870  | 9 99 147  | 41 | 50 55.4 55.0 54 8 54 1 |
| 20 | 9 29 340  | 63 | 9 30 195  | 65    | 0 69 805  | 9 99 145  | 40 | 61 61 61 61            |
| 21 | 9 29 403  | 63 | 9 30 260  | 65    | 0 69 739  | 9 99 142  | 39 | 6 6.4 6 4 6 3 6 3      |
| 22 | 9 29 466  | 62 | 9 30 326  | 65    | 0 69 674  | 9 99 139  | 38 | 7 7.5 7 4 7 4 7 3      |
| 23 | 9 29 528  | 63 | 9 30 391  | 65    | 0 69 608  | 9 99 137  | 37 | 8 8.6 8 5 8 4 8 4      |
| 24 | 9 29 591  | 62 | 9 30 456  | 65    | 0 69 543  | 9 99 134  | 36 | 9 9.7 9 6 9 5 9 4      |
| 25 | 9 29 654  | 62 | 9 30 522  | 65    | 0 69 478  | 9 99 132  | 35 | 10 10.7 10 6 10 6 10 5 |
| 26 | 9 29 716  | 62 | 9 30 587  | 65    | 0 69 413  | 9 99 129  | 34 | 20 21.5 21 3 21 1 21 0 |
| 27 | 9 29 779  | 62 | 9 30 652  | 65    | 0 69 348  | 9 99 127  | 33 | 30 32.2 32 0 31 7 31 5 |
| 28 | 9 29 841  | 62 | 9 30 717  | 64    | 0 69 283  | 9 99 124  | 32 | 40 43.0 42 6 42 3 42 0 |
| 29 | 9 29 903  | 62 | 9 30 781  | 64    | 0 69 218  | 9 99 122  | 31 | 50 53.7 53 3 52 9 52 5 |
| 30 | 9 29 965  | 62 | 9 30 846  | 65    | 0 69 153  | 9 99 119  | 30 | 62 62 61 61            |
| 31 | 9 30 027  | 62 | 9 30 911  | 64    | 0 69 089  | 9 99 116  | 29 | 6 6.2 6 2 6 1 6 1      |
| 32 | 9 30 089  | 62 | 9 30 975  | 64    | 0 69 024  | 9 99 114  | 28 | 7 7.3 7 2 7 2 7 1      |
| 33 | 9 30 151  | 61 | 9 31 040  | 64    | 0 68 960  | 9 99 111  | 27 | 8 8.3 8 2 8 2 8 1      |
| 34 | 9 30 213  | 61 | 9 31 104  | 64    | 0 68 896  | 9 99 109  | 26 | 9 9.4 9 3 9 2 9 1      |
| 35 | 9 30 275  | 61 | 9 31 168  | 64    | 0 68 831  | 9 99 106  | 25 | 10 10.4 10 3 10 2 10 1 |
| 36 | 9 30 336  | 61 | 9 31 232  | 64    | 0 68 767  | 9 99 104  | 24 | 20 20.8 20 6 20 5 20 3 |
| 37 | 9 30 398  | 61 | 9 31 297  | 64    | 0 68 703  | 9 99 101  | 23 | 30 31.2 31 0 30 7 30 5 |
| 38 | 9 30 459  | 61 | 9 31 361  | 63    | 0 68 639  | 9 99 098  | 22 | 40 41.6 41 3 41 0 40 6 |
| 39 | 9 30 520  | 61 | 9 31 424  | 63    | 0 68 575  | 9 99 096  | 21 | 50 52.1 51 6 51 2 50 8 |
| 40 | 9 30 582  | 61 | 9 31 488  | 64    | 0 68 511  | 9 99 093  | 20 | 60 60 60 59            |
| 41 | 9 30 643  | 61 | 9 31 552  | 63    | 0 68 447  | 9 99 091  | 19 | 6 6.0 6 0 6 0 5 9      |
| 42 | 9 30 704  | 61 | 9 31 616  | 63    | 0 68 384  | 9 99 088  | 18 | 7 7.0 7 0 7 0 6 9      |
| 43 | 9 30 765  | 61 | 9 31 679  | 63    | 0 68 320  | 9 99 085  | 17 | 8 8.0 8 0 8 0 7 9      |
| 44 | 9 30 826  | 60 | 9 31 743  | 63    | 0 68 257  | 9 99 083  | 16 | 9 9.1 9 0 9 0 8 9      |
| 45 | 9 30 886  | 61 | 9 31 806  | 63    | 0 68 193  | 9 99 080  | 15 | 10 10.1 10 0 10 0 9 9  |
| 46 | 9 30 947  | 60 | 9 31 869  | 63    | 0 68 130  | 9 99 077  | 14 | 20 20.1 20 0 19 8      |
| 47 | 9 31 008  | 60 | 9 31 933  | 63    | 0 68 067  | 9 99 075  | 13 | 30 30.3 30 0 29 7      |
| 48 | 9 31 068  | 60 | 9 31 996  | 63    | 0 68 004  | 9 99 072  | 12 | 40 40.3 40 0 39 6      |
| 49 | 9 31 129  | 60 | 9 32 059  | 63    | 0 67 941  | 9 99 069  | 11 | 50 50.4 50 0 49 6      |
| 50 | 9 31 189  | 60 | 9 32 122  | 63    | 0 67 878  | 9 99 067  | 10 | 60 60 60 59            |
| 51 | 9 31 249  | 60 | 9 32 185  | 63    | 0 67 815  | 9 99 064  | 9  | 3 3 2 2                |
| 52 | 9 31 309  | 60 | 9 32 248  | 62    | 0 67 752  | 9 99 062  | 8  | 6 6.3 6 3 6 2 6 2      |
| 53 | 9 31 370  | 59 | 9 32 310  | 63    | 0 67 689  | 9 99 059  | 7  | 7 7.3 7 3 7 2 7 1      |
| 54 | 9 31 429  | 59 | 9 32 373  | 63    | 0 67 626  | 9 99 056  | 6  | 8 8.3 8 2 8 2 8 1      |
| 55 | 9 31 489  | 60 | 9 32 436  | 62    | 0 67 564  | 9 99 054  | 5  | 9 9.4 9 3 9 2 9 1      |
| 56 | 9 31 549  | 59 | 9 32 498  | 62    | 0 67 501  | 9 99 051  | 4  | 10 10.4 10 3 10 2 10 1 |
| 57 | 9 31 609  | 60 | 9 32 560  | 62    | 0 67 439  | 9 99 048  | 3  | 20 20.1 20 0 19 8      |
| 58 | 9 31 669  | 59 | 9 32 623  | 62    | 0 67 377  | 9 99 046  | 2  | 30 30.3 30 0 29 7      |
| 59 | 9 31 728  | 59 | 9 32 685  | 62    | 0 67 314  | 9 99 043  | 1  | 40 40.3 40 0 39 6      |
| 60 | 9 31 788  | 59 | 9 32 747  | 62    | 0 67 252  | 9 99 040  | 0  | 50 50.4 50 0 49 6      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |    | P. P.                  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |  |  |  |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|--|--|--|
| 0  | 9.31 788  | 59 | 9.32 747  | 62    | 0.67 252  | 9.99 040  | 60 |       |  |  |  |
| 1  | 9.31 847  | 59 | 9.32 809  | 62    | 0.67 190  | 9.99 038  | 59 |       |  |  |  |
| 2  | 9.31 906  | 59 | 9.32 871  | 62    | 0.67 128  | 9.99 035  | 58 |       |  |  |  |
| 3  | 9.31 966  | 59 | 9.32 933  | 62    | 0.67 066  | 9.99 032  | 57 |       |  |  |  |
| 4  | 9.32 025  | 59 | 9.32 995  | 62    | 0.67 004  | 9.99 029  | 56 |       |  |  |  |
| 5  | 9.32 084  | 59 | 9.33 057  | 61    | 0.66 943  | 9.99 027  | 55 |       |  |  |  |
| 6  | 9.32 143  | 59 | 9.33 118  | 62    | 0.66 881  | 9.99 024  | 54 |       |  |  |  |
| 7  | 9.32 202  | 59 | 9.33 180  | 61    | 0.66 819  | 9.99 021  | 53 |       |  |  |  |
| 8  | 9.32 260  | 58 | 9.33 242  | 61    | 0.66 758  | 9.99 019  | 52 |       |  |  |  |
| 9  | 9.32 319  | 59 | 9.33 303  | 61    | 0.66 696  | 9.99 016  | 51 |       |  |  |  |
| 10 | 9.32 378  | 58 | 9.33 364  | 61    | 0.66 635  | 9.99 013  | 50 |       |  |  |  |
| 11 | 9.32 436  | 58 | 9.33 426  | 61    | 0.66 574  | 9.99 010  | 49 |       |  |  |  |
| 12 | 9.32 495  | 58 | 9.33 487  | 61    | 0.66 513  | 9.99 008  | 48 |       |  |  |  |
| 13 | 9.32 553  | 58 | 9.33 548  | 61    | 0.66 452  | 9.99 005  | 47 |       |  |  |  |
| 14 | 9.32 611  | 58 | 9.33 609  | 60    | 0.66 390  | 9.99 002  | 46 |       |  |  |  |
| 15 | 9.32 670  | 58 | 9.33 670  | 61    | 0.66 330  | 9.98 999  | 45 |       |  |  |  |
| 16 | 9.32 728  | 58 | 9.33 731  | 61    | 0.66 269  | 9.98 997  | 44 |       |  |  |  |
| 17 | 9.32 786  | 58 | 9.33 792  | 60    | 0.66 208  | 9.98 994  | 43 |       |  |  |  |
| 18 | 9.32 844  | 58 | 9.33 852  | 61    | 0.66 147  | 9.98 991  | 42 |       |  |  |  |
| 19 | 9.32 902  | 58 | 9.33 915  | 60    | 0.66 086  | 9.98 988  | 41 |       |  |  |  |
| 20 | 9.32 960  | 57 | 9.33 974  | 60    | 0.66 026  | 9.98 986  | 40 |       |  |  |  |
| 21 | 9.33 017  | 58 | 9.34 034  | 60    | 0.65 965  | 9.98 983  | 39 |       |  |  |  |
| 22 | 9.33 075  | 57 | 9.34 095  | 60    | 0.65 905  | 9.98 980  | 38 |       |  |  |  |
| 23 | 9.33 133  | 57 | 9.34 155  | 60    | 0.65 845  | 9.98 977  | 37 |       |  |  |  |
| 24 | 9.33 190  | 57 | 9.34 215  | 60    | 0.65 784  | 9.98 975  | 36 |       |  |  |  |
| 25 | 9.33 248  | 57 | 9.34 275  | 60    | 0.65 724  | 9.98 972  | 35 |       |  |  |  |
| 26 | 9.33 305  | 57 | 9.34 336  | 60    | 0.65 664  | 9.98 969  | 34 |       |  |  |  |
| 27 | 9.33 362  | 57 | 9.34 396  | 60    | 0.65 604  | 9.98 966  | 33 |       |  |  |  |
| 28 | 9.33 419  | 57 | 9.34 456  | 59    | 0.65 544  | 9.98 963  | 32 |       |  |  |  |
| 29 | 9.33 476  | 57 | 9.34 515  | 60    | 0.65 484  | 9.98 961  | 31 |       |  |  |  |
| 30 | 9.33 533  | 57 | 9.34 575  | 60    | 0.65 424  | 9.98 958  | 30 |       |  |  |  |
| 31 | 9.33 590  | 57 | 9.34 635  | 59    | 0.65 364  | 9.98 955  | 29 |       |  |  |  |
| 32 | 9.33 647  | 57 | 9.34 695  | 59    | 0.65 305  | 9.98 952  | 28 |       |  |  |  |
| 33 | 9.33 704  | 56 | 9.34 754  | 59    | 0.65 245  | 9.98 949  | 27 |       |  |  |  |
| 34 | 9.33 761  | 56 | 9.34 814  | 59    | 0.65 186  | 9.98 947  | 26 |       |  |  |  |
| 35 | 9.33 817  | 56 | 9.34 873  | 59    | 0.65 126  | 9.98 944  | 25 |       |  |  |  |
| 36 | 9.33 874  | 56 | 9.34 933  | 59    | 0.65 067  | 9.98 941  | 24 |       |  |  |  |
| 37 | 9.33 930  | 56 | 9.34 992  | 59    | 0.65 008  | 9.98 938  | 23 |       |  |  |  |
| 38 | 9.33 987  | 56 | 9.35 051  | 59    | 0.64 948  | 9.98 935  | 22 |       |  |  |  |
| 39 | 9.34 043  | 56 | 9.35 110  | 59    | 0.64 889  | 9.98 933  | 21 |       |  |  |  |
| 40 | 9.34 099  | 56 | 9.35 169  | 59    | 0.64 830  | 9.98 930  | 20 |       |  |  |  |
| 41 | 9.34 156  | 56 | 9.35 228  | 59    | 0.64 771  | 9.98 927  | 19 |       |  |  |  |
| 42 | 9.34 212  | 56 | 9.35 287  | 59    | 0.64 712  | 9.98 924  | 18 |       |  |  |  |
| 43 | 9.34 268  | 56 | 9.35 346  | 59    | 0.64 653  | 9.98 921  | 17 |       |  |  |  |
| 44 | 9.34 324  | 55 | 9.35 405  | 58    | 0.64 594  | 9.98 918  | 16 |       |  |  |  |
| 45 | 9.34 379  | 56 | 9.35 464  | 58    | 0.64 536  | 9.98 915  | 15 |       |  |  |  |
| 46 | 9.34 435  | 55 | 9.35 522  | 59    | 0.64 477  | 9.98 913  | 14 |       |  |  |  |
| 47 | 9.34 491  | 56 | 9.35 581  | 58    | 0.64 418  | 9.98 910  | 13 |       |  |  |  |
| 48 | 9.34 547  | 55 | 9.35 640  | 58    | 0.64 360  | 9.98 907  | 12 |       |  |  |  |
| 49 | 9.34 602  | 55 | 9.35 698  | 58    | 0.64 302  | 9.98 904  | 11 |       |  |  |  |
| 50 | 9.34 658  | 55 | 9.35 756  | 58    | 0.64 243  | 9.98 901  | 10 |       |  |  |  |
| 51 | 9.34 713  | 55 | 9.35 815  | 58    | 0.64 185  | 9.98 898  | 9  |       |  |  |  |
| 52 | 9.34 768  | 55 | 9.35 873  | 58    | 0.64 127  | 9.98 895  | 8  |       |  |  |  |
| 53 | 9.34 824  | 55 | 9.35 931  | 58    | 0.64 068  | 9.98 892  | 7  |       |  |  |  |
| 54 | 9.34 879  | 55 | 9.35 989  | 58    | 0.64 010  | 9.98 890  | 6  |       |  |  |  |
| 55 | 9.34 934  | 55 | 9.36 047  | 58    | 0.63 952  | 9.98 887  | 5  |       |  |  |  |
| 56 | 9.34 989  | 55 | 9.36 105  | 57    | 0.63 894  | 9.98 884  | 4  |       |  |  |  |
| 57 | 9.35 044  | 54 | 9.36 163  | 58    | 0.63 837  | 9.98 881  | 3  |       |  |  |  |
| 58 | 9.35 099  | 55 | 9.36 221  | 57    | 0.63 779  | 9.98 878  | 2  |       |  |  |  |
| 59 | 9.35 154  | 55 | 9.36 278  | 57    | 0.63 721  | 9.98 875  | 1  |       |  |  |  |
| 60 | 9.35 209  | 55 | 9.36 336  | 58    | 0.63 663  | 9.98 872  | 0  |       |  |  |  |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | '  |       |  |  |  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|
| 0  | 9.35 209  | 54 | 9.36 336  | 57    | 0.63 663  | 9.98 872  | 60 |       |
| 1  | 9.35 263  | 54 | 9.36 394  | 57    | 0.63 606  | 9.98 869  | 59 |       |
| 2  | 9.35 318  | 54 | 9.36 451  | 57    | 0.63 548  | 9.98 866  | 58 |       |
| 3  | 9.35 372  | 54 | 9.36 509  | 57    | 0.63 491  | 9.98 863  | 57 |       |
| 4  | 9.35 427  | 54 | 9.36 566  | 57    | 0.63 433  | 9.98 860  | 56 |       |
| 5  | 9.35 481  | 54 | 9.36 623  | 57    | 0.63 376  | 9.98 858  | 55 |       |
| 6  | 9.35 536  | 54 | 9.36 681  | 57    | 0.63 319  | 9.98 855  | 54 |       |
| 7  | 9.35 590  | 54 | 9.36 738  | 57    | 0.63 262  | 9.98 852  | 53 |       |
| 8  | 9.35 644  | 54 | 9.36 795  | 57    | 0.63 204  | 9.98 849  | 52 |       |
| 9  | 9.35 698  | 54 | 9.36 852  | 57    | 0.63 147  | 9.98 846  | 51 |       |
| 10 | 9.35 752  | 54 | 9.36 909  | 57    | 0.63 090  | 9.98 843  | 50 |       |
| 11 | 9.35 806  | 54 | 9.36 966  | 57    | 0.63 033  | 9.98 840  | 49 |       |
| 12 | 9.35 860  | 53 | 9.37 023  | 57    | 0.62 977  | 9.98 837  | 48 |       |
| 13 | 9.35 914  | 54 | 9.37 080  | 56    | 0.62 920  | 9.98 834  | 47 |       |
| 14 | 9.35 968  | 53 | 9.37 136  | 57    | 0.62 863  | 9.98 831  | 46 |       |
| 15 | 9.36 021  | 53 | 9.37 193  | 56    | 0.62 806  | 9.98 828  | 45 |       |
| 16 | 9.36 075  | 53 | 9.37 250  | 56    | 0.62 750  | 9.98 825  | 44 |       |
| 17 | 9.36 128  | 53 | 9.37 306  | 56    | 0.62 693  | 9.98 822  | 43 |       |
| 18 | 9.36 182  | 53 | 9.37 363  | 56    | 0.62 637  | 9.98 819  | 42 |       |
| 19 | 9.36 235  | 53 | 9.37 419  | 56    | 0.62 580  | 9.98 816  | 41 |       |
| 20 | 9.36 289  | 53 | 9.37 475  | 56    | 0.62 524  | 9.98 813  | 40 |       |
| 21 | 9.36 342  | 53 | 9.37 532  | 56    | 0.62 468  | 9.98 810  | 39 |       |
| 22 | 9.36 395  | 53 | 9.37 588  | 56    | 0.62 412  | 9.98 807  | 38 |       |
| 23 | 9.36 448  | 53 | 9.37 644  | 56    | 0.62 356  | 9.98 804  | 37 |       |
| 24 | 9.36 501  | 53 | 9.37 700  | 56    | 0.62 299  | 9.98 801  | 36 |       |
| 25 | 9.36 554  | 53 | 9.37 756  | 55    | 0.62 243  | 9.98 798  | 35 |       |
| 26 | 9.36 607  | 53 | 9.37 812  | 56    | 0.62 188  | 9.98 795  | 34 |       |
| 27 | 9.36 660  | 52 | 9.37 868  | 56    | 0.62 132  | 9.98 792  | 33 |       |
| 28 | 9.36 713  | 53 | 9.37 924  | 55    | 0.62 076  | 9.98 789  | 32 |       |
| 29 | 9.36 766  | 52 | 9.37 979  | 56    | 0.62 020  | 9.98 786  | 31 |       |
| 30 | 9.36 818  | 52 | 9.38 035  | 55    | 0.61 964  | 9.98 783  | 30 |       |
| 31 | 9.36 871  | 52 | 9.38 091  | 55    | 0.61 909  | 9.98 780  | 29 |       |
| 32 | 9.36 923  | 52 | 9.38 146  | 55    | 0.61 853  | 9.98 777  | 28 |       |
| 33 | 9.36 976  | 52 | 9.38 202  | 55    | 0.61 798  | 9.98 774  | 27 |       |
| 34 | 9.37 028  | 52 | 9.38 257  | 55    | 0.61 742  | 9.98 771  | 26 |       |
| 35 | 9.37 081  | 52 | 9.38 313  | 55    | 0.61 687  | 9.98 768  | 25 |       |
| 36 | 9.37 133  | 52 | 9.38 368  | 55    | 0.61 632  | 9.98 765  | 24 |       |
| 37 | 9.37 185  | 52 | 9.38 423  | 55    | 0.61 576  | 9.98 762  | 23 |       |
| 38 | 9.37 237  | 52 | 9.38 478  | 55    | 0.61 521  | 9.98 759  | 22 |       |
| 39 | 9.37 289  | 52 | 9.38 533  | 55    | 0.61 466  | 9.98 755  | 21 |       |
| 40 | 9.37 341  | 52 | 9.38 589  | 55    | 0.61 411  | 9.98 752  | 20 |       |
| 41 | 9.37 393  | 51 | 9.38 644  | 54    | 0.61 356  | 9.98 749  | 19 |       |
| 42 | 9.37 445  | 52 | 9.38 698  | 55    | 0.61 301  | 9.98 746  | 18 |       |
| 43 | 9.37 497  | 51 | 9.38 753  | 55    | 0.61 246  | 9.98 743  | 17 |       |
| 44 | 9.37 548  | 52 | 9.38 808  | 54    | 0.61 191  | 9.98 740  | 16 |       |
| 45 | 9.37 600  | 51 | 9.38 863  | 55    | 0.61 137  | 9.98 737  | 15 |       |
| 46 | 9.37 652  | 51 | 9.38 918  | 54    | 0.61 082  | 9.98 734  | 14 |       |
| 47 | 9.37 703  | 51 | 9.38 972  | 54    | 0.61 027  | 9.98 731  | 13 |       |
| 48 | 9.37 755  | 51 | 9.39 027  | 54    | 0.60 973  | 9.98 728  | 12 |       |
| 49 | 9.37 806  | 51 | 9.39 081  | 54    | 0.60 918  | 9.98 725  | 11 |       |
| 50 | 9.37 857  | 51 | 9.39 136  | 54    | 0.60 864  | 9.98 721  | 10 |       |
| 51 | 9.37 909  | 51 | 9.39 190  | 54    | 0.60 809  | 9.98 718  | 9  |       |
| 52 | 9.37 960  | 51 | 9.39 244  | 54    | 0.60 755  | 9.98 715  | 8  |       |
| 53 | 9.38 011  | 51 | 9.39 299  | 54    | 0.60 701  | 9.98 712  | 7  |       |
| 54 | 9.38 062  | 51 | 9.39 353  | 54    | 0.60 647  | 9.98 709  | 6  |       |
| 55 | 9.38 113  | 51 | 9.39 407  | 54    | 0.60 592  | 9.98 706  | 5  |       |
| 56 | 9.38 164  | 50 | 9.39 461  | 54    | 0.60 538  | 9.98 703  | 4  |       |
| 57 | 9.38 215  | 51 | 9.39 515  | 54    | 0.60 484  | 9.98 700  | 3  |       |
| 58 | 9.38 266  | 51 | 9.39 569  | 54    | 0.60 430  | 9.98 696  | 2  |       |
| 59 | 9.38 317  | 51 | 9.39 623  | 53    | 0.60 376  | 9.98 693  | 1  |       |
| 60 | 9.38 367  | 50 | 9.39 677  | 53    | 0.60 323  | 9.98 690  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. |    | P. P. |

|    |      |      |      |      |
|----|------|------|------|------|
|    | 57   | 57   | 56   | 56   |
| 6  | 5.7  | 5.7  | 5.6  | 5.6  |
| 7  | 6.7  | 6.6  | 6.6  | 6.5  |
| 8  | 7.6  | 7.6  | 7.5  | 7.4  |
| 9  | 8.6  | 8.5  | 8.5  | 8.4  |
| 10 | 9.6  | 9.5  | 9.4  | 9.3  |
| 20 | 19.1 | 19.0 | 18.8 | 18.6 |
| 30 | 28.7 | 28.5 | 28.2 | 28.0 |
| 40 | 38.3 | 38.0 | 37.6 | 37.3 |
| 50 | 47.9 | 47.5 | 47.1 | 46.6 |

|    |      |      |      |      |
|----|------|------|------|------|
|    | 55   | 55   | 54   | 54   |
| 6  | 5.5  | 5.5  | 5.4  | 5.4  |
| 7  | 6.5  | 6.4  | 6.3  | 6.3  |
| 8  | 7.4  | 7.3  | 7.2  | 7.2  |
| 9  | 8.3  | 8.2  | 8.2  | 8.1  |
| 10 | 9.2  | 9.1  | 9.1  | 9.0  |
| 20 | 18.5 | 18.3 | 18.1 | 18.0 |
| 30 | 27.7 | 27.5 | 27.2 | 27.0 |
| 40 | 37.0 | 36.6 | 36.3 | 36.0 |
| 50 | 46.2 | 45.8 | 45.4 | 45.0 |

|    |      |      |      |      |
|----|------|------|------|------|
|    | 53   | 53   | 52   | 52   |
| 6  | 5.3  | 5.3  | 5.2  | 5.2  |
| 7  | 6.2  | 6.2  | 6.1  | 6.0  |
| 8  | 7.1  | 7.0  | 7.0  | 6.9  |
| 9  | 8.0  | 7.9  | 7.9  | 7.8  |
| 10 | 8.9  | 8.8  | 8.7  | 8.6  |
| 20 | 17.8 | 17.6 | 17.5 | 17.3 |
| 30 | 26.7 | 26.5 | 26.2 | 26.0 |
| 40 | 35.6 | 35.3 | 35.0 | 34.6 |
| 50 | 44.6 | 44.1 | 43.7 | 43.3 |

|    |      |      |      |  |
|----|------|------|------|--|
|    | 51   | 51   | 50   |  |
| 6  | 5.1  | 5.1  | 5.0  |  |
| 7  | 6.0  | 5.9  | 5.9  |  |
| 8  | 6.8  | 6.8  | 6.7  |  |
| 9  | 7.7  | 7.6  | 7.6  |  |
| 10 | 8.6  | 8.5  | 8.4  |  |
| 20 | 17.1 | 17.0 | 16.8 |  |
| 30 | 25.7 | 25.5 | 25.2 |  |
| 40 | 34.3 | 34.0 | 33.6 |  |
| 50 | 42.9 | 42.5 | 42.1 |  |

|    |     |     |     |  |
|----|-----|-----|-----|--|
|    | 3   | 3   | 2   |  |
| 6  | 0.3 | 0.3 | 0.2 |  |
| 7  | 0.4 | 0.3 | 0.3 |  |
| 8  | 0.4 | 0.4 | 0.3 |  |
| 9  | 0.5 | 0.4 | 0.4 |  |
| 10 | 0.6 | 0.5 | 0.4 |  |
| 20 | 1.1 | 1.0 | 0.8 |  |
| 30 | 1.7 | 1.5 | 1.2 |  |
| 40 | 2.3 | 2.0 | 1.6 |  |
| 50 | 2.9 | 2.5 | 2.1 |  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.38367   | 50 | 9.39677   | 54    | 0.60328   | 9.98690   | 3  | 60 |       |
| 1  | 9.38418   | 50 | 9.39731   | 53    | 0.60269   | 9.98687   | 3  | 59 |       |
| 2  | 9.38468   | 50 | 9.39784   | 53    | 0.60215   | 9.98684   | 3  | 58 |       |
| 3  | 9.38519   | 50 | 9.39838   | 54    | 0.60161   | 9.98681   | 3  | 57 |       |
| 4  | 9.38569   | 50 | 9.39892   | 53    | 0.60108   | 9.98678   | 3  | 56 |       |
| 5  | 9.38620   | 50 | 9.39945   | 53    | 0.60054   | 9.98674   | 3  | 55 |       |
| 6  | 9.38670   | 50 | 9.39999   | 53    | 0.60001   | 9.98671   | 3  | 54 |       |
| 7  | 9.38720   | 50 | 9.40052   | 53    | 0.59947   | 9.98668   | 3  | 53 |       |
| 8  | 9.38771   | 50 | 9.40106   | 53    | 0.59894   | 9.98665   | 3  | 52 |       |
| 9  | 9.38821   | 50 | 9.40159   | 53    | 0.59841   | 9.98662   | 3  | 51 |       |
| 10 | 9.38871   | 50 | 9.40212   | 53    | 0.59787   | 9.98658   | 3  | 50 |       |
| 11 | 9.38921   | 50 | 9.40265   | 53    | 0.59734   | 9.98655   | 3  | 49 |       |
| 12 | 9.38971   | 50 | 9.40318   | 53    | 0.59681   | 9.98652   | 3  | 48 |       |
| 13 | 9.39021   | 50 | 9.40372   | 53    | 0.59628   | 9.98649   | 3  | 47 |       |
| 14 | 9.39071   | 50 | 9.40425   | 53    | 0.59575   | 9.98646   | 3  | 46 |       |
| 15 | 9.39120   | 49 | 9.40478   | 53    | 0.59522   | 9.98642   | 3  | 45 |       |
| 16 | 9.39170   | 49 | 9.40531   | 52    | 0.59469   | 9.98639   | 3  | 44 |       |
| 17 | 9.39220   | 49 | 9.40583   | 53    | 0.59416   | 9.98636   | 3  | 43 |       |
| 18 | 9.39269   | 49 | 9.40636   | 52    | 0.59363   | 9.98633   | 3  | 42 |       |
| 19 | 9.39319   | 49 | 9.40689   | 52    | 0.59311   | 9.98630   | 3  | 41 |       |
| 20 | 9.39368   | 49 | 9.40742   | 53    | 0.59258   | 9.98626   | 3  | 40 |       |
| 21 | 9.39418   | 49 | 9.40794   | 52    | 0.59205   | 9.98623   | 3  | 39 |       |
| 22 | 9.39467   | 49 | 9.40847   | 52    | 0.59153   | 9.98620   | 3  | 38 |       |
| 23 | 9.39516   | 49 | 9.40899   | 52    | 0.59100   | 9.98617   | 3  | 37 |       |
| 24 | 9.39566   | 49 | 9.40952   | 52    | 0.59048   | 9.98613   | 3  | 36 |       |
| 25 | 9.39615   | 49 | 9.41004   | 52    | 0.58995   | 9.98610   | 3  | 35 |       |
| 26 | 9.39664   | 49 | 9.41057   | 52    | 0.58943   | 9.98607   | 3  | 34 |       |
| 27 | 9.39713   | 49 | 9.41109   | 52    | 0.58891   | 9.98604   | 3  | 33 |       |
| 28 | 9.39762   | 49 | 9.41161   | 52    | 0.58838   | 9.98600   | 3  | 32 |       |
| 29 | 9.39811   | 49 | 9.41213   | 52    | 0.58786   | 9.98597   | 3  | 31 |       |
| 30 | 9.39860   | 49 | 9.41266   | 52    | 0.58734   | 9.98594   | 3  | 30 |       |
| 31 | 9.39909   | 48 | 9.41318   | 52    | 0.58682   | 9.98591   | 3  | 29 |       |
| 32 | 9.39957   | 48 | 9.41370   | 52    | 0.58630   | 9.98587   | 3  | 28 |       |
| 33 | 9.40006   | 48 | 9.41422   | 52    | 0.58578   | 9.98584   | 3  | 27 |       |
| 34 | 9.40055   | 48 | 9.41474   | 51    | 0.58526   | 9.98581   | 3  | 26 |       |
| 35 | 9.40103   | 48 | 9.41525   | 52    | 0.58474   | 9.98578   | 3  | 25 |       |
| 36 | 9.40152   | 48 | 9.41577   | 52    | 0.58422   | 9.98574   | 3  | 24 |       |
| 37 | 9.40200   | 48 | 9.41629   | 51    | 0.58370   | 9.98571   | 3  | 23 |       |
| 38 | 9.40249   | 48 | 9.41681   | 51    | 0.58319   | 9.98568   | 3  | 22 |       |
| 39 | 9.40297   | 48 | 9.41732   | 51    | 0.58267   | 9.98564   | 3  | 21 |       |
| 40 | 9.40345   | 48 | 9.41784   | 52    | 0.58216   | 9.98561   | 3  | 20 |       |
| 41 | 9.40394   | 48 | 9.41836   | 51    | 0.58164   | 9.98558   | 3  | 19 |       |
| 42 | 9.40442   | 48 | 9.41887   | 51    | 0.58112   | 9.98554   | 3  | 18 |       |
| 43 | 9.40490   | 48 | 9.41938   | 51    | 0.58061   | 9.98551   | 3  | 17 |       |
| 44 | 9.40538   | 48 | 9.41990   | 51    | 0.58010   | 9.98548   | 3  | 16 |       |
| 45 | 9.40586   | 48 | 9.42041   | 51    | 0.57958   | 9.98544   | 3  | 15 |       |
| 46 | 9.40634   | 48 | 9.42092   | 51    | 0.57907   | 9.98541   | 3  | 14 |       |
| 47 | 9.40682   | 48 | 9.42144   | 51    | 0.57856   | 9.98538   | 3  | 13 |       |
| 48 | 9.40730   | 47 | 9.42195   | 51    | 0.57805   | 9.98534   | 3  | 12 |       |
| 49 | 9.40777   | 47 | 9.42246   | 51    | 0.57752   | 9.98531   | 3  | 11 |       |
| 50 | 9.40825   | 47 | 9.42297   | 51    | 0.57702   | 9.98528   | 3  | 10 |       |
| 51 | 9.40873   | 47 | 9.42348   | 51    | 0.57651   | 9.98524   | 3  | 9  |       |
| 52 | 9.40920   | 47 | 9.42399   | 51    | 0.57600   | 9.98521   | 3  | 8  |       |
| 53 | 9.40968   | 47 | 9.42450   | 50    | 0.57549   | 9.98518   | 3  | 7  |       |
| 54 | 9.41015   | 47 | 9.42501   | 50    | 0.57499   | 9.98514   | 3  | 6  |       |
| 55 | 9.41063   | 47 | 9.42552   | 50    | 0.57448   | 9.98511   | 3  | 5  |       |
| 56 | 9.41110   | 47 | 9.42602   | 51    | 0.57397   | 9.98508   | 3  | 4  |       |
| 57 | 9.41158   | 47 | 9.42653   | 50    | 0.57346   | 9.98504   | 3  | 3  |       |
| 58 | 9.41205   | 47 | 9.42704   | 50    | 0.57296   | 9.98501   | 3  | 2  |       |
| 59 | 9.41252   | 47 | 9.42754   | 50    | 0.57245   | 9.98498   | 3  | 1  |       |
| 60 | 9.41299   | 47 | 9.42805   | 50    | 0.57195   | 9.98494   | 3  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

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**TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.**

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.41 299  |    | 9.42 805  |       | 0.57 195  | 9.98 494  |    | 60 |       |
| 1  | 9.41 346  | 47 | 9.42 856  | 51    | 0.57 144  | 9.98 491  | 59 |    |       |
| 2  | 9.41 394  | 47 | 9.42 906  | 50    | 0.57 094  | 9.98 487  | 58 |    |       |
| 3  | 9.41 441  | 47 | 9.42 956  | 50    | 0.57 043  | 9.98 484  | 57 |    |       |
| 4  | 9.41 488  | 47 | 9.43 007  | 50    | 0.56 993  | 9.98 481  | 56 |    |       |
| 5  | 9.41 534  | 46 | 9.43 051  | 50    | 0.56 942  | 9.98 477  | 55 |    |       |
| 6  | 9.41 581  | 47 | 9.43 107  | 50    | 0.56 892  | 9.98 474  | 54 |    |       |
| 7  | 9.41 628  | 47 | 9.43 157  | 50    | 0.56 842  | 9.98 470  | 53 |    |       |
| 8  | 9.41 675  | 46 | 9.43 208  | 50    | 0.56 792  | 9.98 467  | 52 |    |       |
| 9  | 9.41 721  | 46 | 9.43 258  | 50    | 0.56 742  | 9.98 464  | 51 |    |       |
| 10 | 9.41 768  | 47 | 9.43 308  | 50    | 0.56 692  | 9.98 460  | 50 |    |       |
| 11 | 9.41 815  | 46 | 9.43 358  | 50    | 0.56 642  | 9.98 457  | 49 |    |       |
| 12 | 9.41 861  | 46 | 9.43 408  | 50    | 0.56 592  | 9.98 453  | 48 |    |       |
| 13 | 9.41 908  | 46 | 9.43 458  | 50    | 0.56 542  | 9.98 450  | 47 |    |       |
| 14 | 9.41 954  | 46 | 9.43 508  | 50    | 0.56 492  | 9.98 446  | 46 |    |       |
| 15 | 9.42 000  | 46 | 9.43 557  | 49    | 0.56 442  | 9.98 443  | 45 |    |       |
| 16 | 9.42 047  | 46 | 9.43 607  | 49    | 0.56 392  | 9.98 439  | 44 |    |       |
| 17 | 9.42 093  | 46 | 9.43 657  | 49    | 0.56 343  | 9.98 436  | 43 |    |       |
| 18 | 9.42 139  | 46 | 9.43 706  | 49    | 0.56 293  | 9.98 433  | 42 |    |       |
| 19 | 9.42 185  | 46 | 9.43 756  | 50    | 0.56 243  | 9.98 429  | 41 |    |       |
| 20 | 9.42 232  | 46 | 9.43 806  | 49    | 0.56 194  | 9.98 426  | 40 |    |       |
| 21 | 9.42 278  | 46 | 9.43 855  | 49    | 0.56 144  | 9.98 422  | 39 |    |       |
| 22 | 9.42 324  | 46 | 9.43 905  | 49    | 0.56 095  | 9.98 419  | 38 |    |       |
| 23 | 9.42 369  | 46 | 9.43 954  | 49    | 0.56 045  | 9.98 415  | 37 |    |       |
| 24 | 9.42 415  | 46 | 9.44 003  | 49    | 0.55 996  | 9.98 412  | 36 |    |       |
| 25 | 9.42 461  | 46 | 9.44 053  | 49    | 0.55 947  | 9.98 408  | 35 |    |       |
| 26 | 9.42 507  | 46 | 9.44 102  | 49    | 0.55 898  | 9.98 405  | 34 |    |       |
| 27 | 9.42 553  | 45 | 9.44 151  | 49    | 0.55 848  | 9.98 401  | 33 |    |       |
| 28 | 9.42 598  | 45 | 9.44 200  | 49    | 0.55 799  | 9.98 398  | 32 |    |       |
| 29 | 9.42 644  | 46 | 9.44 249  | 49    | 0.55 750  | 9.98 394  | 31 |    |       |
| 30 | 9.42 690  | 45 | 9.44 299  | 49    | 0.55 701  | 9.98 391  | 30 |    |       |
| 31 | 9.42 735  | 45 | 9.44 348  | 49    | 0.55 652  | 9.98 387  | 29 |    |       |
| 32 | 9.42 781  | 45 | 9.44 397  | 49    | 0.55 603  | 9.98 384  | 28 |    |       |
| 33 | 9.42 826  | 45 | 9.44 446  | 48    | 0.55 554  | 9.98 380  | 27 |    |       |
| 34 | 9.42 871  | 45 | 9.44 494  | 48    | 0.55 505  | 9.98 377  | 26 |    |       |
| 35 | 9.42 917  | 45 | 9.44 543  | 49    | 0.55 456  | 9.98 373  | 25 |    |       |
| 36 | 9.42 962  | 45 | 9.44 592  | 48    | 0.55 407  | 9.98 370  | 24 |    |       |
| 37 | 9.43 007  | 45 | 9.44 641  | 49    | 0.55 359  | 9.98 366  | 23 |    |       |
| 38 | 9.43 052  | 45 | 9.44 690  | 48    | 0.55 310  | 9.98 363  | 22 |    |       |
| 39 | 9.43 098  | 45 | 9.44 738  | 48    | 0.55 261  | 9.98 359  | 21 |    |       |
| 40 | 9.43 143  | 45 | 9.44 787  | 48    | 0.55 213  | 9.98 356  | 20 |    |       |
| 41 | 9.43 188  | 45 | 9.44 835  | 48    | 0.55 164  | 9.98 352  | 19 |    |       |
| 42 | 9.43 233  | 45 | 9.44 884  | 48    | 0.55 116  | 9.98 348  | 18 |    |       |
| 43 | 9.43 278  | 45 | 9.44 932  | 48    | 0.55 067  | 9.98 345  | 17 |    |       |
| 44 | 9.43 322  | 44 | 9.44 981  | 48    | 0.55 019  | 9.98 341  | 16 |    |       |
| 45 | 9.43 367  | 45 | 9.45 029  | 48    | 0.54 970  | 9.98 338  | 15 |    |       |
| 46 | 9.43 412  | 44 | 9.45 077  | 48    | 0.54 922  | 9.98 334  | 14 |    |       |
| 47 | 9.43 457  | 45 | 9.45 126  | 48    | 0.54 874  | 9.98 331  | 13 |    |       |
| 48 | 9.43 501  | 44 | 9.45 174  | 48    | 0.54 825  | 9.98 327  | 12 |    |       |
| 49 | 9.43 546  | 44 | 9.45 222  | 48    | 0.54 777  | 9.98 324  | 11 |    |       |
| 50 | 9.43 591  | 45 | 9.45 270  | 48    | 0.54 729  | 9.98 320  | 10 |    |       |
| 51 | 9.43 635  | 44 | 9.45 318  | 48    | 0.54 681  | 9.98 316  | 9  |    |       |
| 52 | 9.43 680  | 44 | 9.45 367  | 48    | 0.54 633  | 9.98 313  | 8  |    |       |
| 53 | 9.43 724  | 44 | 9.45 415  | 48    | 0.54 585  | 9.98 309  | 7  |    |       |
| 54 | 9.43 768  | 44 | 9.45 463  | 48    | 0.54 537  | 9.98 306  | 6  |    |       |
| 55 | 9.43 813  | 44 | 9.45 510  | 47    | 0.54 489  | 9.98 302  | 5  |    |       |
| 56 | 9.43 857  | 44 | 9.45 558  | 48    | 0.54 441  | 9.98 298  | 4  |    |       |
| 57 | 9.43 901  | 44 | 9.45 606  | 47    | 0.54 393  | 9.98 295  | 3  |    |       |
| 58 | 9.43 945  | 44 | 9.45 654  | 48    | 0.54 346  | 9.98 291  | 2  |    |       |
| 59 | 9.43 989  | 44 | 9.45 702  | 47    | 0.54 298  | 9.98 288  | 1  |    |       |
| 60 | 9.44 034  | 44 | 9.45 749  | 47    | 0.54 256  | 9.98 284  | 0  |    |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------------------|
| 0  | 9.44 034  | 44 | 9.45 749  | 48    | 0.54 250  | 9.98 284  | 60 |                   |
| 1  | 9.44 078  | 44 | 9.45 797  | 47    | 0.54 202  | 9.98 280  | 59 |                   |
| 2  | 9.44 122  | 44 | 9.45 845  | 47    | 0.54 155  | 9.98 277  | 58 |                   |
| 3  | 9.44 166  | 44 | 9.45 892  | 47    | 0.54 107  | 9.98 273  | 57 |                   |
| 4  | 9.44 209  | 43 | 9.45 940  | 47    | 0.54 060  | 9.98 269  | 56 | 48 47 47          |
| 5  | 9.44 253  | 44 | 9.45 987  | 47    | 0.54 012  | 9.98 266  | 55 | 6 4.8 4.7 4.      |
| 6  | 9.44 297  | 44 | 9.46 035  | 47    | 0.53 965  | 9.98 262  | 54 | 7 5.6 5.5 5.      |
| 7  | 9.44 341  | 43 | 9.46 082  | 47    | 0.53 917  | 9.98 258  | 53 | 8 6.4 6.3 6.      |
| 8  | 9.44 384  | 43 | 9.46 129  | 47    | 0.53 870  | 9.98 255  | 52 | 9 7.2 7.1 7.      |
| 9  | 9.44 428  | 44 | 9.46 177  | 47    | 0.53 823  | 9.98 251  | 51 | 10 8.0 7.9 7.     |
| 10 | 9.44 472  | 43 | 9.46 224  | 47    | 0.53 776  | 9.98 247  | 50 | 20 16.0 15.8 15.  |
| 11 | 9.44 515  | 43 | 9.46 271  | 47    | 0.53 728  | 9.98 244  | 49 | 30 24.0 23.7 23.  |
| 12 | 9.44 559  | 43 | 9.46 318  | 47    | 0.53 681  | 9.98 240  | 48 | 40 32.0 31.6 31.  |
| 13 | 9.44 602  | 43 | 9.46 366  | 47    | 0.53 634  | 9.98 236  | 47 | 50 40.0 39.6 39.  |
| 14 | 9.44 646  | 43 | 9.46 413  | 47    | 0.53 587  | 9.98 233  | 46 |                   |
| 15 | 9.44 689  | 43 | 9.46 460  | 47    | 0.53 540  | 9.98 229  | 45 | 46 46 45          |
| 16 | 9.44 732  | 43 | 9.46 507  | 47    | 0.53 493  | 9.98 225  | 44 | 6 4.6 4.6 4.5     |
| 17 | 9.44 776  | 43 | 9.46 554  | 47    | 0.53 446  | 9.98 222  | 43 | 7 5.4 5.3 5.3     |
| 18 | 9.44 819  | 43 | 9.46 601  | 46    | 0.53 399  | 9.98 218  | 42 | 8 6.2 6.1 6.0     |
| 19 | 9.44 862  | 43 | 9.46 647  | 46    | 0.53 352  | 9.98 214  | 41 | 9 7.0 6.9 6.8     |
| 20 | 9.44 905  | 43 | 9.46 694  | 47    | 0.53 305  | 9.98 211  | 40 | 10 7.7 7.6 7.6    |
| 21 | 9.44 948  | 43 | 9.46 741  | 47    | 0.53 258  | 9.98 207  | 39 | 20 15.5 15.3 15.1 |
| 22 | 9.44 991  | 43 | 9.46 788  | 46    | 0.53 212  | 9.98 203  | 38 | 30 23.2 23.0 22.7 |
| 23 | 9.45 034  | 43 | 9.46 834  | 47    | 0.53 165  | 9.98 200  | 37 | 40 31.0 30.6 30.3 |
| 24 | 9.45 077  | 43 | 9.46 881  | 47    | 0.53 118  | 9.98 196  | 36 | 50 38.7 38.3 37.9 |
| 25 | 9.45 120  | 43 | 9.46 928  | 46    | 0.53 072  | 9.98 192  | 35 |                   |
| 26 | 9.45 163  | 43 | 9.46 974  | 46    | 0.53 025  | 9.98 188  | 34 | 44 43 43          |
| 27 | 9.45 206  | 43 | 9.47 021  | 46    | 0.52 979  | 9.98 185  | 33 | 6 4.4 4.3 4.3     |
| 28 | 9.45 249  | 43 | 9.47 067  | 46    | 0.52 932  | 9.98 181  | 32 | 7 5.1 5.1 5.0     |
| 29 | 9.45 291  | 42 | 9.47 114  | 46    | 0.52 886  | 9.98 177  | 31 | 8 5.8 5.8 5.7     |
| 30 | 9.45 334  | 43 | 9.47 160  | 46    | 0.52 839  | 9.98 173  | 30 | 9 6.6 6.5 6.4     |
| 31 | 9.45 377  | 43 | 9.47 207  | 46    | 0.52 793  | 9.98 170  | 29 | 10 7.3 7.2 7.1    |
| 32 | 9.45 419  | 42 | 9.47 253  | 46    | 0.52 747  | 9.98 166  | 28 | 20 14.6 14.5 14.4 |
| 33 | 9.45 462  | 42 | 9.47 299  | 46    | 0.52 700  | 9.98 162  | 27 | 30 22.0 21.7 21.5 |
| 34 | 9.45 504  | 42 | 9.47 345  | 46    | 0.52 654  | 9.98 158  | 26 | 40 29.3 29.0 28.7 |
| 35 | 9.45 547  | 42 | 9.47 392  | 46    | 0.52 608  | 9.98 155  | 25 | 50 36.6 36.2 35.8 |
| 36 | 9.45 589  | 42 | 9.47 438  | 46    | 0.52 562  | 9.98 151  | 24 |                   |
| 37 | 9.45 631  | 42 | 9.47 484  | 46    | 0.52 516  | 9.98 147  | 23 |                   |
| 38 | 9.45 674  | 42 | 9.47 530  | 46    | 0.52 469  | 9.98 143  | 22 | 42 42 41          |
| 39 | 9.45 716  | 42 | 9.47 576  | 46    | 0.52 423  | 9.98 140  | 21 | 6 4.2 4.2 4.1     |
| 40 | 9.45 758  | 42 | 9.47 622  | 46    | 0.52 377  | 9.98 136  | 20 | 7 4.9 4.9 4.8     |
| 41 | 9.45 800  | 42 | 9.47 668  | 45    | 0.52 331  | 9.98 132  | 19 | 8 5.6 5.6 5.5     |
| 42 | 9.45 842  | 42 | 9.47 714  | 46    | 0.52 286  | 9.98 128  | 18 | 9 6.4 6.3 6.2     |
| 43 | 9.45 885  | 42 | 9.47 760  | 46    | 0.52 240  | 9.98 124  | 17 | 10 7.1 7.0 6.9    |
| 44 | 9.45 927  | 42 | 9.47 806  | 46    | 0.52 194  | 9.98 121  | 16 | 20 14.1 14.0 13.8 |
| 45 | 9.45 969  | 42 | 9.47 851  | 45    | 0.52 148  | 9.98 117  | 15 | 30 21.2 21.0 20.7 |
| 46 | 9.46 011  | 41 | 9.47 897  | 45    | 0.52 102  | 9.98 113  | 14 | 40 28.3 28.0 27.6 |
| 47 | 9.46 052  | 42 | 9.47 943  | 46    | 0.52 057  | 9.98 109  | 13 | 50 35.4 35.0 34.6 |
| 48 | 9.46 094  | 42 | 9.47 989  | 45    | 0.52 011  | 9.98 105  | 12 |                   |
| 49 | 9.46 136  | 41 | 9.48 034  | 45    | 0.51 965  | 9.98 102  | 11 |                   |
| 50 | 9.46 178  | 41 | 9.48 080  | 45    | 0.51 920  | 9.98 098  | 10 | 4 3               |
| 51 | 9.46 220  | 41 | 9.48 125  | 45    | 0.51 874  | 9.98 094  | 9  | 6 0.4 0.3         |
| 52 | 9.46 261  | 41 | 9.48 171  | 45    | 0.51 829  | 9.98 090  | 8  | 7 0.4 0.4         |
| 53 | 9.46 303  | 42 | 9.48 216  | 45    | 0.51 783  | 9.98 086  | 7  | 8 0.5 0.4         |
| 54 | 9.46 345  | 41 | 9.48 262  | 45    | 0.51 738  | 9.98 082  | 6  | 9 0.6 0.5         |
| 55 | 9.46 386  | 41 | 9.48 307  | 45    | 0.51 692  | 9.98 079  | 5  | 10 0.6 0.6        |
| 56 | 9.46 428  | 41 | 9.48 353  | 45    | 0.51 647  | 9.98 075  | 4  | 20 1.3 1.1        |
| 57 | 9.46 469  | 41 | 9.48 399  | 45    | 0.51 602  | 9.98 071  | 3  | 30 2.0 1.7        |
| 58 | 9.46 511  | 41 | 9.48 443  | 45    | 0.51 556  | 9.98 067  | 2  | 40 2.6 2.3        |
| 59 | 9.46 552  | 41 | 9.48 488  | 45    | 0.51 511  | 9.98 063  | 1  | 50 3.3 2.9        |
| 60 | 9.46 593  | 41 | 9.48 534  | 45    | 0.51 466  | 9.98 059  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P.             |

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# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.                  |
|----|-----------|----|-----------------|-----------|-----------|----|----|------------------------|
| 0  | 9.46 593  |    | 9.48 534        | 0.51 466  | 9.98 059  | 3  | 60 |                        |
| 1  | 9.46 635  | 41 | 9.48 579        | 0.51 421  | 9.98 056  | 4  | 59 |                        |
| 2  | 9.46 676  | 41 | 9.48 624        | 0.51 376  | 9.98 052  | 4  | 58 |                        |
| 3  | 9.46 717  | 41 | 9.48 669        | 0.51 330  | 9.98 048  | 4  | 57 |                        |
| 4  | 9.46 758  | 41 | 9.48 714        | 0.51 285  | 9.98 044  | 4  | 56 | 45 45 44 44            |
| 5  | 9.46 799  | 41 | 9.48 759        | 0.51 240  | 9.98 040  | 3  | 55 | 6 4 5 4 5 4 4 4        |
| 6  | 9.46 840  | 41 | 9.48 804        | 0.51 195  | 9.98 036  | 4  | 54 | 7 5 3 5 2 5 2 5        |
| 7  | 9.46 881  | 41 | 9.48 849        | 0.51 151  | 9.98 032  | 4  | 53 | 8 6 0 6 0 5 9 5        |
| 8  | 9.46 922  | 41 | 9.48 894        | 0.51 106  | 9.98 028  | 4  | 52 | 9 6 8 6 7 6 7 6        |
| 9  | 9.46 963  | 41 | 9.48 939        | 0.51 061  | 9.98 024  | 4  | 51 | 10 7 6 7 5 7 4 7       |
| 10 | 9.47 004  | 41 | 9.48 984        | 0.51 016  | 9.98 021  | 3  | 50 | 20 15 1 15 0 14 8 14   |
| 11 | 9.47 045  | 41 | 9.49 028        | 0.50 971  | 9.98 017  | 4  | 49 | 30 22 7 22 5 22 2 22 0 |
| 12 | 9.47 086  | 41 | 9.49 073        | 0.50 926  | 9.98 013  | 4  | 48 | 40 30 3 30 0 29 6 29 3 |
| 13 | 9.47 127  | 41 | 9.49 118        | 0.50 882  | 9.98 009  | 4  | 47 | 50 37 9 37 5 37 1 36 6 |
| 14 | 9.47 168  | 41 | 9.49 162        | 0.50 837  | 9.98 005  | 4  | 46 |                        |
| 15 | 9.47 208  | 40 | 9.49 207        | 0.50 792  | 9.98 001  | 4  | 45 | 43 43                  |
| 16 | 9.47 249  | 40 | 9.49 252        | 0.50 748  | 9.97 997  | 3  | 44 | 6 4 3 4 3              |
| 17 | 9.47 290  | 40 | 9.49 296        | 0.50 703  | 9.97 993  | 4  | 43 | 7 5 1 5 0 7            |
| 18 | 9.47 330  | 40 | 9.49 341        | 0.50 659  | 9.97 989  | 4  | 42 | 8 5 8 5 4 5            |
| 19 | 9.47 371  | 40 | 9.49 385        | 0.50 614  | 9.97 985  | 4  | 41 | 9 6 5 6 4 4            |
| 20 | 9.47 411  | 40 | 9.49 430        | 0.50 570  | 9.97 981  | 4  | 40 | 10 7 2 7 1             |
| 21 | 9.47 452  | 40 | 9.49 474        | 0.50 525  | 9.97 977  | 4  | 39 | 20 14 5 14 3           |
| 22 | 9.47 492  | 40 | 9.49 518        | 0.50 481  | 9.97 973  | 4  | 38 | 30 21 7 21 5           |
| 23 | 9.47 532  | 40 | 9.49 563        | 0.50 437  | 9.97 969  | 4  | 37 | 40 29 0 28 6           |
| 24 | 9.47 573  | 40 | 9.49 607        | 0.50 392  | 9.97 966  | 3  | 36 | 50 36 2 35 8           |
| 25 | 9.47 613  | 40 | 9.49 651        | 0.50 348  | 9.97 962  | 4  | 35 |                        |
| 26 | 9.47 653  | 40 | 9.49 695        | 0.50 304  | 9.97 958  | 4  | 34 | 41 41 40 40            |
| 27 | 9.47 694  | 40 | 9.49 740        | 0.50 260  | 9.97 954  | 4  | 33 | 6 4 1 4 0 4 0          |
| 28 | 9.47 734  | 40 | 9.49 784        | 0.50 216  | 9.97 950  | 4  | 32 | 7 4 8 4 7 4 6          |
| 29 | 9.47 774  | 40 | 9.49 828        | 0.50 172  | 9.97 946  | 4  | 31 | 8 5 5 5 4 5 3          |
| 30 | 9.47 814  | 40 | 9.49 872        | 0.50 128  | 9.97 942  | 4  | 30 | 9 6 2 6 1 6 0          |
| 31 | 9.47 854  | 40 | 9.49 916        | 0.50 083  | 9.97 938  | 4  | 29 | 10 6 9 6 8 6 7 6       |
| 32 | 9.47 894  | 40 | 9.49 960        | 0.50 039  | 9.97 934  | 4  | 28 | 20 13 8 13 6 13 5 13 3 |
| 33 | 9.47 934  | 40 | 9.50 004        | 0.49 996  | 9.97 930  | 4  | 27 | 30 20 7 20 5 20 2 20 0 |
| 34 | 9.47 974  | 40 | 9.50 048        | 0.49 952  | 9.97 926  | 4  | 26 | 40 27 6 27 3 27 0 26 6 |
| 35 | 9.48 014  | 40 | 9.50 092        | 0.49 908  | 9.97 922  | 4  | 25 | 50 34 6 34 1 33 7 33 3 |
| 36 | 9.48 054  | 40 | 9.50 136        | 0.49 864  | 9.97 918  | 4  | 24 |                        |
| 37 | 9.48 093  | 39 | 9.50 179        | 0.49 820  | 9.97 914  | 4  | 23 | 39 39 38               |
| 38 | 9.48 133  | 39 | 9.50 223        | 0.49 776  | 9.97 910  | 4  | 22 | 6 3 9 3 9 3 8          |
| 39 | 9.48 173  | 39 | 9.50 267        | 0.49 733  | 9.97 906  | 4  | 21 | 7 4 6 4 5 4 5          |
| 40 | 9.48 213  | 40 | 9.50 311        | 0.49 689  | 9.97 902  | 4  | 20 | 8 5 2 5 2 5 1          |
| 41 | 9.48 252  | 39 | 9.50 354        | 0.49 645  | 9.97 898  | 4  | 19 | 9 5 9 5 8 5 8          |
| 42 | 9.48 292  | 39 | 9.50 398        | 0.49 602  | 9.97 894  | 4  | 18 | 10 6 6 6 5 6 4         |
| 43 | 9.48 331  | 39 | 9.50 442        | 0.49 558  | 9.97 890  | 4  | 17 | 20 13 1 13 0 12 8 12   |
| 44 | 9.48 371  | 39 | 9.50 485        | 0.49 514  | 9.97 886  | 4  | 16 | 30 19 7 19 5 19 2 19 0 |
| 45 | 9.48 410  | 39 | 9.50 529        | 0.49 471  | 9.97 881  | 4  | 15 | 40 26 3 26 0 25 6      |
| 46 | 9.48 450  | 39 | 9.50 572        | 0.49 427  | 9.97 877  | 4  | 14 | 50 32 9 32 5 32 1      |
| 47 | 9.48 489  | 39 | 9.50 616        | 0.49 384  | 9.97 873  | 4  | 13 |                        |
| 48 | 9.48 529  | 39 | 9.50 659        | 0.49 340  | 9.97 869  | 4  | 12 | 4 4 3                  |
| 49 | 9.48 568  | 39 | 9.50 702        | 0.49 297  | 9.97 865  | 4  | 11 | 6 0 4 0 4 0 3          |
| 50 | 9.48 607  | 39 | 9.50 746        | 0.49 254  | 9.97 861  | 4  | 10 | 7 0 5 0 4 0 4          |
| 51 | 9.48 646  | 39 | 9.50 789        | 0.49 210  | 9.97 857  | 4  | 9  | 8 0 6 0 5 0 4          |
| 52 | 9.48 686  | 39 | 9.50 832        | 0.49 167  | 9.97 853  | 4  | 8  | 9 0 7 0 6 0 5          |
| 53 | 9.48 725  | 39 | 9.50 876        | 0.49 124  | 9.97 849  | 4  | 7  | 10 0 7 0 6 0 6         |
| 54 | 9.48 764  | 39 | 9.50 919        | 0.49 081  | 9.97 845  | 4  | 6  | 20 1 5 1 3 1 1         |
| 55 | 9.48 803  | 39 | 9.50 962        | 0.49 038  | 9.97 841  | 4  | 5  | 30 2 2 2 0 1 7         |
| 56 | 9.48 842  | 39 | 9.51 005        | 0.48 994  | 9.97 837  | 4  | 4  | 40 3 0 2 6 2 3         |
| 57 | 9.48 881  | 39 | 9.51 048        | 0.48 951  | 9.97 833  | 4  | 3  | 50 3 7 3 3 2 9         |
| 58 | 9.48 920  | 39 | 9.51 091        | 0.48 908  | 9.97 829  | 4  | 2  |                        |
| 59 | 9.48 959  | 39 | 9.51 134        | 0.48 865  | 9.97 824  | 4  | 1  |                        |
| 60 | 9.48 998  | 38 | 9.51 177        | 0.48 822  | 9.97 820  | 4  | 0  |                        |
|    | Log. Cos. | d. | Log. Cot. c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.                  |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.        |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|--------------|
| 0  | 9.48 998  |    | 9.51 177  |       | 0.48 822  | 9.97 820  |    | 60 |              |
| 1  | 9.49 037  | 39 | 9.51 220  | 43    | 0.48 779  | 9.97 816  | 4  | 59 |              |
| 2  | 9.49 076  | 38 | 9.51 263  | 43    | 0.48 735  | 9.97 812  | 4  | 58 |              |
| 3  | 9.49 114  | 38 | 9.51 306  | 43    | 0.48 693  | 9.97 808  | 4  | 57 |              |
| 4  | 9.49 153  | 39 | 9.51 349  | 43    | 0.48 650  | 9.97 804  | 4  | 56 | 43 42 42     |
| 5  | 9.49 192  | 38 | 9.51 392  | 42    | 0.48 608  | 9.97 800  | 4  | 55 | 6 4.3 4.2    |
| 6  | 9.49 231  | 39 | 9.51 435  | 43    | 0.48 565  | 9.97 796  | 4  | 54 | 7 5.0 4.9    |
| 7  | 9.49 269  | 38 | 9.51 477  | 43    | 0.48 522  | 9.97 792  | 4  | 53 | 8 5.7 5.6    |
| 8  | 9.49 308  | 38 | 9.51 520  | 42    | 0.48 479  | 9.97 787  | 4  | 52 | 9 6.4 6.4    |
| 9  | 9.49 346  | 38 | 9.51 563  | 42    | 0.48 437  | 9.97 783  | 4  | 51 | 10 7.1 7.1   |
| 10 | 9.49 385  | 38 | 9.51 605  | 42    | 0.48 394  | 9.97 779  | 4  | 50 | 20 14.3 14.1 |
| 11 | 9.49 423  | 38 | 9.51 648  | 42    | 0.48 351  | 9.97 775  | 4  | 49 | 30 21.5 21.2 |
| 12 | 9.49 462  | 38 | 9.51 691  | 42    | 0.48 309  | 9.97 771  | 4  | 48 | 40 28.6 28.3 |
| 13 | 9.49 500  | 38 | 9.51 733  | 42    | 0.48 266  | 9.97 767  | 4  | 47 | 50 35.8 35.4 |
| 14 | 9.49 539  | 38 | 9.51 776  | 42    | 0.48 224  | 9.97 763  | 4  | 46 |              |
| 15 | 9.49 577  | 38 | 9.51 818  | 42    | 0.48 181  | 9.97 758  | 4  | 45 | 41 41        |
| 16 | 9.49 615  | 38 | 9.51 861  | 42    | 0.48 139  | 9.97 754  | 4  | 44 | 6 4.1 4.1    |
| 17 | 9.49 653  | 38 | 9.51 905  | 42    | 0.48 096  | 9.97 750  | 4  | 43 | 7 4.8 4.8    |
| 18 | 9.49 692  | 38 | 9.51 946  | 42    | 0.48 054  | 9.97 746  | 4  | 42 | 8 5.5 5.4    |
| 19 | 9.49 730  | 38 | 9.51 988  | 42    | 0.48 012  | 9.97 742  | 4  | 41 | 9 6.2 6.1    |
| 20 | 9.49 768  | 38 | 9.52 030  | 42    | 0.47 969  | 9.97 737  | 4  | 40 | 10 6.9 6.8   |
| 21 | 9.49 806  | 38 | 9.52 073  | 42    | 0.47 927  | 9.97 733  | 4  | 39 | 20 13.8 13.6 |
| 22 | 9.49 844  | 38 | 9.52 115  | 42    | 0.47 885  | 9.97 729  | 4  | 38 | 30 20.7 20.5 |
| 23 | 9.49 882  | 38 | 9.52 157  | 42    | 0.47 842  | 9.97 725  | 4  | 37 | 40 27.6 27.3 |
| 24 | 9.49 920  | 38 | 9.52 199  | 42    | 0.47 800  | 9.97 721  | 4  | 36 | 50 34.6 34.1 |
| 25 | 9.49 958  | 38 | 9.52 241  | 42    | 0.47 758  | 9.97 716  | 4  | 35 |              |
| 26 | 9.49 996  | 37 | 9.52 284  | 42    | 0.47 716  | 9.97 712  | 4  | 34 | 39 38 38     |
| 27 | 9.50 034  | 38 | 9.52 326  | 42    | 0.47 674  | 9.97 708  | 4  | 33 | 6 3.9 3.8    |
| 28 | 9.50 072  | 38 | 9.52 368  | 42    | 0.47 632  | 9.97 704  | 4  | 32 | 7 4.5 4.5    |
| 29 | 9.50 110  | 38 | 9.52 410  | 42    | 0.47 590  | 9.97 700  | 4  | 31 | 8 5.2 5.1    |
| 30 | 9.50 147  | 37 | 9.52 452  | 42    | 0.47 548  | 9.97 695  | 4  | 30 | 9 5.8 5.8    |
| 31 | 9.50 185  | 38 | 9.52 494  | 42    | 0.47 506  | 9.97 691  | 4  | 29 | 10 6.5 6.4   |
| 32 | 9.50 223  | 37 | 9.52 536  | 42    | 0.47 464  | 9.97 687  | 4  | 28 | 20 13.0 12.8 |
| 33 | 9.50 260  | 37 | 9.52 578  | 42    | 0.47 422  | 9.97 683  | 4  | 27 | 30 19.5 19.2 |
| 34 | 9.50 298  | 38 | 9.52 619  | 41    | 0.47 380  | 9.97 678  | 4  | 26 | 40 26.0 25.6 |
| 35 | 9.50 336  | 37 | 9.52 661  | 42    | 0.47 338  | 9.97 674  | 4  | 25 | 50 32.5 32.1 |
| 36 | 9.50 373  | 37 | 9.52 703  | 41    | 0.47 296  | 9.97 670  | 4  | 24 |              |
| 37 | 9.50 411  | 37 | 9.52 745  | 42    | 0.47 255  | 9.97 666  | 4  | 23 | 37 37 36     |
| 38 | 9.50 448  | 37 | 9.52 787  | 41    | 0.47 213  | 9.97 661  | 4  | 22 | 6 3.7 3.7    |
| 39 | 9.50 486  | 37 | 9.52 828  | 41    | 0.47 171  | 9.97 657  | 4  | 21 | 7 4.4 4.3    |
| 40 | 9.50 523  | 37 | 9.52 870  | 42    | 0.47 130  | 9.97 653  | 4  | 20 | 8 5.0 4.9    |
| 41 | 9.50 561  | 37 | 9.52 912  | 41    | 0.47 088  | 9.97 649  | 4  | 19 | 9 5.6 5.5    |
| 42 | 9.50 598  | 37 | 9.52 953  | 41    | 0.47 046  | 9.97 644  | 4  | 18 | 10 6.2 6.1   |
| 43 | 9.50 635  | 37 | 9.52 995  | 41    | 0.47 005  | 9.97 640  | 4  | 17 | 20 12.5 12.3 |
| 44 | 9.50 672  | 37 | 9.53 036  | 41    | 0.46 963  | 9.97 636  | 4  | 16 | 30 18.7 18.5 |
| 45 | 9.50 710  | 37 | 9.53 078  | 41    | 0.46 922  | 9.97 632  | 4  | 15 | 40 25.0 24.6 |
| 46 | 9.50 747  | 37 | 9.53 119  | 41    | 0.46 880  | 9.97 627  | 4  | 14 | 50 31.2 30.8 |
| 47 | 9.50 784  | 37 | 9.53 161  | 41    | 0.46 839  | 9.97 623  | 4  | 13 |              |
| 48 | 9.50 821  | 37 | 9.53 202  | 41    | 0.46 797  | 9.97 619  | 4  | 12 | 4 4          |
| 49 | 9.50 858  | 37 | 9.53 244  | 41    | 0.46 756  | 9.97 614  | 4  | 11 | 6 0.4 0.4    |
| 50 | 9.50 895  | 37 | 9.53 285  | 41    | 0.46 714  | 9.97 610  | 4  | 10 | 7 0.5 0.4    |
| 51 | 9.50 932  | 37 | 9.53 326  | 41    | 0.46 673  | 9.97 606  | 4  | 9  | 8 0.6 0.5    |
| 52 | 9.50 969  | 37 | 9.53 368  | 41    | 0.46 632  | 9.97 601  | 4  | 8  | 9 0.7 0.6    |
| 53 | 9.51 006  | 37 | 9.53 409  | 41    | 0.46 591  | 9.97 597  | 4  | 7  | 10 0.7 0.6   |
| 54 | 9.51 043  | 37 | 9.53 450  | 41    | 0.46 549  | 9.97 593  | 4  | 6  | 20 1.5 1.3   |
| 55 | 9.51 080  | 37 | 9.53 491  | 41    | 0.46 508  | 9.97 588  | 4  | 5  | 30 2.2 2.0   |
| 56 | 9.51 117  | 36 | 9.53 533  | 41    | 0.46 467  | 9.97 584  | 4  | 4  | 40 3.0 2.6   |
| 57 | 9.51 154  | 36 | 9.53 574  | 41    | 0.46 426  | 9.97 580  | 4  | 3  | 50 3.7 3.3   |
| 58 | 9.51 190  | 36 | 9.53 615  | 41    | 0.46 385  | 9.97 575  | 4  | 2  |              |
| 59 | 9.51 227  | 36 | 9.53 656  | 41    | 0.46 344  | 9.97 571  | 4  | 1  |              |
| 60 | 9.51 264  | 36 | 9.53 697  | 41    | 0.46 303  | 9.97 567  | 4  | 0  |              |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.        |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.                |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|----------------------|
| 0  | 9.51 264  | 37 | 9.53 697  | 41    | 0.46 303  | 9.97 567  | 4  | 60 |                      |
| 1  | 9.51 301  | 36 | 9.53 738  | 41    | 0.46 262  | 9.97 522  | 4  | 59 |                      |
| 2  | 9.51 337  | 36 | 9.53 779  | 41    | 0.46 221  | 9.97 558  | 4  | 58 |                      |
| 3  | 9.51 374  | 36 | 9.53 820  | 41    | 0.46 180  | 9.97 554  | 4  | 57 |                      |
| 4  | 9.51 410  | 36 | 9.53 861  | 41    | 0.46 139  | 9.97 549  | 4  | 56 |                      |
| 5  | 9.51 447  | 36 | 9.53 902  | 41    | 0.46 098  | 9.97 545  | 4  | 55 | 41 40 40             |
| 6  | 9.51 483  | 36 | 9.53 943  | 40    | 0.46 057  | 9.97 541  | 4  | 54 | 6' 4.1' 4.0' 4.0     |
| 7  | 9.51 520  | 36 | 9.53 983  | 41    | 0.46 016  | 9.97 536  | 4  | 53 | 7' 4.8' 4.7' 4.6     |
| 8  | 9.51 556  | 36 | 9.54 024  | 41    | 0.45 975  | 9.97 532  | 4  | 52 | 8' 5.4' 5.4' 5.3     |
| 9  | 9.51 593  | 36 | 9.54 065  | 41    | 0.45 934  | 9.97 527  | 4  | 51 | 9' 6.1' 6.1' 6.0     |
| 10 | 9.51 629  | 36 | 9.54 106  | 41    | 0.45 894  | 9.97 523  | 4  | 50 | 10' 6.8' 6.7' 6.6    |
| 11 | 9.51 665  | 36 | 9.54 147  | 41    | 0.45 853  | 9.97 519  | 4  | 49 | 20' 13' 13' 13.3     |
| 12 | 9.51 702  | 36 | 9.54 187  | 40    | 0.45 812  | 9.97 514  | 4  | 48 | 30' 20' 20' 20.0     |
| 13 | 9.51 738  | 36 | 9.54 228  | 41    | 0.45 772  | 9.97 510  | 4  | 47 | 40' 27' 27' 26.6     |
| 14 | 9.51 774  | 36 | 9.54 269  | 41    | 0.45 731  | 9.97 505  | 4  | 46 | 50' 34' 33.7' 33.3   |
| 15 | 9.51 810  | 36 | 9.54 309  | 40    | 0.45 690  | 9.97 501  | 4  | 45 |                      |
| 16 | 9.51 847  | 36 | 9.54 350  | 40    | 0.45 650  | 9.97 497  | 4  | 44 | 39 39                |
| 17 | 9.51 883  | 36 | 9.54 390  | 40    | 0.45 609  | 9.97 492  | 4  | 43 | 6' 3.9' 3.9' 3.9     |
| 18 | 9.51 919  | 36 | 9.54 431  | 40    | 0.45 569  | 9.97 488  | 4  | 42 | 7' 4.6' 4.5' 4.5     |
| 19 | 9.51 955  | 36 | 9.54 471  | 40    | 0.45 528  | 9.97 483  | 4  | 41 | 8' 5.2' 5.2' 5.2     |
| 20 | 9.51 991  | 36 | 9.54 512  | 40    | 0.45 488  | 9.97 479  | 4  | 40 | 9' 5.9' 5.8' 5.8     |
| 21 | 9.52 027  | 36 | 9.54 552  | 40    | 0.45 447  | 9.97 475  | 4  | 39 | 10' 6.6' 6.5' 6.5    |
| 22 | 9.52 063  | 36 | 9.54 593  | 40    | 0.45 407  | 9.97 470  | 4  | 38 | 20' 13' 13.0         |
| 23 | 9.52 099  | 36 | 9.54 633  | 40    | 0.45 367  | 9.97 466  | 4  | 37 | 30' 19.7' 19.5       |
| 24 | 9.52 135  | 36 | 9.54 673  | 40    | 0.45 326  | 9.97 461  | 4  | 36 | 40' 26.3' 26.0       |
| 25 | 9.52 170  | 35 | 9.54 714  | 40    | 0.45 286  | 9.97 457  | 4  | 35 | 50' 32.9' 32.5       |
| 26 | 9.52 206  | 36 | 9.54 754  | 40    | 0.45 246  | 9.97 452  | 4  | 34 |                      |
| 27 | 9.52 242  | 35 | 9.54 794  | 40    | 0.45 205  | 9.97 448  | 4  | 33 | 37 36 36             |
| 28 | 9.52 278  | 36 | 9.54 834  | 40    | 0.45 165  | 9.97 443  | 4  | 32 | 6' 3.7' 3.6' 3.6     |
| 29 | 9.52 314  | 35 | 9.54 874  | 40    | 0.45 125  | 9.97 439  | 4  | 31 | 7' 4.3' 4.2' 4.2     |
| 30 | 9.52 349  | 35 | 9.54 915  | 40    | 0.45 085  | 9.97 434  | 4  | 30 | 8' 4.9' 4.8' 4.8     |
| 31 | 9.52 385  | 36 | 9.54 955  | 40    | 0.45 045  | 9.97 430  | 4  | 29 | 9' 5.5' 5.5' 5.4     |
| 32 | 9.52 421  | 35 | 9.54 995  | 40    | 0.45 005  | 9.97 425  | 4  | 28 | 10' 6.1' 6.1' 6.0    |
| 33 | 9.52 456  | 35 | 9.55 035  | 40    | 0.44 965  | 9.97 421  | 4  | 27 | 20' 12.3' 12.1' 12.0 |
| 34 | 9.52 492  | 35 | 9.55 075  | 40    | 0.44 925  | 9.97 416  | 4  | 26 | 30' 18.5' 18.2' 18.0 |
| 35 | 9.52 527  | 35 | 9.55 115  | 39    | 0.44 884  | 9.97 412  | 4  | 25 | 40' 24.0' 24.0' 24.0 |
| 36 | 9.52 563  | 35 | 9.55 155  | 40    | 0.44 845  | 9.97 407  | 4  | 24 | 50' 30.6' 30.4' 30.0 |
| 37 | 9.52 598  | 35 | 9.55 195  | 40    | 0.44 805  | 9.97 403  | 4  | 23 |                      |
| 38 | 9.52 634  | 35 | 9.55 235  | 40    | 0.44 765  | 9.97 398  | 4  | 22 | 35 35 34             |
| 39 | 9.52 669  | 35 | 9.55 275  | 40    | 0.44 725  | 9.97 394  | 4  | 21 | 6' 3.5' 3.5' 3.4     |
| 40 | 9.52 704  | 35 | 9.55 315  | 40    | 0.44 685  | 9.97 389  | 4  | 20 | 7' 4.1' 4.1' 4.0     |
| 41 | 9.52 740  | 35 | 9.55 355  | 39    | 0.44 645  | 9.97 385  | 4  | 19 | 8' 4.7' 4.6' 4.6     |
| 42 | 9.52 775  | 35 | 9.55 394  | 40    | 0.44 605  | 9.97 380  | 4  | 18 | 9' 5.3' 5.2' 5.2     |
| 43 | 9.52 810  | 35 | 9.55 434  | 39    | 0.44 565  | 9.97 376  | 4  | 17 | 10' 5.9' 5.8' 5.7    |
| 44 | 9.52 846  | 35 | 9.55 474  | 40    | 0.44 526  | 9.97 371  | 4  | 16 | 20' 11.8' 11.6' 11.5 |
| 45 | 9.52 881  | 35 | 9.55 514  | 39    | 0.44 486  | 9.97 367  | 4  | 15 | 30' 17.7' 17.5' 17.2 |
| 46 | 9.52 916  | 35 | 9.55 553  | 40    | 0.44 446  | 9.97 362  | 4  | 14 | 40' 23.6' 23.3' 23.0 |
| 47 | 9.52 951  | 35 | 9.55 593  | 39    | 0.44 406  | 9.97 358  | 4  | 13 | 50' 29.6' 29.1' 28.7 |
| 48 | 9.52 986  | 35 | 9.55 633  | 39    | 0.44 367  | 9.97 353  | 4  | 12 |                      |
| 49 | 9.53 021  | 35 | 9.55 673  | 39    | 0.44 327  | 9.97 349  | 4  | 11 | 5 4                  |
| 50 | 9.53 056  | 35 | 9.55 712  | 39    | 0.44 288  | 9.97 344  | 4  | 10 | 6' 0.5' 0.4' 0.4     |
| 51 | 9.53 091  | 35 | 9.55 751  | 40    | 0.44 248  | 9.97 340  | 4  | 9  | 7' 0.6' 0.5' 0.4     |
| 52 | 9.53 126  | 35 | 9.55 791  | 39    | 0.44 208  | 9.97 335  | 4  | 8  | 8' 0.6' 0.6' 0.5     |
| 53 | 9.53 161  | 35 | 9.55 831  | 39    | 0.44 169  | 9.97 330  | 4  | 7  | 9' 0.7' 0.7' 0.6     |
| 54 | 9.53 196  | 35 | 9.55 870  | 39    | 0.44 129  | 9.97 326  | 4  | 6  | 10' 0.8' 0.7' 0.6    |
| 55 | 9.53 231  | 34 | 9.55 909  | 39    | 0.44 090  | 9.97 321  | 4  | 5  | 20' 1.6' 1.5' 1.3    |
| 56 | 9.53 266  | 35 | 9.55 949  | 39    | 0.44 051  | 9.97 317  | 4  | 4  | 30' 2.5' 2.2' 2.0    |
| 57 | 9.53 301  | 34 | 9.55 988  | 39    | 0.44 011  | 9.97 312  | 4  | 3  | 40' 3.3' 3.0' 2.6    |
| 58 | 9.53 335  | 35 | 9.56 028  | 39    | 0.43 972  | 9.97 308  | 4  | 2  | 50' 4.1' 3.7' 3.3    |
| 59 | 9.53 370  | 34 | 9.56 067  | 39    | 0.43 932  | 9.97 303  | 5  | 1  |                      |
| 60 | 9.53 405  | 34 | 9.56 106  | 39    | 0.43 893  | 9.97 298  | 4  | 0  |                      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.                |

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TABLE VII. -LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.53 405  | 35 | 9.56 106  | 35    | 0.43 893  | 9.97 298  | 4  | 60 |       |
| 1  | 9.53 440  | 34 | 9.56 146  | 39    | 0.43 854  | 9.97 294  | 4  | 59 |       |
| 2  | 9.53 474  | 34 | 9.56 185  | 39    | 0.43 815  | 9.97 289  | 4  | 58 |       |
| 3  | 9.53 509  | 35 | 9.56 224  | 39    | 0.43 775  | 9.97 285  | 4  | 57 |       |
| 4  | 9.53 544  | 34 | 9.56 263  | 39    | 0.43 736  | 9.97 280  | 4  | 56 |       |
| 5  | 9.53 578  | 34 | 9.56 303  | 39    | 0.43 697  | 9.97 275  | 4  | 55 |       |
| 6  | 9.53 613  | 34 | 9.56 342  | 39    | 0.43 658  | 9.97 271  | 4  | 54 |       |
| 7  | 9.53 647  | 34 | 9.56 381  | 39    | 0.43 619  | 9.97 266  | 4  | 53 |       |
| 8  | 9.53 682  | 34 | 9.56 420  | 39    | 0.43 580  | 9.97 261  | 4  | 52 |       |
| 9  | 9.53 716  | 34 | 9.56 459  | 39    | 0.43 540  | 9.97 257  | 4  | 51 |       |
| 10 | 9.53 750  | 34 | 9.56 498  | 39    | 0.43 501  | 9.97 252  | 4  | 50 |       |
| 11 | 9.53 785  | 34 | 9.56 537  | 39    | 0.43 462  | 9.97 248  | 4  | 49 |       |
| 12 | 9.53 819  | 34 | 9.56 576  | 39    | 0.43 423  | 9.97 243  | 4  | 48 |       |
| 13 | 9.53 854  | 34 | 9.56 615  | 38    | 0.43 384  | 9.97 238  | 4  | 47 |       |
| 14 | 9.53 888  | 34 | 9.56 654  | 39    | 0.43 346  | 9.97 234  | 4  | 46 |       |
| 15 | 9.53 923  | 34 | 9.56 693  | 39    | 0.43 307  | 9.97 229  | 4  | 45 |       |
| 16 | 9.53 957  | 34 | 9.56 732  | 39    | 0.43 268  | 9.97 224  | 4  | 44 |       |
| 17 | 9.53 990  | 34 | 9.56 771  | 39    | 0.43 229  | 9.97 220  | 4  | 43 |       |
| 18 | 9.54 025  | 34 | 9.56 810  | 38    | 0.43 190  | 9.97 215  | 4  | 42 |       |
| 19 | 9.54 059  | 34 | 9.56 848  | 39    | 0.43 151  | 9.97 210  | 4  | 41 |       |
| 20 | 9.54 093  | 34 | 9.56 887  | 38    | 0.43 112  | 9.97 206  | 4  | 40 |       |
| 21 | 9.54 127  | 34 | 9.56 926  | 39    | 0.43 074  | 9.97 201  | 4  | 39 |       |
| 22 | 9.54 161  | 34 | 9.56 965  | 38    | 0.43 035  | 9.97 196  | 4  | 38 |       |
| 23 | 9.54 195  | 34 | 9.57 003  | 38    | 0.42 996  | 9.97 191  | 4  | 37 |       |
| 24 | 9.54 229  | 33 | 9.57 042  | 39    | 0.42 958  | 9.97 187  | 4  | 36 |       |
| 25 | 9.54 263  | 34 | 9.57 081  | 38    | 0.42 919  | 9.97 182  | 4  | 35 |       |
| 26 | 9.54 297  | 34 | 9.57 119  | 38    | 0.42 880  | 9.97 177  | 4  | 34 |       |
| 27 | 9.54 331  | 34 | 9.57 158  | 38    | 0.42 842  | 9.97 173  | 4  | 33 |       |
| 28 | 9.54 365  | 33 | 9.57 196  | 38    | 0.42 803  | 9.97 168  | 4  | 32 |       |
| 29 | 9.54 398  | 34 | 9.57 235  | 39    | 0.42 765  | 9.97 163  | 4  | 31 |       |
| 30 | 9.54 432  | 34 | 9.57 274  | 38    | 0.42 726  | 9.97 159  | 4  | 30 |       |
| 31 | 9.54 466  | 34 | 9.57 312  | 38    | 0.42 687  | 9.97 154  | 4  | 29 |       |
| 32 | 9.54 500  | 33 | 9.57 350  | 38    | 0.42 648  | 9.97 149  | 4  | 28 |       |
| 33 | 9.54 534  | 33 | 9.57 389  | 38    | 0.42 611  | 9.97 144  | 4  | 27 |       |
| 34 | 9.54 567  | 33 | 9.57 427  | 38    | 0.42 572  | 9.97 140  | 4  | 26 |       |
| 35 | 9.54 601  | 33 | 9.57 466  | 38    | 0.42 534  | 9.97 135  | 4  | 25 |       |
| 36 | 9.54 634  | 34 | 9.57 504  | 38    | 0.42 495  | 9.97 130  | 4  | 24 |       |
| 37 | 9.54 668  | 33 | 9.57 542  | 38    | 0.42 457  | 9.97 125  | 4  | 23 |       |
| 38 | 9.54 702  | 33 | 9.57 581  | 38    | 0.42 419  | 9.97 121  | 4  | 22 |       |
| 39 | 9.54 735  | 33 | 9.57 619  | 38    | 0.42 380  | 9.97 116  | 4  | 21 |       |
| 40 | 9.54 769  | 33 | 9.57 657  | 38    | 0.42 342  | 9.97 111  | 4  | 20 |       |
| 41 | 9.54 802  | 33 | 9.57 696  | 38    | 0.42 304  | 9.97 106  | 4  | 19 |       |
| 42 | 9.54 836  | 33 | 9.57 734  | 38    | 0.42 266  | 9.97 102  | 4  | 18 |       |
| 43 | 9.54 869  | 33 | 9.57 772  | 38    | 0.42 227  | 9.97 097  | 4  | 17 |       |
| 44 | 9.54 902  | 33 | 9.57 810  | 38    | 0.42 189  | 9.97 092  | 4  | 16 |       |
| 45 | 9.54 936  | 33 | 9.57 848  | 38    | 0.42 151  | 9.97 087  | 4  | 15 |       |
| 46 | 9.54 969  | 33 | 9.57 886  | 38    | 0.42 113  | 9.97 082  | 4  | 14 |       |
| 47 | 9.55 002  | 33 | 9.57 925  | 38    | 0.42 075  | 9.97 078  | 4  | 13 |       |
| 48 | 9.55 036  | 33 | 9.57 963  | 38    | 0.42 037  | 9.97 073  | 4  | 12 |       |
| 49 | 9.55 069  | 33 | 9.58 001  | 38    | 0.41 999  | 9.97 068  | 4  | 11 |       |
| 50 | 9.55 102  | 33 | 9.58 039  | 38    | 0.41 961  | 9.97 063  | 4  | 10 |       |
| 51 | 9.55 135  | 33 | 9.58 077  | 38    | 0.41 923  | 9.97 058  | 4  | 9  |       |
| 52 | 9.55 168  | 33 | 9.58 115  | 38    | 0.41 885  | 9.97 054  | 4  | 8  |       |
| 53 | 9.55 202  | 33 | 9.58 153  | 37    | 0.41 847  | 9.97 049  | 4  | 7  |       |
| 54 | 9.55 235  | 33 | 9.58 190  | 38    | 0.41 809  | 9.97 044  | 4  | 6  |       |
| 55 | 9.55 268  | 33 | 9.58 228  | 38    | 0.41 771  | 9.97 039  | 4  | 5  |       |
| 56 | 9.55 301  | 33 | 9.58 266  | 38    | 0.41 733  | 9.97 034  | 4  | 4  |       |
| 57 | 9.55 334  | 33 | 9.58 304  | 37    | 0.41 695  | 9.97 029  | 4  | 3  |       |
| 58 | 9.55 367  | 33 | 9.58 342  | 38    | 0.41 658  | 9.97 025  | 4  | 2  |       |
| 59 | 9.55 400  | 33 | 9.58 380  | 37    | 0.41 620  | 9.97 020  | 4  | 1  |       |
| 60 | 9.55 433  | 33 | 9.58 417  | 37    | 0.41 582  | 9.97 015  | 4  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.55433   |    | 9.58417   |       | 0.41582   | 9.97015   |    | 60 |       |
| 1  | 9.55486   | 33 | 9.58455   | 38    | 0.41544   | 9.97010   | 4  | 59 |       |
| 2  | 9.55498   | 33 | 9.58493   | 38    | 0.41507   | 9.97005   | 5  | 58 |       |
| 3  | 9.55531   | 33 | 9.58531   | 37    | 0.41469   | 9.97000   | 5  | 57 |       |
| 4  | 9.55584   | 33 | 9.58584   | 37    | 0.41431   | 9.96995   | 5  | 56 |       |
| 5  | 9.55597   | 32 | 9.58606   | 37    | 0.41394   | 9.96991   | 5  | 55 |       |
| 6  | 9.55630   | 32 | 9.58644   | 37    | 0.41356   | 9.96986   | 5  | 54 |       |
| 7  | 9.55662   | 32 | 9.58681   | 37    | 0.41318   | 9.96981   | 5  | 53 |       |
| 8  | 9.55695   | 32 | 9.58719   | 37    | 0.41281   | 9.96976   | 5  | 52 |       |
| 9  | 9.55728   | 32 | 9.58756   | 37    | 0.41243   | 9.96971   | 5  | 51 |       |
| 10 | 9.55760   | 32 | 9.58794   | 37    | 0.41206   | 9.96966   | 5  | 50 |       |
| 11 | 9.55793   | 32 | 9.58831   | 37    | 0.41168   | 9.96961   | 5  | 49 |       |
| 12 | 9.55826   | 32 | 9.58869   | 37    | 0.41131   | 9.96956   | 5  | 48 |       |
| 13 | 9.55858   | 32 | 9.58906   | 37    | 0.41093   | 9.96952   | 5  | 47 |       |
| 14 | 9.55891   | 32 | 9.58944   | 37    | 0.41056   | 9.96947   | 5  | 46 |       |
| 15 | 9.55923   | 32 | 9.58981   | 37    | 0.41018   | 9.96942   | 5  | 45 |       |
| 16 | 9.55956   | 32 | 9.59019   | 37    | 0.40981   | 9.96937   | 5  | 44 |       |
| 17 | 9.55988   | 32 | 9.59056   | 37    | 0.40944   | 9.96932   | 5  | 43 |       |
| 18 | 9.56020   | 32 | 9.59093   | 37    | 0.40906   | 9.96927   | 5  | 42 |       |
| 19 | 9.56053   | 32 | 9.59131   | 37    | 0.40869   | 9.96922   | 4  | 41 |       |
| 20 | 9.56085   | 32 | 9.59168   | 37    | 0.40832   | 9.96917   | 5  | 40 |       |
| 21 | 9.56118   | 32 | 9.59205   | 37    | 0.40794   | 9.96912   | 5  | 39 |       |
| 22 | 9.56150   | 32 | 9.59242   | 37    | 0.40757   | 9.96907   | 5  | 38 |       |
| 23 | 9.56182   | 32 | 9.59280   | 37    | 0.40720   | 9.96902   | 5  | 37 |       |
| 24 | 9.56214   | 32 | 9.59317   | 37    | 0.40683   | 9.96897   | 5  | 36 |       |
| 25 | 9.56247   | 32 | 9.59354   | 37    | 0.40646   | 9.96892   | 5  | 35 |       |
| 26 | 9.56279   | 32 | 9.59391   | 37    | 0.40608   | 9.96887   | 5  | 34 |       |
| 27 | 9.56311   | 32 | 9.59428   | 37    | 0.40571   | 9.96882   | 5  | 33 |       |
| 28 | 9.56343   | 32 | 9.59465   | 37    | 0.40534   | 9.96877   | 5  | 32 |       |
| 29 | 9.56375   | 32 | 9.59502   | 37    | 0.40497   | 9.96873   | 4  | 31 |       |
| 30 | 9.56407   | 32 | 9.59540   | 37    | 0.40460   | 9.96868   | 5  | 30 |       |
| 31 | 9.56439   | 32 | 9.59577   | 37    | 0.40423   | 9.96863   | 5  | 29 |       |
| 32 | 9.56471   | 32 | 9.59614   | 37    | 0.40386   | 9.96858   | 5  | 28 |       |
| 33 | 9.56503   | 32 | 9.59651   | 37    | 0.40349   | 9.96853   | 5  | 27 |       |
| 34 | 9.56535   | 32 | 9.59688   | 37    | 0.40312   | 9.96848   | 5  | 26 |       |
| 35 | 9.56567   | 32 | 9.59724   | 36    | 0.40275   | 9.96843   | 5  | 25 |       |
| 36 | 9.56599   | 32 | 9.59761   | 37    | 0.40238   | 9.96838   | 5  | 24 |       |
| 37 | 9.56631   | 31 | 9.59798   | 37    | 0.40201   | 9.96833   | 5  | 23 |       |
| 38 | 9.56663   | 32 | 9.59835   | 36    | 0.40164   | 9.96828   | 5  | 22 |       |
| 39 | 9.56695   | 32 | 9.59872   | 36    | 0.40128   | 9.96823   | 5  | 21 |       |
| 40 | 9.56727   | 32 | 9.59909   | 37    | 0.40091   | 9.96818   | 5  | 20 |       |
| 41 | 9.56758   | 32 | 9.59946   | 36    | 0.40054   | 9.96813   | 5  | 19 |       |
| 42 | 9.56790   | 32 | 9.59982   | 37    | 0.40017   | 9.96808   | 5  | 18 |       |
| 43 | 9.56822   | 31 | 9.60019   | 36    | 0.39980   | 9.96802   | 5  | 17 |       |
| 44 | 9.56854   | 32 | 9.60056   | 36    | 0.39944   | 9.96797   | 5  | 16 |       |
| 45 | 9.56885   | 31 | 9.60093   | 36    | 0.39907   | 9.96792   | 5  | 15 |       |
| 46 | 9.56917   | 31 | 9.60129   | 37    | 0.39870   | 9.96787   | 5  | 14 |       |
| 47 | 9.56949   | 31 | 9.60166   | 36    | 0.39833   | 9.96782   | 5  | 13 |       |
| 48 | 9.56980   | 31 | 9.60203   | 36    | 0.39797   | 9.96777   | 5  | 12 |       |
| 49 | 9.57012   | 31 | 9.60239   | 36    | 0.39760   | 9.96772   | 5  | 11 |       |
| 50 | 9.57043   | 31 | 9.60276   | 36    | 0.39724   | 9.96767   | 5  | 10 |       |
| 51 | 9.57075   | 31 | 9.60312   | 37    | 0.39687   | 9.96762   | 5  | 9  |       |
| 52 | 9.57106   | 31 | 9.60349   | 36    | 0.39650   | 9.96757   | 5  | 8  |       |
| 53 | 9.57138   | 31 | 9.60386   | 36    | 0.39614   | 9.96752   | 5  | 7  |       |
| 54 | 9.57169   | 31 | 9.60422   | 36    | 0.39577   | 9.96747   | 5  | 6  |       |
| 55 | 9.57201   | 31 | 9.60459   | 36    | 0.39541   | 9.96742   | 5  | 5  |       |
| 56 | 9.57232   | 31 | 9.60495   | 36    | 0.39504   | 9.96737   | 5  | 4  |       |
| 57 | 9.57263   | 31 | 9.60531   | 36    | 0.39468   | 9.96732   | 5  | 3  |       |
| 58 | 9.57295   | 31 | 9.60568   | 36    | 0.39432   | 9.96727   | 5  | 2  |       |
| 59 | 9.57326   | 31 | 9.60604   | 36    | 0.39395   | 9.96721   | 5  | 1  |       |
| 60 | 9.57357   | 31 | 9.60641   | 36    | 0.39358   | 9.96716   | 5  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

|    |      |      |      |    |
|----|------|------|------|----|
|    |      | 38   | 37   | 37 |
| 6  | 3.8  | 3.7  | 3.7  |    |
| 7  | 4.4  | 4.4  | 4.3  |    |
| 8  | 5.0  | 5.0  | 4.9  |    |
| 9  | 5.7  | 5.6  | 5.5  |    |
| 10 | 6.3  | 6.2  | 6.1  |    |
| 20 | 12.6 | 12.5 | 12.3 |    |
| 30 | 19.0 | 18.7 | 18.5 |    |
| 40 | 25.3 | 25.0 | 24.6 |    |
| 50 | 31.6 | 31.2 | 30.8 |    |

|    |      |      |      |  |
|----|------|------|------|--|
|    |      | 36   | 36   |  |
| 6  | 3.6  | 3.6  | 3.6  |  |
| 7  | 4.2  | 4.2  | 4.2  |  |
| 8  | 4.8  | 4.8  | 4.8  |  |
| 9  | 5.5  | 5.4  | 5.4  |  |
| 10 | 6.1  | 6.0  | 6.0  |  |
| 20 | 12.1 | 12.0 | 12.0 |  |
| 30 | 18.2 | 18.0 | 18.0 |  |
| 40 | 24.3 | 24.0 | 24.0 |  |
| 50 | 30.4 | 30.0 | 30.0 |  |

|    |      |      |      |    |
|----|------|------|------|----|
|    |      | 33   | 32   | 32 |
| 6  | 3.3  | 3.2  | 3.2  |    |
| 7  | 3.8  | 3.8  | 3.7  |    |
| 8  | 4.4  | 4.3  | 4.2  |    |
| 9  | 4.9  | 4.9  | 4.8  |    |
| 10 | 5.5  | 5.4  | 5.3  |    |
| 20 | 11.0 | 10.8 | 10.6 |    |
| 30 | 16.5 | 16.2 | 16.0 |    |
| 40 | 22.0 | 21.6 | 21.3 |    |
| 50 | 27.5 | 27.1 | 26.6 |    |

|    |      |      |      |  |
|----|------|------|------|--|
|    |      | 31   | 31   |  |
| 6  | 3.1  | 3.1  | 3.1  |  |
| 7  | 3.7  | 3.6  | 3.6  |  |
| 8  | 4.2  | 4.1  | 4.1  |  |
| 9  | 4.7  | 4.6  | 4.6  |  |
| 10 | 5.2  | 5.1  | 5.1  |  |
| 20 | 10.5 | 10.3 | 10.3 |  |
| 30 | 15.7 | 15.5 | 15.5 |  |
| 40 | 21.0 | 20.6 | 20.6 |  |
| 50 | 26.2 | 25.8 | 25.8 |  |

|    |      |      |      |    |
|----|------|------|------|----|
|    |      | 5    | 5    | 5  |
| 6  | 5.0  | 5.0  | 5.0  | 4  |
| 7  | 5.6  | 5.6  | 5.6  | 5  |
| 8  | 6.2  | 6.0  | 6.0  | 6  |
| 9  | 6.8  | 6.7  | 6.7  | 7  |
| 10 | 7.4  | 7.3  | 7.3  | 8  |
| 20 | 14.1 | 14.1 | 14.1 | 9  |
| 30 | 21.2 | 21.2 | 21.2 | 10 |
| 40 | 28.3 | 28.3 | 28.3 | 11 |
| 50 | 35.4 | 35.4 | 35.4 | 12 |

P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.   |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|---------|
| 0  | 9.57357   |    | 9.60641   |       | 0.39359   | 9.96716   |    | 60 |         |
| 1  | 9.57389   | 31 | 9.60677   | 36    | 0.39322   | 9.96711   | 5  | 59 |         |
| 2  | 9.57420   | 31 | 9.60713   | 36    | 0.39286   | 9.96706   | 5  | 58 |         |
| 3  | 9.57451   | 31 | 9.60750   | 36    | 0.39250   | 9.96701   | 5  | 57 |         |
| 4  | 9.57482   | 31 | 9.60786   | 36    | 0.39213   | 9.96696   | 5  | 56 | 36 36   |
| 5  | 9.57513   | 31 | 9.60822   | 36    | 0.39177   | 9.96691   | 5  | 55 | 6 3.6   |
| 6  | 9.57544   | 31 | 9.60859   | 36    | 0.39141   | 9.96686   | 5  | 54 | 7 4.2   |
| 7  | 9.57576   | 31 | 9.60895   | 36    | 0.39105   | 9.96681   | 5  | 53 | 8 4.8   |
| 8  | 9.57607   | 31 | 9.60931   | 36    | 0.39069   | 9.96675   | 5  | 52 | 9 5.5   |
| 9  | 9.57638   | 31 | 9.60967   | 36    | 0.39032   | 9.96670   | 5  | 51 | 10 6.1  |
| 10 | 9.57669   | 31 | 9.61003   | 36    | 0.38996   | 9.96665   | 5  | 50 | 20 12.1 |
| 11 | 9.57700   | 31 | 9.61039   | 36    | 0.38960   | 9.96660   | 5  | 49 | 30 18.2 |
| 12 | 9.57731   | 31 | 9.61076   | 36    | 0.38924   | 9.96655   | 5  | 48 | 40 24.3 |
| 13 | 9.57762   | 31 | 9.61112   | 36    | 0.38888   | 9.96650   | 5  | 47 | 50 30.4 |
| 14 | 9.57792   | 30 | 9.61148   | 36    | 0.38852   | 9.96644   | 5  | 46 |         |
| 15 | 9.57823   | 31 | 9.61184   | 36    | 0.38816   | 9.96639   | 5  | 45 | 35 35   |
| 16 | 9.57854   | 31 | 9.61220   | 36    | 0.38780   | 9.96634   | 5  | 44 | 6 3.5   |
| 17 | 9.57885   | 31 | 9.61256   | 36    | 0.38744   | 9.96629   | 5  | 43 | 7 4.1   |
| 18 | 9.57916   | 30 | 9.61292   | 36    | 0.38708   | 9.96624   | 5  | 42 | 8 4.7   |
| 19 | 9.57947   | 30 | 9.61328   | 36    | 0.38672   | 9.96619   | 5  | 41 | 9 5.3   |
| 20 | 9.57977   | 30 | 9.61364   | 36    | 0.38636   | 9.96613   | 5  | 40 | 10 5.9  |
| 21 | 9.58008   | 30 | 9.61400   | 36    | 0.38600   | 9.96608   | 5  | 39 | 20 11.8 |
| 22 | 9.58039   | 30 | 9.61436   | 36    | 0.38564   | 9.96603   | 5  | 38 | 30 17.7 |
| 23 | 9.58070   | 31 | 9.61472   | 35    | 0.38528   | 9.96598   | 5  | 37 | 40 23.6 |
| 24 | 9.58100   | 30 | 9.61507   | 36    | 0.38492   | 9.96593   | 5  | 36 | 50 29.6 |
| 25 | 9.58131   | 30 | 9.61543   | 36    | 0.38456   | 9.96587   | 5  | 35 |         |
| 26 | 9.58162   | 31 | 9.61579   | 35    | 0.38420   | 9.96582   | 5  | 34 | 31 31   |
| 27 | 9.58192   | 30 | 9.61615   | 35    | 0.38385   | 9.96577   | 5  | 33 | 6 3.1   |
| 28 | 9.58223   | 30 | 9.61651   | 35    | 0.38349   | 9.96572   | 5  | 32 | 7 3.7   |
| 29 | 9.58253   | 30 | 9.61686   | 35    | 0.38313   | 9.96567   | 5  | 31 | 8 4.2   |
| 30 | 9.58284   | 30 | 9.61722   | 35    | 0.38277   | 9.96561   | 5  | 30 | 9 4.7   |
| 31 | 9.58314   | 30 | 9.61758   | 35    | 0.38242   | 9.96556   | 5  | 29 | 10 5.2  |
| 32 | 9.58345   | 30 | 9.61794   | 35    | 0.38206   | 9.96551   | 5  | 28 | 20 10.5 |
| 33 | 9.58375   | 30 | 9.61829   | 35    | 0.38170   | 9.96546   | 5  | 27 | 30 15.7 |
| 34 | 9.58406   | 30 | 9.61865   | 35    | 0.38135   | 9.96540   | 5  | 26 | 40 21.0 |
| 35 | 9.58436   | 30 | 9.61901   | 36    | 0.38099   | 9.96535   | 5  | 25 | 50 26.2 |
| 36 | 9.58466   | 30 | 9.61936   | 35    | 0.38063   | 9.96530   | 5  | 24 |         |
| 37 | 9.58497   | 30 | 9.61972   | 35    | 0.38028   | 9.96525   | 5  | 23 |         |
| 38 | 9.58527   | 30 | 9.62007   | 35    | 0.37992   | 9.96519   | 5  | 22 | 30 30   |
| 39 | 9.58557   | 30 | 9.62043   | 35    | 0.37957   | 9.96514   | 5  | 21 | 6 3.0   |
| 40 | 9.58587   | 30 | 9.62078   | 35    | 0.37921   | 9.96509   | 5  | 20 | 7 3.5   |
| 41 | 9.58618   | 30 | 9.62114   | 35    | 0.37886   | 9.96503   | 5  | 19 | 8 4.0   |
| 42 | 9.58648   | 30 | 9.62149   | 35    | 0.37850   | 9.96498   | 5  | 18 | 9 4.6   |
| 43 | 9.58678   | 30 | 9.62185   | 35    | 0.37815   | 9.96493   | 5  | 17 | 10 5.1  |
| 44 | 9.58708   | 30 | 9.62220   | 35    | 0.37779   | 9.96488   | 5  | 16 | 20 10.1 |
| 45 | 9.58738   | 30 | 9.62256   | 35    | 0.37744   | 9.96482   | 5  | 15 | 30 15.2 |
| 46 | 9.58769   | 30 | 9.62291   | 35    | 0.37708   | 9.96477   | 5  | 14 | 40 20.3 |
| 47 | 9.58799   | 30 | 9.62327   | 35    | 0.37673   | 9.96472   | 5  | 13 | 50 25.4 |
| 48 | 9.58829   | 30 | 9.62362   | 35    | 0.37637   | 9.96466   | 5  | 12 |         |
| 49 | 9.58859   | 30 | 9.62397   | 35    | 0.37602   | 9.96461   | 5  | 11 | 5 5     |
| 50 | 9.58889   | 30 | 9.62433   | 35    | 0.37567   | 9.96456   | 5  | 10 | 6 5.0   |
| 51 | 9.58919   | 30 | 9.62468   | 35    | 0.37531   | 9.96450   | 5  | 9  | 7 5.6   |
| 52 | 9.58949   | 30 | 9.62503   | 35    | 0.37496   | 9.96445   | 5  | 8  | 8 6.2   |
| 53 | 9.58979   | 30 | 9.62539   | 35    | 0.37461   | 9.96440   | 5  | 7  | 9 6.8   |
| 54 | 9.59009   | 30 | 9.62574   | 35    | 0.37426   | 9.96434   | 5  | 6  | 10 7.4  |
| 55 | 9.59038   | 29 | 9.62609   | 35    | 0.37390   | 9.96429   | 5  | 5  | 20 12.5 |
| 56 | 9.59068   | 30 | 9.62644   | 35    | 0.37355   | 9.96424   | 5  | 4  | 30 17.6 |
| 57 | 9.59098   | 30 | 9.62679   | 35    | 0.37320   | 9.96418   | 5  | 3  | 40 22.7 |
| 58 | 9.59128   | 30 | 9.62715   | 35    | 0.37285   | 9.96413   | 5  | 2  | 50 27.8 |
| 59 | 9.59158   | 30 | 9.62750   | 35    | 0.37250   | 9.96408   | 5  | 1  |         |
| 60 | 9.59188   | 30 | 9.62785   | 35    | 0.37215   | 9.96402   | 5  | 0  |         |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.   |

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# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|           | Log. Sin. | d.        | Log. Tan. c. d. | Log. Cot. | Log. Cos. | d.       |    | P. P. |
|-----------|-----------|-----------|-----------------|-----------|-----------|----------|----|-------|
| 0         | 9.59 188  | 29        | 9.62 785        | 35        | 0.37 215  | 9.96 402 | 60 |       |
| 1         | 9.59 217  | 30        | 9.62 820        | 35        | 0.37 179  | 9.96 397 | 59 |       |
| 2         | 9.59 247  | 29        | 9.62 855        | 35        | 0.37 144  | 9.96 392 | 58 |       |
| 3         | 9.59 277  | 29        | 9.62 890        | 35        | 0.37 109  | 9.96 386 | 57 |       |
| 4         | 9.59 306  | 30        | 9.62 925        | 35        | 0.37 074  | 9.96 381 | 56 |       |
| 5         | 9.59 336  | 29        | 9.62 960        | 35        | 0.37 039  | 9.96 375 | 55 |       |
| 6         | 9.59 366  | 29        | 9.62 995        | 35        | 0.37 004  | 9.96 370 | 54 |       |
| 7         | 9.59 395  | 29        | 9.63 030        | 35        | 0.36 969  | 9.96 365 | 53 |       |
| 8         | 9.59 425  | 29        | 9.63 065        | 35        | 0.36 934  | 9.96 359 | 52 |       |
| 9         | 9.59 454  | 29        | 9.63 100        | 35        | 0.36 899  | 9.96 354 | 51 |       |
| 10        | 9.59 484  | 29        | 9.63 135        | 35        | 0.36 864  | 9.96 349 | 50 |       |
| 11        | 9.59 513  | 29        | 9.63 170        | 35        | 0.36 829  | 9.96 343 | 49 |       |
| 12        | 9.59 543  | 29        | 9.63 205        | 35        | 0.36 794  | 9.96 338 | 48 |       |
| 13        | 9.59 572  | 29        | 9.63 240        | 35        | 0.36 760  | 9.96 332 | 47 |       |
| 14        | 9.59 602  | 29        | 9.63 275        | 35        | 0.36 725  | 9.96 327 | 46 |       |
| 15        | 9.59 631  | 29        | 9.63 310        | 35        | 0.36 690  | 9.96 321 | 45 |       |
| 16        | 9.59 661  | 29        | 9.63 344        | 35        | 0.36 655  | 9.96 316 | 44 |       |
| 17        | 9.59 690  | 29        | 9.63 379        | 35        | 0.36 620  | 9.96 311 | 43 |       |
| 18        | 9.59 719  | 29        | 9.63 414        | 35        | 0.36 585  | 9.96 305 | 42 |       |
| 19        | 9.59 749  | 29        | 9.63 449        | 35        | 0.36 551  | 9.96 300 | 41 |       |
| 20        | 9.59 778  | 29        | 9.63 484        | 35        | 0.36 516  | 9.96 294 | 40 |       |
| 21        | 9.59 807  | 29        | 9.63 518        | 35        | 0.36 481  | 9.96 289 | 39 |       |
| 22        | 9.59 837  | 29        | 9.63 553        | 35        | 0.36 447  | 9.96 283 | 38 |       |
| 23        | 9.59 866  | 29        | 9.63 588        | 35        | 0.36 412  | 9.96 278 | 37 |       |
| 24        | 9.59 895  | 29        | 9.63 622        | 35        | 0.36 377  | 9.96 272 | 36 |       |
| 25        | 9.59 924  | 29        | 9.63 657        | 35        | 0.36 343  | 9.96 267 | 35 |       |
| 26        | 9.59 953  | 29        | 9.63 692        | 35        | 0.36 308  | 9.96 261 | 34 |       |
| 27        | 9.59 982  | 29        | 9.63 726        | 35        | 0.36 273  | 9.96 256 | 33 |       |
| 28        | 9.60 012  | 29        | 9.63 761        | 35        | 0.36 239  | 9.96 251 | 32 |       |
| 29        | 9.60 041  | 29        | 9.63 795        | 35        | 0.36 204  | 9.96 245 | 31 |       |
| 30        | 9.60 070  | 29        | 9.63 830        | 35        | 0.36 170  | 9.96 240 | 30 |       |
| 31        | 9.60 099  | 29        | 9.63 864        | 35        | 0.36 135  | 9.96 234 | 29 |       |
| 32        | 9.60 128  | 29        | 9.63 899        | 35        | 0.36 101  | 9.96 229 | 28 |       |
| 33        | 9.60 157  | 29        | 9.63 933        | 35        | 0.36 066  | 9.96 223 | 27 |       |
| 34        | 9.60 186  | 29        | 9.63 968        | 35        | 0.36 032  | 9.96 218 | 26 |       |
| 35        | 9.60 215  | 29        | 9.64 002        | 35        | 0.35 997  | 9.96 212 | 25 |       |
| 36        | 9.60 244  | 29        | 9.64 037        | 35        | 0.35 963  | 9.96 206 | 24 |       |
| 37        | 9.60 273  | 28        | 9.64 071        | 35        | 0.35 928  | 9.96 201 | 23 |       |
| 38        | 9.60 301  | 29        | 9.64 106        | 35        | 0.35 894  | 9.96 195 | 22 |       |
| 39        | 9.60 330  | 29        | 9.64 140        | 35        | 0.35 859  | 9.96 190 | 21 |       |
| 40        | 9.60 359  | 28        | 9.64 174        | 35        | 0.35 825  | 9.96 184 | 20 |       |
| 41        | 9.60 388  | 29        | 9.64 209        | 35        | 0.35 791  | 9.96 179 | 19 |       |
| 42        | 9.60 417  | 28        | 9.64 243        | 35        | 0.35 756  | 9.96 173 | 18 |       |
| 43        | 9.60 445  | 29        | 9.64 277        | 35        | 0.35 722  | 9.96 168 | 17 |       |
| 44        | 9.60 474  | 28        | 9.64 312        | 35        | 0.35 688  | 9.96 162 | 16 |       |
| 45        | 9.60 503  | 29        | 9.64 346        | 35        | 0.35 653  | 9.96 157 | 15 |       |
| 46        | 9.60 532  | 28        | 9.64 380        | 35        | 0.35 619  | 9.96 151 | 14 |       |
| 47        | 9.60 560  | 29        | 9.64 415        | 35        | 0.35 585  | 9.96 146 | 13 |       |
| 48        | 9.60 589  | 28        | 9.64 449        | 35        | 0.35 551  | 9.96 140 | 12 |       |
| 49        | 9.60 618  | 29        | 9.64 483        | 35        | 0.35 517  | 9.96 134 | 11 |       |
| 50        | 9.60 646  | 28        | 9.64 517        | 35        | 0.35 482  | 9.96 129 | 10 |       |
| 51        | 9.60 675  | 29        | 9.64 551        | 35        | 0.35 448  | 9.96 123 | 9  |       |
| 52        | 9.60 703  | 28        | 9.64 585        | 35        | 0.35 414  | 9.96 118 | 8  |       |
| 53        | 9.60 732  | 29        | 9.64 620        | 35        | 0.35 380  | 9.96 112 | 7  |       |
| 54        | 9.60 760  | 28        | 9.64 654        | 35        | 0.35 346  | 9.96 106 | 6  |       |
| 55        | 9.60 789  | 29        | 9.64 688        | 35        | 0.35 312  | 9.96 101 | 5  |       |
| 56        | 9.60 817  | 28        | 9.64 722        | 35        | 0.35 278  | 9.96 095 | 4  |       |
| 57        | 9.60 846  | 29        | 9.64 756        | 35        | 0.35 244  | 9.96 090 | 3  |       |
| 58        | 9.60 874  | 28        | 9.64 790        | 35        | 0.35 209  | 9.96 084 | 2  |       |
| 59        | 9.60 903  | 29        | 9.64 824        | 35        | 0.35 175  | 9.96 078 | 1  |       |
| 60        | 9.60 931  | 28        | 9.64 858        | 35        | 0.35 141  | 9.96 073 | 0  |       |
| Log. Cos. | d.        | Log. Cot. | c. d.           | Log. Tan. | Log. Sin. | d.       |    | P. P. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------------------|
| 0  | 9.60 931  |    | 9.64 858  |       | 0.35 141  | 9.96 073  |    | 60 |                   |
| 1  | 9.60 959  | 28 | 9.64 892  | 34    | 0.35 107  | 9.96 067  | 6  | 59 |                   |
| 2  | 9.60 988  | 28 | 9.64 926  | 33    | 0.35 073  | 9.96 062  | 6  | 58 |                   |
| 3  | 9.61 016  | 28 | 9.64 960  | 34    | 0.35 040  | 9.96 056  | 6  | 57 |                   |
| 4  | 9.61 044  | 28 | 9.64 994  | 34    | 0.35 006  | 9.96 050  | 6  | 56 |                   |
| 5  | 9.61 073  | 28 | 9.65 028  | 34    | 0.34 972  | 9.96 045  | 6  | 55 | 34 33 33          |
| 6  | 9.61 101  | 28 | 9.65 062  | 34    | 0.34 938  | 9.96 039  | 6  | 54 | 6 3.4 3.3 3.      |
| 7  | 9.61 129  | 28 | 9.65 096  | 33    | 0.34 904  | 9.96 033  | 6  | 53 | 7 3.6 3.9 3.      |
| 8  | 9.61 157  | 28 | 9.65 129  | 34    | 0.34 870  | 9.96 028  | 6  | 52 | 8 4.5 4.4 4.4     |
| 9  | 9.61 186  | 28 | 9.65 163  | 34    | 0.34 836  | 9.96 022  | 6  | 51 | 9 5.1 5.0 4.9     |
| 10 | 9.61 214  | 28 | 9.65 197  | 33    | 0.34 802  | 9.96 016  | 6  | 50 | 10 5.6 5.6 5.5    |
| 11 | 9.61 242  | 28 | 9.65 231  | 34    | 0.34 769  | 9.96 011  | 6  | 49 | 20 11.3 11.1 11.0 |
| 12 | 9.61 270  | 28 | 9.65 265  | 34    | 0.34 735  | 9.96 005  | 6  | 48 | 30 17.0 16.7 16.5 |
| 13 | 9.61 298  | 28 | 9.65 299  | 33    | 0.34 701  | 9.95 999  | 6  | 47 | 40 22.6 22.3 22.0 |
| 14 | 9.61 326  | 28 | 9.65 332  | 34    | 0.34 667  | 9.95 994  | 6  | 46 | 50 28.3 27.9 27.5 |
| 15 | 9.61 354  | 28 | 9.65 366  | 34    | 0.34 633  | 9.95 988  | 6  | 45 |                   |
| 16 | 9.61 382  | 28 | 9.65 400  | 33    | 0.34 600  | 9.95 982  | 6  | 44 |                   |
| 17 | 9.61 410  | 28 | 9.65 433  | 34    | 0.34 566  | 9.95 977  | 6  | 43 |                   |
| 18 | 9.61 438  | 28 | 9.65 467  | 33    | 0.34 532  | 9.95 971  | 6  | 42 |                   |
| 19 | 9.61 466  | 28 | 9.65 501  | 33    | 0.34 499  | 9.95 965  | 6  | 41 |                   |
| 20 | 9.61 494  | 28 | 9.65 535  | 34    | 0.34 465  | 9.95 959  | 6  | 40 | 28 28             |
| 21 | 9.61 522  | 28 | 9.65 568  | 33    | 0.34 431  | 9.95 954  | 6  | 39 | 6 2.8 2.8         |
| 22 | 9.61 550  | 27 | 9.65 602  | 33    | 0.34 398  | 9.95 948  | 6  | 38 | 7 3.3 3.3         |
| 23 | 9.61 578  | 28 | 9.65 635  | 33    | 0.34 364  | 9.95 942  | 6  | 37 | 8 3.8 3.7         |
| 24 | 9.61 606  | 28 | 9.65 669  | 33    | 0.34 331  | 9.95 937  | 6  | 36 | 9 4.3 4.2         |
| 25 | 9.61 634  | 28 | 9.65 703  | 34    | 0.34 297  | 9.95 931  | 6  | 35 | 10 4.7 4.6        |
| 26 | 9.61 661  | 28 | 9.65 736  | 33    | 0.34 263  | 9.95 925  | 6  | 34 | 20 9.5 9.3        |
| 27 | 9.61 689  | 27 | 9.65 770  | 33    | 0.34 230  | 9.95 919  | 6  | 33 | 30 14.2 14.0      |
| 28 | 9.61 717  | 27 | 9.65 803  | 33    | 0.34 196  | 9.95 914  | 6  | 32 | 40 19.0 18.6      |
| 29 | 9.61 745  | 28 | 9.65 837  | 33    | 0.34 163  | 9.95 908  | 6  | 31 | 50 23.7 23.3      |
| 30 | 9.61 772  | 27 | 9.65 870  | 33    | 0.34 129  | 9.95 902  | 6  | 30 |                   |
| 31 | 9.61 800  | 27 | 9.65 904  | 33    | 0.34 096  | 9.95 896  | 6  | 29 |                   |
| 32 | 9.61 828  | 28 | 9.65 937  | 33    | 0.34 062  | 9.95 891  | 6  | 28 |                   |
| 33 | 9.61 856  | 28 | 9.65 971  | 33    | 0.34 029  | 9.95 885  | 6  | 27 |                   |
| 34 | 9.61 883  | 27 | 9.66 004  | 33    | 0.33 996  | 9.95 879  | 6  | 26 | 27 27             |
| 35 | 9.61 911  | 27 | 9.66 037  | 33    | 0.33 962  | 9.95 873  | 5  | 25 | 6 2.7 2.7         |
| 36 | 9.61 938  | 27 | 9.66 071  | 33    | 0.33 929  | 9.95 867  | 5  | 24 | 7 3.2 3.1         |
| 37 | 9.61 966  | 27 | 9.66 104  | 33    | 0.33 895  | 9.95 862  | 5  | 23 | 8 3.6 3.6         |
| 38 | 9.61 994  | 29 | 9.66 137  | 33    | 0.33 862  | 9.95 856  | 5  | 22 | 9 4.1 4.0         |
| 39 | 9.62 021  | 27 | 9.66 171  | 33    | 0.33 829  | 9.95 850  | 5  | 21 | 10 4.6 4.5        |
| 40 | 9.62 049  | 27 | 9.66 204  | 33    | 0.33 795  | 9.95 844  | 6  | 20 | 20 9.1 9.0        |
| 41 | 9.62 076  | 27 | 9.66 237  | 33    | 0.33 762  | 9.95 838  | 6  | 19 | 30 13.7 13.5      |
| 42 | 9.62 104  | 27 | 9.66 271  | 33    | 0.33 729  | 9.95 833  | 6  | 18 | 40 18.3 18.0      |
| 43 | 9.62 131  | 27 | 9.66 304  | 33    | 0.33 696  | 9.95 827  | 6  | 17 | 50 22.9 22.5      |
| 44 | 9.62 158  | 27 | 9.66 337  | 33    | 0.33 662  | 9.95 821  | 5  | 16 |                   |
| 45 | 9.62 186  | 27 | 9.66 370  | 33    | 0.33 629  | 9.95 815  | 6  | 15 |                   |
| 46 | 9.62 213  | 27 | 9.66 404  | 33    | 0.33 596  | 9.95 809  | 6  | 14 |                   |
| 47 | 9.62 241  | 27 | 9.66 437  | 33    | 0.33 563  | 9.95 804  | 6  | 13 |                   |
| 48 | 9.62 268  | 27 | 9.66 470  | 33    | 0.33 529  | 9.95 798  | 6  | 12 | 6 5               |
| 49 | 9.62 295  | 27 | 9.66 503  | 33    | 0.33 496  | 9.95 792  | 6  | 11 | 6 0.6 0.5         |
| 50 | 9.62 323  | 27 | 9.66 536  | 33    | 0.33 463  | 9.95 786  | 5  | 10 | 7 0.7 0.6         |
| 51 | 9.62 350  | 27 | 9.66 570  | 33    | 0.33 430  | 9.95 780  | 6  | 9  | 8 0.8 0.7         |
| 52 | 9.62 377  | 27 | 9.66 603  | 33    | 0.33 397  | 9.95 774  | 6  | 8  | 9 0.9 0.8         |
| 53 | 9.62 404  | 27 | 9.66 636  | 33    | 0.33 364  | 9.95 768  | 6  | 7  | 10 1.0 0.9        |
| 54 | 9.62 432  | 27 | 9.66 669  | 33    | 0.33 331  | 9.95 763  | 5  | 6  | 20 2.0 1.8        |
| 55 | 9.62 459  | 27 | 9.66 702  | 33    | 0.33 298  | 9.95 757  | 6  | 5  | 30 3.0 2.7        |
| 56 | 9.62 486  | 27 | 9.66 735  | 33    | 0.33 265  | 9.95 751  | 6  | 4  | 40 4.0 3.6        |
| 57 | 9.62 513  | 27 | 9.66 768  | 33    | 0.33 232  | 9.95 745  | 6  | 3  | 50 5.0 4.6        |
| 58 | 9.62 540  | 27 | 9.66 801  | 33    | 0.33 198  | 9.95 739  | 5  | 2  |                   |
| 59 | 9.62 567  | 27 | 9.66 834  | 33    | 0.33 165  | 9.95 733  | 6  | 1  |                   |
| 60 | 9.62 595  | 27 | 9.66 867  | 33    | 0.33 132  | 9.95 727  | 6  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.             |

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|
| 0  | 9.62 595  |    | 9.66 867  | 32    | 0.33 132  | 9.95 727  | 6  | 60    |
| 1  | 9.62 622  | 27 | 9.66 900  | 33    | 0.33 100  | 9.95 721  | 5  | 59    |
| 2  | 9.62 649  | 27 | 9.66 933  | 33    | 0.33 067  | 9.95 716  | 6  | 58    |
| 3  | 9.62 676  | 27 | 9.66 966  | 33    | 0.33 034  | 9.95 710  | 6  | 57    |
| 4  | 9.62 703  | 27 | 9.66 999  | 33    | 0.33 001  | 9.95 704  | 6  | 56    |
| 5  | 9.62 730  | 27 | 9.67 032  | 33    | 0.32 968  | 9.95 698  | 6  | 55    |
| 6  | 9.62 757  | 27 | 9.67 065  | 32    | 0.32 935  | 9.95 692  | 6  | 54    |
| 7  | 9.62 784  | 27 | 9.67 097  | 33    | 0.32 902  | 9.95 686  | 6  | 53    |
| 8  | 9.62 811  | 27 | 9.67 130  | 33    | 0.32 869  | 9.95 680  | 6  | 52    |
| 9  | 9.62 838  | 27 | 9.67 163  | 33    | 0.32 836  | 9.95 674  | 6  | 51    |
| 10 | 9.62 864  | 26 | 9.67 196  | 33    | 0.32 803  | 9.95 668  | 6  | 50    |
| 11 | 9.62 891  | 27 | 9.67 229  | 33    | 0.32 771  | 9.95 662  | 6  | 49    |
| 12 | 9.62 918  | 27 | 9.67 262  | 32    | 0.32 738  | 9.95 656  | 6  | 48    |
| 13 | 9.62 945  | 27 | 9.67 294  | 33    | 0.32 705  | 9.95 650  | 6  | 47    |
| 14 | 9.62 972  | 26 | 9.67 327  | 33    | 0.32 672  | 9.95 644  | 6  | 46    |
| 15 | 9.62 999  | 27 | 9.67 360  | 32    | 0.32 640  | 9.95 638  | 6  | 45    |
| 16 | 9.63 025  | 26 | 9.67 393  | 33    | 0.32 607  | 9.95 632  | 5  | 44    |
| 17 | 9.63 052  | 27 | 9.67 425  | 32    | 0.32 574  | 9.95 627  | 6  | 43    |
| 18 | 9.63 079  | 26 | 9.67 458  | 33    | 0.32 541  | 9.95 621  | 6  | 42    |
| 19 | 9.63 106  | 27 | 9.67 491  | 32    | 0.32 509  | 9.95 615  | 6  | 41    |
| 20 | 9.63 132  | 26 | 9.67 523  | 32    | 0.32 476  | 9.95 609  | 6  | 40    |
| 21 | 9.63 159  | 27 | 9.67 556  | 33    | 0.32 443  | 9.95 603  | 6  | 39    |
| 22 | 9.63 186  | 26 | 9.67 589  | 32    | 0.32 411  | 9.95 597  | 6  | 38    |
| 23 | 9.63 212  | 26 | 9.67 621  | 33    | 0.32 378  | 9.95 591  | 6  | 37    |
| 24 | 9.63 239  | 26 | 9.67 654  | 33    | 0.32 345  | 9.95 585  | 6  | 36    |
| 25 | 9.63 266  | 27 | 9.67 687  | 32    | 0.32 313  | 9.95 579  | 6  | 35    |
| 26 | 9.63 292  | 26 | 9.67 719  | 32    | 0.32 280  | 9.95 573  | 6  | 34    |
| 27 | 9.63 319  | 26 | 9.67 752  | 32    | 0.32 248  | 9.95 567  | 6  | 33    |
| 28 | 9.63 345  | 26 | 9.67 784  | 32    | 0.32 215  | 9.95 561  | 6  | 32    |
| 29 | 9.63 372  | 26 | 9.67 817  | 32    | 0.32 183  | 9.95 555  | 6  | 31    |
| 30 | 9.63 398  | 26 | 9.67 849  | 32    | 0.32 150  | 9.95 549  | 6  | 30    |
| 31 | 9.63 425  | 26 | 9.67 882  | 32    | 0.32 118  | 9.95 543  | 6  | 29    |
| 32 | 9.63 451  | 26 | 9.67 914  | 32    | 0.32 085  | 9.95 537  | 6  | 28    |
| 33 | 9.63 478  | 26 | 9.67 947  | 32    | 0.32 053  | 9.95 530  | 6  | 27    |
| 34 | 9.63 504  | 26 | 9.67 979  | 32    | 0.32 020  | 9.95 524  | 6  | 26    |
| 35 | 9.63 530  | 26 | 9.68 012  | 32    | 0.31 988  | 9.95 518  | 6  | 25    |
| 36 | 9.63 557  | 26 | 9.68 044  | 32    | 0.31 955  | 9.95 512  | 6  | 24    |
| 37 | 9.63 583  | 26 | 9.68 077  | 32    | 0.31 923  | 9.95 506  | 6  | 23    |
| 38 | 9.63 609  | 26 | 9.68 109  | 32    | 0.31 891  | 9.95 500  | 6  | 22    |
| 39 | 9.63 636  | 26 | 9.68 141  | 32    | 0.31 858  | 9.95 494  | 6  | 21    |
| 40 | 9.63 662  | 26 | 9.68 174  | 32    | 0.31 826  | 9.95 488  | 6  | 20    |
| 41 | 9.63 688  | 26 | 9.68 206  | 32    | 0.31 793  | 9.95 482  | 6  | 19    |
| 42 | 9.63 715  | 26 | 9.68 238  | 32    | 0.31 761  | 9.95 476  | 6  | 18    |
| 43 | 9.63 741  | 26 | 9.68 271  | 32    | 0.31 729  | 9.95 470  | 6  | 17    |
| 44 | 9.63 767  | 26 | 9.68 303  | 32    | 0.31 696  | 9.95 464  | 6  | 16    |
| 45 | 9.63 793  | 26 | 9.68 335  | 32    | 0.31 664  | 9.95 458  | 6  | 15    |
| 46 | 9.63 819  | 26 | 9.68 368  | 32    | 0.31 632  | 9.95 452  | 6  | 14    |
| 47 | 9.63 846  | 26 | 9.68 400  | 32    | 0.31 600  | 9.95 445  | 6  | 13    |
| 48 | 9.63 872  | 26 | 9.68 432  | 32    | 0.31 567  | 9.95 439  | 6  | 12    |
| 49 | 9.63 898  | 26 | 9.68 464  | 32    | 0.31 535  | 9.95 433  | 6  | 11    |
| 50 | 9.63 924  | 26 | 9.68 497  | 32    | 0.31 503  | 9.95 427  | 6  | 10    |
| 51 | 9.63 950  | 26 | 9.68 529  | 32    | 0.31 471  | 9.95 421  | 6  | 9     |
| 52 | 9.63 976  | 26 | 9.68 561  | 32    | 0.31 439  | 9.95 415  | 6  | 8     |
| 53 | 9.64 002  | 26 | 9.68 593  | 32    | 0.31 406  | 9.95 409  | 6  | 7     |
| 54 | 9.64 028  | 26 | 9.68 625  | 32    | 0.31 374  | 9.95 403  | 6  | 6     |
| 55 | 9.64 054  | 26 | 9.68 657  | 32    | 0.31 342  | 9.95 397  | 6  | 5     |
| 56 | 9.64 080  | 26 | 9.68 690  | 32    | 0.31 310  | 9.95 390  | 6  | 4     |
| 57 | 9.64 106  | 26 | 9.68 722  | 32    | 0.31 278  | 9.95 384  | 6  | 3     |
| 58 | 9.64 132  | 26 | 9.68 754  | 32    | 0.31 246  | 9.95 378  | 6  | 2     |
| 59 | 9.64 158  | 25 | 9.68 786  | 32    | 0.31 214  | 9.95 372  | 6  | 1     |
| 60 | 9.64 184  | 25 | 9.68 818  | 32    | 0.31 182  | 9.95 366  | 6  | 0     |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P. |

**TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.**

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.        |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|--------------|
| 0  | 9.64 184  |    | 9.68 818  | 32    | 0.31 182  | 9.95 366  | 6  | 60 |              |
| 1  | 9.64 210  | 26 | 9.68 850  | 32    | 0.31 150  | 9.95 360  | 6  | 59 |              |
| 2  | 9.64 236  | 26 | 9.68 882  | 32    | 0.31 117  | 9.95 353  | 6  | 58 |              |
| 3  | 9.64 262  | 26 | 9.68 914  | 32    | 0.31 085  | 9.95 347  | 6  | 57 |              |
| 4  | 9.64 287  | 25 | 9.68 946  | 32    | 0.31 053  | 9.95 341  | 6  | 56 |              |
| 5  | 9.64 313  | 26 | 9.68 978  | 32    | 0.31 021  | 9.95 335  | 6  | 55 | 32 32        |
| 6  | 9.64 339  | 26 | 9.69 010  | 32    | 0.30 989  | 9.95 329  | 6  | 54 | 6 3.2 3.2    |
| 7  | 9.64 365  | 25 | 9.69 042  | 32    | 0.30 957  | 9.95 323  | 6  | 53 | 7 3.8 3.7    |
| 8  | 9.64 391  | 26 | 9.69 074  | 31    | 0.30 926  | 9.95 316  | 6  | 52 | 8 4.3 4.2    |
| 9  | 9.64 416  | 25 | 9.69 106  | 32    | 0.30 894  | 9.95 310  | 6  | 51 | 9 4.9 4.8    |
| 10 | 9.64 442  | 26 | 9.69 138  | 32    | 0.30 862  | 9.95 304  | 6  | 50 | 10 5.4 5.3   |
| 11 | 9.64 468  | 25 | 9.69 170  | 32    | 0.30 830  | 9.95 298  | 6  | 49 | 20 10.8 10.6 |
| 12 | 9.64 493  | 26 | 9.69 202  | 32    | 0.30 798  | 9.95 292  | 6  | 48 | 30 16.2 16.0 |
| 13 | 9.64 519  | 25 | 9.69 234  | 31    | 0.30 766  | 9.95 285  | 6  | 47 | 40 21.6 21.3 |
| 14 | 9.64 545  | 25 | 9.69 265  | 32    | 0.30 734  | 9.95 279  | 6  | 46 | 50 27.1 26.6 |
| 15 | 9.64 570  | 25 | 9.69 297  | 32    | 0.30 702  | 9.95 273  | 6  | 45 |              |
| 16 | 9.64 596  | 25 | 9.69 329  | 32    | 0.30 670  | 9.95 267  | 6  | 44 |              |
| 17 | 9.64 622  | 26 | 9.69 361  | 31    | 0.30 639  | 9.95 260  | 6  | 43 |              |
| 18 | 9.64 647  | 25 | 9.69 393  | 32    | 0.30 607  | 9.95 254  | 6  | 42 |              |
| 19 | 9.64 673  | 25 | 9.69 425  | 32    | 0.30 575  | 9.95 248  | 6  | 41 |              |
| 20 | 9.64 698  | 25 | 9.69 456  | 31    | 0.30 543  | 9.95 242  | 6  | 40 | 31 31        |
| 21 | 9.64 724  | 25 | 9.69 488  | 32    | 0.30 511  | 9.95 235  | 6  | 39 | 6 3.1 3.1    |
| 22 | 9.64 749  | 25 | 9.69 520  | 32    | 0.30 480  | 9.95 229  | 6  | 38 | 7 3.7 3.6    |
| 23 | 9.64 775  | 25 | 9.69 552  | 32    | 0.30 448  | 9.95 223  | 6  | 37 | 8 4.2 4.1    |
| 24 | 9.64 800  | 25 | 9.69 583  | 31    | 0.30 416  | 9.95 217  | 6  | 36 | 9 4.7 4.6    |
| 25 | 9.64 826  | 25 | 9.69 615  | 32    | 0.30 384  | 9.95 210  | 6  | 35 | 10 5.2 5.1   |
| 26 | 9.64 851  | 25 | 9.69 647  | 31    | 0.30 353  | 9.95 204  | 6  | 34 | 20 10.5 10.3 |
| 27 | 9.64 876  | 25 | 9.69 678  | 32    | 0.30 321  | 9.95 198  | 6  | 33 | 30 15.7 15.5 |
| 28 | 9.64 902  | 25 | 9.69 710  | 32    | 0.30 289  | 9.95 191  | 6  | 32 | 40 21.0 20.6 |
| 29 | 9.64 927  | 25 | 9.69 742  | 31    | 0.30 258  | 9.95 185  | 6  | 31 | 50 26.2 25.8 |
| 30 | 9.64 952  | 25 | 9.69 773  | 31    | 0.30 226  | 9.95 179  | 6  | 30 |              |
| 31 | 9.64 978  | 25 | 9.69 805  | 32    | 0.30 194  | 9.95 173  | 6  | 29 |              |
| 32 | 9.65 003  | 25 | 9.69 837  | 31    | 0.30 163  | 9.95 166  | 6  | 28 |              |
| 33 | 9.65 028  | 25 | 9.69 868  | 31    | 0.30 131  | 9.95 160  | 6  | 27 | 26 25 25     |
| 34 | 9.65 054  | 25 | 9.69 900  | 31    | 0.30 100  | 9.95 154  | 6  | 26 | 6 2.6 2.5    |
| 35 | 9.65 079  | 25 | 9.69 931  | 31    | 0.30 068  | 9.95 147  | 6  | 25 | 7 3.0 3.0    |
| 36 | 9.65 104  | 25 | 9.69 963  | 31    | 0.30 037  | 9.95 141  | 6  | 24 | 8 3.4 3.4    |
| 37 | 9.65 129  | 25 | 9.69 994  | 31    | 0.30 005  | 9.95 135  | 6  | 23 | 9 3.9 3.8    |
| 38 | 9.65 155  | 25 | 9.70 026  | 32    | 0.29 973  | 9.95 128  | 6  | 22 | 10 4.3 4.2   |
| 39 | 9.65 180  | 25 | 9.70 058  | 32    | 0.29 942  | 9.95 122  | 6  | 21 | 20 8.6 8.5   |
| 40 | 9.65 205  | 25 | 9.70 089  | 31    | 0.29 910  | 9.95 116  | 6  | 20 | 30 13.0 12.7 |
| 41 | 9.65 230  | 25 | 9.70 121  | 31    | 0.29 879  | 9.95 109  | 6  | 19 | 40 17.3 17.0 |
| 42 | 9.65 255  | 25 | 9.70 152  | 31    | 0.29 847  | 9.95 103  | 6  | 18 | 50 21.6 21.2 |
| 43 | 9.65 280  | 25 | 9.70 183  | 31    | 0.29 816  | 9.95 097  | 6  | 17 |              |
| 44 | 9.65 305  | 25 | 9.70 215  | 31    | 0.29 785  | 9.95 090  | 6  | 16 |              |
| 45 | 9.65 331  | 25 | 9.70 246  | 31    | 0.29 753  | 9.95 084  | 6  | 15 |              |
| 46 | 9.65 356  | 25 | 9.70 278  | 31    | 0.29 722  | 9.95 078  | 6  | 14 |              |
| 47 | 9.65 381  | 25 | 9.70 309  | 31    | 0.29 690  | 9.95 071  | 6  | 13 |              |
| 48 | 9.65 406  | 25 | 9.70 341  | 31    | 0.29 659  | 9.95 065  | 6  | 12 | 24 6 6       |
| 49 | 9.65 431  | 25 | 9.70 372  | 31    | 0.29 628  | 9.95 058  | 6  | 11 | 6 2.4 0.6    |
| 50 | 9.65 456  | 25 | 9.70 403  | 31    | 0.29 596  | 9.95 052  | 6  | 10 | 7 2.8 0.7    |
| 51 | 9.65 481  | 25 | 9.70 435  | 31    | 0.29 565  | 9.95 046  | 6  | 9  | 8 3.2 0.8    |
| 52 | 9.65 506  | 24 | 9.70 466  | 31    | 0.29 533  | 9.95 039  | 6  | 8  | 9 3.7 1.0    |
| 53 | 9.65 530  | 25 | 9.70 497  | 31    | 0.29 502  | 9.95 033  | 6  | 7  | 10 4.1 1.1   |
| 54 | 9.65 555  | 25 | 9.70 529  | 31    | 0.29 471  | 9.95 026  | 6  | 6  | 20 8.1 2.2   |
| 55 | 9.65 580  | 25 | 9.70 560  | 31    | 0.29 439  | 9.95 020  | 6  | 5  | 30 12.2 3.2  |
| 56 | 9.65 605  | 24 | 9.70 591  | 31    | 0.29 408  | 9.95 014  | 6  | 4  | 40 16.3 4.3  |
| 57 | 9.65 630  | 25 | 9.70 623  | 31    | 0.29 377  | 9.95 007  | 6  | 3  | 50 20.4 5.4  |
| 58 | 9.65 655  | 25 | 9.70 654  | 31    | 0.29 346  | 9.95 001  | 6  | 2  |              |
| 59 | 9.65 680  | 24 | 9.70 685  | 31    | 0.29 314  | 9.94 994  | 6  | 1  |              |
| 60 | 9.65 704  |    | 9.70 716  | 31    | 0.29 283  | 9.94 988  | 6  | 0  |              |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.        |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot  | Log. Cos. | d. |    | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------------------|
| 0  | 9.65 704  |    | 9.70 716  |       | 0.29 283  | 9.94 988  |    | 60 |                   |
| 1  | 9.65 729  | 25 | 9.70 748  | 31    | 0.29 252  | 9.94 981  | 6  | 59 |                   |
| 2  | 9.65 754  | 24 | 9.70 779  | 31    | 0.29 221  | 9.94 975  | 6  | 58 |                   |
| 3  | 9.65 779  | 25 | 9.70 810  | 31    | 0.29 190  | 9.94 969  | 6  | 57 |                   |
| 4  | 9.65 803  | 24 | 9.70 841  | 31    | 0.29 158  | 9.94 962  | 6  | 56 |                   |
| 5  | 9.65 828  | 25 | 9.70 872  | 31    | 0.29 127  | 9.94 956  | 6  | 55 |                   |
| 6  | 9.65 853  | 24 | 9.70 902  | 31    | 0.29 096  | 9.94 949  | 6  | 54 |                   |
| 7  | 9.65 878  | 25 | 9.70 935  | 31    | 0.29 065  | 9.94 943  | 6  | 53 | 31 31 30          |
| 8  | 9.65 902  | 24 | 9.70 966  | 31    | 0.29 034  | 9.94 936  | 6  | 52 | 6 3.1 3.1 3.0     |
| 9  | 9.65 927  | 24 | 9.70 997  | 31    | 0.29 003  | 9.94 930  | 6  | 51 | 7 3.7 3.6 3.0     |
| 10 | 9.65 951  | 25 | 9.71 028  | 31    | 0.28 972  | 9.94 923  | 6  | 50 | 8 4.2 4.1 4.0     |
| 11 | 9.65 976  | 24 | 9.71 059  | 31    | 0.28 940  | 9.94 917  | 6  | 49 | 9 4.7 4.6 4.6     |
| 12 | 9.66 001  | 24 | 9.71 090  | 31    | 0.28 909  | 9.94 910  | 10 | 48 | 5 5.2 5.1 5.1     |
| 13 | 9.66 025  | 24 | 9.71 121  | 31    | 0.28 878  | 9.94 904  | 20 | 47 | 10 5.7 5.6 5.5    |
| 14 | 9.66 050  | 24 | 9.71 152  | 31    | 0.28 847  | 9.94 897  | 30 | 46 | 15 7 15.5 15.2    |
| 15 | 9.66 074  | 24 | 9.71 183  | 31    | 0.28 816  | 9.94 891  | 40 | 45 | 21 0 20.6 20.3    |
| 16 | 9.66 099  | 24 | 9.71 214  | 31    | 0.28 785  | 9.94 884  | 50 | 44 | 26 2.25 8 25.4    |
| 17 | 9.66 123  | 24 | 9.71 245  | 31    | 0.28 754  | 9.94 878  |    | 43 |                   |
| 18 | 9.66 148  | 24 | 9.71 275  | 31    | 0.28 723  | 9.94 871  |    | 42 |                   |
| 19 | 9.66 172  | 24 | 9.71 307  | 31    | 0.28 692  | 9.94 865  |    | 41 |                   |
| 20 | 9.66 197  | 24 | 9.71 338  | 31    | 0.28 661  | 9.94 858  |    | 40 | 25                |
| 21 | 9.66 221  | 24 | 9.71 369  | 31    | 0.28 630  | 9.94 852  |    | 39 | 6 2.5             |
| 22 | 9.66 246  | 24 | 9.71 400  | 31    | 0.28 599  | 9.94 845  |    | 38 | 7 2.9             |
| 23 | 9.66 270  | 24 | 9.71 431  | 31    | 0.28 568  | 9.94 839  |    | 37 | 8 3.3             |
| 24 | 9.66 294  | 24 | 9.71 462  | 31    | 0.28 537  | 9.94 832  |    | 36 | 9 3.7             |
| 25 | 9.66 319  | 24 | 9.71 493  | 31    | 0.28 506  | 9.94 825  | 7  | 35 | 10 4.1            |
| 26 | 9.66 343  | 24 | 9.71 524  | 31    | 0.28 476  | 9.94 819  | 6  | 34 | 20 8.3            |
| 27 | 9.66 367  | 24 | 9.71 555  | 31    | 0.28 445  | 9.94 812  | 6  | 33 | 30 12.5           |
| 28 | 9.66 392  | 24 | 9.71 586  | 31    | 0.28 414  | 9.94 806  | 6  | 32 | 40 16.6           |
| 29 | 9.66 416  | 24 | 9.71 617  | 31    | 0.28 383  | 9.94 799  | 6  | 31 | 50 20.8           |
| 30 | 9.66 440  | 24 | 9.71 647  | 31    | 0.28 352  | 9.94 793  | 6  | 30 |                   |
| 31 | 9.66 465  | 24 | 9.71 678  | 31    | 0.28 321  | 9.94 786  | 6  | 29 |                   |
| 32 | 9.66 489  | 24 | 9.71 709  | 31    | 0.28 290  | 9.94 779  | 6  | 28 |                   |
| 33 | 9.66 513  | 24 | 9.71 740  | 31    | 0.28 260  | 9.94 773  | 6  | 27 | 24 24 23          |
| 34 | 9.66 537  | 24 | 9.71 771  | 31    | 0.28 229  | 9.94 766  | 6  | 26 | 6 2.4 2.4 2.3     |
| 35 | 9.66 561  | 24 | 9.71 801  | 31    | 0.28 198  | 9.94 760  | 6  | 25 | 7 2.8 2.8 2.7     |
| 36 | 9.66 586  | 24 | 9.71 832  | 31    | 0.28 167  | 9.94 753  | 6  | 24 | 8 3.2 3.2 3.1     |
| 37 | 9.66 610  | 24 | 9.71 863  | 31    | 0.28 136  | 9.94 746  | 6  | 23 | 9 3.7 3.6 3.5     |
| 38 | 9.66 634  | 24 | 9.71 894  | 31    | 0.28 106  | 9.94 740  | 6  | 22 | 10 4.1 4.0 3.9    |
| 39 | 9.66 658  | 24 | 9.71 925  | 31    | 0.28 075  | 9.94 733  | 6  | 21 | 20 8.1 8.0 7.8    |
| 40 | 9.66 682  | 24 | 9.71 955  | 31    | 0.28 044  | 9.94 727  | 6  | 20 | 30 12.2 12.0 11.7 |
| 41 | 9.66 706  | 24 | 9.71 986  | 31    | 0.28 014  | 9.94 720  | 6  | 19 | 40 16.3 16.0 15.6 |
| 42 | 9.66 730  | 24 | 9.72 017  | 31    | 0.27 983  | 9.94 713  | 6  | 18 | 50 20.4 20.0 19.6 |
| 43 | 9.66 754  | 24 | 9.72 047  | 31    | 0.27 952  | 9.94 707  | 6  | 17 |                   |
| 44 | 9.66 778  | 24 | 9.72 078  | 31    | 0.27 921  | 9.94 700  | 6  | 16 |                   |
| 45 | 9.66 802  | 24 | 9.72 109  | 31    | 0.27 891  | 9.94 693  | 7  | 15 |                   |
| 46 | 9.66 826  | 24 | 9.72 139  | 31    | 0.27 860  | 9.94 687  | 6  | 14 | 7 6 6             |
| 47 | 9.66 850  | 24 | 9.72 170  | 31    | 0.27 830  | 9.94 680  | 6  | 13 | 6 7 0 6 0 6       |
| 48 | 9.66 874  | 24 | 9.72 201  | 31    | 0.27 799  | 9.94 674  | 6  | 12 | 7 0 8 0 7 0 7     |
| 49 | 9.66 898  | 24 | 9.72 231  | 31    | 0.27 768  | 9.94 667  | 6  | 11 | 8 0 9 0 8 0 8     |
| 50 | 9.66 922  | 24 | 9.72 262  | 31    | 0.27 738  | 9.94 660  | 6  | 10 | 9 1 0 1 0 0 9     |
| 51 | 9.66 946  | 24 | 9.72 292  | 31    | 0.27 707  | 9.94 654  | 6  | 9  | 10 1 1 1 1 1 0    |
| 52 | 9.66 970  | 23 | 9.72 323  | 31    | 0.27 677  | 9.94 647  | 6  | 8  | 20 2 3 2 1 2 0    |
| 53 | 9.66 994  | 24 | 9.72 354  | 31    | 0.27 646  | 9.94 640  | 6  | 7  | 30 3 5 3 2 3 0    |
| 54 | 9.67 018  | 24 | 9.72 384  | 31    | 0.27 615  | 9.94 633  | 6  | 6  | 40 4 6 4 3 4 0    |
| 55 | 9.67 042  | 24 | 9.72 415  | 31    | 0.27 585  | 9.94 627  | 6  | 5  | 50 5 8 5 4 5 0    |
| 56 | 9.67 066  | 23 | 9.72 445  | 31    | 0.27 554  | 9.94 620  | 6  | 4  |                   |
| 57 | 9.67 089  | 24 | 9.72 476  | 31    | 0.27 524  | 9.94 613  | 6  | 3  |                   |
| 58 | 9.67 113  | 23 | 9.72 506  | 31    | 0.27 493  | 9.94 607  | 6  | 2  |                   |
| 59 | 9.67 137  | 24 | 9.72 537  | 31    | 0.27 463  | 9.94 600  | 6  | 1  |                   |
| 60 | 9.67 161  | 24 | 9.72 567  | 31    | 0.27 432  | 9.94 593  | 6  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot  | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.             |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.          |
|----|-----------|----|-----------------|-----------|-----------|----|----|----------------|
| 0  | 9.67 161  |    | 9.72 567        | 0.27 432  | 9.94 593  | 60 |    |                |
| 1  | 9.67 184  | 23 | 9.72 598        | 0.27 402  | 9.94 587  | 59 |    |                |
| 2  | 9.67 208  | 24 | 9.72 628        | 0.27 371  | 9.94 580  | 58 |    |                |
| 3  | 9.67 232  | 23 | 9.72 659        | 0.27 341  | 9.94 573  | 57 |    |                |
| 4  | 9.67 256  | 24 | 9.72 689        | 0.27 311  | 9.94 566  | 56 |    |                |
| 5  | 9.67 279  | 23 | 9.72 719        | 0.27 280  | 9.94 560  | 55 |    | 30 20 29       |
| 6  | 9.67 303  | 23 | 9.72 750        | 0.27 250  | 9.94 553  | 54 | 6  | 3.0 3.0 2.4    |
| 7  | 9.67 327  | 24 | 9.72 780        | 0.27 219  | 9.94 546  | 53 | 7  | 3.5 3.5 3.4    |
| 8  | 9.67 350  | 23 | 9.72 811        | 0.27 189  | 9.94 539  | 52 | 8  | 4.0 4.0 3.9    |
| 9  | 9.67 374  | 23 | 9.72 841        | 0.27 159  | 9.94 533  | 51 | 9  | 4.6 4.5 4.4    |
| 10 | 9.67 397  | 23 | 9.72 871        | 0.27 128  | 9.94 526  | 50 | 10 | 5.1 5.0 4.9    |
| 11 | 9.67 421  | 24 | 9.72 902        | 0.27 098  | 9.94 519  | 49 | 20 | 10.1 10.0 9.8  |
| 12 | 9.67 445  | 23 | 9.72 932        | 0.27 067  | 9.94 512  | 48 | 30 | 15.2 15.0 14.7 |
| 13 | 9.67 468  | 23 | 9.72 962        | 0.27 037  | 9.94 506  | 47 | 40 | 20.3 20.0 19.6 |
| 14 | 9.67 492  | 23 | 9.72 993        | 0.27 007  | 9.94 499  | 46 | 50 | 25.4 25.0 24.6 |
| 15 | 9.67 515  | 23 | 9.73 023        | 0.26 976  | 9.94 492  | 45 |    |                |
| 16 | 9.67 539  | 23 | 9.73 053        | 0.26 946  | 9.94 485  | 44 |    |                |
| 17 | 9.67 562  | 23 | 9.73 084        | 0.26 916  | 9.94 478  | 43 |    |                |
| 18 | 9.67 586  | 23 | 9.73 114        | 0.26 886  | 9.94 472  | 42 |    |                |
| 19 | 9.67 609  | 23 | 9.73 144        | 0.26 855  | 9.94 465  | 41 |    |                |
| 20 | 9.67 633  | 23 | 9.73 174        | 0.26 825  | 9.94 458  | 40 |    | 24             |
| 21 | 9.67 656  | 23 | 9.73 205        | 0.26 795  | 9.94 451  | 39 | 6  | 2.4            |
| 22 | 9.67 679  | 23 | 9.73 235        | 0.26 765  | 9.94 444  | 38 | 7  | 2.8            |
| 23 | 9.67 703  | 23 | 9.73 265        | 0.26 734  | 9.94 437  | 37 | 8  | 3.2            |
| 24 | 9.67 726  | 23 | 9.73 295        | 0.26 704  | 9.94 431  | 36 | 9  | 3.6            |
| 25 | 9.67 750  | 23 | 9.73 325        | 0.26 674  | 9.94 424  | 35 | 10 | 4.0            |
| 26 | 9.67 773  | 23 | 9.73 356        | 0.26 644  | 9.94 417  | 34 | 20 | 8.0            |
| 27 | 9.67 796  | 23 | 9.73 386        | 0.26 614  | 9.94 410  | 33 | 30 | 12.0           |
| 28 | 9.67 819  | 23 | 9.73 416        | 0.26 584  | 9.94 403  | 32 | 40 | 16.0           |
| 29 | 9.67 843  | 23 | 9.73 446        | 0.26 553  | 9.94 396  | 31 | 50 | 20.0           |
| 30 | 9.67 866  | 23 | 9.73 476        | 0.26 523  | 9.94 390  | 30 |    |                |
| 31 | 9.67 889  | 23 | 9.73 506        | 0.26 493  | 9.94 383  | 29 |    |                |
| 32 | 9.67 913  | 23 | 9.73 536        | 0.26 463  | 9.94 376  | 28 |    |                |
| 33 | 9.67 936  | 23 | 9.73 567        | 0.26 433  | 9.94 369  | 27 |    |                |
| 34 | 9.67 959  | 23 | 9.73 597        | 0.26 403  | 9.94 362  | 26 |    |                |
| 35 | 9.67 982  | 23 | 9.73 627        | 0.26 373  | 9.94 355  | 25 |    |                |
| 36 | 9.68 005  | 23 | 9.73 657        | 0.26 343  | 9.94 348  | 24 |    |                |
| 37 | 9.68 029  | 23 | 9.73 687        | 0.26 313  | 9.94 341  | 23 |    |                |
| 38 | 9.68 052  | 23 | 9.73 717        | 0.26 283  | 9.94 335  | 22 |    |                |
| 39 | 9.68 075  | 23 | 9.73 747        | 0.26 253  | 9.94 328  | 21 |    |                |
| 40 | 9.68 098  | 23 | 9.73 777        | 0.26 223  | 9.94 321  | 20 |    |                |
| 41 | 9.68 121  | 23 | 9.73 807        | 0.26 193  | 9.94 314  | 19 |    |                |
| 42 | 9.68 144  | 23 | 9.73 837        | 0.26 163  | 9.94 307  | 18 |    |                |
| 43 | 9.68 167  | 23 | 9.73 867        | 0.26 133  | 9.94 300  | 17 |    |                |
| 44 | 9.68 190  | 23 | 9.73 897        | 0.26 103  | 9.94 293  | 16 |    |                |
| 45 | 9.68 213  | 23 | 9.73 927        | 0.26 073  | 9.94 286  | 15 |    |                |
| 46 | 9.68 236  | 23 | 9.73 957        | 0.26 043  | 9.94 279  | 14 |    |                |
| 47 | 9.68 259  | 23 | 9.73 987        | 0.26 013  | 9.94 272  | 13 |    |                |
| 48 | 9.68 282  | 23 | 9.74 017        | 0.25 983  | 9.94 265  | 12 |    |                |
| 49 | 9.68 305  | 23 | 9.74 047        | 0.25 953  | 9.94 258  | 11 |    |                |
| 50 | 9.68 328  | 23 | 9.74 076        | 0.25 923  | 9.94 251  | 10 |    |                |
| 51 | 9.68 351  | 23 | 9.74 106        | 0.25 893  | 9.94 245  | 9  |    |                |
| 52 | 9.68 374  | 22 | 9.74 136        | 0.25 863  | 9.94 238  | 8  |    |                |
| 53 | 9.68 397  | 23 | 9.74 166        | 0.25 833  | 9.94 231  | 7  |    |                |
| 54 | 9.68 420  | 23 | 9.74 196        | 0.25 804  | 9.94 224  | 6  |    |                |
| 55 | 9.68 443  | 23 | 9.74 226        | 0.25 774  | 9.94 217  | 5  |    |                |
| 56 | 9.68 466  | 23 | 9.74 256        | 0.25 744  | 9.94 210  | 4  |    |                |
| 57 | 9.68 489  | 22 | 9.74 286        | 0.25 714  | 9.94 203  | 3  |    |                |
| 58 | 9.68 511  | 23 | 9.74 315        | 0.25 684  | 9.94 196  | 2  |    |                |
| 59 | 9.68 534  | 23 | 9.74 345        | 0.25 654  | 9.94 189  | 1  |    |                |
| 60 | 9.68 557  | 22 | 9.74 375        | 0.25 625  | 9.94 182  | 0  |    |                |
|    | Log. Cos. | d. | Log. Cot. c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.          |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.        |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|--------------|
| 0  | 9.68 557  |    | 9.74 375  |       | 0.25 625  | 9.94 182  |    | 60 |              |
| 1  | 9.68 580  | 23 | 9.74 405  | 30    | 0.25 595  | 9.94 175  | 7  | 59 |              |
| 2  | 9.68 602  | 22 | 9.74 435  | 29    | 0.25 565  | 9.94 168  | 7  | 58 |              |
| 3  | 9.68 625  | 23 | 9.74 464  | 30    | 0.25 535  | 9.94 161  | 7  | 57 |              |
| 4  | 9.68 648  | 22 | 9.74 494  | 29    | 0.25 505  | 9.94 154  | 7  | 56 |              |
| 5  | 9.68 671  | 23 | 9.74 524  | 30    | 0.25 476  | 9.94 147  | 7  | 55 | 30 29 29     |
| 6  | 9.68 693  | 22 | 9.74 554  | 29    | 0.25 446  | 9.94 140  | 7  | 54 | 6 3.0 2.9    |
| 7  | 9.68 716  | 23 | 9.74 583  | 30    | 0.25 416  | 9.94 133  | 7  | 53 | 7 3.5 3.4    |
| 8  | 9.68 739  | 22 | 9.74 613  | 29    | 0.25 387  | 9.94 126  | 7  | 52 | 8 4.0 3.9    |
| 9  | 9.68 761  | 23 | 9.74 643  | 30    | 0.25 357  | 9.94 118  | 7  | 51 | 9 4.5 4.4    |
| 10 | 9.68 784  | 22 | 9.74 672  | 29    | 0.25 327  | 9.94 111  | 7  | 50 | 10 5.0 4.9   |
| 11 | 9.68 807  | 23 | 9.74 702  | 30    | 0.25 297  | 9.94 104  | 7  | 49 | 20 10.0 9.8  |
| 12 | 9.68 829  | 22 | 9.74 732  | 29    | 0.25 268  | 9.94 97   | 7  | 48 | 30 15.0 14.7 |
| 13 | 9.68 852  | 23 | 9.74 761  | 30    | 0.25 238  | 9.94 90   | 7  | 47 | 40 20.0 19.6 |
| 14 | 9.68 874  | 22 | 9.74 791  | 29    | 0.25 208  | 9.94 83   | 7  | 46 | 50 25.0 24.6 |
| 15 | 9.68 897  | 23 | 9.74 821  | 30    | 0.25 179  | 9.94 76   | 7  | 45 |              |
| 16 | 9.68 920  | 22 | 9.74 850  | 29    | 0.25 149  | 9.94 69   | 7  | 44 |              |
| 17 | 9.68 942  | 23 | 9.74 880  | 30    | 0.25 120  | 9.94 62   | 7  | 43 |              |
| 18 | 9.68 965  | 22 | 9.74 909  | 29    | 0.25 090  | 9.94 55   | 7  | 42 |              |
| 19 | 9.68 987  | 23 | 9.74 939  | 30    | 0.25 060  | 9.94 48   | 7  | 41 |              |
| 20 | 9.69 010  | 22 | 9.74 969  | 29    | 0.25 031  | 9.94 41   | 7  | 40 | 23           |
| 21 | 9.69 032  | 23 | 9.74 998  | 30    | 0.25 001  | 9.94 34   | 7  | 39 | 6 2.3        |
| 22 | 9.69 055  | 22 | 9.75 028  | 29    | 0.24 972  | 9.94 27   | 7  | 38 | 7 2.7        |
| 23 | 9.69 077  | 23 | 9.75 057  | 30    | 0.24 942  | 9.94 20   | 7  | 37 | 8 3.0        |
| 24 | 9.69 099  | 22 | 9.75 087  | 29    | 0.24 913  | 9.94 12   | 7  | 36 | 9 3.4        |
| 25 | 9.69 122  | 23 | 9.75 116  | 30    | 0.24 883  | 9.94 05   | 7  | 35 | 10 3.8       |
| 26 | 9.69 144  | 22 | 9.75 146  | 29    | 0.24 854  | 9.93 98   | 7  | 34 | 20 7.6       |
| 27 | 9.69 167  | 23 | 9.75 175  | 30    | 0.24 824  | 9.93 91   | 7  | 33 | 30 11.5      |
| 28 | 9.69 189  | 22 | 9.75 205  | 29    | 0.24 795  | 9.93 84   | 7  | 32 | 40 15.3      |
| 29 | 9.69 211  | 23 | 9.75 234  | 30    | 0.24 765  | 9.93 77   | 7  | 31 | 50 19.1      |
| 30 | 9.69 234  | 22 | 9.75 264  | 29    | 0.24 736  | 9.93 69   | 7  | 30 |              |
| 31 | 9.69 256  | 23 | 9.75 293  | 30    | 0.24 706  | 9.93 62   | 7  | 29 |              |
| 32 | 9.69 278  | 22 | 9.75 323  | 29    | 0.24 677  | 9.93 55   | 7  | 28 |              |
| 33 | 9.69 301  | 23 | 9.75 352  | 30    | 0.24 647  | 9.93 48   | 7  | 27 |              |
| 34 | 9.69 323  | 22 | 9.75 382  | 29    | 0.24 618  | 9.93 41   | 7  | 26 | 25 22 21     |
| 35 | 9.69 345  | 23 | 9.75 411  | 30    | 0.24 588  | 9.93 34   | 7  | 25 | 6 2.2 2.1    |
| 36 | 9.69 367  | 22 | 9.75 441  | 29    | 0.24 558  | 9.93 26   | 7  | 24 | 7 2.6 2.5    |
| 37 | 9.69 390  | 23 | 9.75 470  | 30    | 0.24 529  | 9.93 19   | 7  | 23 | 8 3.0 2.9    |
| 38 | 9.69 412  | 22 | 9.75 499  | 29    | 0.24 500  | 9.93 12   | 7  | 22 | 9 3.4 3.3    |
| 39 | 9.69 434  | 23 | 9.75 528  | 30    | 0.24 471  | 9.93 05   | 7  | 21 | 10 3.7 3.6   |
| 40 | 9.69 456  | 22 | 9.75 558  | 29    | 0.24 441  | 9.93 98   | 7  | 20 | 20 7.5 7.4   |
| 41 | 9.69 478  | 23 | 9.75 588  | 30    | 0.24 412  | 9.93 91   | 7  | 19 | 30 11.2 11.0 |
| 42 | 9.69 500  | 22 | 9.75 617  | 29    | 0.24 383  | 9.93 83   | 7  | 18 | 40 15.0 14.8 |
| 43 | 9.69 523  | 23 | 9.75 646  | 30    | 0.24 353  | 9.93 76   | 7  | 17 | 50 18.7 18.5 |
| 44 | 9.69 545  | 22 | 9.75 676  | 29    | 0.24 324  | 9.93 69   | 7  | 16 |              |
| 45 | 9.69 567  | 23 | 9.75 705  | 30    | 0.24 295  | 9.93 62   | 7  | 15 |              |
| 46 | 9.69 589  | 22 | 9.75 734  | 29    | 0.24 265  | 9.93 55   | 7  | 14 |              |
| 47 | 9.69 611  | 23 | 9.75 764  | 30    | 0.24 236  | 9.93 47   | 7  | 13 | 7 7          |
| 48 | 9.69 633  | 22 | 9.75 793  | 29    | 0.24 207  | 9.93 40   | 7  | 12 | 6 0.7 0.7    |
| 49 | 9.69 655  | 23 | 9.75 822  | 30    | 0.24 177  | 9.93 33   | 7  | 11 | 7 0.9 0.8    |
| 50 | 9.69 677  | 22 | 9.75 851  | 29    | 0.24 148  | 9.93 26   | 7  | 10 | 8 1.0 0.9    |
| 51 | 9.69 699  | 23 | 9.75 881  | 30    | 0.24 119  | 9.93 18   | 7  | 9  | 9 1.1 1.0    |
| 52 | 9.69 721  | 22 | 9.75 910  | 29    | 0.24 090  | 9.93 11   | 7  | 8  | 10 1.2 1.1   |
| 53 | 9.69 743  | 23 | 9.75 939  | 30    | 0.24 060  | 9.93 04   | 7  | 7  | 20 2.5 2.3   |
| 54 | 9.69 765  | 22 | 9.75 968  | 29    | 0.24 031  | 9.93 96   | 7  | 6  | 30 3.7 3.5   |
| 55 | 9.69 787  | 23 | 9.75 998  | 30    | 0.24 002  | 9.93 89   | 7  | 5  | 40 5.0 4.8   |
| 56 | 9.69 809  | 22 | 9.76 027  | 29    | 0.23 973  | 9.93 82   | 7  | 4  | 50 6.2 5.8   |
| 57 | 9.69 831  | 23 | 9.76 056  | 30    | 0.23 943  | 9.93 75   | 7  | 3  |              |
| 58 | 9.69 853  | 22 | 9.76 085  | 29    | 0.23 914  | 9.93 67   | 7  | 2  |              |
| 59 | 9.69 875  | 23 | 9.76 115  | 30    | 0.23 885  | 9.93 60   | 7  | 1  |              |
| 60 | 9.96 897  | 22 | 9.76 144  | 29    | 0.23 856  | 9.93 53   | 7  | 0  |              |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.        |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |  | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|--|-------|
| 0  | 9.69 897  | 22 | 9.76 144  | 29    | 0.23 856  | 9.93 753  | 60 |  |       |
| 1  | 9.69 919  | 21 | 9.76 173  | 29    | 0.23 827  | 9.93 746  | 59 |  |       |
| 2  | 9.69 940  | 22 | 9.76 202  | 29    | 0.23 797  | 9.93 738  | 58 |  |       |
| 3  | 9.69 962  | 22 | 9.76 231  | 29    | 0.23 768  | 9.93 731  | 57 |  |       |
| 4  | 9.69 984  | 22 | 9.76 260  | 29    | 0.23 739  | 9.93 724  | 56 |  |       |
| 5  | 9.70 006  | 21 | 9.76 289  | 29    | 0.23 710  | 9.93 716  | 55 |  |       |
| 6  | 9.70 028  | 22 | 9.76 319  | 29    | 0.23 681  | 9.93 709  | 54 |  |       |
| 7  | 9.70 050  | 22 | 9.76 348  | 29    | 0.23 652  | 9.93 702  | 53 |  |       |
| 8  | 9.70 071  | 22 | 9.76 377  | 29    | 0.23 623  | 9.93 694  | 52 |  |       |
| 9  | 9.70 093  | 21 | 9.76 406  | 29    | 0.23 594  | 9.93 687  | 51 |  |       |
| 10 | 9.70 115  | 22 | 9.76 435  | 29    | 0.23 565  | 9.93 680  | 50 |  |       |
| 11 | 9.70 137  | 21 | 9.76 464  | 29    | 0.23 535  | 9.93 672  | 49 |  |       |
| 12 | 9.70 158  | 21 | 9.76 493  | 29    | 0.23 506  | 9.93 665  | 48 |  |       |
| 13 | 9.70 180  | 22 | 9.76 522  | 29    | 0.23 477  | 9.93 658  | 47 |  |       |
| 14 | 9.70 202  | 21 | 9.76 551  | 29    | 0.23 448  | 9.93 650  | 46 |  |       |
| 15 | 9.70 223  | 21 | 9.76 580  | 29    | 0.23 419  | 9.93 643  | 45 |  |       |
| 16 | 9.70 245  | 22 | 9.76 609  | 29    | 0.23 390  | 9.93 635  | 44 |  |       |
| 17 | 9.70 267  | 21 | 9.76 638  | 29    | 0.23 361  | 9.93 628  | 43 |  |       |
| 18 | 9.70 288  | 21 | 9.76 667  | 29    | 0.23 332  | 9.93 621  | 42 |  |       |
| 19 | 9.70 310  | 21 | 9.76 696  | 29    | 0.23 303  | 9.93 613  | 41 |  |       |
| 20 | 9.70 331  | 22 | 9.76 725  | 29    | 0.23 274  | 9.93 606  | 40 |  |       |
| 21 | 9.70 353  | 21 | 9.76 754  | 29    | 0.23 245  | 9.93 599  | 39 |  |       |
| 22 | 9.70 375  | 21 | 9.76 783  | 29    | 0.23 216  | 9.93 591  | 38 |  |       |
| 23 | 9.70 396  | 21 | 9.76 812  | 29    | 0.23 187  | 9.93 584  | 37 |  |       |
| 24 | 9.70 418  | 21 | 9.76 841  | 29    | 0.23 158  | 9.93 576  | 36 |  |       |
| 25 | 9.70 439  | 21 | 9.76 870  | 29    | 0.23 129  | 9.93 568  | 35 |  |       |
| 26 | 9.70 461  | 21 | 9.76 899  | 28    | 0.23 101  | 9.93 562  | 34 |  |       |
| 27 | 9.70 482  | 21 | 9.76 928  | 29    | 0.23 072  | 9.93 554  | 33 |  |       |
| 28 | 9.70 504  | 21 | 9.76 957  | 29    | 0.23 043  | 9.93 547  | 32 |  |       |
| 29 | 9.70 525  | 21 | 9.76 986  | 29    | 0.23 014  | 9.93 539  | 31 |  |       |
| 30 | 9.70 547  | 21 | 9.77 015  | 29    | 0.22 985  | 9.93 532  | 30 |  |       |
| 31 | 9.70 568  | 21 | 9.77 043  | 28    | 0.22 956  | 9.93 524  | 29 |  |       |
| 32 | 9.70 590  | 21 | 9.77 072  | 29    | 0.22 927  | 9.93 517  | 28 |  |       |
| 33 | 9.70 611  | 21 | 9.77 101  | 29    | 0.22 898  | 9.93 509  | 27 |  |       |
| 34 | 9.70 632  | 21 | 9.77 130  | 29    | 0.22 869  | 9.93 502  | 26 |  |       |
| 35 | 9.70 654  | 21 | 9.77 159  | 28    | 0.22 841  | 9.93 495  | 25 |  |       |
| 36 | 9.70 675  | 21 | 9.77 188  | 29    | 0.22 812  | 9.93 487  | 24 |  |       |
| 37 | 9.70 696  | 21 | 9.77 217  | 28    | 0.22 783  | 9.93 480  | 23 |  |       |
| 38 | 9.70 718  | 21 | 9.77 245  | 28    | 0.22 754  | 9.93 472  | 22 |  |       |
| 39 | 9.70 739  | 21 | 9.77 274  | 29    | 0.22 725  | 9.93 465  | 21 |  |       |
| 40 | 9.70 760  | 21 | 9.77 303  | 29    | 0.22 696  | 9.93 457  | 20 |  |       |
| 41 | 9.70 782  | 21 | 9.77 332  | 29    | 0.22 668  | 9.93 450  | 19 |  |       |
| 42 | 9.70 803  | 21 | 9.77 361  | 28    | 0.22 639  | 9.93 442  | 18 |  |       |
| 43 | 9.70 824  | 21 | 9.77 389  | 29    | 0.22 610  | 9.93 435  | 17 |  |       |
| 44 | 9.70 846  | 21 | 9.77 418  | 28    | 0.22 581  | 9.93 427  | 16 |  |       |
| 45 | 9.70 867  | 21 | 9.77 447  | 29    | 0.22 553  | 9.93 420  | 15 |  |       |
| 46 | 9.70 888  | 21 | 9.77 476  | 28    | 0.22 524  | 9.93 412  | 14 |  |       |
| 47 | 9.70 909  | 21 | 9.77 504  | 29    | 0.22 495  | 9.93 405  | 13 |  |       |
| 48 | 9.70 930  | 21 | 9.77 533  | 28    | 0.22 466  | 9.93 397  | 12 |  |       |
| 49 | 9.70 952  | 21 | 9.77 562  | 28    | 0.22 438  | 9.93 390  | 11 |  |       |
| 50 | 9.70 973  | 21 | 9.77 591  | 29    | 0.22 409  | 9.93 382  | 10 |  |       |
| 51 | 9.70 994  | 21 | 9.77 619  | 28    | 0.22 380  | 9.93 374  | 9  |  |       |
| 52 | 9.71 015  | 21 | 9.77 648  | 29    | 0.22 352  | 9.93 367  | 8  |  |       |
| 53 | 9.71 036  | 21 | 9.77 677  | 28    | 0.22 323  | 9.93 359  | 7  |  |       |
| 54 | 9.71 057  | 21 | 9.77 705  | 28    | 0.22 294  | 9.93 352  | 6  |  |       |
| 55 | 9.71 078  | 21 | 9.77 734  | 29    | 0.22 266  | 9.93 344  | 5  |  |       |
| 56 | 9.71 099  | 21 | 9.77 763  | 28    | 0.22 237  | 9.93 337  | 4  |  |       |
| 57 | 9.71 121  | 21 | 9.77 791  | 28    | 0.22 208  | 9.93 329  | 3  |  |       |
| 58 | 9.71 142  | 21 | 9.77 820  | 29    | 0.22 180  | 9.93 321  | 2  |  |       |
| 59 | 9.71 163  | 21 | 9.77 849  | 28    | 0.22 151  | 9.93 314  | 1  |  |       |
| 60 | 9.71 184  | 21 | 9.77 877  | 28    | 0.22 122  | 9.93 306  | 0  |  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |  | P. P. |

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# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS. AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c.d. | Log. Cot. | Log. Cos. | d. | P. P. |
|----|-----------|----|-----------|------|-----------|-----------|----|-------|
| 0  | 9.71 184  | 21 | 9.77 877  | 28   | 0.22 122  | 9.93 306  | 7  | 60    |
| 1  | 9.71 205  | 21 | 9.77 908  | 28   | 0.22 094  | 9.93 299  | 7  | 59    |
| 2  | 9.71 226  | 21 | 9.77 934  | 28   | 0.22 065  | 9.93 291  | 7  | 58    |
| 3  | 9.71 247  | 21 | 9.77 963  | 28   | 0.22 037  | 9.93 284  | 7  | 57    |
| 4  | 9.71 268  | 21 | 9.77 992  | 29   | 0.22 008  | 9.93 276  | 8  | 56    |
| 5  | 9.71 289  | 21 | 9.78 020  | 28   | 0.21 979  | 9.93 268  | 7  | 55    |
| 6  | 9.71 310  | 21 | 9.78 049  | 28   | 0.21 951  | 9.93 261  | 7  | 54    |
| 7  | 9.71 331  | 20 | 9.78 077  | 28   | 0.21 922  | 9.93 253  | 7  | 53    |
| 8  | 9.71 351  | 21 | 9.78 106  | 28   | 0.21 894  | 9.93 245  | 8  | 52    |
| 9  | 9.71 372  | 21 | 9.78 134  | 28   | 0.21 865  | 9.93 238  | 7  | 51    |
| 10 | 9.71 393  | 21 | 9.78 163  | 28   | 0.21 837  | 9.93 230  | 7  | 50    |
| 11 | 9.71 414  | 21 | 9.78 191  | 28   | 0.21 808  | 9.93 223  | 7  | 49    |
| 12 | 9.71 435  | 21 | 9.78 220  | 28   | 0.21 780  | 9.93 215  | 8  | 48    |
| 13 | 9.71 456  | 21 | 9.78 248  | 28   | 0.21 751  | 9.93 207  | 7  | 47    |
| 14 | 9.71 477  | 21 | 9.78 277  | 28   | 0.21 723  | 9.93 200  | 7  | 46    |
| 15 | 9.71 498  | 21 | 9.78 305  | 28   | 0.21 694  | 9.93 192  | 8  | 45    |
| 16 | 9.71 518  | 20 | 9.78 334  | 28   | 0.21 666  | 9.93 184  | 7  | 44    |
| 17 | 9.71 539  | 20 | 9.78 362  | 28   | 0.21 637  | 9.93 177  | 8  | 43    |
| 18 | 9.71 560  | 21 | 9.78 391  | 28   | 0.21 609  | 9.93 169  | 7  | 42    |
| 19 | 9.71 581  | 20 | 9.78 419  | 28   | 0.21 580  | 9.93 161  | 8  | 41    |
| 20 | 9.71 601  | 21 | 9.78 448  | 28   | 0.21 552  | 9.93 153  | 7  | 40    |
| 21 | 9.71 622  | 20 | 9.78 476  | 28   | 0.21 523  | 9.93 146  | 7  | 39    |
| 22 | 9.71 643  | 21 | 9.78 505  | 28   | 0.21 495  | 9.93 138  | 8  | 38    |
| 23 | 9.71 664  | 20 | 9.78 533  | 28   | 0.21 467  | 9.93 130  | 7  | 37    |
| 24 | 9.71 684  | 21 | 9.78 561  | 28   | 0.21 438  | 9.93 123  | 8  | 36    |
| 25 | 9.71 705  | 20 | 9.78 590  | 28   | 0.21 410  | 9.93 115  | 7  | 35    |
| 26 | 9.71 726  | 20 | 9.78 618  | 28   | 0.21 381  | 9.93 107  | 7  | 34    |
| 27 | 9.71 746  | 21 | 9.78 647  | 28   | 0.21 353  | 9.93 100  | 8  | 33    |
| 28 | 9.71 767  | 20 | 9.78 675  | 28   | 0.21 325  | 9.93 092  | 7  | 32    |
| 29 | 9.71 788  | 20 | 9.78 703  | 28   | 0.21 296  | 9.93 084  | 8  | 31    |
| 30 | 9.71 808  | 20 | 9.78 732  | 28   | 0.21 268  | 9.93 076  | 7  | 30    |
| 31 | 9.71 829  | 20 | 9.78 760  | 28   | 0.21 239  | 9.93 069  | 8  | 29    |
| 32 | 9.71 849  | 21 | 9.78 788  | 28   | 0.21 211  | 9.93 061  | 7  | 28    |
| 33 | 9.71 870  | 20 | 9.78 817  | 28   | 0.21 183  | 9.93 053  | 8  | 27    |
| 34 | 9.71 891  | 20 | 9.78 845  | 28   | 0.21 154  | 9.93 045  | 7  | 26    |
| 35 | 9.71 911  | 20 | 9.78 873  | 28   | 0.21 126  | 9.93 038  | 8  | 25    |
| 36 | 9.71 932  | 20 | 9.78 902  | 28   | 0.21 098  | 9.93 030  | 7  | 24    |
| 37 | 9.71 952  | 20 | 9.78 930  | 28   | 0.21 070  | 9.93 022  | 8  | 23    |
| 38 | 9.71 973  | 20 | 9.78 958  | 28   | 0.21 041  | 9.93 014  | 7  | 22    |
| 39 | 9.71 993  | 20 | 9.78 987  | 28   | 0.21 013  | 9.93 006  | 8  | 21    |
| 40 | 9.72 014  | 20 | 9.79 015  | 28   | 0.20 985  | 9.92 999  | 7  | 20    |
| 41 | 9.72 034  | 20 | 9.79 043  | 28   | 0.20 956  | 9.92 991  | 8  | 19    |
| 42 | 9.72 055  | 20 | 9.79 071  | 28   | 0.20 928  | 9.92 983  | 7  | 18    |
| 43 | 9.72 075  | 20 | 9.79 100  | 28   | 0.20 900  | 9.92 975  | 8  | 17    |
| 44 | 9.72 096  | 20 | 9.79 128  | 28   | 0.20 872  | 9.92 967  | 7  | 16    |
| 45 | 9.72 116  | 20 | 9.79 156  | 28   | 0.20 843  | 9.92 960  | 8  | 15    |
| 46 | 9.72 136  | 20 | 9.79 184  | 28   | 0.20 815  | 9.92 952  | 7  | 14    |
| 47 | 9.72 157  | 20 | 9.79 213  | 28   | 0.20 787  | 9.92 944  | 8  | 13    |
| 48 | 9.72 177  | 20 | 9.79 241  | 28   | 0.20 759  | 9.92 936  | 7  | 12    |
| 49 | 9.72 198  | 20 | 9.79 269  | 28   | 0.20 731  | 9.92 928  | 8  | 11    |
| 50 | 9.72 218  | 20 | 9.79 297  | 28   | 0.20 702  | 9.92 920  | 7  | 10    |
| 51 | 9.72 238  | 20 | 9.79 325  | 28   | 0.20 674  | 9.92 913  | 8  | 9     |
| 52 | 9.72 259  | 20 | 9.79 354  | 28   | 0.20 646  | 9.92 905  | 7  | 8     |
| 53 | 9.72 279  | 20 | 9.79 382  | 28   | 0.20 618  | 9.92 897  | 8  | 7     |
| 54 | 9.72 299  | 20 | 9.79 410  | 28   | 0.20 590  | 9.92 889  | 7  | 6     |
| 55 | 9.72 319  | 20 | 9.79 438  | 28   | 0.20 561  | 9.92 881  | 8  | 5     |
| 56 | 9.72 340  | 20 | 9.79 466  | 28   | 0.20 532  | 9.92 873  | 7  | 4     |
| 57 | 9.72 360  | 20 | 9.79 494  | 28   | 0.20 505  | 9.92 865  | 8  | 3     |
| 58 | 9.72 380  | 20 | 9.79 522  | 28   | 0.20 477  | 9.92 858  | 7  | 2     |
| 59 | 9.72 400  | 20 | 9.79 551  | 28   | 0.20 449  | 9.92 850  | 8  | 1     |
| 60 | 9.72 421  | 20 | 9.79 579  | 28   | 0.20 421  | 9.92 842  | 7  | 0     |
|    | Log. Cos. | d. | Log. Cot. | c.d. | Log. Tan. | Log. Sin. | d. | P. P. |

|    |      |      |      |
|----|------|------|------|
|    | 29   | 28   | 28   |
| 6  | 2.9  | 2.8  | 2.8  |
| 7  | 3.4  | 3.3  | 3.2  |
| 8  | 3.8  | 3.8  | 3.7  |
| 9  | 4.3  | 4.3  | 4.2  |
| 10 | 4.8  | 4.7  | 4.6  |
| 20 | 8.6  | 9.5  | 9.3  |
| 30 | 14.5 | 14.2 | 14.0 |
| 40 | 19.3 | 19.0 | 18.6 |
| 50 | 24.1 | 23.7 | 23.3 |

|    |      |      |      |
|----|------|------|------|
|    | 21   | 20   | 20   |
| 6  | 2.1  | 2.0  | 2.0  |
| 7  | 2.4  | 2.4  | 2.3  |
| 8  | 2.8  | 2.7  | 2.6  |
| 9  | 3.1  | 3.1  | 3.0  |
| 10 | 3.5  | 3.4  | 3.3  |
| 20 | 7.0  | 6.8  | 6.6  |
| 30 | 10.5 | 10.2 | 10.0 |
| 40 | 14.0 | 13.6 | 13.3 |
| 50 | 17.5 | 17.1 | 16.6 |

|     |   |     |
|-----|---|-----|
|     | 8 | 7   |
| 60  | 8 | 0.7 |
| 70  | 9 | 0.9 |
| 81  | 0 | 1.0 |
| 91  | 2 | 1.1 |
| 101 | 3 | 1.2 |
| 20  | 2 | 2.5 |
| 30  | 4 | 3.7 |
| 40  | 5 | 5.0 |
| 50  | 6 | 6.2 |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.72 421  | 20 | 9.79 579  | 28    | 0.20 421  | 9.92 842  | 8  | 60 |       |
| 1  | 9.72 441  | 20 | 9.79 607  | 28    | 0.20 393  | 9.92 834  | 7  | 59 |       |
| 2  | 9.72 461  | 20 | 9.79 635  | 28    | 0.20 365  | 9.92 826  | 8  | 58 |       |
| 3  | 9.72 481  | 20 | 9.79 663  | 28    | 0.20 337  | 9.92 818  | 8  | 57 |       |
| 4  | 9.72 501  | 20 | 9.79 691  | 28    | 0.20 308  | 9.92 810  | 8  | 56 |       |
| 5  | 9.72 522  | 20 | 9.79 719  | 28    | 0.20 280  | 9.92 802  | 8  | 55 |       |
| 6  | 9.72 542  | 20 | 9.79 747  | 28    | 0.20 252  | 9.92 794  | 8  | 54 |       |
| 7  | 9.72 562  | 20 | 9.79 775  | 28    | 0.20 224  | 9.92 786  | 8  | 53 |       |
| 8  | 9.72 582  | 20 | 9.79 803  | 28    | 0.20 196  | 9.92 778  | 8  | 52 |       |
| 9  | 9.72 602  | 20 | 9.79 831  | 28    | 0.20 168  | 9.92 771  | 7  | 51 |       |
| 10 | 9.72 622  | 20 | 9.79 859  | 28    | 0.20 140  | 9.92 763  | 8  | 50 |       |
| 11 | 9.72 642  | 20 | 9.79 887  | 28    | 0.20 112  | 9.92 755  | 8  | 49 |       |
| 12 | 9.72 662  | 20 | 9.79 915  | 28    | 0.20 084  | 9.92 747  | 8  | 48 |       |
| 13 | 9.72 682  | 20 | 9.79 943  | 28    | 0.20 056  | 9.92 739  | 8  | 47 |       |
| 14 | 9.72 702  | 20 | 9.79 971  | 28    | 0.20 028  | 9.92 731  | 8  | 46 |       |
| 15 | 9.72 723  | 20 | 9.79 999  | 28    | 0.20 000  | 9.92 723  | 8  | 45 |       |
| 16 | 9.72 743  | 20 | 9.80 027  | 28    | 0.19 972  | 9.92 715  | 8  | 44 |       |
| 17 | 9.72 763  | 20 | 9.80 055  | 28    | 0.19 944  | 9.92 707  | 8  | 43 |       |
| 18 | 9.72 783  | 20 | 9.80 083  | 28    | 0.19 916  | 9.92 699  | 8  | 42 |       |
| 19 | 9.72 802  | 19 | 9.80 111  | 28    | 0.19 888  | 9.92 691  | 8  | 41 |       |
| 20 | 9.72 822  | 20 | 9.80 139  | 28    | 0.19 860  | 9.92 683  | 8  | 40 |       |
| 21 | 9.72 842  | 20 | 9.80 167  | 28    | 0.19 832  | 9.92 675  | 8  | 39 |       |
| 22 | 9.72 862  | 20 | 9.80 195  | 28    | 0.19 804  | 9.92 667  | 8  | 38 |       |
| 23 | 9.72 882  | 20 | 9.80 223  | 28    | 0.19 776  | 9.92 659  | 8  | 37 |       |
| 24 | 9.72 902  | 20 | 9.80 251  | 28    | 0.19 748  | 9.92 651  | 8  | 36 |       |
| 25 | 9.72 922  | 20 | 9.80 279  | 27    | 0.19 721  | 9.92 643  | 8  | 35 |       |
| 26 | 9.72 942  | 19 | 9.80 307  | 28    | 0.19 693  | 9.92 635  | 8  | 34 |       |
| 27 | 9.72 962  | 20 | 9.80 335  | 28    | 0.19 665  | 9.92 627  | 8  | 33 |       |
| 28 | 9.72 982  | 20 | 9.80 363  | 28    | 0.19 637  | 9.92 619  | 8  | 32 |       |
| 29 | 9.73 002  | 20 | 9.80 391  | 28    | 0.19 609  | 9.92 611  | 8  | 31 |       |
| 30 | 9.73 021  | 19 | 9.80 418  | 27    | 0.19 581  | 9.92 603  | 8  | 30 |       |
| 31 | 9.73 041  | 20 | 9.80 446  | 28    | 0.19 553  | 9.92 595  | 8  | 29 |       |
| 32 | 9.73 061  | 20 | 9.80 474  | 28    | 0.19 525  | 9.92 587  | 8  | 28 |       |
| 33 | 9.73 081  | 19 | 9.80 502  | 28    | 0.19 497  | 9.92 579  | 8  | 27 |       |
| 34 | 9.73 101  | 20 | 9.80 530  | 27    | 0.19 470  | 9.92 570  | 8  | 26 |       |
| 35 | 9.73 120  | 19 | 9.80 558  | 28    | 0.19 442  | 9.92 562  | 8  | 25 |       |
| 36 | 9.73 140  | 20 | 9.80 586  | 28    | 0.19 414  | 9.92 554  | 8  | 24 |       |
| 37 | 9.73 160  | 19 | 9.80 613  | 27    | 0.19 386  | 9.92 546  | 8  | 23 |       |
| 38 | 9.73 180  | 20 | 9.80 641  | 28    | 0.19 358  | 9.92 538  | 8  | 22 |       |
| 39 | 9.73 199  | 19 | 9.80 669  | 28    | 0.19 330  | 9.92 530  | 8  | 21 |       |
| 40 | 9.73 219  | 20 | 9.80 697  | 27    | 0.19 303  | 9.92 522  | 8  | 20 |       |
| 41 | 9.73 239  | 19 | 9.80 725  | 28    | 0.19 275  | 9.92 514  | 8  | 19 |       |
| 42 | 9.73 258  | 20 | 9.80 752  | 27    | 0.19 247  | 9.92 506  | 8  | 18 |       |
| 43 | 9.73 278  | 19 | 9.80 780  | 28    | 0.19 219  | 9.92 498  | 8  | 17 |       |
| 44 | 9.73 298  | 20 | 9.80 808  | 28    | 0.19 191  | 9.92 489  | 8  | 16 |       |
| 45 | 9.73 317  | 19 | 9.80 836  | 27    | 0.19 164  | 9.92 481  | 8  | 15 |       |
| 46 | 9.73 337  | 20 | 9.80 864  | 28    | 0.19 136  | 9.92 473  | 8  | 14 |       |
| 47 | 9.73 357  | 19 | 9.80 891  | 27    | 0.19 108  | 9.92 465  | 8  | 13 |       |
| 48 | 9.73 376  | 20 | 9.80 919  | 28    | 0.19 080  | 9.92 457  | 8  | 12 |       |
| 49 | 9.73 396  | 19 | 9.80 947  | 27    | 0.19 053  | 9.92 449  | 8  | 11 |       |
| 50 | 9.73 415  | 20 | 9.80 975  | 28    | 0.19 025  | 9.92 441  | 8  | 10 |       |
| 51 | 9.73 435  | 19 | 9.81 002  | 27    | 0.18 997  | 9.92 433  | 8  | 9  |       |
| 52 | 9.73 455  | 20 | 9.81 030  | 28    | 0.18 970  | 9.92 424  | 8  | 8  |       |
| 53 | 9.73 474  | 19 | 9.81 058  | 27    | 0.18 942  | 9.92 416  | 8  | 7  |       |
| 54 | 9.73 494  | 20 | 9.81 085  | 28    | 0.18 914  | 9.92 408  | 8  | 6  |       |
| 55 | 9.73 513  | 19 | 9.81 113  | 28    | 0.18 886  | 9.92 400  | 8  | 5  |       |
| 56 | 9.73 533  | 20 | 9.81 141  | 27    | 0.18 859  | 9.92 392  | 8  | 4  |       |
| 57 | 9.73 552  | 19 | 9.81 168  | 28    | 0.18 831  | 9.92 383  | 8  | 3  |       |
| 58 | 9.73 572  | 20 | 9.81 196  | 27    | 0.18 803  | 9.92 375  | 8  | 2  |       |
| 59 | 9.73 591  | 19 | 9.81 224  | 28    | 0.18 776  | 9.92 367  | 8  | 1  |       |
| 60 | 9.73 611  | 20 | 9.81 251  | 27    | 0.18 748  | 9.92 359  | 8  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

|    |      |      |      |
|----|------|------|------|
|    | 28   | 28   | 27   |
| 6  | 2.8  | 2.8  | 2.7  |
| 7  | 3.3  | 3.2  | 3.2  |
| 8  | 3.8  | 3.7  | 3.6  |
| 9  | 4.3  | 4.2  | 4.1  |
| 10 | 4.7  | 4.6  | 4.6  |
| 20 | 9.5  | 9.3  | 9.1  |
| 30 | 14.2 | 14.0 | 13.7 |
| 40 | 19.0 | 18.6 | 18.3 |
| 50 | 23.7 | 23.3 | 22.9 |

|    |      |      |      |
|----|------|------|------|
|    | 20   | 20   | 19   |
| 6  | 2.0  | 2.0  | 1.9  |
| 7  | 2.4  | 2.3  | 2.3  |
| 8  | 2.7  | 2.6  | 2.6  |
| 9  | 3.1  | 3.0  | 2.9  |
| 10 | 3.4  | 3.3  | 3.2  |
| 20 | 6.8  | 6.6  | 6.5  |
| 30 | 10.2 | 10.0 | 9.7  |
| 40 | 13.6 | 13.3 | 13.0 |
| 50 | 17.1 | 16.6 | 16.2 |

|    |     |     |     |
|----|-----|-----|-----|
|    | 8   | 8   | 7   |
| 6  | 0.8 | 0.8 | 0.7 |
| 7  | 1.0 | 0.9 | 0.9 |
| 8  | 1.1 | 1.0 | 1.0 |
| 9  | 1.3 | 1.2 | 1.1 |
| 10 | 1.4 | 1.3 | 1.2 |
| 20 | 2.8 | 2.6 | 2.5 |
| 30 | 4.2 | 4.0 | 3.7 |
| 40 | 5.6 | 5.3 | 5.0 |
| 50 | 7.1 | 6.6 | 6.2 |



# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P.             |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------------------|
| 0  | 9.73 611  | 19 | 9.81 251  |       | 0.18 748  | 9.92 359  | 60 |                   |
| 1  | 9.73 630  | 19 | 9.81 279  | 28    | 0.18 720  | 9.92 351  | 59 |                   |
| 2  | 9.73 650  | 19 | 9.81 307  | 27    | 0.18 693  | 9.92 342  | 58 |                   |
| 3  | 9.73 669  | 19 | 9.81 334  | 27    | 0.18 665  | 9.92 334  | 57 |                   |
| 4  | 9.73 688  | 19 | 9.81 362  | 28    | 0.18 637  | 9.92 326  | 56 |                   |
| 5  | 9.73 708  | 19 | 9.81 390  | 27    | 0.18 610  | 9.92 318  | 55 |                   |
| 6  | 9.73 727  | 19 | 9.81 417  | 27    | 0.18 582  | 9.92 310  | 54 |                   |
| 7  | 9.73 746  | 19 | 9.81 445  | 27    | 0.18 555  | 9.92 301  | 53 |                   |
| 8  | 9.73 766  | 19 | 9.81 473  | 28    | 0.18 527  | 9.92 293  | 52 |                   |
| 9  | 9.73 785  | 19 | 9.81 500  | 27    | 0.18 499  | 9.92 285  | 51 |                   |
| 10 | 9.73 805  | 19 | 9.81 528  | 27    | 0.18 472  | 9.92 277  | 50 | 28 27 27          |
| 11 | 9.73 824  | 19 | 9.81 555  | 27    | 0.18 444  | 9.92 268  | 49 | 6 2.8 2.7 2.7     |
| 12 | 9.73 843  | 19 | 9.81 583  | 27    | 0.18 417  | 9.92 260  | 48 | 7 3.2 3.2 3.1     |
| 13 | 9.73 862  | 19 | 9.81 610  | 27    | 0.18 389  | 9.92 252  | 47 | 8 3.7 3.6 3.6     |
| 14 | 9.73 882  | 19 | 9.81 638  | 27    | 0.18 362  | 9.92 244  | 46 | 9 4.2 4.1 4.0     |
| 15 | 9.73 901  | 19 | 9.81 666  | 28    | 0.18 334  | 9.92 235  | 45 | 10 4.6 4.6 4.5    |
| 16 | 9.73 920  | 19 | 9.81 693  | 27    | 0.18 306  | 9.92 227  | 44 | 20 9.3 9.1 9.0    |
| 17 | 9.73 940  | 19 | 9.81 721  | 27    | 0.18 279  | 9.92 219  | 43 | 30 14.0 13.7 13.5 |
| 18 | 9.73 959  | 19 | 9.81 748  | 27    | 0.18 251  | 9.92 210  | 42 | 40 18.6 18.3 18.0 |
| 19 | 9.73 978  | 19 | 9.81 776  | 27    | 0.18 224  | 9.92 202  | 41 | 50 23.3 22.9 22.5 |
| 20 | 9.73 997  | 19 | 9.81 803  | 27    | 0.18 196  | 9.92 194  | 40 |                   |
| 21 | 9.74 016  | 19 | 9.81 831  | 27    | 0.18 169  | 9.92 185  | 39 |                   |
| 22 | 9.74 036  | 19 | 9.81 858  | 27    | 0.18 141  | 9.92 177  | 38 |                   |
| 23 | 9.74 055  | 19 | 9.81 886  | 27    | 0.18 114  | 9.92 169  | 37 |                   |
| 24 | 9.74 074  | 19 | 9.81 913  | 27    | 0.18 086  | 9.92 160  | 36 |                   |
| 25 | 9.74 093  | 19 | 9.81 941  | 27    | 0.18 059  | 9.92 152  | 35 |                   |
| 26 | 9.74 112  | 19 | 9.81 968  | 27    | 0.18 031  | 9.92 144  | 34 | 19 19 18          |
| 27 | 9.74 131  | 19 | 9.81 996  | 27    | 0.18 004  | 9.92 135  | 33 | 6 1.9 1.9 1.8     |
| 28 | 9.74 151  | 19 | 9.82 023  | 27    | 0.17 976  | 9.92 127  | 32 | 7 2.3 2.2 2.1     |
| 29 | 9.74 170  | 19 | 9.82 051  | 27    | 0.17 949  | 9.92 119  | 31 | 8 2.6 2.5 2.4     |
| 30 | 9.74 189  | 19 | 9.82 078  | 27    | 0.17 921  | 9.92 110  | 30 | 9 2.9 2.8 2.8     |
| 31 | 9.74 208  | 19 | 9.82 105  | 27    | 0.17 894  | 9.92 102  | 29 | 10 3.2 3.1 3.1    |
| 32 | 9.74 227  | 19 | 9.82 133  | 27    | 0.17 867  | 9.92 094  | 28 | 20 6.5 6.3 6.1    |
| 33 | 9.74 246  | 19 | 9.82 160  | 27    | 0.17 839  | 9.92 085  | 27 | 30 9.7 9.5 9.2    |
| 34 | 9.74 265  | 19 | 9.82 188  | 27    | 0.17 812  | 9.92 077  | 26 | 40 13.0 12.6 12.3 |
| 35 | 9.74 284  | 19 | 9.82 215  | 27    | 0.17 784  | 9.92 069  | 25 | 50 16.2 15.8 15.4 |
| 36 | 9.74 303  | 19 | 9.82 243  | 27    | 0.17 757  | 9.92 060  | 24 |                   |
| 37 | 9.74 322  | 19 | 9.82 270  | 27    | 0.17 729  | 9.92 052  | 23 |                   |
| 38 | 9.74 341  | 19 | 9.82 297  | 27    | 0.17 702  | 9.92 043  | 22 |                   |
| 39 | 9.74 360  | 18 | 9.82 325  | 27    | 0.17 675  | 9.92 035  | 21 |                   |
| 40 | 9.74 379  | 19 | 9.82 352  | 27    | 0.17 647  | 9.92 027  | 20 |                   |
| 41 | 9.74 398  | 19 | 9.82 380  | 27    | 0.17 620  | 9.92 018  | 19 |                   |
| 42 | 9.74 417  | 19 | 9.82 407  | 27    | 0.17 593  | 9.92 010  | 18 |                   |
| 43 | 9.74 436  | 19 | 9.82 434  | 27    | 0.17 565  | 9.92 001  | 17 | 8 8               |
| 44 | 9.74 455  | 19 | 9.82 462  | 27    | 0.17 538  | 9.91 993  | 16 | 6 0.8 0.8 0.8     |
| 45 | 9.74 474  | 19 | 9.82 489  | 27    | 0.17 510  | 9.91 984  | 15 | 7 1.0 0.9 0.9     |
| 46 | 9.74 493  | 18 | 9.82 516  | 27    | 0.17 483  | 9.91 976  | 14 | 8 1.1 1.0 1.0     |
| 47 | 9.74 511  | 19 | 9.82 544  | 27    | 0.17 456  | 9.91 967  | 13 | 9 1.3 1.2 1.2     |
| 48 | 9.74 530  | 19 | 9.82 571  | 27    | 0.17 428  | 9.91 959  | 12 | 10 1.4 1.3 1.3    |
| 49 | 9.74 549  | 19 | 9.82 598  | 27    | 0.17 401  | 9.91 951  | 11 | 20 2.6 2.6 2.6    |
| 50 | 9.74 568  | 18 | 9.82 626  | 27    | 0.17 374  | 9.91 942  | 10 | 30 4.2 4.0 4.0    |
| 51 | 9.74 587  | 19 | 9.82 653  | 27    | 0.17 347  | 9.91 934  | 9  | 40 5.6 5.3 5.3    |
| 52 | 9.74 606  | 19 | 9.82 680  | 27    | 0.17 319  | 9.91 925  | 8  | 50 7.1 6.6 6.6    |
| 53 | 9.74 625  | 18 | 9.82 708  | 27    | 0.17 292  | 9.91 917  | 7  |                   |
| 54 | 9.74 643  | 19 | 9.82 735  | 27    | 0.17 265  | 9.91 908  | 6  |                   |
| 55 | 9.74 662  | 19 | 9.82 762  | 27    | 0.17 237  | 9.91 900  | 5  |                   |
| 56 | 9.74 681  | 18 | 9.82 789  | 27    | 0.17 210  | 9.91 891  | 4  |                   |
| 57 | 9.74 700  | 18 | 9.82 817  | 27    | 0.17 183  | 9.91 883  | 3  |                   |
| 58 | 9.74 718  | 19 | 9.82 844  | 27    | 0.17 156  | 9.91 874  | 2  |                   |
| 59 | 9.74 737  | 19 | 9.82 871  | 27    | 0.17 128  | 9.91 866  | 1  |                   |
| 60 | 9.74 756  | 18 | 9.82 898  | 27    | 0.17 101  | 9.91 857  | 0  |                   |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P.             |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.74 736  |    | 9.82 898  | 27    | 0.17 101  | 9.91 857  |    | 60 |       |
| 1  | 9.74 775  | 19 | 9.82 926  | 27    | 0.17 074  | 9.91 849  | 8  | 59 |       |
| 2  | 9.74 793  | 18 | 9.82 953  | 27    | 0.17 047  | 9.91 840  | 8  | 58 |       |
| 3  | 9.74 812  | 19 | 9.82 980  | 27    | 0.17 019  | 9.91 832  | 8  | 57 |       |
| 4  | 9.74 831  | 18 | 9.83 007  | 27    | 0.16 992  | 9.91 823  | 8  | 56 |       |
| 5  | 9.74 849  | 18 | 9.83 035  | 27    | 0.16 965  | 9.91 814  | 9  | 55 |       |
| 6  | 9.74 868  | 19 | 9.83 062  | 27    | 0.16 938  | 9.91 806  | 8  | 54 |       |
| 7  | 9.74 887  | 18 | 9.83 089  | 27    | 0.16 910  | 9.91 797  | 8  | 53 |       |
| 8  | 9.74 905  | 18 | 9.83 116  | 27    | 0.16 883  | 9.91 789  | 8  | 52 |       |
| 9  | 9.74 924  | 19 | 9.83 143  | 27    | 0.16 856  | 9.91 780  | 8  | 51 |       |
| 10 | 9.74 943  | 18 | 9.83 171  | 27    | 0.16 829  | 9.91 772  | 8  | 50 |       |
| 11 | 9.74 961  | 18 | 9.83 198  | 27    | 0.16 802  | 9.91 763  | 8  | 49 |       |
| 12 | 9.74 980  | 18 | 9.83 225  | 27    | 0.16 774  | 9.91 755  | 8  | 48 |       |
| 13 | 9.74 998  | 18 | 9.83 252  | 27    | 0.16 747  | 9.91 746  | 9  | 47 |       |
| 14 | 9.75 017  | 19 | 9.83 279  | 27    | 0.16 720  | 9.91 737  | 8  | 46 |       |
| 15 | 9.75 036  | 18 | 9.83 307  | 27    | 0.16 693  | 9.91 729  | 8  | 45 |       |
| 16 | 9.75 054  | 18 | 9.83 334  | 27    | 0.16 666  | 9.91 720  | 8  | 44 |       |
| 17 | 9.75 073  | 18 | 9.83 361  | 27    | 0.16 639  | 9.91 712  | 8  | 43 |       |
| 18 | 9.75 091  | 18 | 9.83 388  | 27    | 0.16 612  | 9.91 703  | 8  | 42 |       |
| 19 | 9.75 110  | 18 | 9.83 415  | 27    | 0.16 584  | 9.91 694  | 8  | 41 |       |
| 20 | 9.75 128  | 18 | 9.83 442  | 27    | 0.16 557  | 9.91 686  | 8  | 40 |       |
| 21 | 9.75 147  | 18 | 9.83 469  | 27    | 0.16 530  | 9.91 677  | 8  | 39 |       |
| 22 | 9.75 165  | 18 | 9.83 496  | 27    | 0.16 503  | 9.91 668  | 8  | 38 |       |
| 23 | 9.75 184  | 18 | 9.83 524  | 27    | 0.16 476  | 9.91 660  | 8  | 37 |       |
| 24 | 9.75 202  | 18 | 9.83 551  | 27    | 0.16 449  | 9.91 651  | 8  | 36 |       |
| 25 | 9.75 221  | 18 | 9.83 578  | 27    | 0.16 422  | 9.91 642  | 9  | 35 |       |
| 26 | 9.75 239  | 18 | 9.83 605  | 27    | 0.16 395  | 9.91 634  | 8  | 34 |       |
| 27 | 9.75 257  | 18 | 9.83 632  | 27    | 0.16 368  | 9.91 625  | 8  | 33 |       |
| 28 | 9.75 276  | 18 | 9.83 659  | 27    | 0.16 340  | 9.91 616  | 8  | 32 |       |
| 29 | 9.75 294  | 18 | 9.83 686  | 27    | 0.16 313  | 9.91 608  | 8  | 31 |       |
| 30 | 9.75 313  | 18 | 9.83 713  | 27    | 0.16 285  | 9.91 599  | 8  | 30 |       |
| 31 | 9.75 331  | 18 | 9.83 740  | 27    | 0.16 259  | 9.91 590  | 8  | 29 |       |
| 32 | 9.75 349  | 18 | 9.83 767  | 27    | 0.16 232  | 9.91 582  | 8  | 28 |       |
| 33 | 9.75 368  | 18 | 9.83 794  | 27    | 0.16 205  | 9.91 573  | 8  | 27 |       |
| 34 | 9.75 386  | 18 | 9.83 821  | 27    | 0.16 178  | 9.91 564  | 8  | 26 |       |
| 35 | 9.75 404  | 18 | 9.83 848  | 27    | 0.16 151  | 9.91 556  | 8  | 25 |       |
| 36 | 9.75 423  | 18 | 9.83 875  | 27    | 0.16 124  | 9.91 547  | 8  | 24 |       |
| 37 | 9.75 441  | 18 | 9.83 902  | 27    | 0.16 097  | 9.91 538  | 8  | 23 |       |
| 38 | 9.75 459  | 18 | 9.83 929  | 27    | 0.16 070  | 9.91 529  | 8  | 22 |       |
| 39 | 9.75 478  | 18 | 9.83 957  | 27    | 0.16 042  | 9.91 521  | 8  | 21 |       |
| 40 | 9.75 496  | 18 | 9.83 984  | 27    | 0.16 016  | 9.91 512  | 8  | 20 |       |
| 41 | 9.75 514  | 18 | 9.84 011  | 27    | 0.15 989  | 9.91 503  | 8  | 19 |       |
| 42 | 9.75 532  | 18 | 9.84 038  | 27    | 0.15 962  | 9.91 495  | 8  | 18 |       |
| 43 | 9.75 551  | 18 | 9.84 065  | 27    | 0.15 935  | 9.91 486  | 8  | 17 |       |
| 44 | 9.75 569  | 18 | 9.84 091  | 26    | 0.15 908  | 9.91 477  | 8  | 16 |       |
| 45 | 9.75 587  | 18 | 9.84 118  | 27    | 0.15 881  | 9.91 468  | 8  | 15 |       |
| 46 | 9.75 605  | 18 | 9.84 145  | 27    | 0.15 854  | 9.91 460  | 8  | 14 |       |
| 47 | 9.75 623  | 18 | 9.84 172  | 27    | 0.15 827  | 9.91 451  | 8  | 13 |       |
| 48 | 9.75 642  | 18 | 9.84 199  | 27    | 0.15 800  | 9.91 442  | 8  | 12 |       |
| 49 | 9.75 660  | 18 | 9.84 226  | 27    | 0.15 773  | 9.91 433  | 8  | 11 |       |
| 50 | 9.75 678  | 18 | 9.84 253  | 27    | 0.15 746  | 9.91 424  | 9  | 10 |       |
| 51 | 9.75 696  | 18 | 9.84 280  | 27    | 0.15 719  | 9.91 416  | 8  | 9  |       |
| 52 | 9.75 714  | 18 | 9.84 307  | 27    | 0.15 692  | 9.91 407  | 8  | 8  |       |
| 53 | 9.75 732  | 18 | 9.84 334  | 27    | 0.15 665  | 9.91 398  | 8  | 7  |       |
| 54 | 9.75 750  | 18 | 9.84 361  | 26    | 0.15 639  | 9.91 389  | 8  | 6  |       |
| 55 | 9.75 769  | 18 | 9.84 388  | 27    | 0.15 612  | 9.91 380  | 8  | 5  |       |
| 56 | 9.75 787  | 18 | 9.84 415  | 27    | 0.15 585  | 9.91 372  | 8  | 4  |       |
| 57 | 9.75 805  | 18 | 9.84 442  | 27    | 0.15 558  | 9.91 363  | 8  | 3  |       |
| 58 | 9.75 823  | 18 | 9.84 469  | 27    | 0.15 531  | 9.91 354  | 8  | 2  |       |
| 59 | 9.75 841  | 18 | 9.84 496  | 27    | 0.15 504  | 9.91 345  | 8  | 1  |       |
| 60 | 9.75 859  | 18 | 9.84 522  | 26    | 0.15 477  | 9.91 336  | 9  | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

|    |      |      |      |
|----|------|------|------|
| 6  | 2.7  | 2.7  | 2.6  |
| 7  | 3.2  | 3.1  | 3.1  |
| 8  | 3.6  | 3.6  | 3.5  |
| 9  | 4.1  | 4.0  | 4.0  |
| 10 | 4.6  | 4.5  | 4.4  |
| 20 | 9.1  | 9.0  | 8.8  |
| 30 | 13.7 | 13.5 | 13.2 |
| 40 | 18.3 | 18.0 | 17.6 |
| 50 | 22.9 | 22.5 | 22.1 |

|    |      |      |
|----|------|------|
| 19 | 15   | 18   |
| 6  | 1.9  | 1.8  |
| 7  | 2.2  | 2.1  |
| 8  | 2.5  | 2.4  |
| 9  | 2.8  | 2.8  |
| 10 | 3.1  | 3.0  |
| 20 | 6.3  | 6.0  |
| 30 | 9.5  | 9.0  |
| 40 | 12.6 | 12.0 |
| 50 | 15.8 | 15.0 |

|     |     |     |
|-----|-----|-----|
| 60  | 9.9 | 9.8 |
| 71  | 0.1 | 0.1 |
| 81  | 2.1 | 2.1 |
| 91  | 3.1 | 3.1 |
| 101 | 4.1 | 4.1 |
| 203 | 0.2 | 0.2 |
| 304 | 5.4 | 5.4 |
| 406 | 6.5 | 6.5 |
| 507 | 7.5 | 7.1 |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. | P. P. |         |      |
|----|-----------|----|-----------|-------|-----------|-----------|----|-------|---------|------|
| 0  | 9.75 859  | 18 | 9.84 521  | 27    | 0.15 477  | 9.91 336  | 9  | 60    |         |      |
| 1  | 9.75 877  | 18 | 9.84 549  | 27    | 0.15 450  | 9.91 327  | 9  | 59    |         |      |
| 2  | 9.75 895  | 18 | 9.84 576  | 27    | 0.15 423  | 9.91 318  | 9  | 58    |         |      |
| 3  | 9.75 913  | 18 | 9.84 603  | 27    | 0.15 396  | 9.91 310  | 9  | 57    |         |      |
| 4  | 9.75 931  | 18 | 9.84 630  | 27    | 0.15 370  | 9.91 301  | 9  | 56    |         |      |
| 5  | 9.75 949  | 18 | 9.84 657  | 27    | 0.15 343  | 9.91 292  | 9  | 55    |         |      |
| 6  | 9.75 967  | 18 | 9.84 684  | 27    | 0.15 316  | 9.91 283  | 9  | 54    |         |      |
| 7  | 9.75 985  | 18 | 9.84 711  | 27    | 0.15 289  | 9.91 274  | 9  | 53    |         |      |
| 8  | 9.76 003  | 18 | 9.84 737  | 27    | 0.15 262  | 9.91 265  | 9  | 52    |         |      |
| 9  | 9.76 021  | 18 | 9.84 764  | 27    | 0.15 235  | 9.91 256  | 9  | 51    |         |      |
| 10 | 9.76 039  | 18 | 9.84 791  | 27    | 0.15 208  | 9.91 247  | 9  | 50    | 27      | 26   |
| 11 | 9.76 057  | 18 | 9.84 818  | 27    | 0.15 182  | 9.91 239  | 9  | 49    | 6' 2.7  | 2.6  |
| 12 | 9.76 075  | 18 | 9.84 845  | 27    | 0.15 155  | 9.91 230  | 9  | 48    | 7 3.1   | 3.1  |
| 13 | 9.76 092  | 17 | 9.84 871  | 26    | 0.15 128  | 9.91 221  | 9  | 47    | 8 3.6   | 3.5  |
| 14 | 9.76 110  | 18 | 9.84 898  | 27    | 0.15 101  | 9.91 212  | 9  | 46    | 9 4.0   | 4.0  |
| 15 | 9.76 128  | 18 | 9.84 925  | 27    | 0.15 074  | 9.91 203  | 9  | 45    | 10 4.5  | 4.4  |
| 16 | 9.76 146  | 18 | 9.84 952  | 27    | 0.15 048  | 9.91 194  | 9  | 44    | 20 9 0  | 8.8  |
| 17 | 9.76 164  | 17 | 9.84 979  | 26    | 0.15 021  | 9.91 185  | 9  | 43    | 30 13.5 | 13.2 |
| 18 | 9.76 182  | 18 | 9.85 005  | 27    | 0.14 994  | 9.91 176  | 9  | 42    | 40 18.0 | 17.6 |
| 19 | 9.76 200  | 18 | 9.85 032  | 27    | 0.14 967  | 9.91 167  | 9  | 41    | 50 22.5 | 22.1 |
| 20 | 9.76 217  | 17 | 9.85 059  | 26    | 0.14 940  | 9.91 158  | 9  | 40    |         |      |
| 21 | 9.76 235  | 18 | 9.85 086  | 27    | 0.14 914  | 9.91 149  | 9  | 39    |         |      |
| 22 | 9.76 253  | 18 | 9.85 113  | 27    | 0.14 887  | 9.91 140  | 9  | 38    |         |      |
| 23 | 9.76 271  | 17 | 9.85 139  | 26    | 0.14 860  | 9.91 131  | 9  | 37    |         |      |
| 24 | 9.76 289  | 18 | 9.85 166  | 27    | 0.14 833  | 9.91 122  | 9  | 36    |         |      |
| 25 | 9.76 306  | 17 | 9.85 193  | 26    | 0.14 807  | 9.91 113  | 9  | 35    |         |      |
| 26 | 9.76 324  | 18 | 9.85 220  | 27    | 0.14 780  | 9.91 104  | 9  | 34    | 18      | 17   |
| 27 | 9.76 342  | 18 | 9.85 246  | 27    | 0.14 753  | 9.91 095  | 9  | 33    | 6 1.8   | 1.7  |
| 28 | 9.76 360  | 17 | 9.85 273  | 26    | 0.14 726  | 9.91 086  | 9  | 32    | 7 2.1   | 2.0  |
| 29 | 9.76 377  | 18 | 9.85 300  | 27    | 0.14 700  | 9.91 077  | 9  | 31    | 8 2.4   | 2.3  |
| 30 | 9.76 395  | 17 | 9.85 327  | 26    | 0.14 673  | 9.91 068  | 9  | 30    | 9 2.7   | 2.6  |
| 31 | 9.76 413  | 18 | 9.85 353  | 27    | 0.14 646  | 9.91 059  | 9  | 29    | 10 3.0  | 2.9  |
| 32 | 9.76 431  | 18 | 9.85 380  | 27    | 0.14 620  | 9.91 050  | 9  | 28    | 20 6.0  | 5.8  |
| 33 | 9.76 448  | 17 | 9.85 407  | 26    | 0.14 593  | 9.91 041  | 9  | 27    | 30 9.0  | 8.7  |
| 34 | 9.76 466  | 18 | 9.85 433  | 27    | 0.14 566  | 9.91 032  | 9  | 26    | 40 12.0 | 11.6 |
| 35 | 9.76 484  | 17 | 9.85 460  | 26    | 0.14 539  | 9.91 023  | 9  | 25    | 50 15.0 | 14.1 |
| 36 | 9.76 501  | 18 | 9.85 487  | 27    | 0.14 513  | 9.91 014  | 9  | 24    |         |      |
| 37 | 9.76 519  | 17 | 9.85 513  | 26    | 0.14 486  | 9.91 005  | 9  | 23    |         |      |
| 38 | 9.76 536  | 18 | 9.85 540  | 27    | 0.14 459  | 9.90 996  | 9  | 22    |         |      |
| 39 | 9.76 554  | 17 | 9.85 567  | 26    | 0.14 433  | 9.90 987  | 9  | 21    |         |      |
| 40 | 9.76 572  | 18 | 9.85 594  | 27    | 0.14 406  | 9.90 978  | 9  | 20    |         |      |
| 41 | 9.76 589  | 17 | 9.85 620  | 26    | 0.14 379  | 9.90 969  | 9  | 19    |         |      |
| 42 | 9.76 607  | 18 | 9.85 647  | 27    | 0.14 353  | 9.90 960  | 9  | 18    |         |      |
| 43 | 9.76 624  | 17 | 9.85 673  | 26    | 0.14 326  | 9.90 951  | 9  | 17    |         |      |
| 44 | 9.76 642  | 18 | 9.85 700  | 27    | 0.14 299  | 9.90 942  | 9  | 16    |         |      |
| 45 | 9.76 660  | 17 | 9.85 727  | 26    | 0.14 273  | 9.90 933  | 9  | 15    | 9       | 8    |
| 46 | 9.76 677  | 18 | 9.85 753  | 27    | 0.14 246  | 9.90 923  | 9  | 14    | 6' 0.0  | 0.8  |
| 47 | 9.76 695  | 17 | 9.85 780  | 26    | 0.14 219  | 9.90 914  | 9  | 13    | 7 1.1   | 1.0  |
| 48 | 9.76 712  | 18 | 9.85 807  | 27    | 0.14 193  | 9.90 905  | 9  | 12    | 8 1.2   | 1.1  |
| 49 | 9.76 730  | 17 | 9.85 833  | 26    | 0.14 166  | 9.90 896  | 9  | 11    | 9 1.4   | 1.3  |
| 50 | 9.76 747  | 18 | 9.85 860  | 27    | 0.14 140  | 9.90 887  | 9  | 10    | 10 1.6  | 1.4  |
| 51 | 9.76 765  | 17 | 9.85 887  | 26    | 0.14 113  | 9.90 878  | 9  | 9     | 20 3.1  | 3.3  |
| 52 | 9.76 782  | 18 | 9.85 913  | 27    | 0.14 086  | 9.90 869  | 9  | 8     | 30 4.1  | 4.4  |
| 53 | 9.76 800  | 17 | 9.85 940  | 26    | 0.14 060  | 9.90 860  | 9  | 7     | 40 5.1  | 5.6  |
| 54 | 9.76 817  | 18 | 9.85 966  | 27    | 0.14 033  | 9.90 850  | 9  | 6     | 50 7.9  | 7.1  |
| 55 | 9.76 835  | 17 | 9.85 993  | 26    | 0.14 007  | 9.90 841  | 9  | 5     |         |      |
| 56 | 9.76 852  | 18 | 9.86 020  | 27    | 0.13 980  | 9.90 832  | 9  | 4     |         |      |
| 57 | 9.76 869  | 17 | 9.86 046  | 26    | 0.13 953  | 9.90 823  | 9  | 3     |         |      |
| 58 | 9.76 887  | 18 | 9.86 073  | 27    | 0.13 927  | 9.90 814  | 9  | 2     |         |      |
| 59 | 9.76 904  | 17 | 9.86 099  | 26    | 0.13 900  | 9.90 805  | 9  | 1     |         |      |
| 60 | 9.76 922  | 17 | 9.86 126  | 26    | 0.13 874  | 9.90 796  | 9  | 0     |         |      |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. | P. P. |         |      |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |  | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|--|-------|
| 0  | 9.76 922  | 17 | 9.86 126  | 26    | 0.13 874  | 9.90 796  | 60 |  |       |
| 1  | 9.76 939  | 17 | 9.86 152  | 26    | 0.13 847  | 9.90 786  | 59 |  |       |
| 2  | 9.76 956  | 17 | 9.86 179  | 27    | 0.13 821  | 9.90 777  | 58 |  |       |
| 3  | 9.76 974  | 17 | 9.86 206  | 26    | 0.13 794  | 9.90 768  | 57 |  |       |
| 4  | 9.76 991  | 17 | 9.86 232  | 26    | 0.13 767  | 9.90 759  | 56 |  |       |
| 5  | 9.77 008  | 17 | 9.86 259  | 26    | 0.13 741  | 9.90 750  | 55 |  |       |
| 6  | 9.77 026  | 17 | 9.86 285  | 26    | 0.13 714  | 9.90 740  | 54 |  |       |
| 7  | 9.77 043  | 17 | 9.86 312  | 26    | 0.13 688  | 9.90 731  | 53 |  |       |
| 8  | 9.77 060  | 17 | 9.86 338  | 26    | 0.13 661  | 9.90 722  | 52 |  |       |
| 9  | 9.77 078  | 17 | 9.86 365  | 26    | 0.13 635  | 9.90 713  | 51 |  |       |
| 10 | 9.77 095  | 17 | 9.86 391  | 26    | 0.13 608  | 9.90 703  | 50 |  |       |
| 11 | 9.77 112  | 17 | 9.86 418  | 26    | 0.13 582  | 9.90 694  | 49 |  |       |
| 12 | 9.77 130  | 17 | 9.86 444  | 26    | 0.13 555  | 9.90 685  | 48 |  |       |
| 13 | 9.77 147  | 17 | 9.86 471  | 26    | 0.13 529  | 9.90 676  | 47 |  |       |
| 14 | 9.77 164  | 17 | 9.86 497  | 26    | 0.13 502  | 9.90 666  | 46 |  |       |
| 15 | 9.77 181  | 17 | 9.86 524  | 26    | 0.13 476  | 9.90 657  | 45 |  |       |
| 16 | 9.77 198  | 17 | 9.86 550  | 26    | 0.13 449  | 9.90 648  | 44 |  |       |
| 17 | 9.77 216  | 17 | 9.86 577  | 26    | 0.13 423  | 9.90 639  | 43 |  |       |
| 18 | 9.77 233  | 17 | 9.86 603  | 26    | 0.13 396  | 9.90 629  | 42 |  |       |
| 19 | 9.77 250  | 17 | 9.86 630  | 26    | 0.13 370  | 9.90 620  | 41 |  |       |
| 20 | 9.77 267  | 17 | 9.86 656  | 26    | 0.13 343  | 9.90 611  | 40 |  |       |
| 21 | 9.77 284  | 17 | 9.86 683  | 26    | 0.13 317  | 9.90 602  | 39 |  |       |
| 22 | 9.77 302  | 17 | 9.86 709  | 26    | 0.13 290  | 9.90 592  | 38 |  |       |
| 23 | 9.77 319  | 17 | 9.86 736  | 26    | 0.13 264  | 9.90 583  | 37 |  |       |
| 24 | 9.77 336  | 17 | 9.86 762  | 26    | 0.13 237  | 9.90 574  | 36 |  |       |
| 25 | 9.77 353  | 17 | 9.86 788  | 26    | 0.13 211  | 9.90 564  | 35 |  |       |
| 26 | 9.77 370  | 17 | 9.86 815  | 26    | 0.13 185  | 9.90 555  | 34 |  |       |
| 27 | 9.77 387  | 17 | 9.86 841  | 26    | 0.13 158  | 9.90 546  | 33 |  |       |
| 28 | 9.77 404  | 17 | 9.86 868  | 26    | 0.13 132  | 9.90 536  | 32 |  |       |
| 29 | 9.77 421  | 17 | 9.86 894  | 26    | 0.13 105  | 9.90 527  | 31 |  |       |
| 30 | 9.77 439  | 17 | 9.86 921  | 26    | 0.13 079  | 9.90 518  | 30 |  |       |
| 31 | 9.77 456  | 17 | 9.86 947  | 26    | 0.13 052  | 9.90 508  | 29 |  |       |
| 32 | 9.77 473  | 17 | 9.86 973  | 26    | 0.13 026  | 9.90 499  | 28 |  |       |
| 33 | 9.77 490  | 17 | 9.87 000  | 26    | 0.13 000  | 9.90 490  | 27 |  |       |
| 34 | 9.77 507  | 17 | 9.87 026  | 26    | 0.12 973  | 9.90 480  | 26 |  |       |
| 35 | 9.77 524  | 17 | 9.87 053  | 26    | 0.12 947  | 9.90 471  | 25 |  |       |
| 36 | 9.77 541  | 17 | 9.87 079  | 26    | 0.12 920  | 9.90 461  | 24 |  |       |
| 37 | 9.77 558  | 17 | 9.87 105  | 26    | 0.12 894  | 9.90 452  | 23 |  |       |
| 38 | 9.77 575  | 17 | 9.87 132  | 26    | 0.12 868  | 9.90 443  | 22 |  |       |
| 39 | 9.77 592  | 17 | 9.87 158  | 26    | 0.12 841  | 9.90 433  | 21 |  |       |
| 40 | 9.77 609  | 17 | 9.87 185  | 26    | 0.12 815  | 9.90 424  | 20 |  |       |
| 41 | 9.77 626  | 17 | 9.87 211  | 26    | 0.12 789  | 9.90 414  | 19 |  |       |
| 42 | 9.77 643  | 17 | 9.87 237  | 26    | 0.12 762  | 9.90 405  | 18 |  |       |
| 43 | 9.77 660  | 17 | 9.87 264  | 26    | 0.12 736  | 9.90 396  | 17 |  |       |
| 44 | 9.77 677  | 17 | 9.87 290  | 26    | 0.12 709  | 9.90 386  | 16 |  |       |
| 45 | 9.77 693  | 16 | 9.87 316  | 26    | 0.12 683  | 9.90 377  | 15 |  |       |
| 46 | 9.77 710  | 17 | 9.87 343  | 26    | 0.12 657  | 9.90 367  | 14 |  |       |
| 47 | 9.77 727  | 17 | 9.87 369  | 26    | 0.12 630  | 9.90 358  | 13 |  |       |
| 48 | 9.77 744  | 17 | 9.87 395  | 26    | 0.12 604  | 9.90 348  | 12 |  |       |
| 49 | 9.77 761  | 16 | 9.87 422  | 26    | 0.12 578  | 9.90 339  | 11 |  |       |
| 50 | 9.77 778  | 17 | 9.87 448  | 26    | 0.12 551  | 9.90 330  | 10 |  |       |
| 51 | 9.77 795  | 17 | 9.87 474  | 26    | 0.12 525  | 9.90 320  | 9  |  |       |
| 52 | 9.77 812  | 16 | 9.87 501  | 26    | 0.12 499  | 9.90 311  | 8  |  |       |
| 53 | 9.77 828  | 17 | 9.87 527  | 26    | 0.12 473  | 9.90 301  | 7  |  |       |
| 54 | 9.77 845  | 17 | 9.87 553  | 26    | 0.12 446  | 9.90 292  | 6  |  |       |
| 55 | 9.77 862  | 16 | 9.87 580  | 26    | 0.12 420  | 9.90 282  | 5  |  |       |
| 56 | 9.77 879  | 17 | 9.87 606  | 26    | 0.12 393  | 9.90 273  | 4  |  |       |
| 57 | 9.77 896  | 17 | 9.87 632  | 26    | 0.12 367  | 9.90 263  | 3  |  |       |
| 58 | 9.77 913  | 16 | 9.87 659  | 26    | 0.12 341  | 9.90 254  | 2  |  |       |
| 59 | 9.77 930  | 17 | 9.87 685  | 26    | 0.12 315  | 9.90 244  | 1  |  |       |
| 60 | 9.77 946  | 17 | 9.87 711  | 26    | 0.12 288  | 9.90 235  | 0  |  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |  | P. P. |

|    |      |      |      |
|----|------|------|------|
|    | 27   | 26   | 26   |
| 6  | 2.7  | 2.6  | 2.6  |
| 7  | 3.1  | 3.1  | 3.0  |
| 8  | 3.6  | 3.5  | 3.4  |
| 9  | 4.0  | 4.0  | 3.9  |
| 10 | 4.5  | 4.4  | 4.3  |
| 20 | 9.0  | 8.8  | 8.6  |
| 30 | 13.5 | 13.2 | 13.0 |
| 40 | 18.0 | 17.6 | 17.3 |
| 50 | 22.5 | 22.1 | 21.6 |

|    |      |      |      |
|----|------|------|------|
|    | 17   | 17   | 16   |
| 6  | 1.7  | 1.7  | 1.6  |
| 7  | 2.0  | 2.0  | 1.9  |
| 8  | 2.3  | 2.2  | 2.2  |
| 9  | 2.6  | 2.5  | 2.5  |
| 10 | 2.9  | 2.8  | 2.7  |
| 20 | 5.8  | 5.6  | 5.5  |
| 30 | 8.7  | 8.5  | 8.2  |
| 40 | 11.6 | 11.3 | 11.0 |
| 50 | 14.6 | 14.1 | 13.7 |

|    |     |     |
|----|-----|-----|
|    | 9   | 9   |
| 6  | 0.9 | 0.9 |
| 7  | 1.1 | 1.0 |
| 8  | 1.2 | 1.2 |
| 9  | 1.4 | 1.3 |
| 10 | 1.6 | 1.5 |
| 20 | 3.1 | 3.0 |
| 30 | 4.7 | 4.5 |
| 40 | 6.3 | 6.0 |
| 50 | 7.9 | 7.5 |

# TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.77 946  | 16 | 9.87 711  | 26    | 0.12 288  | 9.90 235  | 10 | 60 |       |
| 1  | 9.77 963  | 17 | 9.87 737  | 26    | 0.12 262  | 9.90 225  | 10 | 59 |       |
| 2  | 9.77 980  | 16 | 9.87 764  | 26    | 0.12 236  | 9.90 216  | 10 | 58 |       |
| 3  | 9.77 996  | 17 | 9.87 790  | 26    | 0.12 209  | 9.90 206  | 10 | 57 |       |
| 4  | 9.78 013  | 16 | 9.87 816  | 26    | 0.12 183  | 9.90 196  | 10 | 56 |       |
| 5  | 9.78 030  | 16 | 9.87 843  | 26    | 0.12 157  | 9.90 187  | 10 | 55 |       |
| 6  | 9.78 046  | 17 | 9.87 869  | 26    | 0.12 131  | 9.90 177  | 10 | 54 |       |
| 7  | 9.78 063  | 16 | 9.87 895  | 26    | 0.12 104  | 9.90 168  | 10 | 53 |       |
| 8  | 9.78 080  | 17 | 9.87 921  | 26    | 0.12 078  | 9.90 158  | 10 | 52 |       |
| 9  | 9.78 097  | 16 | 9.87 948  | 26    | 0.12 052  | 9.90 149  | 10 | 51 |       |
| 10 | 9.78 113  | 16 | 9.87 974  | 26    | 0.12 026  | 9.90 139  | 10 | 50 |       |
| 11 | 9.78 130  | 17 | 9.88 000  | 26    | 0.11 999  | 9.90 130  | 10 | 49 |       |
| 12 | 9.78 147  | 16 | 9.88 026  | 26    | 0.11 973  | 9.90 120  | 10 | 48 |       |
| 13 | 9.78 163  | 16 | 9.88 053  | 26    | 0.11 947  | 9.90 110  | 10 | 47 |       |
| 14 | 9.78 180  | 16 | 9.88 079  | 26    | 0.11 921  | 9.90 101  | 10 | 46 |       |
| 15 | 9.78 196  | 16 | 9.88 105  | 26    | 0.11 895  | 9.90 091  | 10 | 45 |       |
| 16 | 9.78 213  | 17 | 9.88 131  | 26    | 0.11 868  | 9.90 082  | 10 | 44 |       |
| 17 | 9.78 230  | 16 | 9.88 157  | 26    | 0.11 842  | 9.90 072  | 10 | 43 |       |
| 18 | 9.78 246  | 16 | 9.88 184  | 26    | 0.11 816  | 9.90 062  | 10 | 42 |       |
| 19 | 9.78 263  | 16 | 9.88 210  | 26    | 0.11 790  | 9.90 053  | 10 | 41 |       |
| 20 | 9.78 279  | 16 | 9.88 236  | 26    | 0.11 763  | 9.90 043  | 10 | 40 |       |
| 21 | 9.78 296  | 16 | 9.88 262  | 26    | 0.11 737  | 9.90 033  | 10 | 39 |       |
| 22 | 9.78 312  | 16 | 9.88 288  | 26    | 0.11 711  | 9.90 024  | 10 | 38 |       |
| 23 | 9.78 329  | 17 | 9.88 315  | 26    | 0.11 685  | 9.90 014  | 10 | 37 |       |
| 24 | 9.78 346  | 16 | 9.88 341  | 26    | 0.11 659  | 9.90 004  | 10 | 36 |       |
| 25 | 9.78 362  | 16 | 9.88 367  | 26    | 0.11 633  | 9.89 995  | 10 | 35 |       |
| 26 | 9.78 379  | 16 | 9.88 393  | 26    | 0.11 606  | 9.89 985  | 10 | 34 |       |
| 27 | 9.78 395  | 16 | 9.88 419  | 26    | 0.11 580  | 9.89 975  | 10 | 33 |       |
| 28 | 9.78 412  | 16 | 9.88 445  | 26    | 0.11 554  | 9.89 966  | 10 | 32 |       |
| 29 | 9.78 428  | 16 | 9.88 472  | 26    | 0.11 528  | 9.89 956  | 10 | 31 |       |
| 30 | 9.78 444  | 16 | 9.88 498  | 26    | 0.11 502  | 9.89 946  | 10 | 30 |       |
| 31 | 9.78 461  | 16 | 9.88 524  | 26    | 0.11 476  | 9.89 937  | 10 | 29 |       |
| 32 | 9.78 477  | 16 | 9.88 550  | 26    | 0.11 449  | 9.89 927  | 10 | 28 |       |
| 33 | 9.78 494  | 16 | 9.88 576  | 26    | 0.11 423  | 9.89 917  | 10 | 27 |       |
| 34 | 9.78 510  | 16 | 9.88 602  | 26    | 0.11 397  | 9.89 908  | 10 | 26 |       |
| 35 | 9.78 527  | 16 | 9.88 629  | 26    | 0.11 371  | 9.89 898  | 10 | 25 |       |
| 36 | 9.78 543  | 16 | 9.88 655  | 26    | 0.11 345  | 9.89 888  | 10 | 24 |       |
| 37 | 9.78 559  | 16 | 9.88 681  | 26    | 0.11 319  | 9.89 878  | 10 | 23 |       |
| 38 | 9.78 576  | 16 | 9.88 707  | 26    | 0.11 293  | 9.89 869  | 10 | 22 |       |
| 39 | 9.78 592  | 16 | 9.88 733  | 26    | 0.11 266  | 9.89 859  | 10 | 21 |       |
| 40 | 9.78 609  | 16 | 9.88 759  | 26    | 0.11 240  | 9.89 849  | 10 | 20 |       |
| 41 | 9.78 625  | 16 | 9.88 785  | 26    | 0.11 214  | 9.89 839  | 10 | 19 |       |
| 42 | 9.78 641  | 16 | 9.88 811  | 26    | 0.11 188  | 9.89 830  | 10 | 18 |       |
| 43 | 9.78 658  | 16 | 9.88 838  | 26    | 0.11 162  | 9.89 820  | 10 | 17 |       |
| 44 | 9.78 674  | 16 | 9.88 864  | 26    | 0.11 136  | 9.89 810  | 10 | 16 |       |
| 45 | 9.78 690  | 16 | 9.88 890  | 26    | 0.11 110  | 9.89 800  | 10 | 15 |       |
| 46 | 9.78 707  | 16 | 9.88 916  | 26    | 0.11 084  | 9.89 791  | 10 | 14 |       |
| 47 | 9.78 723  | 16 | 9.88 942  | 26    | 0.11 058  | 9.89 781  | 10 | 13 |       |
| 48 | 9.78 739  | 16 | 9.88 968  | 26    | 0.11 032  | 9.89 771  | 10 | 12 |       |
| 49 | 9.78 755  | 16 | 9.88 994  | 26    | 0.11 005  | 9.89 761  | 10 | 11 |       |
| 50 | 9.78 772  | 16 | 9.89 020  | 26    | 0.10 979  | 9.89 751  | 10 | 10 |       |
| 51 | 9.78 788  | 16 | 9.89 046  | 26    | 0.10 953  | 9.89 742  | 10 | 9  |       |
| 52 | 9.78 804  | 16 | 9.89 072  | 26    | 0.10 927  | 9.89 732  | 10 | 8  |       |
| 53 | 9.78 821  | 16 | 9.89 098  | 26    | 0.10 901  | 9.89 722  | 10 | 7  |       |
| 54 | 9.78 837  | 16 | 9.89 124  | 26    | 0.10 875  | 9.89 712  | 10 | 6  |       |
| 55 | 9.78 853  | 16 | 9.89 150  | 26    | 0.10 849  | 9.89 702  | 10 | 5  |       |
| 56 | 9.78 869  | 16 | 9.89 177  | 26    | 0.10 823  | 9.89 692  | 10 | 4  |       |
| 57 | 9.78 885  | 16 | 9.89 203  | 26    | 0.10 797  | 9.89 683  | 10 | 3  |       |
| 58 | 9.78 902  | 16 | 9.89 229  | 26    | 0.10 771  | 9.89 673  | 10 | 2  |       |
| 59 | 9.78 918  | 16 | 9.89 255  | 26    | 0.10 745  | 9.89 663  | 10 | 1  |       |
| 60 | 9.78 934  | 16 | 9.89 281  | 26    | 0.10 719  | 9.89 653  | 10 | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

26 26  
6 2.6 2.6  
7 3.1 3.0  
8 3.5 3.4  
9 4.0 3.9  
10 4.4 4.3  
20 8.3 8.6  
30 13.2 13.0  
40 17.6 17.3  
50 22.1 21.6

17 16 16  
6 1.7 1.6 1.6  
7 2.0 1.9 1.8  
8 2.2 2.2 2.1  
9 2.5 2.5 2.4  
10 2.8 2.7 2.6  
20 5.6 5.5 5.3  
30 8.5 8.2 8.0  
40 11.3 11.0 10.6  
50 14.1 13.7 13.3

10 9  
6 1.0 0.9  
7 1.1 1.1  
8 1.3 1.2  
9 1.5 1.4  
10 1.6 1.6  
20 3.3 3.1  
30 5.0 4.7  
40 6.6 6.3  
50 8.3 7.9

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.        |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|--------------|
| 0  | 9.78 934  | 16 | 9.89 281  | 26    | 0.10 719  | 9.89 653  | 9  | 60 |              |
| 1  | 9.78 950  | 16 | 9.89 307  | 26    | 0.10 693  | 9.89 643  | 10 | 59 |              |
| 2  | 9.78 966  | 16 | 9.89 333  | 26    | 0.10 667  | 9.89 633  | 10 | 58 |              |
| 3  | 9.78 982  | 16 | 9.89 359  | 26    | 0.10 641  | 9.89 623  | 10 | 57 |              |
| 4  | 9.78 999  | 16 | 9.89 385  | 26    | 0.10 615  | 9.89 613  | 10 | 56 |              |
| 5  | 9.79 015  | 16 | 9.89 411  | 26    | 0.10 589  | 9.89 604  | 9  | 55 |              |
| 6  | 9.79 031  | 16 | 9.89 437  | 26    | 0.10 563  | 9.89 594  | 10 | 54 |              |
| 7  | 9.79 047  | 16 | 9.89 463  | 26    | 0.10 537  | 9.89 584  | 10 | 53 |              |
| 8  | 9.79 063  | 16 | 9.89 489  | 26    | 0.10 511  | 9.89 574  | 10 | 52 |              |
| 9  | 9.79 079  | 16 | 9.89 515  | 26    | 0.10 485  | 9.89 564  | 10 | 51 | 26 25        |
| 10 | 9.79 095  | 16 | 9.89 541  | 26    | 0.10 459  | 9.89 554  | 10 | 50 | 6 2.6 2.5    |
| 11 | 9.79 111  | 16 | 9.89 567  | 26    | 0.10 433  | 9.89 544  | 9  | 49 | 7 3.0 3.0    |
| 12 | 9.79 127  | 16 | 9.89 593  | 26    | 0.10 407  | 9.89 534  | 10 | 48 | 8 3.4 3.4    |
| 13 | 9.79 143  | 16 | 9.89 619  | 26    | 0.10 381  | 9.89 524  | 10 | 47 | 9 3.9 3.8    |
| 14 | 9.79 159  | 16 | 9.89 645  | 26    | 0.10 355  | 9.89 514  | 10 | 46 | 10 4.3 4.2   |
| 15 | 9.79 175  | 16 | 9.89 671  | 26    | 0.10 329  | 9.89 504  | 10 | 45 | 20 8.6 8.5   |
| 16 | 9.79 191  | 16 | 9.89 697  | 26    | 0.10 303  | 9.89 494  | 10 | 44 | 30 13.0 12.7 |
| 17 | 9.79 207  | 16 | 9.89 723  | 26    | 0.10 277  | 9.89 484  | 10 | 43 | 40 17.3 17.0 |
| 18 | 9.79 223  | 16 | 9.89 749  | 26    | 0.10 251  | 9.89 474  | 10 | 42 | 50 21.6 21.2 |
| 19 | 9.79 239  | 16 | 9.89 775  | 26    | 0.10 225  | 9.89 464  | 10 | 41 |              |
| 20 | 9.79 255  | 16 | 9.89 801  | 26    | 0.10 199  | 9.89 454  | 10 | 40 |              |
| 21 | 9.79 271  | 16 | 9.89 827  | 26    | 0.10 173  | 9.89 444  | 10 | 39 |              |
| 22 | 9.79 287  | 16 | 9.89 853  | 26    | 0.10 147  | 9.89 434  | 10 | 38 |              |
| 23 | 9.79 303  | 16 | 9.89 879  | 26    | 0.10 121  | 9.89 424  | 10 | 37 |              |
| 24 | 9.79 319  | 16 | 9.89 905  | 26    | 0.10 095  | 9.89 414  | 10 | 36 |              |
| 25 | 9.79 335  | 16 | 9.89 931  | 26    | 0.10 069  | 9.89 404  | 10 | 35 |              |
| 26 | 9.79 351  | 16 | 9.89 957  | 26    | 0.10 043  | 9.89 394  | 10 | 34 |              |
| 27 | 9.79 367  | 16 | 9.89 982  | 25    | 0.10 017  | 9.89 384  | 10 | 33 | 16 16 15     |
| 28 | 9.79 383  | 15 | 9.90 008  | 26    | 0.09 991  | 9.89 374  | 10 | 32 | 6 1.6 1.6    |
| 29 | 9.79 399  | 16 | 9.90 034  | 26    | 0.09 965  | 9.89 364  | 10 | 31 | 7 1.9 1.8    |
| 30 | 9.79 415  | 16 | 9.90 060  | 26    | 0.09 939  | 9.89 354  | 10 | 30 | 8 2.2 2.1    |
| 31 | 9.79 431  | 15 | 9.90 086  | 26    | 0.09 913  | 9.89 344  | 10 | 29 | 9 2.5 2.4    |
| 32 | 9.79 446  | 16 | 9.90 112  | 26    | 0.09 887  | 9.89 334  | 10 | 28 | 10 2.7 2.6   |
| 33 | 9.79 462  | 16 | 9.90 138  | 26    | 0.09 861  | 9.89 324  | 10 | 27 | 20 5.5 5.3   |
| 34 | 9.79 478  | 16 | 9.90 164  | 25    | 0.09 836  | 9.89 314  | 10 | 26 | 30 8.2 8.0   |
| 35 | 9.79 494  | 15 | 9.90 190  | 26    | 0.09 810  | 9.89 304  | 10 | 25 | 40 11.0 10.6 |
| 36 | 9.79 510  | 16 | 9.90 216  | 26    | 0.09 784  | 9.89 294  | 10 | 24 | 50 13.7 13.3 |
| 37 | 9.79 526  | 16 | 9.90 242  | 26    | 0.09 758  | 9.89 284  | 10 | 23 |              |
| 38 | 9.79 541  | 16 | 9.90 268  | 26    | 0.09 732  | 9.89 274  | 10 | 22 |              |
| 39 | 9.79 557  | 16 | 9.90 294  | 26    | 0.09 706  | 9.89 264  | 10 | 21 |              |
| 40 | 9.79 573  | 15 | 9.90 319  | 25    | 0.09 680  | 9.89 253  | 10 | 20 |              |
| 41 | 9.79 589  | 16 | 9.90 345  | 26    | 0.09 654  | 9.89 243  | 10 | 19 |              |
| 42 | 9.79 605  | 15 | 9.90 371  | 26    | 0.09 628  | 9.89 233  | 10 | 18 |              |
| 43 | 9.79 620  | 16 | 9.90 397  | 26    | 0.09 602  | 9.89 223  | 10 | 17 |              |
| 44 | 9.79 636  | 16 | 9.90 423  | 25    | 0.09 577  | 9.89 213  | 10 | 16 |              |
| 45 | 9.79 652  | 15 | 9.90 449  | 26    | 0.09 551  | 9.89 203  | 10 | 15 | 10 10 9      |
| 46 | 9.79 668  | 16 | 9.90 475  | 26    | 0.09 525  | 9.89 193  | 10 | 14 | 6 1.0 1.0    |
| 47 | 9.79 683  | 16 | 9.90 501  | 26    | 0.09 499  | 9.89 182  | 10 | 13 | 7 1.2 1.1    |
| 48 | 9.79 699  | 15 | 9.90 526  | 25    | 0.09 473  | 9.89 172  | 10 | 12 | 8 1.4 1.3    |
| 49 | 9.79 715  | 15 | 9.90 552  | 26    | 0.09 447  | 9.89 162  | 10 | 11 | 9 1.6 1.5    |
| 50 | 9.79 730  | 15 | 9.90 578  | 26    | 0.09 421  | 9.89 152  | 10 | 10 | 10 1.7 1.6   |
| 51 | 9.79 746  | 16 | 9.90 604  | 25    | 0.09 395  | 9.89 142  | 10 | 9  | 20 3.5 3.3   |
| 52 | 9.79 762  | 15 | 9.90 630  | 26    | 0.09 370  | 9.89 132  | 10 | 8  | 30 5.2 5.0   |
| 53 | 9.79 777  | 15 | 9.90 656  | 26    | 0.09 344  | 9.89 121  | 10 | 7  | 40 7.0 6.6   |
| 54 | 9.79 793  | 16 | 9.90 682  | 26    | 0.09 318  | 9.89 111  | 10 | 6  | 50 8.7 8.3   |
| 55 | 9.79 809  | 15 | 9.90 707  | 25    | 0.09 292  | 9.89 101  | 10 | 5  |              |
| 56 | 9.79 824  | 15 | 9.90 733  | 26    | 0.09 266  | 9.89 091  | 10 | 4  |              |
| 57 | 9.79 840  | 16 | 9.90 759  | 26    | 0.09 240  | 9.89 081  | 10 | 3  |              |
| 58 | 9.79 856  | 15 | 9.90 785  | 26    | 0.09 214  | 9.89 070  | 10 | 2  |              |
| 59 | 9.79 871  | 15 | 9.90 811  | 25    | 0.09 189  | 9.89 060  | 10 | 1  |              |
| 60 | 9.79 887  | 15 | 9.90 837  | 26    | 0.09 163  | 9.89 050  | 10 | 0  |              |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.        |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P.        |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|--------------|
| 0  | 9.79 887  | 16 | 9.90 837  | 26    | 0.09 163  | 9.89 050  | 10 | 60 |              |
| 1  | 9.79 903  | 15 | 9.90 863  | 25    | 0.09 137  | 9.89 040  | 10 | 59 |              |
| 2  | 9.79 918  | 15 | 9.90 888  | 26    | 0.09 111  | 9.89 030  | 10 | 58 |              |
| 3  | 9.79 934  | 15 | 9.90 914  | 25    | 0.09 085  | 9.89 019  | 10 | 57 |              |
| 4  | 9.79 949  | 15 | 9.90 940  | 25    | 0.09 060  | 9.89 009  | 10 | 56 |              |
| 5  | 9.79 965  | 15 | 9.90 966  | 26    | 0.09 034  | 9.88 999  | 10 | 55 |              |
| 6  | 9.79 980  | 15 | 9.90 992  | 25    | 0.09 008  | 9.88 989  | 10 | 54 |              |
| 7  | 9.79 996  | 15 | 9.91 017  | 26    | 0.08 982  | 9.88 978  | 10 | 53 |              |
| 8  | 9.80 011  | 15 | 9.91 043  | 26    | 0.08 956  | 9.88 968  | 10 | 52 |              |
| 9  | 9.80 027  | 15 | 9.91 069  | 26    | 0.08 930  | 9.88 958  | 10 | 51 |              |
| 10 | 9.80 042  | 15 | 9.91 095  | 25    | 0.08 905  | 9.88 947  | 10 | 50 | 26 25        |
| 11 | 9.80 058  | 15 | 9.91 121  | 26    | 0.08 879  | 9.88 937  | 10 | 49 | 6 2.6 2.5    |
| 12 | 9.80 073  | 15 | 9.91 146  | 25    | 0.08 853  | 9.88 927  | 10 | 48 | 7 3.0 3.0    |
| 13 | 9.80 089  | 15 | 9.91 172  | 26    | 0.08 827  | 9.88 917  | 10 | 47 | 8 3.4 3.4    |
| 14 | 9.80 104  | 15 | 9.91 198  | 25    | 0.08 802  | 9.88 906  | 10 | 46 | 9 3.9 3.8    |
| 15 | 9.80 120  | 15 | 9.91 224  | 26    | 0.08 776  | 9.88 896  | 10 | 45 | 10 4.3 4.2   |
| 16 | 9.80 135  | 15 | 9.91 250  | 25    | 0.08 750  | 9.88 886  | 10 | 44 | 20 8.6 8.5   |
| 17 | 9.80 151  | 15 | 9.91 275  | 26    | 0.08 724  | 9.88 875  | 10 | 43 | 30 13.0 12.7 |
| 18 | 9.80 166  | 15 | 9.91 301  | 25    | 0.08 698  | 9.88 865  | 10 | 42 | 40 17.3 17.0 |
| 19 | 9.80 182  | 15 | 9.91 327  | 25    | 0.08 673  | 9.88 855  | 10 | 41 | 50 21.6 21.2 |
| 20 | 9.80 197  | 15 | 9.91 353  | 26    | 0.08 647  | 9.88 844  | 10 | 40 |              |
| 21 | 9.80 213  | 15 | 9.91 378  | 25    | 0.08 621  | 9.88 834  | 10 | 39 |              |
| 22 | 9.80 228  | 15 | 9.91 404  | 26    | 0.08 595  | 9.88 823  | 10 | 38 |              |
| 23 | 9.80 243  | 15 | 9.91 430  | 25    | 0.08 570  | 9.88 813  | 10 | 37 |              |
| 24 | 9.80 259  | 15 | 9.91 456  | 26    | 0.08 544  | 9.88 803  | 10 | 36 |              |
| 25 | 9.80 274  | 15 | 9.91 481  | 25    | 0.08 518  | 9.88 792  | 10 | 35 |              |
| 26 | 9.80 289  | 15 | 9.91 507  | 26    | 0.08 492  | 9.88 782  | 10 | 34 | 16 15 15     |
| 27 | 9.80 305  | 15 | 9.91 533  | 25    | 0.08 467  | 9.88 772  | 10 | 33 | 6 1.6 1.5    |
| 28 | 9.80 320  | 15 | 9.91 559  | 26    | 0.08 441  | 9.88 761  | 10 | 32 | 7 1.8 1.7    |
| 29 | 9.80 335  | 15 | 9.91 584  | 25    | 0.08 415  | 9.88 751  | 10 | 31 | 8 2.1 2.0    |
| 30 | 9.80 351  | 15 | 9.91 610  | 26    | 0.08 389  | 9.88 740  | 10 | 30 | 9 2.4 2.3    |
| 31 | 9.80 366  | 15 | 9.91 636  | 25    | 0.08 364  | 9.88 730  | 10 | 29 | 10 2.6 2.5   |
| 32 | 9.80 381  | 15 | 9.91 662  | 26    | 0.08 338  | 9.88 720  | 10 | 28 | 20 5.3 5.1   |
| 33 | 9.80 397  | 15 | 9.91 687  | 25    | 0.08 312  | 9.88 709  | 10 | 27 | 30 8.0 7.7   |
| 34 | 9.80 412  | 15 | 9.91 713  | 26    | 0.08 286  | 9.88 699  | 10 | 26 | 40 10.6 10.3 |
| 35 | 9.80 427  | 15 | 9.91 739  | 25    | 0.08 261  | 9.88 688  | 10 | 25 | 50 13.3 12.9 |
| 36 | 9.80 443  | 15 | 9.91 765  | 26    | 0.08 235  | 9.88 678  | 10 | 24 |              |
| 37 | 9.80 458  | 15 | 9.91 790  | 25    | 0.08 209  | 9.88 667  | 10 | 23 |              |
| 38 | 9.80 473  | 15 | 9.91 816  | 26    | 0.08 183  | 9.88 657  | 10 | 22 |              |
| 39 | 9.80 488  | 15 | 9.91 842  | 25    | 0.08 158  | 9.88 646  | 10 | 21 |              |
| 40 | 9.80 504  | 15 | 9.91 867  | 26    | 0.08 132  | 9.88 636  | 10 | 20 |              |
| 41 | 9.80 519  | 15 | 9.91 893  | 25    | 0.08 106  | 9.88 625  | 10 | 19 |              |
| 42 | 9.80 534  | 15 | 9.91 919  | 26    | 0.08 081  | 9.88 615  | 10 | 18 |              |
| 43 | 9.80 549  | 15 | 9.91 945  | 25    | 0.08 055  | 9.88 604  | 10 | 17 |              |
| 44 | 9.80 564  | 15 | 9.91 970  | 26    | 0.08 029  | 9.88 594  | 10 | 16 | 11 10 10     |
| 45 | 9.80 580  | 15 | 9.91 996  | 25    | 0.08 004  | 9.88 583  | 10 | 15 | 6 1.1 1.0    |
| 46 | 9.80 595  | 15 | 9.92 022  | 26    | 0.07 978  | 9.88 573  | 10 | 14 | 7 1.3 1.2    |
| 47 | 9.80 610  | 15 | 9.92 047  | 25    | 0.07 952  | 9.88 562  | 10 | 13 | 8 1.4 1.4    |
| 48 | 9.80 625  | 15 | 9.92 073  | 26    | 0.07 926  | 9.88 552  | 10 | 12 | 9 1.6 1.6    |
| 49 | 9.80 640  | 15 | 9.92 099  | 25    | 0.07 901  | 9.88 541  | 10 | 11 | 10 1.8 1.7   |
| 50 | 9.80 655  | 15 | 9.92 124  | 26    | 0.07 875  | 9.88 531  | 10 | 10 | 20 3.6 3.5   |
| 51 | 9.80 671  | 15 | 9.92 150  | 25    | 0.07 849  | 9.88 520  | 10 | 9  | 30 5.5 5.2   |
| 52 | 9.80 686  | 15 | 9.92 176  | 26    | 0.07 824  | 9.88 510  | 10 | 8  | 40 7.3 7.0   |
| 53 | 9.80 701  | 15 | 9.92 201  | 25    | 0.07 798  | 9.88 499  | 10 | 7  | 50 9.1 8.7   |
| 54 | 9.80 716  | 15 | 9.92 227  | 26    | 0.07 772  | 9.88 489  | 10 | 6  |              |
| 55 | 9.80 731  | 15 | 9.92 253  | 25    | 0.07 747  | 9.88 478  | 10 | 5  |              |
| 56 | 9.80 746  | 15 | 9.92 278  | 26    | 0.07 721  | 9.88 467  | 10 | 4  |              |
| 57 | 9.80 761  | 15 | 9.92 304  | 25    | 0.07 695  | 9.88 457  | 10 | 3  |              |
| 58 | 9.80 776  | 15 | 9.92 330  | 26    | 0.07 670  | 9.88 446  | 10 | 2  |              |
| 59 | 9.80 791  | 15 | 9.92 355  | 25    | 0.07 644  | 9.88 436  | 10 | 1  |              |
| 60 | 9.80 806  | 15 | 9.92 381  | 26    | 0.07 618  | 9.88 425  | 10 | 0  |              |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P.        |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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| '  | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.80 806  | 15 | 9.92 381  | 25    | 0.07 618  | 9.88 425  | 10 | 60 |       |
| 1  | 9.80 822  | 15 | 9.92 407  | 25    | 0.07 593  | 9.88 415  | 11 | 59 |       |
| 2  | 9.80 837  | 15 | 9.92 432  | 26    | 0.07 567  | 9.88 404  | 10 | 58 |       |
| 3  | 9.80 852  | 15 | 9.92 458  | 25    | 0.07 541  | 9.88 393  | 10 | 57 |       |
| 4  | 9.80 867  | 15 | 9.92 484  | 25    | 0.07 516  | 9.88 383  | 10 | 56 |       |
| 5  | 9.80 882  | 15 | 9.92 509  | 25    | 0.07 490  | 9.88 372  | 11 | 55 |       |
| 6  | 9.80 897  | 15 | 9.92 535  | 26    | 0.07 465  | 9.88 361  | 10 | 54 |       |
| 7  | 9.80 912  | 15 | 9.92 561  | 25    | 0.07 439  | 9.88 351  | 10 | 53 |       |
| 8  | 9.80 927  | 15 | 9.92 586  | 25    | 0.07 413  | 9.88 340  | 11 | 52 |       |
| 9  | 9.80 942  | 15 | 9.92 612  | 25    | 0.07 388  | 9.88 329  | 10 | 51 |       |
| 10 | 9.80 957  | 15 | 9.92 638  | 26    | 0.07 362  | 9.88 319  | 10 | 50 |       |
| 11 | 9.80 972  | 15 | 9.92 663  | 25    | 0.07 336  | 9.88 308  | 11 | 49 |       |
| 12 | 9.80 987  | 15 | 9.92 689  | 25    | 0.07 311  | 9.88 297  | 11 | 48 |       |
| 13 | 9.81 001  | 14 | 9.92 714  | 26    | 0.07 285  | 9.88 287  | 10 | 47 |       |
| 14 | 9.81 016  | 15 | 9.92 740  | 26    | 0.07 259  | 9.88 276  | 10 | 46 |       |
| 15 | 9.81 031  | 15 | 9.92 766  | 25    | 0.07 234  | 9.88 265  | 11 | 45 |       |
| 16 | 9.81 046  | 15 | 9.92 791  | 25    | 0.07 208  | 9.88 255  | 10 | 44 |       |
| 17 | 9.81 061  | 15 | 9.92 817  | 25    | 0.07 183  | 9.88 244  | 11 | 43 |       |
| 18 | 9.81 076  | 15 | 9.92 842  | 25    | 0.07 157  | 9.88 233  | 11 | 42 |       |
| 19 | 9.81 091  | 14 | 9.92 868  | 26    | 0.07 131  | 9.88 223  | 10 | 41 |       |
| 20 | 9.81 106  | 15 | 9.92 894  | 25    | 0.07 106  | 9.88 212  | 11 | 40 |       |
| 21 | 9.81 121  | 15 | 9.92 919  | 25    | 0.07 080  | 9.88 201  | 10 | 39 |       |
| 22 | 9.81 136  | 15 | 9.92 945  | 25    | 0.07 055  | 9.88 190  | 11 | 38 |       |
| 23 | 9.81 150  | 14 | 9.92 971  | 26    | 0.07 029  | 9.88 180  | 10 | 37 |       |
| 24 | 9.81 165  | 15 | 9.92 996  | 25    | 0.07 003  | 9.88 169  | 11 | 36 |       |
| 25 | 9.81 180  | 15 | 9.93 022  | 25    | 0.06 979  | 9.88 158  | 10 | 35 |       |
| 26 | 9.81 195  | 14 | 9.93 047  | 26    | 0.06 952  | 9.88 147  | 11 | 34 |       |
| 27 | 9.81 210  | 15 | 9.93 073  | 25    | 0.06 927  | 9.88 137  | 10 | 33 |       |
| 28 | 9.81 225  | 15 | 9.93 098  | 25    | 0.06 901  | 9.88 126  | 11 | 32 |       |
| 29 | 9.81 239  | 14 | 9.93 124  | 26    | 0.06 875  | 9.88 115  | 10 | 31 |       |
| 30 | 9.81 254  | 15 | 9.93 150  | 25    | 0.06 850  | 9.88 104  | 11 | 30 |       |
| 31 | 9.81 269  | 14 | 9.93 175  | 26    | 0.06 824  | 9.88 094  | 10 | 29 |       |
| 32 | 9.81 284  | 15 | 9.93 201  | 25    | 0.06 799  | 9.88 083  | 11 | 28 |       |
| 33 | 9.81 299  | 15 | 9.93 226  | 25    | 0.06 773  | 9.88 072  | 10 | 27 |       |
| 34 | 9.81 313  | 14 | 9.93 252  | 26    | 0.06 748  | 9.88 061  | 11 | 26 |       |
| 35 | 9.81 328  | 15 | 9.93 278  | 25    | 0.06 722  | 9.88 050  | 11 | 25 |       |
| 36 | 9.81 343  | 14 | 9.93 303  | 26    | 0.06 696  | 9.88 039  | 10 | 24 |       |
| 37 | 9.81 358  | 15 | 9.93 329  | 25    | 0.06 671  | 9.88 029  | 11 | 23 |       |
| 38 | 9.81 372  | 14 | 9.93 354  | 26    | 0.06 645  | 9.88 018  | 10 | 22 |       |
| 39 | 9.81 387  | 15 | 9.93 380  | 25    | 0.06 620  | 9.88 007  | 11 | 21 |       |
| 40 | 9.81 402  | 15 | 9.93 405  | 25    | 0.06 594  | 9.87 996  | 10 | 20 |       |
| 41 | 9.81 416  | 14 | 9.93 431  | 26    | 0.06 569  | 9.87 985  | 11 | 19 |       |
| 42 | 9.81 431  | 15 | 9.93 456  | 25    | 0.06 543  | 9.87 974  | 10 | 18 |       |
| 43 | 9.81 446  | 14 | 9.93 482  | 26    | 0.06 518  | 9.87 963  | 11 | 17 |       |
| 44 | 9.81 460  | 15 | 9.93 508  | 25    | 0.06 492  | 9.87 953  | 10 | 16 |       |
| 45 | 9.81 475  | 14 | 9.93 533  | 26    | 0.06 466  | 9.87 942  | 11 | 15 |       |
| 46 | 9.81 490  | 15 | 9.93 559  | 25    | 0.06 441  | 9.87 931  | 10 | 14 |       |
| 47 | 9.81 504  | 14 | 9.93 584  | 26    | 0.06 415  | 9.87 920  | 11 | 13 |       |
| 48 | 9.81 519  | 15 | 9.93 610  | 25    | 0.06 390  | 9.87 909  | 10 | 12 |       |
| 49 | 9.81 534  | 14 | 9.93 635  | 26    | 0.06 364  | 9.87 898  | 11 | 11 |       |
| 50 | 9.81 548  | 15 | 9.93 661  | 25    | 0.06 339  | 9.87 887  | 10 | 10 |       |
| 51 | 9.81 563  | 14 | 9.93 686  | 26    | 0.06 313  | 9.87 876  | 11 | 9  |       |
| 52 | 9.81 578  | 15 | 9.93 712  | 25    | 0.06 288  | 9.87 865  | 10 | 8  |       |
| 53 | 9.81 592  | 14 | 9.93 737  | 26    | 0.06 262  | 9.87 854  | 11 | 7  |       |
| 54 | 9.81 607  | 15 | 9.93 763  | 25    | 0.06 237  | 9.87 844  | 10 | 6  |       |
| 55 | 9.81 621  | 14 | 9.93 788  | 26    | 0.06 211  | 9.87 833  | 11 | 5  |       |
| 56 | 9.81 636  | 15 | 9.93 814  | 25    | 0.06 186  | 9.87 822  | 10 | 4  |       |
| 57 | 9.81 650  | 14 | 9.93 840  | 26    | 0.06 160  | 9.87 811  | 11 | 3  |       |
| 58 | 9.81 665  | 15 | 9.93 865  | 25    | 0.06 134  | 9.87 800  | 10 | 2  |       |
| 59 | 9.81 680  | 14 | 9.93 891  | 26    | 0.06 109  | 9.87 789  | 11 | 1  |       |
| 60 | 9.81 694  | 15 | 9.93 916  | 25    | 0.06 083  | 9.87 778  | 10 | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

26 25  
6 2.6 2.5  
7 3.0 3.0  
8 3.4 3.4  
9 3.9 3.8  
10 4.3 4.2  
20 8.6 8.5  
30 13.0 12.7  
40 17.3 17.0  
50 21.6 21.2

15 15 14  
6 1.5 1.5 1.4  
7 1.8 1.7 1.6  
8 2.0 2.0 1.8  
9 2.3 2.2 2.0  
10 2.6 2.5 2.2  
20 5.1 5.0 4.4  
30 7.7 7.5 7.0  
40 10.3 10.0 9.6  
50 12.9 12.5 12.1

11 10  
6 1.1 1.0  
7 1.3 1.2  
8 1.4 1.4  
9 1.6 1.6  
10 1.8 1.7  
20 3.6 3.5  
30 5.5 5.2  
40 7.3 7.0  
50 9.1 8.7

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.81694   | 14 | 9.93916   | 25    | 0.06083   | 9.87778   | 11 | 60 |       |
| 1  | 9.81709   | 14 | 9.93942   | 25    | 0.06058   | 9.87767   | 11 | 59 |       |
| 2  | 9.81723   | 14 | 9.93967   | 25    | 0.06032   | 9.87756   | 11 | 58 |       |
| 3  | 9.81738   | 14 | 9.93993   | 25    | 0.06007   | 9.87745   | 11 | 57 |       |
| 4  | 9.81752   | 14 | 9.94018   | 25    | 0.05981   | 9.87734   | 11 | 56 |       |
| 5  | 9.81767   | 14 | 9.94044   | 25    | 0.05956   | 9.87723   | 11 | 55 |       |
| 6  | 9.81781   | 14 | 9.94069   | 25    | 0.05930   | 9.87712   | 11 | 54 |       |
| 7  | 9.81796   | 14 | 9.94095   | 25    | 0.05905   | 9.87701   | 11 | 53 |       |
| 8  | 9.81810   | 14 | 9.94120   | 25    | 0.05879   | 9.87690   | 11 | 52 |       |
| 9  | 9.81824   | 14 | 9.94146   | 25    | 0.05854   | 9.87679   | 11 | 51 |       |
| 10 | 9.81839   | 14 | 9.94171   | 25    | 0.05828   | 9.87668   | 11 | 50 |       |
| 11 | 9.81855   | 14 | 9.94197   | 25    | 0.05803   | 9.87657   | 11 | 49 |       |
| 12 | 9.81868   | 14 | 9.94222   | 25    | 0.05777   | 9.87645   | 11 | 48 |       |
| 13 | 9.81882   | 14 | 9.94248   | 25    | 0.05752   | 9.87634   | 11 | 47 |       |
| 14 | 9.81897   | 14 | 9.94273   | 25    | 0.05726   | 9.87623   | 11 | 46 |       |
| 15 | 9.81911   | 14 | 9.94299   | 25    | 0.05701   | 9.87612   | 11 | 45 |       |
| 16 | 9.81925   | 14 | 9.94324   | 25    | 0.05675   | 9.87601   | 11 | 44 |       |
| 17 | 9.81940   | 14 | 9.94350   | 25    | 0.05650   | 9.87590   | 11 | 43 |       |
| 18 | 9.81954   | 14 | 9.94375   | 25    | 0.05625   | 9.87579   | 11 | 42 |       |
| 19 | 9.81969   | 14 | 9.94400   | 25    | 0.05600   | 9.87568   | 11 | 41 |       |
| 20 | 9.81983   | 14 | 9.94426   | 25    | 0.05574   | 9.87557   | 11 | 40 |       |
| 21 | 9.81997   | 14 | 9.94451   | 25    | 0.05548   | 9.87546   | 11 | 39 |       |
| 22 | 9.82012   | 14 | 9.94477   | 25    | 0.05523   | 9.87535   | 11 | 38 |       |
| 23 | 9.82026   | 14 | 9.94502   | 25    | 0.05497   | 9.87523   | 11 | 37 |       |
| 24 | 9.82040   | 14 | 9.94528   | 25    | 0.05472   | 9.87512   | 11 | 36 |       |
| 25 | 9.82055   | 14 | 9.94553   | 25    | 0.05446   | 9.87501   | 11 | 35 |       |
| 26 | 9.82069   | 14 | 9.94579   | 25    | 0.05421   | 9.87290   | 11 | 34 |       |
| 27 | 9.82083   | 14 | 9.94604   | 25    | 0.05395   | 9.87479   | 11 | 33 |       |
| 28 | 9.82098   | 14 | 9.94630   | 25    | 0.05370   | 9.87468   | 11 | 32 |       |
| 29 | 9.82112   | 14 | 9.94655   | 25    | 0.05344   | 9.87457   | 11 | 31 |       |
| 30 | 9.82126   | 14 | 9.94681   | 25    | 0.05319   | 9.87445   | 11 | 30 |       |
| 31 | 9.82140   | 14 | 9.94706   | 25    | 0.05293   | 9.87434   | 11 | 29 |       |
| 32 | 9.82155   | 14 | 9.94732   | 25    | 0.05268   | 9.87423   | 11 | 28 |       |
| 33 | 9.82169   | 14 | 9.94757   | 25    | 0.05243   | 9.87412   | 11 | 27 |       |
| 34 | 9.82183   | 14 | 9.94782   | 25    | 0.05217   | 9.87401   | 11 | 26 |       |
| 35 | 9.82197   | 14 | 9.94808   | 25    | 0.05192   | 9.87389   | 11 | 25 |       |
| 36 | 9.82212   | 14 | 9.94833   | 25    | 0.05166   | 9.87378   | 11 | 24 |       |
| 37 | 9.82226   | 14 | 9.94859   | 25    | 0.05141   | 9.87367   | 11 | 23 |       |
| 38 | 9.82240   | 14 | 9.94884   | 25    | 0.05115   | 9.87356   | 11 | 22 |       |
| 39 | 9.82254   | 14 | 9.94910   | 25    | 0.05090   | 9.87345   | 11 | 21 |       |
| 40 | 9.82269   | 14 | 9.94935   | 25    | 0.05064   | 9.87333   | 11 | 20 |       |
| 41 | 9.82283   | 14 | 9.94961   | 25    | 0.05039   | 9.87322   | 11 | 19 |       |
| 42 | 9.82297   | 14 | 9.94986   | 25    | 0.05014   | 9.87311   | 11 | 18 |       |
| 43 | 9.82311   | 14 | 9.95011   | 25    | 0.04988   | 9.87300   | 11 | 17 |       |
| 44 | 9.82325   | 14 | 9.95037   | 25    | 0.04963   | 9.87288   | 11 | 16 |       |
| 45 | 9.82339   | 14 | 9.95062   | 25    | 0.04937   | 9.87277   | 11 | 15 |       |
| 46 | 9.82354   | 14 | 9.95088   | 25    | 0.04912   | 9.87266   | 11 | 14 |       |
| 47 | 9.82368   | 14 | 9.95113   | 25    | 0.04886   | 9.87254   | 11 | 13 |       |
| 48 | 9.82382   | 14 | 9.95139   | 25    | 0.04861   | 9.87243   | 11 | 12 |       |
| 49 | 9.82396   | 14 | 9.95164   | 25    | 0.04836   | 9.87232   | 11 | 11 |       |
| 50 | 9.82410   | 14 | 9.95189   | 25    | 0.04810   | 9.87221   | 11 | 10 |       |
| 51 | 9.82424   | 14 | 9.95215   | 25    | 0.04785   | 9.87209   | 11 | 9  |       |
| 52 | 9.82438   | 14 | 9.95240   | 25    | 0.04759   | 9.87198   | 11 | 8  |       |
| 53 | 9.82452   | 14 | 9.95266   | 25    | 0.04734   | 9.87187   | 11 | 7  |       |
| 54 | 9.82467   | 14 | 9.95291   | 25    | 0.04708   | 9.87175   | 11 | 6  |       |
| 55 | 9.82481   | 14 | 9.95316   | 25    | 0.04683   | 9.87164   | 11 | 5  |       |
| 56 | 9.82495   | 14 | 9.95342   | 25    | 0.04658   | 9.87153   | 11 | 4  |       |
| 57 | 9.82509   | 14 | 9.95367   | 25    | 0.04632   | 9.87141   | 11 | 3  |       |
| 58 | 9.82523   | 14 | 9.95393   | 25    | 0.04607   | 9.87130   | 11 | 2  |       |
| 59 | 9.82537   | 14 | 9.95418   | 25    | 0.04581   | 9.87118   | 11 | 1  |       |
| 60 | 9.82551   | 14 | 9.95443   | 25    | 0.04556   | 9.87107   | 11 | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

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TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.82 551  | 14 | 9.95 443  | 25    | 0.04 556  | 9.87 107  | 11 | 60 |       |
| 1  | 9.82 565  | 14 | 9.95 469  | 25    | 0.04 531  | 9.87 096  | 11 | 59 |       |
| 2  | 9.82 579  | 14 | 9.95 494  | 25    | 0.04 505  | 9.87 084  | 11 | 58 |       |
| 3  | 9.82 593  | 14 | 9.95 520  | 25    | 0.04 480  | 9.87 073  | 11 | 57 |       |
| 4  | 9.82 607  | 14 | 9.95 545  | 25    | 0.04 454  | 9.87 062  | 11 | 56 |       |
| 5  | 9.82 621  | 14 | 9.95 571  | 25    | 0.04 429  | 9.87 050  | 11 | 55 |       |
| 6  | 9.82 635  | 14 | 9.95 596  | 25    | 0.04 404  | 9.87 039  | 11 | 54 |       |
| 7  | 9.82 649  | 14 | 9.95 621  | 25    | 0.04 378  | 9.87 027  | 11 | 53 |       |
| 8  | 9.82 663  | 14 | 9.95 647  | 25    | 0.04 353  | 9.87 016  | 11 | 52 |       |
| 9  | 9.82 677  | 14 | 9.95 672  | 25    | 0.04 327  | 9.87 004  | 11 | 51 |       |
| 10 | 9.82 691  | 14 | 9.95 697  | 25    | 0.04 302  | 9.86 993  | 11 | 50 |       |
| 11 | 9.82 705  | 14 | 9.95 723  | 25    | 0.04 277  | 9.86 982  | 11 | 49 |       |
| 12 | 9.82 719  | 14 | 9.95 748  | 25    | 0.04 251  | 9.86 970  | 11 | 48 |       |
| 13 | 9.82 733  | 13 | 9.95 774  | 25    | 0.04 226  | 9.86 959  | 11 | 47 |       |
| 14 | 9.82 746  | 13 | 9.95 799  | 25    | 0.04 200  | 9.86 947  | 11 | 46 |       |
| 15 | 9.82 760  | 14 | 9.95 824  | 25    | 0.04 175  | 9.86 936  | 11 | 45 |       |
| 16 | 9.82 774  | 14 | 9.95 850  | 25    | 0.04 150  | 9.86 924  | 11 | 44 |       |
| 17 | 9.82 788  | 14 | 9.95 875  | 25    | 0.04 124  | 9.86 913  | 11 | 43 |       |
| 18 | 9.82 802  | 13 | 9.95 901  | 25    | 0.04 099  | 9.86 901  | 11 | 42 |       |
| 19 | 9.82 816  | 14 | 9.95 926  | 25    | 0.04 074  | 9.86 890  | 11 | 41 |       |
| 20 | 9.82 830  | 14 | 9.95 951  | 25    | 0.04 048  | 9.86 878  | 11 | 40 |       |
| 21 | 9.82 844  | 14 | 9.95 977  | 25    | 0.04 023  | 9.86 867  | 11 | 39 |       |
| 22 | 9.82 858  | 13 | 9.96 002  | 25    | 0.03 997  | 9.86 855  | 11 | 38 |       |
| 23 | 9.82 871  | 14 | 9.96 027  | 25    | 0.03 972  | 9.86 844  | 11 | 37 |       |
| 24 | 9.82 885  | 14 | 9.96 053  | 25    | 0.03 947  | 9.86 832  | 11 | 36 |       |
| 25 | 9.82 899  | 13 | 9.96 078  | 25    | 0.03 921  | 9.86 821  | 11 | 35 |       |
| 26 | 9.82 913  | 14 | 9.96 104  | 25    | 0.03 896  | 9.86 809  | 11 | 34 |       |
| 27 | 9.82 927  | 13 | 9.96 129  | 25    | 0.03 871  | 9.86 798  | 12 | 33 |       |
| 28 | 9.82 940  | 14 | 9.96 154  | 25    | 0.03 845  | 9.86 786  | 11 | 32 |       |
| 29 | 9.82 954  | 14 | 9.96 180  | 25    | 0.03 820  | 9.86 774  | 11 | 31 |       |
| 30 | 9.82 968  | 13 | 9.96 205  | 25    | 0.03 795  | 9.86 763  | 11 | 30 |       |
| 31 | 9.82 982  | 14 | 9.96 230  | 25    | 0.03 769  | 9.86 751  | 11 | 29 |       |
| 32 | 9.82 996  | 13 | 9.96 256  | 25    | 0.03 744  | 9.86 740  | 11 | 28 |       |
| 33 | 9.83 009  | 14 | 9.96 281  | 25    | 0.03 718  | 9.86 728  | 11 | 27 |       |
| 34 | 9.83 023  | 13 | 9.96 306  | 25    | 0.03 693  | 9.86 716  | 12 | 26 |       |
| 35 | 9.83 037  | 14 | 9.96 332  | 25    | 0.03 668  | 9.86 705  | 11 | 25 |       |
| 36 | 9.83 051  | 13 | 9.96 357  | 25    | 0.03 642  | 9.86 693  | 11 | 24 |       |
| 37 | 9.83 064  | 14 | 9.96 383  | 25    | 0.03 617  | 9.86 682  | 11 | 23 |       |
| 38 | 9.83 078  | 13 | 9.96 408  | 25    | 0.03 592  | 9.86 670  | 12 | 22 |       |
| 39 | 9.83 092  | 14 | 9.96 433  | 25    | 0.03 566  | 9.86 658  | 11 | 21 |       |
| 40 | 9.83 106  | 13 | 9.96 459  | 25    | 0.03 541  | 9.86 647  | 11 | 20 |       |
| 41 | 9.83 119  | 14 | 9.96 484  | 25    | 0.03 516  | 9.86 635  | 12 | 19 |       |
| 42 | 9.83 133  | 13 | 9.96 509  | 25    | 0.03 490  | 9.86 623  | 11 | 18 |       |
| 43 | 9.83 147  | 14 | 9.96 535  | 25    | 0.03 465  | 9.86 612  | 11 | 17 |       |
| 44 | 9.83 160  | 13 | 9.96 560  | 25    | 0.03 440  | 9.86 600  | 11 | 16 |       |
| 45 | 9.83 174  | 14 | 9.96 585  | 25    | 0.03 414  | 9.86 588  | 12 | 15 |       |
| 46 | 9.83 188  | 13 | 9.96 611  | 25    | 0.03 389  | 9.86 577  | 11 | 14 |       |
| 47 | 9.83 201  | 14 | 9.96 636  | 25    | 0.03 364  | 9.86 565  | 12 | 13 |       |
| 48 | 9.83 215  | 13 | 9.96 661  | 25    | 0.03 338  | 9.86 553  | 11 | 12 |       |
| 49 | 9.83 229  | 14 | 9.96 687  | 25    | 0.03 313  | 9.86 542  | 11 | 11 |       |
| 50 | 9.83 242  | 13 | 9.96 712  | 25    | 0.03 287  | 9.86 530  | 12 | 10 |       |
| 51 | 9.83 256  | 14 | 9.96 737  | 25    | 0.03 262  | 9.86 518  | 11 | 9  |       |
| 52 | 9.83 269  | 13 | 9.96 763  | 25    | 0.03 237  | 9.86 507  | 12 | 8  |       |
| 53 | 9.83 283  | 14 | 9.96 788  | 25    | 0.03 211  | 9.86 495  | 11 | 7  |       |
| 54 | 9.83 297  | 13 | 9.96 813  | 25    | 0.03 186  | 9.86 483  | 11 | 6  |       |
| 55 | 9.83 310  | 14 | 9.96 839  | 25    | 0.03 161  | 9.86 471  | 12 | 5  |       |
| 56 | 9.83 324  | 13 | 9.96 864  | 25    | 0.03 135  | 9.86 460  | 11 | 4  |       |
| 57 | 9.83 337  | 14 | 9.96 889  | 25    | 0.03 110  | 9.86 448  | 12 | 3  |       |
| 58 | 9.83 351  | 13 | 9.96 915  | 25    | 0.03 085  | 9.86 436  | 11 | 2  |       |
| 59 | 9.83 365  | 14 | 9.96 940  | 25    | 0.03 059  | 9.86 424  | 12 | 1  |       |
| 60 | 9.83 378  | 13 | 9.96 965  | 25    | 0.03 034  | 9.86 412  | 12 | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

|    |      |      |
|----|------|------|
|    | 25   | 25   |
| 6  | 2.5  | 2.5  |
| 7  | 3.0  | 2.9  |
| 8  | 3.4  | 3.3  |
| 9  | 3.8  | 3.7  |
| 10 | 4.2  | 4.1  |
| 20 | 8.5  | 8.3  |
| 30 | 12.7 | 12.5 |
| 40 | 17.0 | 16.6 |
| 50 | 21.2 | 20.8 |

|    |      |      |
|----|------|------|
|    | 14   | 13   |
| 6  | 1.4  | 1.3  |
| 7  | 1.6  | 1.6  |
| 8  | 1.8  | 1.8  |
| 9  | 2.1  | 2.0  |
| 10 | 2.3  | 2.2  |
| 20 | 4.6  | 4.5  |
| 30 | 7.0  | 6.7  |
| 40 | 9.3  | 9.0  |
| 50 | 11.6 | 11.2 |

|    |      |     |     |
|----|------|-----|-----|
|    | 12   | 11  | 11  |
| 6  | 1.2  | 1.1 | 1.1 |
| 7  | 1.4  | 1.3 | 1.3 |
| 8  | 1.6  | 1.5 | 1.4 |
| 9  | 1.8  | 1.7 | 1.6 |
| 10 | 2.0  | 1.9 | 1.8 |
| 20 | 4.0  | 3.8 | 3.6 |
| 30 | 6.0  | 5.7 | 5.5 |
| 40 | 8.0  | 7.6 | 7.3 |
| 50 | 10.0 | 9.6 | 9.1 |

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3°

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|           | Log. Sin. | d.        | Log. Tan. | c. d.     | Log. Cot. | Log. Cos. | d. |       | P. P.        |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----|-------|--------------|
| 0         | 9 83 378  | 1         | 9 96 965  | 25        | 0 03 034  | 9 86 412  | 11 | 60    |              |
| 1         | 9 83 392  | 1         | 9 96 991  | 25        | 0 03 009  | 9 86 401  | 12 | 59    |              |
| 2         | 9 83 405  | 1         | 9 97 016  | 25        | 0 02 984  | 9 86 389  | 12 | 58    |              |
| 3         | 9 83 419  | 1         | 9 97 041  | 25        | 0 02 958  | 9 86 377  | 12 | 57    |              |
| 4         | 9 83 432  | 1         | 9 97 067  | 25        | 0 02 933  | 9 86 365  | 12 | 56    |              |
| 5         | 9 83 446  | 1         | 9 97 092  | 25        | 0 02 908  | 9 86 354  | 12 | 55    |              |
| 6         | 9 83 459  | 1         | 9 97 117  | 25        | 0 02 882  | 9 86 342  | 12 | 54    |              |
| 7         | 9 83 473  | 1         | 9 97 143  | 25        | 0 02 857  | 9 86 330  | 12 | 53    |              |
| 8         | 9 83 486  | 1         | 9 97 168  | 25        | 0 02 832  | 9 86 318  | 12 | 52    |              |
| 9         | 9 83 500  | 1         | 9 97 193  | 25        | 0 02 806  | 9 86 306  | 12 | 51    | 25 25        |
| 0         | 9 83 513  | 1         | 9 97 219  | 25        | 0 02 781  | 9 86 294  | 12 | 50    | 6 2.5 2.5    |
| 1         | 9 83 527  | 1         | 9 97 244  | 25        | 0 02 756  | 9 86 282  | 12 | 49    | 7 3.0 2.9    |
| 2         | 9 83 540  | 1         | 9 97 269  | 25        | 0 02 730  | 9 86 271  | 12 | 48    | 8 3.4 3.3    |
| 3         | 9 83 554  | 1         | 9 97 295  | 25        | 0 02 705  | 9 86 259  | 12 | 47    | 9 3.8 3.7    |
| 4         | 9 83 567  | 1         | 9 97 320  | 25        | 0 02 680  | 9 86 247  | 12 | 46    | 10 4.2 4.1   |
| 5         | 9 83 580  | 1         | 9 97 345  | 25        | 0 02 654  | 9 86 235  | 12 | 45    | 20 8.5 8.3   |
| 6         | 9 83 594  | 1         | 9 97 370  | 25        | 0 02 629  | 9 86 223  | 12 | 44    | 30 12.7 12.5 |
| 7         | 9 83 607  | 1         | 9 97 396  | 25        | 0 02 604  | 9 86 211  | 12 | 43    | 40 17.0 16.6 |
| 8         | 9 83 621  | 1         | 9 97 421  | 25        | 0 02 578  | 9 86 199  | 12 | 42    | 50 21.2 20.8 |
| 9         | 9 83 634  | 1         | 9 97 445  | 25        | 0 02 553  | 9 86 187  | 12 | 41    |              |
| 0         | 9 83 647  | 13        | 9 97 472  | 25        | 0 02 528  | 9 86 176  | 12 | 40    |              |
| 1         | 9 83 661  | 13        | 9 97 497  | 25        | 0 02 502  | 9 86 164  | 12 | 39    |              |
| 2         | 9 83 674  | 13        | 9 97 522  | 25        | 0 02 477  | 9 86 152  | 12 | 38    |              |
| 3         | 9 83 688  | 13        | 9 97 548  | 25        | 0 02 452  | 9 86 140  | 12 | 37    |              |
| 4         | 9 83 701  | 13        | 9 97 573  | 25        | 0 02 427  | 9 86 128  | 12 | 36    |              |
| 5         | 9 83 714  | 13        | 9 97 598  | 25        | 0 02 401  | 9 86 116  | 12 | 35    |              |
| 6         | 9 83 728  | 13        | 9 97 624  | 25        | 0 02 376  | 9 86 104  | 12 | 34    | 13 13        |
| 7         | 9 83 741  | 13        | 9 97 649  | 25        | 0 02 351  | 9 86 092  | 12 | 33    | 6 1.3 1.3    |
| 8         | 9 83 754  | 13        | 9 97 674  | 25        | 0 02 325  | 9 86 080  | 12 | 32    | 7 1.6 1.5    |
| 9         | 9 83 768  | 13        | 9 97 699  | 25        | 0 02 300  | 9 86 068  | 12 | 31    | 8 1.8 1.7    |
| 0         | 9 83 781  | 13        | 9 97 725  | 25        | 0 02 275  | 9 86 056  | 12 | 30    | 9 2.0 1.9    |
| 1         | 9 83 794  | 13        | 9 97 750  | 25        | 0 02 249  | 9 86 044  | 12 | 29    | 10 2.2 2.1   |
| 2         | 9 83 808  | 13        | 9 97 775  | 25        | 0 02 224  | 9 86 032  | 12 | 28    | 20 4.5 4.3   |
| 3         | 9 83 821  | 13        | 9 97 801  | 25        | 0 02 199  | 9 86 020  | 12 | 27    | 30 6.7 6.5   |
| 4         | 9 83 834  | 13        | 9 97 826  | 25        | 0 02 174  | 9 86 008  | 12 | 26    | 40 9.0 8.6   |
| 5         | 9 83 847  | 13        | 9 97 851  | 25        | 0 02 148  | 9 85 996  | 12 | 25    | 50 11.2 10.8 |
| 6         | 9 83 861  | 13        | 9 97 877  | 25        | 0 02 123  | 9 85 984  | 12 | 24    |              |
| 7         | 9 83 874  | 13        | 9 97 902  | 25        | 0 02 098  | 9 85 972  | 12 | 23    |              |
| 8         | 9 83 887  | 13        | 9 97 927  | 25        | 0 02 072  | 9 85 960  | 12 | 22    |              |
| 9         | 9 83 900  | 13        | 9 97 952  | 25        | 0 02 047  | 9 85 948  | 12 | 21    |              |
| 0         | 9 83 914  | 13        | 9 97 978  | 25        | 0 02 022  | 9 85 936  | 12 | 20    |              |
| 1         | 9 83 927  | 13        | 9 98 003  | 25        | 0 01 996  | 9 85 924  | 12 | 19    |              |
| 2         | 9 83 940  | 13        | 9 98 028  | 25        | 0 01 971  | 9 85 912  | 12 | 18    |              |
| 3         | 9 83 953  | 13        | 9 98 054  | 25        | 0 01 946  | 9 85 900  | 12 | 17    |              |
| 4         | 9 83 967  | 13        | 9 98 079  | 25        | 0 01 921  | 9 85 887  | 12 | 16    |              |
| 5         | 9 83 980  | 13        | 9 98 104  | 25        | 0 01 895  | 9 85 875  | 12 | 15    | 12 12 11     |
| 6         | 9 83 993  | 13        | 9 98 129  | 25        | 0 01 870  | 9 85 863  | 12 | 14    | 6 1.2 1.1    |
| 7         | 9 84 006  | 13        | 9 98 155  | 25        | 0 01 845  | 9 85 851  | 12 | 13    | 7 1.4 1.3    |
| 8         | 9 84 019  | 13        | 9 98 180  | 25        | 0 01 819  | 9 85 839  | 12 | 12    | 8 1.6 1.5    |
| 9         | 9 84 033  | 13        | 9 98 205  | 25        | 0 01 794  | 9 85 827  | 12 | 11    | 9 1.9 1.8    |
| 0         | 9 84 046  | 13        | 9 98 231  | 25        | 0 01 769  | 9 85 815  | 12 | 10    | 10 2.1 2.0   |
| 1         | 9 84 059  | 13        | 9 98 256  | 25        | 0 01 744  | 9 85 803  | 12 | 9     | 20 4.1 4.0   |
| 2         | 9 84 072  | 13        | 9 98 281  | 25        | 0 01 718  | 9 85 791  | 12 | 8     | 30 6.2 6.0   |
| 3         | 9 84 085  | 13        | 9 98 306  | 25        | 0 01 693  | 9 85 778  | 12 | 7     | 40 8.3 8.0   |
| 4         | 9 84 098  | 13        | 9 98 332  | 25        | 0 01 668  | 9 85 766  | 12 | 6     | 50 10.4 10.0 |
| 5         | 9 84 111  | 13        | 9 98 357  | 25        | 0 01 642  | 9 85 754  | 12 | 5     |              |
| 6         | 9 84 124  | 13        | 9 98 382  | 25        | 0 01 617  | 9 85 742  | 12 | 4     |              |
| 7         | 9 84 138  | 13        | 9 98 408  | 25        | 0 01 592  | 9 85 730  | 12 | 3     |              |
| 8         | 9 84 151  | 13        | 9 98 433  | 25        | 0 01 567  | 9 85 718  | 12 | 2     |              |
| 9         | 9 84 164  | 13        | 9 98 458  | 25        | 0 01 541  | 9 85 705  | 12 | 1     |              |
| 0         | 9 84 177  | 13        | 9 98 483  | 25        | 0 01 516  | 9 85 693  | 12 | 0     |              |
| Log. Cos. | d.        | Log. Cot. | c. d.     | Log. Tan. | Log. Sin. | d.        |    | P. P. |              |

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS,  
AND COTANGENTS.

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|    | Log. Sin. | d. | Log. Tan. | c. d. | Log. Cot. | Log. Cos. | d. |    | P. P. |
|----|-----------|----|-----------|-------|-----------|-----------|----|----|-------|
| 0  | 9.84 177  | 13 | 9.98 483  | 25    | 0.01 516  | 9.85 693  | 12 | 60 |       |
| 1  | 9.84 190  | 13 | 9.98 509  | 25    | 0.01 491  | 9.85 681  | 12 | 59 |       |
| 2  | 9.84 203  | 13 | 9.98 534  | 25    | 0.01 465  | 9.85 669  | 12 | 58 |       |
| 3  | 9.84 216  | 13 | 9.98 559  | 25    | 0.01 440  | 9.85 657  | 12 | 57 |       |
| 4  | 9.84 229  | 13 | 9.98 585  | 25    | 0.01 415  | 9.85 644  | 12 | 56 |       |
| 5  | 9.84 242  | 13 | 9.98 610  | 25    | 0.01 390  | 9.85 632  | 12 | 55 |       |
| 6  | 9.84 255  | 13 | 9.98 635  | 25    | 0.01 364  | 9.85 620  | 12 | 54 |       |
| 7  | 9.84 268  | 13 | 9.98 660  | 25    | 0.01 339  | 9.85 608  | 12 | 53 |       |
| 8  | 9.84 281  | 13 | 9.98 686  | 25    | 0.01 314  | 9.85 595  | 12 | 52 |       |
| 9  | 9.84 294  | 13 | 9.98 711  | 25    | 0.01 289  | 9.85 583  | 12 | 51 |       |
| 10 | 9.84 307  | 13 | 9.98 736  | 25    | 0.01 263  | 9.85 571  | 12 | 50 |       |
| 11 | 9.84 320  | 13 | 9.98 762  | 25    | 0.01 238  | 9.85 559  | 12 | 49 |       |
| 12 | 9.84 333  | 13 | 9.98 787  | 25    | 0.01 213  | 9.85 546  | 12 | 48 |       |
| 13 | 9.84 346  | 13 | 9.98 812  | 25    | 0.01 187  | 9.85 534  | 12 | 47 |       |
| 14 | 9.84 359  | 13 | 9.98 837  | 25    | 0.01 162  | 9.85 522  | 12 | 46 |       |
| 15 | 9.84 372  | 13 | 9.98 863  | 25    | 0.01 137  | 9.85 509  | 12 | 45 |       |
| 16 | 9.84 385  | 13 | 9.98 888  | 25    | 0.01 112  | 9.85 497  | 12 | 44 |       |
| 17 | 9.84 398  | 13 | 9.98 913  | 25    | 0.01 085  | 9.85 485  | 12 | 43 |       |
| 18 | 9.84 411  | 13 | 9.98 938  | 25    | 0.01 061  | 9.85 472  | 12 | 42 |       |
| 19 | 9.84 424  | 13 | 9.98 964  | 25    | 0.01 036  | 9.85 460  | 12 | 41 |       |
| 20 | 9.84 437  | 13 | 9.98 989  | 25    | 0.01 010  | 9.85 448  | 12 | 40 |       |
| 21 | 9.84 450  | 13 | 9.99 014  | 25    | 0.00 985  | 9.85 435  | 12 | 39 |       |
| 22 | 9.84 463  | 13 | 9.99 040  | 25    | 0.00 960  | 9.85 423  | 12 | 38 |       |
| 23 | 9.84 476  | 13 | 9.99 065  | 25    | 0.00 935  | 9.85 411  | 12 | 37 |       |
| 24 | 9.84 489  | 13 | 9.99 090  | 25    | 0.00 909  | 9.85 398  | 12 | 36 |       |
| 25 | 9.84 502  | 12 | 9.99 115  | 25    | 0.00 884  | 9.85 386  | 12 | 35 |       |
| 26 | 9.84 514  | 13 | 9.99 141  | 25    | 0.00 859  | 9.85 374  | 12 | 34 |       |
| 27 | 9.84 527  | 13 | 9.99 166  | 25    | 0.00 834  | 9.85 361  | 12 | 33 |       |
| 28 | 9.84 540  | 13 | 9.99 191  | 25    | 0.00 808  | 9.85 349  | 12 | 32 |       |
| 29 | 9.84 553  | 13 | 9.99 216  | 25    | 0.00 783  | 9.85 336  | 12 | 31 |       |
| 30 | 9.84 566  | 12 | 9.99 242  | 25    | 0.00 758  | 9.85 324  | 12 | 30 |       |
| 31 | 9.84 579  | 13 | 9.99 267  | 25    | 0.00 733  | 9.85 312  | 12 | 29 |       |
| 32 | 9.84 592  | 12 | 9.99 292  | 25    | 0.00 707  | 9.85 299  | 12 | 28 |       |
| 33 | 9.84 604  | 13 | 9.99 318  | 25    | 0.00 682  | 9.85 287  | 12 | 27 |       |
| 34 | 9.84 617  | 13 | 9.99 343  | 25    | 0.00 657  | 9.85 274  | 12 | 26 |       |
| 35 | 9.84 630  | 13 | 9.99 368  | 25    | 0.00 631  | 9.85 262  | 12 | 25 |       |
| 36 | 9.84 643  | 13 | 9.99 393  | 25    | 0.00 606  | 9.85 249  | 12 | 24 |       |
| 37 | 9.84 656  | 13 | 9.99 419  | 25    | 0.00 581  | 9.85 237  | 12 | 23 |       |
| 38 | 9.84 669  | 13 | 9.99 444  | 25    | 0.00 556  | 9.85 224  | 12 | 22 |       |
| 39 | 9.84 681  | 12 | 9.99 469  | 25    | 0.00 530  | 9.85 212  | 12 | 21 |       |
| 40 | 9.84 694  | 13 | 9.99 494  | 25    | 0.00 505  | 9.85 199  | 12 | 20 |       |
| 41 | 9.84 707  | 13 | 9.99 520  | 25    | 0.00 480  | 9.85 187  | 12 | 19 |       |
| 42 | 9.84 720  | 13 | 9.99 545  | 25    | 0.00 455  | 9.85 174  | 12 | 18 |       |
| 43 | 9.84 732  | 13 | 9.99 570  | 25    | 0.00 429  | 9.85 162  | 12 | 17 |       |
| 44 | 9.84 745  | 12 | 9.99 595  | 25    | 0.00 404  | 9.85 149  | 12 | 16 |       |
| 45 | 9.84 758  | 13 | 9.99 621  | 25    | 0.00 379  | 9.85 137  | 12 | 15 |       |
| 46 | 9.84 771  | 12 | 9.99 646  | 25    | 0.00 353  | 9.85 124  | 12 | 14 |       |
| 47 | 9.84 783  | 13 | 9.99 671  | 25    | 0.00 328  | 9.85 112  | 12 | 13 |       |
| 48 | 9.84 796  | 13 | 9.99 697  | 25    | 0.00 303  | 9.85 099  | 12 | 12 |       |
| 49 | 9.84 809  | 12 | 9.99 722  | 25    | 0.00 278  | 9.85 087  | 12 | 11 |       |
| 50 | 9.84 822  | 13 | 9.99 747  | 25    | 0.00 252  | 9.85 074  | 12 | 10 |       |
| 51 | 9.84 834  | 12 | 9.99 772  | 25    | 0.00 227  | 9.85 062  | 12 | 9  |       |
| 52 | 9.84 847  | 13 | 9.99 798  | 25    | 0.00 202  | 9.85 049  | 12 | 8  |       |
| 53 | 9.84 860  | 13 | 9.99 823  | 25    | 0.00 177  | 9.85 037  | 12 | 7  |       |
| 54 | 9.84 872  | 12 | 9.99 848  | 25    | 0.00 151  | 9.85 024  | 13 | 6  |       |
| 55 | 9.84 885  | 13 | 9.99 873  | 25    | 0.00 126  | 9.85 011  | 12 | 5  |       |
| 56 | 9.84 898  | 12 | 9.99 899  | 25    | 0.00 101  | 9.84 999  | 12 | 4  |       |
| 57 | 9.84 910  | 13 | 9.99 924  | 25    | 0.00 076  | 9.84 986  | 12 | 3  |       |
| 58 | 9.84 923  | 13 | 9.99 949  | 25    | 0.00 050  | 9.84 974  | 12 | 2  |       |
| 59 | 9.84 936  | 12 | 9.99 974  | 25    | 0.00 025  | 9.84 961  | 13 | 1  |       |
| 60 | 9.84 948  | 12 | 0.00 000  | 25    | 0.00 000  | 9.84 948  | 12 | 0  |       |
|    | Log. Cos. | d. | Log. Cot. | c. d. | Log. Tan. | Log. Sin. | d. |    | P. P. |

25 25  
6 2.5 2.5  
7 3.0 2.9  
8 3.4 3.3  
9 3.8 3.7  
10 4.2 4.1  
20 8.5 8.3  
30 12.7 12.5  
40 17.0 16.6  
50 21.2 20.8

13 13  
6 1.3 1.3  
7 1.6 1.5  
8 1.8 1.7  
9 2.0 1.9  
10 2.2 2.1  
20 4.5 4.3  
30 6.7 6.5  
40 9.0 8.6  
50 11.2 10.8

12 12  
6 1.2 1.2  
7 1.4 1.4  
8 1.6 1.6  
9 1.9 1.8  
10 2.1 2.0  
20 4.1 4.0  
30 6.2 6.0  
40 8.3 8.0  
50 10.4 10.0

## TABLE VIII.

LOGARITHMIC VERSED SINES AND EXTERNAL  
SECANTS.

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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL

0°

SECANTS.

1°

|    | Log. Vers. | D     | Log. Exsec. | D     | Log. Vers. | D    | Log. Exsec. | D    |
|----|------------|-------|-------------|-------|------------|------|-------------|------|
| 0  | — ∞        |       | — ∞         |       | 6.18271    | 1435 | 6.18278     | 1436 |
| 1  | 2.62642    | 60206 | 2.62642     | 60206 | .19707     | 1412 | .19714      | 1412 |
| 2  | 3.22848    | 35218 | 3.22848     | 35218 | .21119     | 1389 | .21126      | 1390 |
| 3  | 3.58066    | 24987 | 3.58066     | 24987 | .22509     | 1368 | .22516      | 1368 |
| 4  | 3.83054    | 19382 | 3.83054     | 19382 | .23877     | 1346 | .23884      | 1347 |
| 5  | 4.02436    | 15836 | 4.02436     | 15836 | 6.25223    | 1326 | 6.25231     | 1326 |
| 6  | .18272     | 13389 | .18272      | 13389 | .26549     | 1306 | .26557      | 1306 |
| 7  | .31662     | 11598 | .31662      | 11598 | .27856     | 1286 | .27864      | 1287 |
| 8  | .43260     | 10230 | .43260      | 10230 | .29142     | 1268 | .29151      | 1268 |
| 9  | .53430     | 9151  | .53491      | 9151  | .30410     | 1250 | .30419      | 1250 |
| 10 | 4.62342    | 8278  | 4.62642     | 8279  | 6.31660    | 1232 | 6.31669     | 1232 |
| 11 | .70920     | 7558  | .70921      | 7557  | .32892     | 1214 | .32901      | 1215 |
| 12 | .78478     | 6953  | .78478      | 6952  | .34107     | 1198 | .34116      | 1198 |
| 13 | .85431     | 6437  | .85431      | 6437  | .35305     | 1182 | .35315      | 1182 |
| 14 | .91838     | 5992  | .91868      | 5993  | .36487     | 1166 | .36497      | 1166 |
| 15 | 4.97860    | 5605  | 4.97861     | 5605  | 6.37653    | 1150 | 6.37663     | 1151 |
| 16 | 5.03466    | 5266  | 5.03466     | 5266  | .38803     | 1135 | .38814      | 1135 |
| 17 | .08732     | 4964  | .08732      | 4964  | .39938     | 1121 | .39949      | 1121 |
| 18 | .13696     | 4696  | .13697      | 4696  | .41059     | 1106 | .41070      | 1106 |
| 19 | .18393     | 4455  | .18393      | 4456  | .42165     | 1093 | .42177      | 1093 |
| 20 | 5.22848    | 4238  | 5.22849     | 4238  | 6.43258    | 1078 | 6.43270     | 1079 |
| 21 | .27086     | 4040  | .27087      | 4040  | .44337     | 1066 | .44349      | 1066 |
| 22 | .31126     | 3861  | .31127      | 3861  | .45403     | 1052 | .45415      | 1053 |
| 23 | .34987     | 3697  | .34988      | 3697  | .46455     | 1040 | .46468      | 1040 |
| 24 | .38684     | 3545  | .38685      | 3545  | .47496     | 1028 | .47509      | 1028 |
| 25 | 5.42230    | 3406  | 5.42231     | 3407  | 6.48524    | 1016 | 6.48537     | 1015 |
| 26 | .45636     | 3278  | .45638      | 3278  | .49539     | 1004 | .49553      | 1004 |
| 27 | .48915     | 3158  | .48916      | 3159  | .50544     | 992  | .50557      | 993  |
| 28 | .52073     | 3048  | .52075      | 3048  | .51536     | 981  | .51550      | 982  |
| 29 | .55121     | 2944  | .55123      | 2945  | .52518     | 970  | .52532      | 970  |
| 30 | 5.58066    | 2848  | 5.58068     | 2848  | 6.53488    | 960  | 6.53503     | 960  |
| 31 | .60914     | 2757  | .60916      | 2758  | .54448     | 949  | .54463      | 950  |
| 32 | .63672     | 2672  | .63674      | 2672  | .55397     | 939  | .55413      | 939  |
| 33 | .66344     | 2593  | .66346      | 2593  | .56336     | 929  | .56352      | 929  |
| 34 | .68937     | 2518  | .68940      | 2517  | .57265     | 919  | .57281      | 919  |
| 35 | 5.71455    | 2447  | 5.71457     | 2447  | 6.58184    | 909  | 6.58201     | 909  |
| 36 | .73902     | 2379  | .73904      | 2380  | .59093     | 900  | .59110      | 900  |
| 37 | .76282     | 2316  | .76234      | 2316  | .59993     | 891  | .60011      | 891  |
| 38 | .78598     | 2256  | .78601      | 2256  | .60884     | 882  | .60902      | 882  |
| 39 | .80854     | 2199  | .80857      | 2199  | .61766     | 872  | .61784      | 873  |
| 40 | 5.83053    | 2145  | 5.83056     | 2145  | 6.62639    | 864  | 6.62657     | 864  |
| 41 | .85198     | 2093  | .85201      | 2093  | .63503     | 855  | .63522      | 856  |
| 42 | .87291     | 2044  | .87295      | 2043  | .64359     | 847  | .64378      | 848  |
| 43 | .89335     | 1996  | .89338      | 1997  | .65206     | 839  | .65226      | 839  |
| 44 | .91332     | 1952  | .91335      | 1952  | .66045     | 831  | .66065      | 831  |
| 45 | 5.93284    | 1909  | 5.93288     | 1909  | 6.66876    | 823  | 6.66897     | 823  |
| 46 | .95193     | 1868  | .95197      | 1868  | .67700     | 815  | .67720      | 816  |
| 47 | .97061     | 1829  | .97065      | 1829  | .68515     | 808  | .68536      | 808  |
| 48 | 5.98890    | 1790  | 5.98894     | 1791  | .69323     | 800  | .69345      | 800  |
| 49 | 6.00680    | 1755  | 6.00685     | 1755  | .70124     | 793  | .70145      | 794  |
| 50 | 6.02435    | 1720  | 6.02440     | 1720  | 6.70917    | 786  | 6.70939     | 786  |
| 51 | .04155     | 1686  | .04160      | 1687  | .71703     | 779  | .71725      | 779  |
| 52 | .05842     | 1654  | .05847      | 1654  | .72482     | 772  | .72505      | 772  |
| 53 | .07496     | 1623  | .07501      | 1623  | .73254     | 765  | .73277      | 765  |
| 54 | .09120     | 1594  | .09125      | 1594  | .74019     | 758  | .74043      | 759  |
| 55 | 6.10714    | 1565  | 6.10719     | 1565  | 6.74777    | 752  | 6.74802     | 752  |
| 56 | .12279     | 1537  | .12284      | 1537  | .75529     | 745  | .75554      | 746  |
| 57 | .13816     | 1511  | .13822      | 1511  | .76275     | 739  | .76300      | 739  |
| 58 | .15327     | 1484  | .15333      | 1485  | .77014     | 733  | .77040      | 733  |
| 59 | .16811     | 1460  | .16818      | 1460  | .77747     | 726  | .77773      | 727  |
| 60 | 6.18271    |       | 6.18278     |       | 6.78474    |      | 6.78500     |      |
|    | Log. Vers. | D     | Log. Exsec. | D     | Log. Vers. | D    | Log. Exsec. | D    |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

2°

3°

|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    |
|----|------------|-----|-------------|-----|------------|-----|-------------|-----|----|
| 0  | 6.78474    | 721 | 6.78500     | 721 | 7.13687    | 481 | 7.13746     | 481 | 0  |
| 1  | .79195     | 714 | .79221      | 715 | .14168     | 478 | .14228      | 479 | 1  |
| 2  | .79909     | 709 | .79937      | 709 | .14646     | 475 | .14707      | 476 | 2  |
| 3  | .80618     | 703 | .80646      | 703 | .15122     | 473 | .15183      | 474 | 3  |
| 4  | .81322     | 697 | .81350      | 698 | .15595     | 470 | .15657      | 471 | 4  |
| 5  | 6.82019    | 692 | 6.82048     | 692 | 7.16066    | 468 | 7.16129     | 469 | 5  |
| 6  | .82711     | 686 | .82740      | 687 | .16534     | 466 | .16598      | 466 | 6  |
| 7  | .83398     | 681 | .83427      | 682 | .17000     | 463 | .17064      | 464 | 7  |
| 8  | .84079     | 676 | .84109      | 676 | .17463     | 460 | .17528      | 461 | 8  |
| 9  | .84755     | 670 | .84785      | 671 | .17923     | 458 | .17989      | 459 | 9  |
| 10 | 6.85425    | 665 | 6.85457     | 666 | 7.18382    | 455 | 7.18448     | 456 | 10 |
| 11 | .86091     | 660 | .86123      | 660 | .18837     | 453 | .18905      | 454 | 11 |
| 12 | .86751     | 655 | .86783      | 656 | .19291     | 451 | .19359      | 452 | 12 |
| 13 | .87407     | 650 | .87439      | 651 | .19742     | 448 | .19811      | 449 | 13 |
| 14 | .88057     | 646 | .88090      | 646 | .20191     | 446 | .20260      | 447 | 14 |
| 15 | 6.88703    | 641 | 6.88737     | 641 | 7.20637    | 444 | 7.20707     | 445 | 15 |
| 16 | .89344     | 636 | .89378      | 636 | .21081     | 442 | .21152      | 442 | 16 |
| 17 | .89980     | 631 | .90015      | 632 | .21523     | 440 | .21595      | 440 | 17 |
| 18 | .90612     | 627 | .90647      | 628 | .21963     | 437 | .22035      | 438 | 18 |
| 19 | .91239     | 622 | .91275      | 623 | .22400     | 435 | .22473      | 436 | 19 |
| 20 | 6.91862    | 618 | 6.91898     | 618 | 7.22838    | 433 | 7.22909     | 434 | 20 |
| 21 | .92480     | 613 | .92516      | 614 | .23269     | 431 | .23343      | 431 | 21 |
| 22 | .93093     | 609 | .93131      | 610 | .23700     | 429 | .23775      | 429 | 22 |
| 23 | .93703     | 605 | .93741      | 605 | .24129     | 426 | .24204      | 427 | 23 |
| 24 | .94308     | 601 | .94346      | 601 | .24555     | 424 | .24632      | 425 | 24 |
| 25 | 6.94909    | 597 | 6.94948     | 597 | 7.24980    | 422 | 7.25057     | 423 | 25 |
| 26 | .95506     | 592 | .95545      | 593 | .25402     | 420 | .25480      | 421 | 26 |
| 27 | .96099     | 589 | .96139      | 589 | .25823     | 418 | .25902      | 419 | 27 |
| 28 | .96688     | 584 | .96728      | 585 | .26241     | 416 | .26321      | 417 | 28 |
| 29 | .97272     | 581 | .97313      | 581 | .26658     | 414 | .26738      | 415 | 29 |
| 30 | 6.97853    | 577 | 6.97895     | 577 | 7.27072    | 412 | 7.27153     | 413 | 30 |
| 31 | .98430     | 573 | .98472      | 574 | .27485     | 410 | .27567      | 411 | 31 |
| 32 | .99004     | 569 | .99046      | 570 | .27895     | 409 | .27978      | 409 | 32 |
| 33 | 6.99573    | 565 | 6.99616     | 566 | .28304     | 406 | .28387      | 407 | 33 |
| 34 | 7.00139    | 562 | 7.00182     | 563 | .28711     | 405 | .28795      | 405 | 34 |
| 35 | 7.00701    | 558 | 7.00745     | 559 | 7.29116    | 402 | 7.29200     | 404 | 35 |
| 36 | .01259     | 555 | .01304      | 555 | .29518     | 401 | .29604      | 402 | 36 |
| 37 | .01814     | 551 | .01860      | 552 | .29919     | 399 | .30006      | 400 | 37 |
| 38 | .02366     | 548 | .02412      | 548 | .30319     | 397 | .30406      | 398 | 38 |
| 39 | .02914     | 544 | .02960      | 545 | .30716     | 395 | .30804      | 396 | 39 |
| 40 | 7.03458    | 541 | 7.03505     | 541 | 7.31112    | 393 | 7.31201     | 394 | 40 |
| 41 | .03999     | 537 | .04047      | 538 | .31505     | 392 | .31595      | 393 | 41 |
| 42 | .04537     | 534 | .04585      | 535 | .31897     | 390 | .31988      | 391 | 42 |
| 43 | .05071     | 531 | .05120      | 531 | .32288     | 388 | .32379      | 389 | 43 |
| 44 | .05603     | 527 | .05652      | 528 | .32676     | 386 | .32768      | 388 | 44 |
| 45 | 7.06130    | 525 | 7.06180     | 525 | 7.33063    | 385 | 7.33156     | 385 | 45 |
| 46 | .06655     | 521 | .06706      | 522 | .33448     | 383 | .33542      | 384 | 46 |
| 47 | .07177     | 518 | .07228      | 519 | .33831     | 382 | .33926      | 382 | 47 |
| 48 | .07695     | 515 | .07747      | 516 | .34213     | 380 | .34309      | 380 | 48 |
| 49 | .08211     | 512 | .08263      | 513 | .34593     | 378 | .34689      | 379 | 49 |
| 50 | 7.08723    | 509 | 7.08776     | 509 | 7.34971    | 377 | 7.35069     | 377 | 50 |
| 51 | .09232     | 506 | .09286      | 507 | .35348     | 375 | .35446      | 376 | 51 |
| 52 | .09739     | 503 | .09793      | 503 | .35723     | 373 | .35822      | 374 | 52 |
| 53 | .10242     | 500 | .10297      | 501 | .36097     | 371 | .36196      | 373 | 53 |
| 54 | .10743     | 497 | .10798      | 498 | .36468     | 370 | .36569      | 371 | 54 |
| 55 | 7.11240    | 495 | 7.11297     | 495 | 7.36839    | 368 | 7.36940     | 369 | 55 |
| 56 | .11735     | 492 | .11792      | 493 | .37207     | 367 | .37310      | 368 | 56 |
| 57 | .12227     | 489 | .12285      | 490 | .37574     | 366 | .37678      | 366 | 57 |
| 58 | .12716     | 486 | .12775      | 487 | .37940     | 364 | .38044      | 365 | 58 |
| 59 | .13203     | 484 | .13262      | 484 | .38304     | 362 | .38409      | 363 | 59 |
| 60 | 7.13687    | 484 | 7.13746     | 484 | 7.38667    | 362 | 7.38773     | 363 | 60 |
|    | Log. Vers. | D   | Log. Exsec. | D   | Log. Vers. | D   | Log. Exsec. | D   |    |

TABLE VIII. — LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D   | Log. Exs. | D   | Lg. Vers. | D   | Log. Exs. | D   |    | P. P.          |
|----|-----------|-----|-----------|-----|-----------|-----|-----------|-----|----|----------------|
| 0  | 7.88667   | 361 | 7.38773   | 361 | 7.58039   | 289 | 7.58204   | 290 | 0  | 360 350        |
| 1  | .39028    | 359 | .39134    | 360 | .58328    | 287 | .58494    | 289 | 1  | 6 36.0 35.0    |
| 2  | .39387    | 358 | .39495    | 359 | .58615    | 287 | .58783    | 288 | 2  | 7 42.0 40.0    |
| 3  | .39745    | 356 | .39854    | 357 | .58902    | 286 | .59071    | 287 | 3  | 8 48.0 46.0    |
| 4  | .40102    | 355 | .40211    | 356 | .59188    | 285 | .59358    | 286 | 4  | 9 54.0 51.5    |
| 5  | 7.40457   | 353 | 7.40567   | 354 | 7.59473   | 284 | 7.59645   | 285 | 5  | 10 60.0 58.3   |
| 6  | .40810    | 352 | .40922    | 353 | .59758    | 283 | .59930    | 284 | 6  | 20 120.0 116.0 |
| 7  | .41163    | 350 | .41275    | 352 | .60041    | 282 | .60214    | 283 | 7  | 30 180.0 175.0 |
| 8  | .41513    | 349 | .41627    | 350 | .60323    | 281 | .60496    | 282 | 8  | 40 240.0 233.3 |
| 9  | .41863    | 348 | .41977    | 349 | .60604    | 280 | .60780    | 281 | 9  | 50 300.0 291.6 |
| 10 | 7.42211   | 346 | 7.42326   | 347 | 7.60885   | 279 | 7.61062   | 280 | 10 | 330 320        |
| 11 | .42557    | 345 | .42673    | 346 | .61164    | 279 | .61342    | 280 | 11 | 6 33.0 32.0    |
| 12 | .42903    | 343 | .43019    | 345 | .61443    | 277 | .61622    | 278 | 12 | 7 38.5 37.3    |
| 13 | .43246    | 342 | .43364    | 343 | .61721    | 277 | .61901    | 278 | 13 | 8 44.0 42.6    |
| 14 | .43589    | 341 | .43708    | 342 | .61998    | 276 | .62179    | 277 | 14 | 9 49.5 48.0    |
| 15 | 7.43930   | 339 | 7.44050   | 340 | 7.62274   | 275 | 7.62456   | 276 | 15 | 10 55.0 53.3   |
| 16 | .44270    | 338 | .44390    | 339 | .62549    | 274 | .62733    | 275 | 16 | 20 110.0 106.6 |
| 17 | .44608    | 337 | .44730    | 338 | .62823    | 273 | .63008    | 274 | 17 | 30 165.0 160.0 |
| 18 | .44946    | 335 | .45068    | 337 | .63096    | 272 | .63282    | 273 | 18 | 40 220.0 213.3 |
| 19 | .45281    | 334 | .45405    | 335 | .63369    | 272 | .63556    | 273 | 19 | 50 275.0 266.6 |
| 20 | 7.45616   | 333 | 7.45740   | 334 | 7.63641   | 270 | 7.63829   | 272 | 20 | 300 290        |
| 21 | .45949    | 332 | .46075    | 333 | .63911    | 270 | .64101    | 271 | 21 | 6 30.0 29.0    |
| 22 | .46281    | 330 | .46407    | 332 | .64181    | 269 | .64372    | 270 | 22 | 7 35.0 33.8    |
| 23 | .46612    | 329 | .46739    | 330 | .64451    | 268 | .64643    | 269 | 23 | 8 40.0 38.6    |
| 24 | .46941    | 328 | .47070    | 329 | .64719    | 267 | .64912    | 268 | 24 | 9 45.0 43.5    |
| 25 | 7.47270   | 327 | 7.47399   | 328 | 7.64986   | 266 | 7.65181   | 267 | 25 | 10 50.0 48.3   |
| 26 | .47597    | 325 | .47727    | 327 | .65253    | 266 | .65449    | 267 | 26 | 20 100.0 96.9  |
| 27 | .47922    | 324 | .48054    | 325 | .65519    | 265 | .65716    | 266 | 27 | 30 150.0 145.0 |
| 28 | .48247    | 323 | .48379    | 324 | .65784    | 264 | .65982    | 265 | 28 | 40 200.0 193.3 |
| 29 | .48570    | 322 | .48703    | 323 | .66048    | 263 | .66247    | 264 | 29 | 50 250.0 241.6 |
| 30 | 7.48892   | 321 | 7.49026   | 322 | 7.66311   | 263 | 7.66512   | 264 | 30 | 270 260        |
| 31 | .49213    | 320 | .49348    | 321 | .66574    | 261 | .66776    | 263 | 31 | 6 27.0 26.0    |
| 32 | .49533    | 318 | .49669    | 319 | .66836    | 261 | .67039    | 262 | 32 | 7 31.5 30.3    |
| 33 | .49852    | 317 | .49989    | 318 | .67097    | 260 | .67301    | 261 | 33 | 8 36.0 34.6    |
| 34 | .50169    | 316 | .50307    | 317 | .67357    | 259 | .67562    | 260 | 34 | 9 40.5 39.0    |
| 35 | 7.50485   | 315 | 7.50624   | 316 | 7.67617   | 258 | 7.67823   | 259 | 35 | 10 45.0 43.3   |
| 36 | .50800    | 314 | .50941    | 315 | .67875    | 258 | .68083    | 259 | 36 | 20 90.0 86.6   |
| 37 | .51114    | 313 | .51256    | 313 | .68133    | 257 | .68342    | 258 | 37 | 30 135.0 130.0 |
| 38 | .51427    | 311 | .51569    | 313 | .68390    | 256 | .68601    | 257 | 38 | 40 180.0 173.3 |
| 39 | .51739    | 311 | .51882    | 311 | .68647    | 255 | .68858    | 256 | 39 | 50 225.0 216.6 |
| 40 | 7.52050   | 309 | 7.52194   | 310 | 7.68902   | 255 | 7.69115   | 256 | 40 | 240 230        |
| 41 | .52359    | 308 | .52504    | 309 | .69157    | 254 | .69371    | 255 | 41 | 6 24.0 23.0    |
| 42 | .52667    | 307 | .52814    | 308 | .69411    | 253 | .69627    | 254 | 42 | 7 28.0 26.8    |
| 43 | .52975    | 306 | .53122    | 307 | .69665    | 252 | .69881    | 253 | 43 | 8 32.0 30.6    |
| 44 | .53281    | 305 | .53429    | 306 | .69917    | 252 | .70135    | 253 | 44 | 9 36.0 34.5    |
| 45 | 7.53586   | 304 | 7.53735   | 305 | 7.70169   | 251 | 7.70388   | 252 | 45 | 10 40.0 38.3   |
| 46 | .53890    | 303 | .54041    | 304 | .70421    | 250 | .70641    | 251 | 46 | 20 80.0 76.6   |
| 47 | .54193    | 302 | .54345    | 303 | .70671    | 250 | .70893    | 251 | 47 | 30 120.0 115.0 |
| 48 | .54495    | 300 | .54648    | 302 | .70921    | 249 | .71144    | 250 | 48 | 40 160.0 153.3 |
| 49 | .54796    | 300 | .54950    | 301 | .71170    | 248 | .71394    | 249 | 49 | 50 200.0 191.6 |
| 50 | 7.55096   | 299 | 7.55251   | 299 | 7.71418   | 247 | 7.71644   | 248 | 50 | 210 200        |
| 51 | .55395    | 297 | .55550    | 299 | .71666    | 247 | .71892    | 248 | 51 | 6 21.0 20.0    |
| 52 | .55692    | 297 | .55846    | 298 | .71913    | 246 | .72141    | 247 | 52 | 7 24.5 23.3    |
| 53 | .55989    | 295 | .56147    | 296 | .72159    | 245 | .72388    | 246 | 53 | 8 28.0 26.6    |
| 54 | .56285    | 295 | .56444    | 296 | .72404    | 245 | .72635    | 246 | 54 | 9 31.5 30.0    |
| 55 | 7.56580   | 293 | 7.56740   | 295 | 7.72649   | 244 | 7.72881   | 245 | 55 | 10 35.0 33.3   |
| 56 | .56873    | 293 | .57035    | 294 | .72893    | 243 | .73126    | 244 | 56 | 20 70.0 66.6   |
| 57 | .57166    | 292 | .57329    | 292 | .73137    | 242 | .73371    | 243 | 57 | 30 105.0 100.0 |
| 58 | .57458    | 290 | .57621    | 292 | .73379    | 242 | .73615    | 243 | 58 | 40 140.0 133.3 |
| 59 | .57749    | 290 | .57913    | 291 | .73621    | 241 | .73859    | 242 | 59 | 50 175.0 166.6 |
| 60 | 7.58039   | 290 | 7.58204   | 291 | 7.73863   | 241 | 7.74101   | 242 | 60 |                |
|    | Lg. Vers. | D   | Log. Exs. | D   | Lg. Vers. | D   | Log. Exs. | D   |    | P. P.          |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

6°

7°

|    | Lg. Vers. | D   | Log. Exs. | D   | Lg. Vers. | D   | Log. Exs. | D   |    | P. P.        |
|----|-----------|-----|-----------|-----|-----------|-----|-----------|-----|----|--------------|
| 0  | 7.73863   | 241 | 7.74101   | 242 | 7.87238   | 206 | 7.87563   | 208 | 0  | 180 9 9      |
| 1  | .74104    | 240 | .74343    | 241 | .87444    | 205 | .87771    | 207 | 1  | 6 18.0 0.9   |
| 2  | .74344    | 239 | .74585    | 241 | .87650    | 205 | .87978    | 207 | 2  | 7 21.0 1.1   |
| 3  | .74583    | 239 | .74826    | 240 | .87855    | 204 | .88185    | 206 | 3  | 8 24.0 1.2   |
| 4  | .74822    | 238 | .75066    | 239 | .88060    | 204 | .88391    | 206 | 4  | 9 27.0 1.4   |
| 5  | 7.75060   | 237 | 7.75305   | 239 | 7.88264   | 204 | 7.88597   | 205 | 5  | 10 30.0 1.6  |
| 6  | .75297    | 236 | .75544    | 238 | .88468    | 203 | .88803    | 205 | 6  | 20 60.0 3.1  |
| 7  | .75534    | 236 | .75782    | 237 | .88672    | 203 | .89008    | 204 | 7  | 30 90.0 4.7  |
| 8  | .75770    | 235 | .76019    | 237 | .88875    | 202 | .89212    | 204 | 8  | 40 120.0 6.3 |
| 9  | .76006    | 234 | .76256    | 236 | .89077    | 202 | .89416    | 203 | 9  | 50 150.0 7.9 |
| 10 | 7.76240   | 234 | 7.76492   | 235 | 7.89279   | 201 | 7.89620   | 203 | 10 | 8 8 7        |
| 11 | .76475    | 233 | .76728    | 235 | .89481    | 201 | .89823    | 202 | 11 | 6 0.8 0.8    |
| 12 | .76708    | 233 | .76963    | 234 | .89682    | 200 | .90025    | 202 | 12 | 7 1.0 0.9    |
| 13 | .76941    | 232 | .77197    | 233 | .89882    | 200 | .90228    | 201 | 13 | 8 1.1 1.0    |
| 14 | .77173    | 232 | .77431    | 233 | .90082    | 199 | .90429    | 201 | 14 | 9 1.3 1.2    |
| 15 | 7.77405   | 231 | 7.77664   | 232 | 7.90282   | 199 | 7.90630   | 201 | 15 | 10 1.4 1.3   |
| 16 | .77636    | 230 | .77897    | 231 | .90481    | 198 | .90831    | 200 | 16 | 20 2.8 2.6   |
| 17 | .77867    | 230 | .78128    | 231 | .90680    | 198 | .91032    | 199 | 17 | 30 4.2 4.0   |
| 18 | .78097    | 229 | .78360    | 230 | .90878    | 197 | .91231    | 199 | 18 | 40 5.6 5.3   |
| 19 | .78326    | 228 | .78590    | 230 | .91076    | 197 | .91431    | 199 | 19 | 50 7.1 6.6   |
| 20 | 7.78554   | 228 | 7.78820   | 229 | 7.91273   | 197 | 7.91630   | 198 | 20 | 7 6 6        |
| 21 | .78783    | 227 | .79050    | 229 | .91470    | 196 | .91828    | 198 | 21 | 6 0.7 0.6    |
| 22 | .79010    | 227 | .79279    | 228 | .91667    | 196 | .92027    | 197 | 22 | 7 0.8 0.7    |
| 23 | .79237    | 226 | .79507    | 228 | .91863    | 195 | .92224    | 197 | 23 | 8 0.9 0.8    |
| 24 | .79463    | 225 | .79735    | 227 | .92058    | 195 | .92421    | 197 | 24 | 9 1.0 1.0    |
| 25 | 7.79689   | 225 | 7.79962   | 226 | 7.92253   | 195 | 7.92618   | 197 | 25 | 10 1.1 1.1   |
| 26 | .79914    | 224 | .80188    | 226 | .92448    | 194 | .92815    | 196 | 26 | 20 2.3 2.0   |
| 27 | .80138    | 224 | .80414    | 225 | .92642    | 194 | .93010    | 195 | 27 | 30 3.5 3.2   |
| 28 | .80362    | 223 | .80639    | 225 | .92836    | 193 | .93206    | 195 | 28 | 40 4.6 4.4   |
| 29 | .80586    | 222 | .80864    | 224 | .93029    | 193 | .93401    | 195 | 29 | 50 5.8 5.5   |
| 30 | 7.80808   | 222 | 7.81088   | 224 | 7.93222   | 192 | 7.93596   | 194 | 30 | 5 5 4        |
| 31 | .81031    | 221 | .81312    | 224 | .93415    | 192 | .93790    | 194 | 31 | 6 0.5 0.5    |
| 32 | .81252    | 221 | .81535    | 223 | .93607    | 191 | .93984    | 193 | 32 | 7 0.6 0.6    |
| 33 | .81473    | 220 | .81758    | 222 | .93799    | 191 | .94177    | 193 | 33 | 8 0.7 0.7    |
| 34 | .81694    | 220 | .81980    | 221 | .93990    | 190 | .94370    | 192 | 34 | 9 0.8 0.7    |
| 35 | 7.81914   | 219 | 7.82201   | 221 | 7.94181   | 190 | 7.94562   | 192 | 35 | 10 0.9 0.8   |
| 36 | .82133    | 219 | .82422    | 220 | .94371    | 190 | .94754    | 192 | 36 | 20 1.8 1.5   |
| 37 | .82352    | 218 | .82642    | 219 | .94561    | 189 | .94946    | 191 | 37 | 30 2.7 2.2   |
| 38 | .82570    | 217 | .82862    | 219 | .94751    | 189 | .95137    | 191 | 38 | 40 3.6 3.3   |
| 39 | .82788    | 217 | .83081    | 219 | .94940    | 189 | .95328    | 191 | 39 | 50 4.6 4.4   |
| 40 | 7.83005   | 217 | 7.83300   | 219 | 7.95129   | 188 | 7.95519   | 190 | 40 | 3 3 2        |
| 41 | .83222    | 216 | .83518    | 218 | .95317    | 187 | .95709    | 189 | 41 | 6 0.3 0.3    |
| 42 | .83438    | 215 | .83735    | 217 | .95505    | 187 | .95898    | 189 | 42 | 7 0.4 0.3    |
| 43 | .83653    | 215 | .83952    | 216 | .95693    | 187 | .96088    | 188 | 43 | 8 0.4 0.4    |
| 44 | .83868    | 214 | .84169    | 216 | .95880    | 186 | .96276    | 188 | 44 | 9 0.5 0.4    |
| 45 | 7.84083   | 214 | 7.84385   | 215 | 7.96066   | 186 | 7.96465   | 188 | 45 | 10 0.6 0.5   |
| 46 | .84297    | 213 | .84600    | 215 | .96253    | 186 | .96653    | 188 | 46 | 20 1.1 1.0   |
| 47 | .84510    | 213 | .84815    | 214 | .96439    | 185 | .96841    | 187 | 47 | 30 1.7 1.5   |
| 48 | .84723    | 212 | .85030    | 213 | .96624    | 185 | .97028    | 187 | 48 | 40 2.3 2.0   |
| 49 | .84935    | 212 | .85243    | 213 | .96809    | 184 | .97215    | 186 | 49 | 50 2.9 2.5   |
| 50 | 7.85147   | 211 | 7.85457   | 213 | 7.96994   | 184 | 7.97401   | 186 | 50 | 1 1 0        |
| 51 | .85359    | 211 | .85670    | 212 | .97178    | 184 | .97587    | 185 | 51 | 6 0.1 0.1    |
| 52 | .85570    | 210 | .85882    | 211 | .97362    | 183 | .97773    | 185 | 52 | 7 0.2 0.1    |
| 53 | .85780    | 210 | .86094    | 211 | .97546    | 183 | .97958    | 184 | 53 | 8 0.2 0.1    |
| 54 | .85990    | 209 | .86305    | 211 | .97729    | 183 | .98143    | 184 | 54 | 9 0.2 0.1    |
| 55 | 7.86199   | 209 | 7.86516   | 210 | 7.97912   | 182 | 7.98327   | 184 | 55 | 10 0.2 0.1   |
| 56 | .86408    | 208 | .86726    | 210 | .98094    | 182 | .98512    | 183 | 56 | 20 0.5 0.3   |
| 57 | .86616    | 208 | .86936    | 209 | .98276    | 182 | .98695    | 183 | 57 | 30 0.7 0.5   |
| 58 | .86824    | 207 | .87146    | 208 | .98458    | 181 | .98879    | 183 | 58 | 40 1.0 0.6   |
| 59 | .87031    | 206 | .87354    | 208 | .98639    | 181 | .99062    | 182 | 59 | 50 1.2 0.8   |
| 60 | 7.87238   | 206 | 7.87563   | 208 | 7.98820   | 181 | 7.99244   | 182 | 60 | 6 0.1 0.1    |
|    | Lg. Vers. | D   | Log. Exs. | D   | Lg. Vers. | D   | Log. Exs. | D   |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

8°

9°

| '  | 8°        |     |           |     | 9°        |     |           |     | '  | P. P. |       |       |       |
|----|-----------|-----|-----------|-----|-----------|-----|-----------|-----|----|-------|-------|-------|-------|
|    | Lg. Vers. | D   | Log. Exs. | D   | Lg. Vers. | D   | Log. Exs. | D   |    | 150   | 170   | 190   | 210   |
| 0  | 7.98820   | 180 | 7.99244   | 182 | 8.09031   | 160 | 8.09569   | 162 | 0  | 6     | 18.0  | 17.0  | 16.0  |
| 1  | .99000    | 180 | .99427    | 182 | .09192    | 160 | .09732    | 162 | 1  | 7     | 21.0  | 19.8  | 18.6  |
| 2  | .99180    | 179 | .99609    | 181 | .09352    | 160 | .09894    | 162 | 2  | 8     | 24.0  | 22.6  | 21.4  |
| 3  | .99360    | 179 | .99790    | 181 | .09512    | 160 | .10056    | 162 | 3  | 9     | 27.0  | 25.5  | 24.3  |
| 4  | .99539    | 179 | 7.99971   | 180 | .09671    | 159 | .10217    | 161 | 4  | 10    | 30.0  | 28.3  | 27.1  |
| 5  | 7.99718   | 178 | 8.00152   | 180 | 8.09830   | 159 | 8.10378   | 161 | 5  | 20    | 60.0  | 56.6  | 53.2  |
| 6  | 7.99897   | 178 | .00332    | 180 | .09989    | 159 | .10539    | 161 | 6  | 30    | 90.0  | 85.0  | 80.0  |
| 7  | 8.00075   | 177 | .00512    | 180 | .10148    | 158 | .10700    | 160 | 7  | 40    | 120.0 | 113.3 | 106.7 |
| 8  | .00253    | 178 | .00692    | 179 | .10306    | 158 | .10860    | 160 | 8  | 50    | 150.0 | 141.6 | 133.4 |
| 9  | .00431    | 177 | .00871    | 178 | .10464    | 157 | .11020    | 160 | 9  |       |       |       |       |
| 10 | 8.00608   | 176 | 8.01050   | 178 | 8.10622   | 157 | 8.11180   | 160 | 10 |       | 150   | 140   |       |
| 11 | .00784    | 176 | .01229    | 178 | .10779    | 157 | .11340    | 159 | 11 | 6     | 15.0  | 14.0  |       |
| 12 | .00961    | 176 | .01407    | 178 | .10936    | 157 | .11499    | 159 | 12 | 7     | 17.5  | 16.3  |       |
| 13 | .01137    | 176 | .01585    | 177 | .11093    | 156 | .11658    | 158 | 13 | 8     | 20.0  | 18.6  |       |
| 14 | .01313    | 175 | .01763    | 177 | .11250    | 156 | .11816    | 158 | 14 | 9     | 22.5  | 21.0  |       |
| 15 | 8.01488   | 175 | 8.01940   | 177 | 8.11406   | 156 | 8.11975   | 158 | 15 | 10    | 25.0  | 23.3  |       |
| 16 | .01663    | 175 | .02117    | 176 | .11562    | 155 | .12133    | 158 | 16 | 20    | 50.0  | 46.6  |       |
| 17 | .01838    | 174 | .02293    | 176 | .11718    | 155 | .12291    | 157 | 17 | 30    | 75.0  | 70.0  |       |
| 18 | .02012    | 174 | .02469    | 175 | .11873    | 155 | .12448    | 157 | 18 | 40    | 100.0 | 93.3  |       |
| 19 | .02186    | 173 | .02645    | 175 | .12029    | 155 | .12605    | 157 | 19 | 50    | 125.0 | 116.6 |       |
| 20 | 8.02359   | 173 | 8.02820   | 175 | 8.12184   | 154 | 8.12762   | 157 | 20 |       | 9     | 9     | 8     |
| 21 | .02533    | 173 | .02995    | 175 | .12338    | 154 | .12919    | 157 | 21 | 6     | 0.9   | 0.9   | 0.8   |
| 22 | .02706    | 172 | .03170    | 174 | .12492    | 154 | .13075    | 156 | 22 | 7     | 1.1   | 1.0   | 1.0   |
| 23 | .02878    | 172 | .03345    | 174 | .12647    | 153 | .13232    | 156 | 23 | 8     | 1.2   | 1.2   | 1.1   |
| 24 | .03050    | 172 | .03519    | 173 | .12800    | 153 | .13387    | 155 | 24 | 9     | 1.4   | 1.3   | 1.3   |
| 25 | 8.03222   | 171 | 8.03692   | 173 | 8.12954   | 153 | 8.13543   | 156 | 25 | 10    | 1.6   | 1.5   | 1.4   |
| 26 | .03394    | 171 | .03866    | 173 | .13107    | 153 | .13698    | 155 | 26 | 20    | 3.1   | 3.0   | 2.8   |
| 27 | .03565    | 171 | .04039    | 173 | .13260    | 152 | .13854    | 154 | 27 | 30    | 4.7   | 4.5   | 4.2   |
| 28 | .03736    | 170 | .04212    | 172 | .13413    | 152 | .14008    | 154 | 28 | 40    | 6.3   | 6.0   | 5.6   |
| 29 | .03906    | 170 | .04384    | 172 | .13565    | 152 | .14163    | 154 | 29 | 50    | 7.9   | 7.5   | 7.1   |
| 30 | 8.04076   | 169 | 8.04556   | 172 | 8.13717   | 152 | 8.14317   | 154 | 30 |       | 8     | 7     | 7     |
| 31 | .04246    | 169 | .04728    | 171 | .13869    | 152 | .14471    | 153 | 31 | 6     | 0.8   | 0.7   | 0.7   |
| 32 | .04416    | 169 | .04899    | 171 | .14021    | 151 | .14625    | 153 | 32 | 7     | 0.9   | 0.9   | 0.8   |
| 33 | .04585    | 169 | .05070    | 170 | .14172    | 151 | .14778    | 153 | 33 | 8     | 1.0   | 1.0   | 0.9   |
| 34 | .04754    | 168 | .05241    | 170 | .14323    | 151 | .14932    | 153 | 34 | 9     | 1.2   | 1.1   | 1.0   |
| 35 | 8.04922   | 168 | 8.05411   | 170 | 8.14474   | 151 | 8.15085   | 153 | 35 | 10    | 1.3   | 1.2   | 1.1   |
| 36 | .05090    | 168 | .05581    | 170 | .14625    | 150 | .15237    | 152 | 36 | 20    | 2.6   | 2.5   | 2.3   |
| 37 | .05258    | 167 | .05751    | 169 | .14775    | 150 | .15390    | 152 | 37 | 30    | 4.0   | 3.7   | 3.5   |
| 38 | .05426    | 167 | .05921    | 169 | .14925    | 149 | .15542    | 152 | 38 | 40    | 5.3   | 5.0   | 4.6   |
| 39 | .05593    | 167 | .06090    | 169 | .15075    | 150 | .15694    | 152 | 39 | 50    | 6.6   | 6.2   | 5.8   |
| 40 | 8.05760   | 166 | 8.06259   | 168 | 8.15225   | 149 | 8.15846   | 152 | 40 |       | 6     | 6     |       |
| 41 | .05926    | 166 | .06427    | 168 | .15374    | 149 | .15997    | 151 | 41 | 6     | 0.6   | 0.6   |       |
| 42 | .06093    | 166 | .06595    | 168 | .15523    | 149 | .16148    | 151 | 42 | 7     | 0.7   | 0.7   |       |
| 43 | .06259    | 165 | .06763    | 167 | .15672    | 148 | .16299    | 151 | 43 | 8     | 0.8   | 0.8   |       |
| 44 | .06424    | 165 | .06931    | 167 | .15820    | 148 | .16450    | 150 | 44 | 9     | 1.0   | 0.9   |       |
| 45 | 8.06589   | 165 | 8.07098   | 167 | 8.15968   | 148 | 8.16600   | 150 | 45 | 10    | 1.1   | 1.0   |       |
| 46 | .06754    | 165 | .07265    | 166 | .16116    | 148 | .16750    | 150 | 46 | 20    | 2.1   | 2.0   |       |
| 47 | .06919    | 164 | .07431    | 166 | .16264    | 147 | .16900    | 149 | 47 | 30    | 3.2   | 3.0   |       |
| 48 | .07083    | 164 | .07598    | 166 | .16412    | 147 | .17050    | 149 | 48 | 40    | 4.3   | 4.0   |       |
| 49 | .07247    | 164 | .07764    | 165 | .16559    | 147 | .17199    | 149 | 49 | 50    | 5.4   | 5.0   |       |
| 50 | 8.07411   | 163 | 8.07929   | 165 | 8.16706   | 146 | 8.17349   | 148 | 50 |       | 5     | 5     |       |
| 51 | .07575    | 163 | .08095    | 165 | .16852    | 146 | .17497    | 148 | 51 | 6     | 0.5   | 0.5   |       |
| 52 | .07738    | 162 | .08260    | 164 | .16999    | 146 | .17646    | 148 | 52 | 7     | 0.6   | 0.6   |       |
| 53 | .07900    | 162 | .08424    | 164 | .17145    | 146 | .17795    | 148 | 53 | 8     | 0.7   | 0.7   |       |
| 54 | .08063    | 162 | .08589    | 164 | .17291    | 145 | .17943    | 148 | 54 | 9     | 0.8   | 0.8   |       |
| 55 | 8.08225   | 161 | 8.08753   | 163 | 8.17437   | 145 | 8.18091   | 148 | 55 | 10    | 0.9   | 0.8   |       |
| 56 | .08387    | 162 | .08917    | 164 | .17582    | 145 | .18238    | 147 | 56 | 20    | 1.8   | 1.6   |       |
| 57 | .08549    | 161 | .09081    | 163 | .17728    | 145 | .18386    | 147 | 57 | 30    | 2.7   | 2.5   |       |
| 58 | .08710    | 161 | .09244    | 163 | .17873    | 144 | .18533    | 147 | 58 | 40    | 3.6   | 3.3   |       |
| 59 | .08871    | 160 | .09407    | 162 | .18017    | 144 | .18680    | 146 | 59 | 50    | 4.6   | 4.1   |       |
| 60 | 8.09031   |     | 8.09569   |     | 8.18162   |     | 8.18827   |     | 60 |       |       |       |       |
| '  | 8°        |     |           |     | 9°        |     |           |     | '  | P. P. |       |       |       |
|    | Lg. Vers. | D   | Log. Exs. | D   | Lg. Vers. | D   | Log. Exs. | D   |    | 150   | 170   | 190   | 210   |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

| 10°       |         |     |         |     |         |     |         |     |    | 11°   |  |  |  |  |  |  |  |  |  |
|-----------|---------|-----|---------|-----|---------|-----|---------|-----|----|-------|--|--|--|--|--|--|--|--|--|
|           |         |     |         |     |         |     |         |     |    |       |  |  |  |  |  |  |  |  |  |
| Lg. Vers. |         |     |         |     |         |     |         |     |    | P. P. |  |  |  |  |  |  |  |  |  |
| D         |         |     |         |     |         |     |         |     |    |       |  |  |  |  |  |  |  |  |  |
| 0         | 8.18162 | 144 | 8.18827 | 146 | 8.26417 | 131 | 8.27223 | 133 | 0  |       |  |  |  |  |  |  |  |  |  |
| 1         | .18306  | 144 | .18973  | 146 | .26548  | 131 | .27356  | 133 | 1  |       |  |  |  |  |  |  |  |  |  |
| 2         | .18450  | 144 | .19120  | 146 | .26679  | 131 | .27490  | 133 | 2  |       |  |  |  |  |  |  |  |  |  |
| 3         | .18594  | 143 | .19266  | 145 | .26810  | 131 | .27623  | 133 | 3  |       |  |  |  |  |  |  |  |  |  |
| 4         | .18738  | 143 | .19411  | 145 | .26941  | 130 | .27756  | 133 | 4  |       |  |  |  |  |  |  |  |  |  |
| 5         | 8.18881 | 143 | 8.19557 | 145 | 8.27071 | 130 | 8.27889 | 132 | 5  |       |  |  |  |  |  |  |  |  |  |
| 6         | .19024  | 142 | .19702  | 145 | .27201  | 130 | .28021  | 132 | 6  |       |  |  |  |  |  |  |  |  |  |
| 7         | .19167  | 142 | .19847  | 145 | .27331  | 130 | .28153  | 132 | 7  |       |  |  |  |  |  |  |  |  |  |
| 8         | .19309  | 142 | .19992  | 144 | .27461  | 129 | .28286  | 132 | 8  |       |  |  |  |  |  |  |  |  |  |
| 9         | .19452  | 142 | .20137  | 144 | .27590  | 129 | .28418  | 132 | 9  |       |  |  |  |  |  |  |  |  |  |
| 10        | 8.19594 | 142 | 8.20281 | 144 | 8.27719 | 129 | 8.28550 | 132 | 10 |       |  |  |  |  |  |  |  |  |  |
| 11        | .19736  | 142 | .20425  | 144 | .27849  | 128 | .28681  | 131 | 11 |       |  |  |  |  |  |  |  |  |  |
| 12        | .19878  | 141 | .20569  | 144 | .27977  | 129 | .28813  | 131 | 12 |       |  |  |  |  |  |  |  |  |  |
| 13        | .20019  | 141 | .20713  | 143 | .28106  | 128 | .28944  | 131 | 13 |       |  |  |  |  |  |  |  |  |  |
| 14        | .20160  | 141 | .20857  | 143 | .28235  | 128 | .29075  | 131 | 14 |       |  |  |  |  |  |  |  |  |  |
| 15        | 8.20301 | 140 | 8.21003 | 143 | 8.28363 | 128 | 8.29206 | 130 | 15 |       |  |  |  |  |  |  |  |  |  |
| 16        | .20442  | 140 | .21143  | 143 | .28491  | 128 | .29336  | 130 | 16 |       |  |  |  |  |  |  |  |  |  |
| 17        | .20582  | 140 | .21286  | 142 | .28619  | 128 | .29467  | 130 | 17 |       |  |  |  |  |  |  |  |  |  |
| 18        | .20723  | 140 | .21428  | 142 | .28747  | 127 | .29597  | 130 | 18 |       |  |  |  |  |  |  |  |  |  |
| 19        | .20863  | 140 | .21571  | 142 | .28875  | 127 | .29727  | 130 | 19 |       |  |  |  |  |  |  |  |  |  |
| 20        | 8.21003 | 139 | 8.21713 | 142 | 8.29012 | 127 | 8.29857 | 130 | 20 |       |  |  |  |  |  |  |  |  |  |
| 1         | .21142  | 139 | .21855  | 141 | .29129  | 127 | .29987  | 129 | 21 |       |  |  |  |  |  |  |  |  |  |
| 2         | .21282  | 139 | .21996  | 141 | .29256  | 127 | .30117  | 129 | 22 |       |  |  |  |  |  |  |  |  |  |
| 3         | .21421  | 139 | .22138  | 141 | .29383  | 126 | .30246  | 129 | 23 |       |  |  |  |  |  |  |  |  |  |
| 4         | .21560  | 138 | .22279  | 141 | .29510  | 126 | .30375  | 129 | 24 |       |  |  |  |  |  |  |  |  |  |
| 5         | 8.21698 | 138 | 8.22420 | 141 | 8.29636 | 125 | 8.30504 | 129 | 25 |       |  |  |  |  |  |  |  |  |  |
| 6         | .21837  | 138 | .22561  | 140 | .29763  | 126 | .30633  | 129 | 26 |       |  |  |  |  |  |  |  |  |  |
| 7         | .21975  | 138 | .22701  | 140 | .29889  | 126 | .30762  | 128 | 27 |       |  |  |  |  |  |  |  |  |  |
| 8         | .22113  | 137 | .22842  | 140 | .30015  | 125 | .30890  | 128 | 28 |       |  |  |  |  |  |  |  |  |  |
| 9         | .22251  | 137 | .22982  | 140 | .30140  | 125 | .31019  | 128 | 29 |       |  |  |  |  |  |  |  |  |  |
| 10        | 8.22389 | 137 | 8.23122 | 140 | 8.30266 | 125 | 8.31147 | 128 | 30 |       |  |  |  |  |  |  |  |  |  |
| 1         | .22526  | 137 | .23262  | 140 | .30391  | 125 | .31275  | 128 | 31 |       |  |  |  |  |  |  |  |  |  |
| 2         | .22663  | 137 | .23401  | 139 | .30516  | 125 | .31402  | 127 | 32 |       |  |  |  |  |  |  |  |  |  |
| 3         | .22800  | 136 | .23540  | 139 | .30642  | 125 | .31530  | 127 | 33 |       |  |  |  |  |  |  |  |  |  |
| 4         | .22937  | 137 | .23679  | 139 | .30766  | 124 | .31657  | 127 | 34 |       |  |  |  |  |  |  |  |  |  |
| 5         | 8.23073 | 136 | 8.23818 | 139 | 8.30891 | 124 | 8.31785 | 127 | 35 |       |  |  |  |  |  |  |  |  |  |
| 6         | .23209  | 136 | .23957  | 138 | .31015  | 124 | .31912  | 127 | 36 |       |  |  |  |  |  |  |  |  |  |
| 7         | .23346  | 136 | .24095  | 138 | .31140  | 124 | .32039  | 126 | 37 |       |  |  |  |  |  |  |  |  |  |
| 8         | .23481  | 135 | .24234  | 138 | .31264  | 124 | .32165  | 126 | 38 |       |  |  |  |  |  |  |  |  |  |
| 9         | .23617  | 136 | .24372  | 137 | .31388  | 123 | .32292  | 126 | 39 |       |  |  |  |  |  |  |  |  |  |
| 10        | 8.23752 | 135 | 8.24509 | 137 | 8.31511 | 124 | 8.32418 | 126 | 40 |       |  |  |  |  |  |  |  |  |  |
| 1         | .23888  | 135 | .24647  | 137 | .31635  | 123 | .32544  | 126 | 41 |       |  |  |  |  |  |  |  |  |  |
| 2         | .24023  | 135 | .24784  | 137 | .31758  | 123 | .32670  | 126 | 42 |       |  |  |  |  |  |  |  |  |  |
| 3         | .24158  | 135 | .24922  | 137 | .31882  | 123 | .32796  | 125 | 43 |       |  |  |  |  |  |  |  |  |  |
| 4         | .24292  | 134 | .25059  | 137 | .32005  | 123 | .32922  | 125 | 44 |       |  |  |  |  |  |  |  |  |  |
| 5         | 8.24426 | 134 | 8.25195 | 136 | 8.32128 | 123 | 8.33047 | 125 | 45 |       |  |  |  |  |  |  |  |  |  |
| 6         | .24561  | 134 | .25332  | 136 | .32250  | 122 | .33173  | 125 | 46 |       |  |  |  |  |  |  |  |  |  |
| 7         | .24695  | 133 | .25468  | 136 | .32373  | 122 | .33298  | 125 | 47 |       |  |  |  |  |  |  |  |  |  |
| 8         | .24828  | 133 | .25604  | 136 | .32495  | 122 | .33423  | 125 | 48 |       |  |  |  |  |  |  |  |  |  |
| 9         | .24962  | 133 | .25740  | 136 | .32617  | 122 | .33547  | 124 | 49 |       |  |  |  |  |  |  |  |  |  |
| 10        | 8.25095 | 133 | 8.25876 | 135 | 8.32739 | 122 | 8.33672 | 124 | 50 |       |  |  |  |  |  |  |  |  |  |
| 1         | .25228  | 133 | .26012  | 135 | .32861  | 121 | .33797  | 124 | 51 |       |  |  |  |  |  |  |  |  |  |
| 2         | .25361  | 132 | .26147  | 135 | .32983  | 121 | .33921  | 124 | 52 |       |  |  |  |  |  |  |  |  |  |
| 3         | .25494  | 133 | .26282  | 135 | .33104  | 121 | .34045  | 123 | 53 |       |  |  |  |  |  |  |  |  |  |
| 4         | .25627  | 132 | .26417  | 134 | .33225  | 121 | .34169  | 124 | 54 |       |  |  |  |  |  |  |  |  |  |
| 5         | 8.25759 | 132 | 8.26552 | 134 | 8.33347 | 121 | 8.34293 | 124 | 55 |       |  |  |  |  |  |  |  |  |  |
| 6         | .25891  | 132 | .26686  | 134 | .33468  | 120 | .34417  | 123 | 56 |       |  |  |  |  |  |  |  |  |  |
| 7         | .26023  | 132 | .26821  | 134 | .33588  | 120 | .34540  | 123 | 57 |       |  |  |  |  |  |  |  |  |  |
| 8         | .26155  | 131 | .26955  | 134 | .33709  | 120 | .34663  | 123 | 58 |       |  |  |  |  |  |  |  |  |  |
| 9         | .26286  | 131 | .27089  | 134 | .33829  | 120 | .34786  | 123 | 59 |       |  |  |  |  |  |  |  |  |  |
| 10        | 8.26417 | 131 | 8.27223 | 134 | 8.33950 | 120 | 8.34909 | 123 | 60 |       |  |  |  |  |  |  |  |  |  |
| Lg. Vers. |         |     |         |     |         |     |         |     |    | P. P. |  |  |  |  |  |  |  |  |  |
| D         |         |     |         |     |         |     |         |     |    |       |  |  |  |  |  |  |  |  |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

| 12° |           |     |           |     |         | 13°       |         |           |     |     |       |      |      |      |       |      |       |
|-----|-----------|-----|-----------|-----|---------|-----------|---------|-----------|-----|-----|-------|------|------|------|-------|------|-------|
|     |           |     | 12°       |     |         |           |         |           | 13° |     |       |      |      |      | P. P. |      |       |
|     | Lg. Vers. | D   | Log. Exs. | D   |         | Lg. Vers. | D       | Log. Exs. | D   |     |       |      |      |      |       |      |       |
| 0   | 8.33950   | 120 | 8.34909   | 123 | 8.40875 | 110       | 8.42002 | 113       | 0   | 120 | 119   | 118  | 117  | 116  | 115   | 114  | 113   |
| 1   | .34070    | 120 | .35032    | 122 | .40985  | 110       | .42116  | 113       | 1   | 6   | 12.0  | 11.9 | 11.8 | 11.7 | 11.6  | 11.5 | 11.4  |
| 2   | .34190    | 119 | .35155    | 122 | .41096  | 110       | .42229  | 113       | 2   | 7   | 14.0  | 13.9 | 13.8 | 13.7 | 13.6  | 13.5 | 13.4  |
| 3   | .34309    | 119 | .35277    | 122 | .41206  | 110       | .42343  | 113       | 3   | 8   | 16.0  | 15.8 | 15.7 | 15.6 | 15.5  | 15.4 | 15.3  |
| 4   | .34429    | 120 | .35399    | 122 | .41317  | 110       | .42456  | 113       | 4   | 9   | 18.0  | 17.8 | 17.7 | 17.6 | 17.5  | 17.4 | 17.3  |
| 5   | 8.34549   | 119 | 8.35522   | 122 | 8.41427 | 110       | 8.42569 | 113       | 5   | 10  | 20.0  | 19.8 | 19.7 | 19.6 | 19.5  | 19.4 | 19.3  |
| 6   | .34668    | 119 | .35644    | 121 | .41537  | 110       | .42682  | 113       | 6   | 20  | 40.0  | 39.6 | 39.5 | 39.4 | 39.3  | 39.2 | 39.1  |
| 7   | .34787    | 119 | .35765    | 122 | .41647  | 109       | .42795  | 113       | 7   | 30  | 60.0  | 59.5 | 59.4 | 59.3 | 59.2  | 59.1 | 59.0  |
| 8   | .34906    | 119 | .35887    | 121 | .41757  | 110       | .42908  | 112       | 8   | 40  | 80.0  | 79.3 | 79.2 | 79.1 | 79.0  | 78.9 | 78.8  |
| 9   | .35025    | 119 | .36009    | 121 | .41867  | 109       | .43021  | 112       | 9   | 50  | 100.0 | 99.1 | 99.0 | 98.9 | 98.8  | 98.7 | 98.6  |
| 10  | 8.35143   | 118 | 8.36130   | 121 | 8.41976 | 109       | 8.43133 | 112       | 10  | 117 | 116   | 115  | 114  | 113  | 112   | 111  | 110   |
| 11  | .35262    | 118 | .36251    | 121 | .42086  | 109       | .43246  | 112       | 11  | 6   | 11.7  | 11.6 | 11.5 | 11.4 | 11.3  | 11.2 | 11.1  |
| 12  | .35380    | 118 | .36372    | 121 | .42195  | 109       | .43358  | 112       | 12  | 7   | 13.6  | 13.5 | 13.4 | 13.3 | 13.2  | 13.1 | 13.0  |
| 13  | .35498    | 118 | .36493    | 121 | .42304  | 109       | .43470  | 112       | 13  | 8   | 15.6  | 15.5 | 15.4 | 15.3 | 15.2  | 15.1 | 15.0  |
| 14  | .35616    | 117 | .36614    | 120 | .42413  | 109       | .43582  | 112       | 14  | 9   | 17.5  | 17.4 | 17.3 | 17.2 | 17.1  | 17.0 | 16.9  |
| 15  | 8.35734   | 118 | 8.36734   | 120 | 8.42522 | 108       | 8.43694 | 111       | 15  | 10  | 19.5  | 19.3 | 19.2 | 19.1 | 19.0  | 18.9 | 18.8  |
| 16  | .35852    | 117 | .36855    | 120 | .42630  | 109       | .43805  | 111       | 16  | 20  | 39.0  | 38.6 | 38.5 | 38.4 | 38.3  | 38.2 | 38.1  |
| 17  | .35969    | 117 | .36975    | 120 | .42739  | 108       | .43917  | 111       | 17  | 30  | 58.5  | 58.0 | 57.9 | 57.8 | 57.7  | 57.6 | 57.5  |
| 18  | .36086    | 117 | .37095    | 120 | .42847  | 108       | .44028  | 111       | 18  | 40  | 78.0  | 77.3 | 77.2 | 77.1 | 77.0  | 76.9 | 76.8  |
| 19  | .36204    | 117 | .37215    | 120 | .42956  | 108       | .44139  | 111       | 19  | 50  | 97.5  | 96.6 | 96.5 | 96.4 | 96.3  | 96.2 | 96.1  |
| 20  | 8.36321   | 116 | 8.37335   | 119 | 8.43064 | 108       | 8.44251 | 111       | 20  | 114 | 113   | 112  | 111  | 110  | 109   | 108  | 107   |
| 21  | .36437    | 117 | .37454    | 119 | .43172  | 108       | .44362  | 111       | 21  | 6   | 11.4  | 11.3 | 11.2 | 11.1 | 11.0  | 10.9 | 10.8  |
| 22  | .36554    | 116 | .37574    | 119 | .43280  | 108       | .44473  | 110       | 22  | 7   | 13.3  | 13.2 | 13.1 | 13.0 | 12.9  | 12.8 | 12.7  |
| 23  | .36671    | 116 | .37693    | 119 | .43388  | 107       | .44583  | 110       | 23  | 8   | 15.2  | 15.0 | 14.9 | 14.8 | 14.7  | 14.6 | 14.5  |
| 24  | .36787    | 116 | .37812    | 119 | .43495  | 107       | .44694  | 110       | 24  | 9   | 17.1  | 16.0 | 15.9 | 15.8 | 15.7  | 15.6 | 15.5  |
| 25  | 8.36903   | 116 | 8.37931   | 118 | 8.43603 | 107       | 8.44804 | 110       | 25  | 10  | 19.0  | 18.8 | 18.7 | 18.6 | 18.5  | 18.4 | 18.3  |
| 26  | .37019    | 116 | .38050    | 119 | .43710  | 107       | .44915  | 110       | 26  | 20  | 38.0  | 37.6 | 37.5 | 37.4 | 37.3  | 37.2 | 37.1  |
| 27  | .37135    | 115 | .38169    | 118 | .43817  | 107       | .45025  | 110       | 27  | 30  | 57.0  | 56.5 | 56.4 | 56.3 | 56.2  | 56.1 | 56.0  |
| 28  | .37251    | 115 | .38287    | 118 | .43924  | 107       | .45135  | 109       | 28  | 40  | 76.0  | 75.3 | 75.2 | 75.1 | 75.0  | 74.9 | 74.8  |
| 29  | .37366    | 115 | .38406    | 118 | .44031  | 106       | .45245  | 110       | 29  | 50  | 95.0  | 94.1 | 94.0 | 93.9 | 93.8  | 93.7 | 93.6  |
| 30  | 8.37482   | 115 | 8.38524   | 118 | 8.44138 | 107       | 8.45355 | 110       | 30  | 111 | 110   | 109  | 108  | 107  | 106   | 105  | 104   |
| 31  | .37597    | 115 | .38642    | 118 | .44245  | 106       | .45465  | 110       | 31  | 6   | 11.1  | 11.0 | 10.9 | 10.8 | 10.7  | 10.6 | 10.5  |
| 32  | .37712    | 115 | .38760    | 118 | .44351  | 106       | .45574  | 109       | 32  | 7   | 12.9  | 12.8 | 12.7 | 12.6 | 12.5  | 12.4 | 12.3  |
| 33  | .37827    | 115 | .38877    | 117 | .44458  | 106       | .45684  | 109       | 33  | 8   | 14.8  | 14.6 | 14.5 | 14.4 | 14.3  | 14.2 | 14.1  |
| 34  | .37942    | 114 | .38995    | 117 | .44564  | 106       | .45793  | 109       | 34  | 9   | 16.6  | 16.5 | 16.4 | 16.3 | 16.2  | 16.1 | 16.0  |
| 35  | 8.38057   | 114 | 8.39113   | 117 | 8.44670 | 106       | 8.45902 | 109       | 35  | 10  | 18.5  | 18.3 | 18.2 | 18.1 | 18.0  | 17.9 | 17.8  |
| 36  | .38171    | 114 | .39230    | 117 | .44776  | 105       | .46011  | 109       | 36  | 20  | 37.0  | 36.6 | 36.5 | 36.4 | 36.3  | 36.2 | 36.1  |
| 37  | .38286    | 114 | .39347    | 117 | .44882  | 106       | .46120  | 108       | 37  | 30  | 55.5  | 55.0 | 54.9 | 54.8 | 54.7  | 54.6 | 54.5  |
| 38  | .38400    | 114 | .39444    | 117 | .44988  | 105       | .46229  | 109       | 38  | 40  | 74.0  | 73.3 | 73.2 | 73.1 | 73.0  | 72.9 | 72.8  |
| 39  | .38514    | 114 | .39591    | 116 | .45093  | 105       | .46338  | 108       | 39  | 50  | 92.5  | 91.6 | 91.5 | 91.4 | 91.3  | 91.2 | 91.1  |
| 40  | 8.38628   | 113 | 8.39698   | 116 | 8.45199 | 105       | 8.46446 | 108       | 40  | 108 | 107   | 106  | 105  | 104  | 103   | 102  | 101   |
| 41  | .38741    | 114 | .39814    | 116 | .45304  | 105       | .46555  | 108       | 41  | 6   | 10.8  | 10.7 | 10.6 | 10.5 | 10.4  | 10.3 | 10.2  |
| 42  | .38855    | 113 | .39931    | 116 | .45409  | 105       | .46663  | 108       | 42  | 7   | 12.6  | 12.5 | 12.4 | 12.3 | 12.2  | 12.1 | 12.0  |
| 43  | .38969    | 113 | .40045    | 116 | .45514  | 105       | .46771  | 108       | 43  | 8   | 14.4  | 14.2 | 14.1 | 14.0 | 13.9  | 13.8 | 13.7  |
| 44  | .39082    | 113 | .40165    | 116 | .45619  | 105       | .46879  | 108       | 44  | 9   | 16.2  | 16.0 | 15.9 | 15.8 | 15.7  | 15.6 | 15.5  |
| 45  | 8.39195   | 113 | 8.40275   | 116 | 8.45724 | 104       | 8.46987 | 107       | 45  | 10  | 18.0  | 17.8 | 17.7 | 17.6 | 17.5  | 17.4 | 17.3  |
| 46  | .39308    | 113 | .40395    | 115 | .45829  | 105       | .47095  | 108       | 46  | 20  | 36.0  | 35.6 | 35.5 | 35.4 | 35.3  | 35.2 | 35.1  |
| 47  | .39421    | 113 | .40511    | 115 | .45934  | 104       | .47202  | 107       | 47  | 30  | 54.0  | 53.5 | 53.4 | 53.3 | 53.2  | 53.1 | 53.0  |
| 48  | .39534    | 112 | .40626    | 115 | .46039  | 104       | .47310  | 107       | 48  | 40  | 72.0  | 71.3 | 71.2 | 71.1 | 71.0  | 70.9 | 70.8  |
| 49  | .39646    | 112 | .40742    | 115 | .46142  | 104       | .47417  | 107       | 49  | 50  | 90.0  | 89.1 | 89.0 | 88.9 | 88.8  | 88.7 | 88.6  |
| 50  | 8.39758   | 112 | 8.40857   | 115 | 8.46247 | 104       | 8.47525 | 107       | 50  | 105 | 104   | 103  | 102  | 101  | 100   | 99   | 98    |
| 51  | .39871    | 112 | .40972    | 115 | .46351  | 104       | .47632  | 107       | 51  | 6   | 10.5  | 10.4 | 10.3 | 10.2 | 10.1  | 10.0 | 9.9   |
| 52  | .39983    | 112 | .41087    | 115 | .46455  | 103       | .47739  | 107       | 52  | 7   | 12.2  | 12.1 | 12.0 | 11.9 | 11.8  | 11.7 | 11.6  |
| 53  | .40095    | 112 | .41202    | 114 | .46558  | 104       | .47846  | 106       | 53  | 8   | 14.0  | 13.8 | 13.7 | 13.6 | 13.5  | 13.4 | 13.3  |
| 54  | .40207    | 111 | .41317    | 114 | .46662  | 103       | .47953  | 107       | 54  | 9   | 15.7  | 15.6 | 15.5 | 15.4 | 15.3  | 15.2 | 15.1  |
| 55  | 8.40318   | 111 | 8.41431   | 114 | 8.46766 | 103       | 8.48080 | 106       | 55  | 10  | 17.5  | 17.3 | 17.2 | 17.1 | 17.0  | 16.9 | 16.8  |
| 56  | .40430    | 111 | .41546    | 114 | .46869  | 103       | .48166  | 106       | 56  | 20  | 35.0  | 34.6 | 34.5 | 34.4 | 34.3  | 34.2 | 34.1  |
| 57  | .40541    | 111 | .41660    | 114 | .46972  | 103       | .48273  | 106       | 57  | 30  | 52.5  | 52.0 | 51.9 | 51.8 | 51.7  | 51.6 | 51.5  |
| 58  | .40652    | 111 | .41774    | 114 | .47076  | 103       | .48379  | 106       | 58  | 40  | 70.0  | 69.3 | 69.2 | 69.1 | 69.0  | 68.9 | 68.8  |
| 59  | .40764    | 111 | .41888    | 114 | .47179  | 103       | .48485  | 106       | 59  | 50  | 87.5  | 86.6 | 86.5 | 86.4 | 86.3  | 86.2 | 86.1  |
| 60  | 8.40875   | 111 | 8.42005   | 114 | 8.47282 | 103       | 8.48591 | 106       | 60  | 100 | 100.0 | 99.1 | 99.0 | 98.9 | 98.8  | 98.7 | 98.6  |
|     | Lg. Vers. | D   | Log. Exs. | D   |         | Lg. Vers. | D       | Log. Exs. | D   |     |       |      |      |      |       |      | P. P. |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

14°

15°

| 14°       |         |           |         | 15°       |         |           |         | P. P. |    |  |  |
|-----------|---------|-----------|---------|-----------|---------|-----------|---------|-------|----|--|--|
| Lg. Vers. | D       | Log. Exs. | D       | Lg. Vers. | D       | Log. Exs. | D       |       |    |  |  |
| 0         | 8.47282 | 102       | 8.48591 | 106       | 8.53242 | 96        | 8.54748 | 95    | 0  |  |  |
| 1         | .47384  | 103       | .48697  | 106       | .53338  | 95        | .54847  | 99    | 1  |  |  |
| 2         | .47487  | 102       | .48803  | 105       | .53434  | 96        | .54946  | 99    | 2  |  |  |
| 3         | .47590  | 102       | .48909  | 105       | .53530  | 95        | .55045  | 99    | 3  |  |  |
| 4         | .47692  | 102       | .49014  | 105       | .53625  | 95        | .55144  | 99    | 4  |  |  |
| 5         | 8.47795 | 102       | 8.49120 | 105       | 8.53721 | 95        | 8.55243 | 99    | 5  |  |  |
| 6         | .47897  | 102       | .49225  | 105       | .53816  | 95        | .55342  | 99    | 6  |  |  |
| 7         | .47999  | 102       | .49331  | 105       | .53911  | 95        | .55441  | 98    | 7  |  |  |
| 8         | .48101  | 102       | .49436  | 105       | .54007  | 95        | .55539  | 98    | 8  |  |  |
| 9         | .48203  | 102       | .49541  | 105       | .54102  | 95        | .55638  | 98    | 9  |  |  |
| 10        | 8.48304 | 101       | 8.49646 | 105       | 8.54197 | 95        | 8.55736 | 98    | 10 |  |  |
| 11        | .48406  | 101       | .49750  | 104       | .54291  | 94        | .55834  | 98    | 11 |  |  |
| 12        | .48507  | 101       | .49855  | 105       | .54386  | 95        | .55933  | 98    | 12 |  |  |
| 13        | .48609  | 101       | .49960  | 104       | .54481  | 94        | .56031  | 98    | 13 |  |  |
| 14        | .48710  | 101       | .50064  | 104       | .54575  | 94        | .56129  | 98    | 14 |  |  |
| 15        | 8.48811 | 101       | 8.50168 | 104       | 8.54670 | 94        | 8.56226 | 97    | 15 |  |  |
| 16        | .48912  | 101       | .50273  | 104       | .54764  | 94        | .56324  | 98    | 16 |  |  |
| 17        | .49013  | 100       | .50377  | 104       | .54858  | 94        | .56422  | 97    | 17 |  |  |
| 18        | .49114  | 101       | .50481  | 104       | .54952  | 94        | .56519  | 97    | 18 |  |  |
| 19        | .49215  | 100       | .50585  | 103       | .55046  | 94        | .56617  | 97    | 19 |  |  |
| 20        | 8.49315 | 100       | 8.50688 | 104       | 8.55140 | 93        | 8.56714 | 97    | 20 |  |  |
| 21        | .49415  | 100       | .50792  | 103       | .55234  | 94        | .56812  | 97    | 21 |  |  |
| 22        | .49516  | 100       | .50896  | 103       | .55328  | 93        | .56909  | 97    | 22 |  |  |
| 23        | .49616  | 100       | .50999  | 103       | .55421  | 93        | .57006  | 97    | 23 |  |  |
| 24        | .49718  | 100       | .51102  | 103       | .55515  | 93        | .57103  | 97    | 24 |  |  |
| 25        | 8.49816 | 100       | 8.51205 | 103       | 8.55608 | 93        | 8.57200 | 96    | 25 |  |  |
| 26        | .49916  | 99        | .51309  | 103       | .55701  | 93        | .57296  | 97    | 26 |  |  |
| 27        | .50015  | 100       | .51412  | 102       | .55795  | 93        | .57393  | 96    | 27 |  |  |
| 28        | .50115  | 99        | .51514  | 103       | .55888  | 93        | .57490  | 96    | 28 |  |  |
| 29        | .50215  | 99        | .51617  | 102       | .55981  | 93        | .57586  | 96    | 29 |  |  |
| 30        | 8.50314 | 99        | 8.51720 | 102       | 8.56074 | 92        | 8.57682 | 96    | 30 |  |  |
| 31        | .50411  | 99        | .51822  | 102       | .56166  | 92        | .57779  | 96    | 31 |  |  |
| 32        | .50512  | 99        | .51925  | 102       | .56259  | 92        | .57875  | 96    | 32 |  |  |
| 33        | .50611  | 99        | .52027  | 102       | .56352  | 92        | .57971  | 96    | 33 |  |  |
| 34        | .50710  | 98        | .52129  | 102       | .56444  | 92        | .58067  | 95    | 34 |  |  |
| 35        | 8.50809 | 98        | 8.52231 | 102       | 8.56536 | 92        | 8.58163 | 95    | 35 |  |  |
| 36        | .50908  | 98        | .52333  | 102       | .56629  | 92        | .58259  | 95    | 36 |  |  |
| 37        | .51006  | 98        | .52435  | 101       | .56721  | 92        | .58354  | 95    | 37 |  |  |
| 38        | .51105  | 98        | .52537  | 101       | .56813  | 92        | .58450  | 95    | 38 |  |  |
| 39        | .51203  | 98        | .52638  | 101       | .56905  | 92        | .58546  | 95    | 39 |  |  |
| 40        | 8.51301 | 98        | 8.52740 | 101       | 8.56997 | 92        | 8.58641 | 95    | 40 |  |  |
| 41        | .51399  | 98        | .52841  | 101       | .57089  | 91        | .58736  | 95    | 41 |  |  |
| 42        | .51497  | 98        | .52943  | 101       | .57180  | 91        | .58832  | 95    | 42 |  |  |
| 43        | .51595  | 97        | .53044  | 101       | .57272  | 91        | .58927  | 95    | 43 |  |  |
| 44        | .51693  | 97        | .53145  | 101       | .57363  | 91        | .59022  | 95    | 44 |  |  |
| 45        | 8.51791 | 97        | 8.53246 | 101       | 8.57455 | 91        | 8.59117 | 95    | 45 |  |  |
| 46        | .51888  | 97        | .53347  | 101       | .57546  | 91        | .59211  | 95    | 46 |  |  |
| 47        | .51986  | 97        | .53448  | 100       | .57637  | 91        | .59306  | 95    | 47 |  |  |
| 48        | .52083  | 97        | .53548  | 100       | .57728  | 91        | .59400  | 94    | 48 |  |  |
| 49        | .52180  | 97        | .53649  | 100       | .57819  | 91        | .59495  | 94    | 49 |  |  |
| 50        | 8.52277 | 97        | 8.53749 | 100       | 8.57910 | 90        | 8.59588 | 94    | 50 |  |  |
| 51        | .52374  | 97        | .53850  | 100       | .58001  | 91        | .59684  | 94    | 51 |  |  |
| 52        | .52471  | 96        | .53950  | 100       | .58092  | 90        | .59779  | 94    | 52 |  |  |
| 53        | .52568  | 97        | .54050  | 100       | .58182  | 90        | .59873  | 94    | 53 |  |  |
| 54        | .52665  | 96        | .54150  | 100       | .58273  | 90        | .59967  | 94    | 54 |  |  |
| 55        | 8.52761 | 96        | 8.54250 | 100       | 8.58363 | 90        | 8.60061 | 94    | 55 |  |  |
| 56        | .52858  | 96        | .54350  | 99        | .58453  | 90        | .60155  | 94    | 56 |  |  |
| 57        | .52954  | 96        | .54449  | 100       | .58544  | 90        | .60249  | 95    | 57 |  |  |
| 58        | .53050  | 96        | .54549  | 99        | .58634  | 90        | .60342  | 94    | 58 |  |  |
| 59        | .53146  | 96        | .54649  | 99        | .58724  | 90        | .60436  | 93    | 59 |  |  |
| 60        | 8.53242 | 96        | 8.54748 | 99        | 8.58814 | 90        | 8.60530 | 93    | 60 |  |  |
| Lg. Vers. | D       | Log. Exs. | D       | Lg. Vers. | D       | Log. Exs. | D       | P. P. |    |  |  |

103 102 101

6 10.3 10.2 10.1  
7 12.0 11.9 11.8  
8 13.7 13.6 13.4  
9 15.4 15.3 15.1  
10 17.1 17.0 16.8  
20 34.3 34.0 33.6  
30 51.5 51.0 50.5  
40 68.6 68.0 67.3  
50 85.8 85.0 84.1

100 99 98

6 10.0 9.9 9.8  
7 11.6 11.5 11.4  
8 13.3 13.2 13.0  
9 15.0 14.8 14.7  
10 16.7 16.5 16.3  
20 33.3 33.0 32.6  
30 50.0 49.5 49.0  
40 66.6 66.0 65.3  
50 83.3 82.5 82.6

97 96 95

6 9.7 9.6 9.5  
7 11.3 11.2 11.1  
8 12.9 12.8 12.6  
9 14.5 14.4 14.2  
10 16.1 16.0 15.8  
20 32.3 32.0 31.6  
30 48.5 48.0 47.5  
40 64.6 64.0 63.3  
50 80.8 80.0 79.1

91 93 92

6 9.4 9.3 9.2  
7 10.9 10.8 10.7  
8 12.5 12.4 12.3  
9 14.1 13.9 13.8  
10 15.6 15.5 15.3  
20 31.3 31.0 30.6  
30 47.0 46.5 46.0  
40 62.0 62.0 61.3  
50 78.3 77.5 77.6

91 90 0

6 9.1 9.0 0.0  
7 10.6 10.5 0.0  
8 12.1 12.0 0.0  
9 13.6 13.5 0.1  
10 15.1 15.0 0.1  
20 30.3 30.0 0.1  
30 45.5 45.0 0.2  
40 60.6 60.0 0.3  
50 75.8 75.0 0.4

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANT

16°

17°

|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------------------|
| 0  | 8.58814   | 90 | 8.60530   | 93 | 8.64043   | 84 | 8.65984   | 88 | 0  | 93 92 91          |
| 1  | .58904    | 89 | .60623    | 93 | .64128    | 84 | .66072    | 88 | 1  | 6 9.8 9.2 9.1     |
| 2  | .58993    | 89 | .60716    | 93 | .64212    | 84 | .66160    | 88 | 2  | 7 10.8 10.7 10.6  |
| 3  | .59083    | 89 | .60810    | 93 | .64296    | 84 | .66248    | 88 | 3  | 8 12.4 12.2 12.1  |
| 4  | .59173    | 89 | .60903    | 93 | .64381    | 84 | .66336    | 88 | 4  | 9 13.9 13.8 13.6  |
| 5  | 8.59262   | 89 | 8.60996   | 93 | 8.64465   | 84 | 8.66425   | 88 | 5  | 10 15.5 15.3 15.1 |
| 6  | .59351    | 89 | .61089    | 93 | .64549    | 84 | .66512    | 87 | 6  | 20 31.0 30.6 30.5 |
| 7  | .59441    | 89 | .61182    | 92 | .64633    | 84 | .66600    | 88 | 7  | 30 46.5 46.0 45.5 |
| 8  | .59530    | 89 | .61275    | 93 | .64717    | 84 | .66688    | 88 | 8  | 40 62.0 61.3 60.6 |
| 9  | .59619    | 89 | .61368    | 93 | .64801    | 84 | .66776    | 87 | 9  | 50 77.5 76.6 75.8 |
| 10 | 8.59708   | 89 | 8.61460   | 92 | 8.64884   | 83 | 8.66863   | 87 | 10 | 90 89 88          |
| 11 | .59797    | 89 | .61553    | 92 | .64968    | 83 | .66951    | 88 | 11 | 6 9.0 8.9 8.8     |
| 12 | .59886    | 88 | .61645    | 92 | .65052    | 83 | .67039    | 87 | 12 | 7 10.5 10.4 10.2  |
| 13 | .59974    | 89 | .61738    | 92 | .65135    | 83 | .67126    | 87 | 13 | 8 12.0 11.8 11.7  |
| 14 | .60063    | 88 | .61830    | 92 | .65218    | 83 | .67213    | 87 | 14 | 9 13.5 13.3 13.2  |
| 15 | 8.60152   | 88 | 8.61922   | 92 | 8.65302   | 83 | 8.67301   | 87 | 15 | 10 15.0 14.8 14.6 |
| 16 | .60240    | 88 | .62014    | 92 | .65385    | 83 | .67388    | 87 | 16 | 20 30.0 29.6 29.3 |
| 17 | .60328    | 88 | .62106    | 92 | .65468    | 83 | .67475    | 87 | 17 | 30 45.0 44.5 44.0 |
| 18 | .60417    | 88 | .62198    | 92 | .65551    | 83 | .67562    | 87 | 18 | 40 60.0 59.3 58.6 |
| 19 | .60505    | 88 | .62290    | 91 | .65634    | 83 | .67649    | 87 | 19 | 50 75.0 74.1 73.3 |
| 20 | 8.60593   | 88 | 8.62382   | 92 | 8.65717   | 83 | 8.67736   | 87 | 20 | 87 86 85          |
| 21 | .60681    | 88 | .62474    | 91 | .65800    | 83 | .67822    | 87 | 21 | 6 8.7 8.6 8.5     |
| 22 | .60769    | 88 | .62565    | 91 | .65883    | 83 | .67909    | 86 | 22 | 7 10.1 10.0 9.9   |
| 23 | .60857    | 87 | .62657    | 91 | .65966    | 83 | .67996    | 86 | 23 | 8 11.6 11.4 11.3  |
| 24 | .60944    | 87 | .62748    | 91 | .66048    | 83 | .68082    | 86 | 24 | 9 13.0 12.9 12.7  |
| 25 | 8.61032   | 87 | 8.62840   | 91 | 8.66131   | 83 | 8.68169   | 86 | 25 | 10 14.5 14.3 14.1 |
| 26 | .61119    | 87 | .62931    | 91 | .66213    | 82 | .68255    | 86 | 26 | 20 29.0 28.6 28.3 |
| 27 | .61207    | 87 | .63022    | 91 | .66295    | 82 | .68341    | 86 | 27 | 30 43.5 43.0 42.5 |
| 28 | .61294    | 87 | .63113    | 91 | .66378    | 82 | .68428    | 86 | 28 | 40 58.0 57.3 56.6 |
| 29 | .61381    | 87 | .63204    | 91 | .66460    | 82 | .68514    | 86 | 29 | 50 72.5 71.6 70.8 |
| 30 | 8.61469   | 87 | 8.63295   | 90 | 8.66542   | 82 | 8.68600   | 86 | 30 | 84 83 82          |
| 31 | .61556    | 87 | .63386    | 91 | .66624    | 82 | .68686    | 86 | 31 | 6 8.4 8.3 8.2     |
| 32 | .61643    | 87 | .63477    | 90 | .66706    | 82 | .68772    | 86 | 32 | 7 9.8 9.7 9.5     |
| 33 | .61730    | 86 | .63567    | 90 | .66788    | 82 | .68858    | 86 | 33 | 8 11.2 11.0 10.9  |
| 34 | .61816    | 86 | .63658    | 90 | .66870    | 82 | .68944    | 86 | 34 | 9 12.6 12.4 12.3  |
| 35 | 8.61903   | 87 | 8.63748   | 90 | 8.66951   | 81 | 8.69029   | 85 | 35 | 10 14.0 13.8 13.6 |
| 36 | .61990    | 86 | .63839    | 90 | .67033    | 82 | .69115    | 85 | 36 | 20 28.0 27.6 27.3 |
| 37 | .62076    | 86 | .63929    | 90 | .67115    | 81 | .69201    | 85 | 37 | 30 42.0 41.5 41.0 |
| 38 | .62163    | 86 | .64019    | 90 | .67196    | 81 | .69286    | 85 | 38 | 40 56.0 55.3 54.6 |
| 39 | .62249    | 86 | .64109    | 90 | .67277    | 81 | .69372    | 85 | 39 | 50 70.0 69.1 68.3 |
| 40 | 8.62336   | 86 | 8.64199   | 90 | 8.67359   | 81 | 8.69457   | 85 | 40 | 81 80 79          |
| 41 | .62422    | 86 | .64289    | 90 | .67440    | 81 | .69542    | 85 | 41 | 6 8.1 8.0 7.9     |
| 42 | .62508    | 86 | .64379    | 90 | .67521    | 81 | .69627    | 85 | 42 | 7 9.4 9.3 9.2     |
| 43 | .62594    | 86 | .64469    | 89 | .67602    | 81 | .69712    | 85 | 43 | 8 10.8 10.6 10.5  |
| 44 | .62680    | 86 | .64559    | 90 | .67683    | 81 | .69798    | 85 | 44 | 9 12.1 12.0 11.8  |
| 45 | 8.62766   | 86 | 8.64649   | 89 | 8.67764   | 81 | 8.69883   | 85 | 45 | 10 13.5 13.3 13.1 |
| 46 | .62852    | 85 | .64738    | 89 | .67845    | 80 | .69967    | 85 | 46 | 20 27.0 26.6 26.3 |
| 47 | .62937    | 85 | .64828    | 89 | .67926    | 81 | .70052    | 85 | 47 | 30 40.5 40.0 39.5 |
| 48 | .63023    | 85 | .64917    | 89 | .68007    | 80 | .70137    | 85 | 48 | 40 54.0 53.3 52.6 |
| 49 | .63108    | 85 | .65006    | 89 | .68087    | 80 | .70222    | 84 | 49 | 50 67.5 66.6 65.8 |
| 50 | 8.63194   | 85 | 8.65096   | 89 | 8.68168   | 80 | 8.70306   | 84 | 50 | 80 79 78          |
| 51 | .63279    | 85 | .65185    | 89 | .68248    | 80 | .70391    | 84 | 51 | 6 8.0 7.9 7.8     |
| 52 | .63364    | 85 | .65274    | 89 | .68329    | 80 | .70475    | 84 | 52 | 7 9.4 9.3 9.2     |
| 53 | .63449    | 85 | .65363    | 89 | .68409    | 80 | .70560    | 84 | 53 | 8 10.8 10.6 10.5  |
| 54 | .63534    | 85 | .65452    | 89 | .68489    | 80 | .70644    | 84 | 54 | 9 12.1 12.0 11.8  |
| 55 | 8.63619   | 85 | 8.65541   | 88 | 8.68569   | 80 | 8.70728   | 84 | 55 | 10 13.5 13.3 13.1 |
| 56 | .63704    | 85 | .65629    | 88 | .68650    | 80 | .70813    | 84 | 56 | 20 27.0 26.6 26.3 |
| 57 | .63789    | 84 | .65718    | 89 | .68730    | 80 | .70897    | 84 | 57 | 30 40.5 40.0 39.5 |
| 58 | .63874    | 85 | .65807    | 88 | .68810    | 79 | .70981    | 84 | 58 | 40 54.0 53.3 52.6 |
| 59 | .63959    | 84 | .65895    | 88 | .68889    | 80 | .71065    | 84 | 59 | 50 67.5 66.6 65.8 |
| 60 | 8.64043   | 84 | 8.65984   | 88 | 8.68969   | 80 | 8.71149   | 84 | 60 | 80 79 78          |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

| 18° |           |    |           | 19° |           |    |           |    |    |         |      |
|-----|-----------|----|-----------|-----|-----------|----|-----------|----|----|---------|------|
|     | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.   |      |
| 0   | 8.68909   | 79 | 8.71149   | 83  | 8.73625   | 75 | 8.76058   | 79 | C  |         |      |
| 1   | .69049    | 80 | .71232    | 84  | .73700    | 75 | .76137    | 80 | 1  |         |      |
| 2   | .69129    | 79 | .71316    | 83  | .73775    | 75 | .76217    | 79 | 2  |         |      |
| 3   | .69208    | 79 | .71400    | 84  | .73851    | 75 | .76297    | 79 | 3  |         |      |
| 4   | .69288    | 79 | .71484    | 83  | .73926    | 75 | .76376    | 79 | 4  |         |      |
| 5   | 8.69367   | 79 | 8.71567   | 83  | 8.74001   | 75 | 8.76456   | 80 | 5  | 84      | 83   |
| 6   | .69446    | 79 | .71651    | 83  | .74076    | 75 | .76536    | 79 | 6  | 8.4     | 8.3  |
| 7   | .69526    | 79 | .71734    | 83  | .74151    | 75 | .76615    | 79 | 7  | 9.8     | 9.7  |
| 8   | .69605    | 79 | .71817    | 83  | .74226    | 75 | .76694    | 79 | 8  | 11.2    | 11.0 |
| 9   | .69684    | 79 | .71901    | 83  | .74301    | 75 | .76774    | 79 | 9  | 12.6    | 12.4 |
| 10  | 8.69763   | 79 | 8.71984   | 83  | 8.74376   | 75 | 8.76853   | 79 | 10 | 14.0    | 13.8 |
| 11  | .69842    | 79 | .72067    | 83  | .74451    | 75 | .76932    | 79 | 11 | 28.0    | 27.6 |
| 12  | .69921    | 79 | .72150    | 83  | .74526    | 75 | .77011    | 79 | 12 | 30.42.0 | 41.5 |
| 13  | .70000    | 78 | .72233    | 83  | .74600    | 74 | .77090    | 79 | 13 | 40.56.0 | 55.3 |
| 14  | .70079    | 79 | .72316    | 83  | .74675    | 74 | .77169    | 79 | 14 | 50.70.0 | 69.1 |
| 15  | 8.70157   | 78 | 8.72399   | 82  | 8.74749   | 74 | 8.77248   | 79 | 15 |         | 81   |
| 16  | .70236    | 78 | .72481    | 83  | .74824    | 74 | .77327    | 79 | 16 | 6       | 8.1  |
| 17  | .70314    | 78 | .72564    | 83  | .74898    | 74 | .77406    | 79 | 17 | 7       | 9.4  |
| 18  | .70393    | 78 | .72647    | 82  | .74973    | 74 | .77485    | 79 | 18 | 8       | 10.8 |
| 19  | .70471    | 78 | .72729    | 82  | .75047    | 74 | .77563    | 79 | 19 | 9       | 12.1 |
| 20  | 8.70550   | 78 | 8.72812   | 82  | 8.75121   | 74 | 8.77642   | 79 | 20 | 10      | 13.5 |
| 21  | .70628    | 78 | .72894    | 82  | .75195    | 74 | .77720    | 79 | 21 | 20      | 27.0 |
| 22  | .70706    | 78 | .72977    | 82  | .75269    | 74 | .77799    | 79 | 22 | 30      | 40.5 |
| 23  | .70784    | 78 | .73059    | 82  | .75343    | 74 | .77877    | 79 | 23 | 40      | 54.0 |
| 24  | .70862    | 78 | .73141    | 82  | .75417    | 74 | .77956    | 79 | 24 | 50      | 67.5 |
| 25  | 8.70940   | 78 | 8.73223   | 82  | 8.75491   | 73 | 8.78034   | 79 | 25 |         | 81   |
| 26  | .71018    | 77 | .73306    | 82  | .75565    | 74 | .78112    | 79 | 26 | 6       | 8.1  |
| 27  | .71096    | 77 | .73388    | 82  | .75639    | 73 | .78191    | 79 | 27 | 7       | 9.4  |
| 28  | .71174    | 77 | .73470    | 81  | .75712    | 73 | .78269    | 79 | 28 | 8       | 10.8 |
| 29  | .71251    | 77 | .73551    | 82  | .75786    | 74 | .78347    | 79 | 29 | 9       | 12.1 |
| 30  | 8.71329   | 77 | 8.73633   | 82  | 8.75860   | 73 | 8.78425   | 79 | 30 | 10      | 13.5 |
| 31  | .71406    | 77 | .73715    | 81  | .75933    | 73 | .78503    | 79 | 31 | 20      | 27.0 |
| 32  | .71484    | 77 | .73797    | 81  | .76006    | 73 | .78581    | 79 | 32 | 30      | 40.5 |
| 33  | .71561    | 77 | .73878    | 81  | .76080    | 73 | .78659    | 79 | 33 | 40      | 54.0 |
| 34  | .71639    | 77 | .73960    | 81  | .76153    | 73 | .78736    | 79 | 34 | 50      | 67.5 |
| 35  | 8.71716   | 77 | 8.74041   | 81  | 8.76226   | 73 | 8.78814   | 79 | 35 |         | 81   |
| 36  | .71793    | 77 | .74123    | 81  | .76300    | 73 | .78892    | 79 | 36 | 6       | 8.1  |
| 37  | .71870    | 77 | .74204    | 81  | .76373    | 73 | .78969    | 79 | 37 | 7       | 9.4  |
| 38  | .71947    | 77 | .74286    | 81  | .76446    | 73 | .79047    | 79 | 38 | 8       | 10.8 |
| 39  | .72024    | 77 | .74367    | 81  | .76519    | 73 | .79124    | 79 | 39 | 9       | 12.1 |
| 40  | 8.72101   | 77 | 8.74448   | 81  | 8.76592   | 72 | 8.79202   | 79 | 40 | 10      | 13.5 |
| 41  | .72178    | 77 | .74529    | 81  | .76664    | 73 | .79279    | 79 | 41 | 20      | 27.0 |
| 42  | .72255    | 77 | .74610    | 81  | .76737    | 72 | .79357    | 79 | 42 | 30      | 40.5 |
| 43  | .72331    | 76 | .74691    | 80  | .76810    | 73 | .79434    | 79 | 43 | 40      | 54.0 |
| 44  | .72408    | 77 | .74772    | 81  | .76883    | 72 | .79511    | 79 | 44 | 50      | 67.5 |
| 45  | 8.72485   | 76 | 8.74853   | 81  | 8.76955   | 72 | 8.79588   | 79 | 45 |         | 81   |
| 46  | .72561    | 76 | .74934    | 80  | .77028    | 72 | .79665    | 79 | 46 | 6       | 8.1  |
| 47  | .72637    | 76 | .75014    | 80  | .77100    | 72 | .79742    | 79 | 47 | 7       | 9.4  |
| 48  | .72714    | 76 | .75095    | 80  | .77173    | 72 | .79819    | 79 | 48 | 8       | 10.8 |
| 49  | .72790    | 76 | .75175    | 80  | .77245    | 72 | .79896    | 79 | 49 | 9       | 12.1 |
| 50  | 8.72866   | 76 | 8.75256   | 80  | 8.77317   | 72 | 8.79973   | 79 | 50 | 10      | 13.5 |
| 51  | .72942    | 76 | .75336    | 80  | .77390    | 72 | .80050    | 79 | 51 | 20      | 27.0 |
| 52  | .73018    | 76 | .75417    | 80  | .77462    | 72 | .80126    | 79 | 52 | 30      | 40.5 |
| 53  | .73094    | 76 | .75497    | 80  | .77534    | 72 | .80203    | 79 | 53 | 40      | 54.0 |
| 54  | .73170    | 76 | .75577    | 80  | .77606    | 72 | .80280    | 79 | 54 | 50      | 67.5 |
| 55  | 8.73246   | 76 | 8.75658   | 80  | 8.77678   | 72 | 8.80356   | 79 | 55 |         | 81   |
| 56  | .73322    | 75 | .75738    | 80  | .77750    | 72 | .80433    | 79 | 56 | 6       | 8.1  |
| 57  | .73398    | 75 | .75818    | 80  | .77822    | 71 | .80509    | 79 | 57 | 7       | 9.4  |
| 58  | .73473    | 76 | .75898    | 80  | .77893    | 72 | .80586    | 79 | 58 | 8       | 10.8 |
| 59  | .73549    | 75 | .75978    | 80  | .77965    | 71 | .80662    | 79 | 59 | 9       | 12.1 |
| 60  | 8.73625   | 75 | 8.76058   | 80  | 8.78037   | 71 | 8.80738   | 79 | 60 | 10      | 13.5 |
|     | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.   |      |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

20°

21°

|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------------------|
| 0  | 8.78037   | 71 | 8.80736   | 76 | 8.82220   | 68 | 8.85214   | 73 | 0  |                   |
| 1  | .78108    | 71 | .80814    | 76 | .82297    | 68 | .85287    | 73 | 1  |                   |
| 2  | .78180    | 71 | .80891    | 76 | .82366    | 68 | .85360    | 72 | 2  |                   |
| 3  | .78251    | 71 | .80967    | 76 | .82434    | 68 | .85433    | 73 | 3  |                   |
| 4  | .78323    | 71 | .81043    | 76 | .82502    | 67 | .85506    | 73 | 4  | 76 75 74          |
| 5  | 8.78394   | 71 | 8.81119   | 76 | 8.82569   | 68 | 8.85579   | 72 | 5  | 6 7.6 7.5 7.4     |
| 6  | .78466    | 71 | .81195    | 76 | .82637    | 68 | .85651    | 72 | 6  | 7 8.8 8.7 8.6     |
| 7  | .78537    | 71 | .81271    | 75 | .82705    | 68 | .85724    | 73 | 7  | 8 10.1 10.0 9.9   |
| 8  | .78608    | 71 | .81346    | 76 | .82773    | 67 | .85797    | 72 | 8  | 9 11.4 11.2 11.1  |
| 9  | .78679    | 71 | .81422    | 75 | .82841    | 68 | .85869    | 72 | 9  | 10 12.6 12.5 12.4 |
| 10 | 8.78750   | 71 | 8.81496   | 75 | 8.82908   | 67 | 8.85942   | 72 | 10 | 20 25.3 25.0 21.6 |
| 11 | .78821    | 71 | .81573    | 76 | .82976    | 67 | .86014    | 72 | 11 | 30 38.0 37.5 37.0 |
| 12 | .78892    | 71 | .81649    | 75 | .83043    | 67 | .86087    | 72 | 12 | 40 50.6 50.0 49.4 |
| 13 | .78963    | 71 | .81725    | 75 | .83111    | 67 | .86159    | 72 | 13 | 50 63.3 62.5 61.6 |
| 14 | .79034    | 70 | .81800    | 75 | .83178    | 67 | .86231    | 72 | 14 |                   |
| 15 | 8.79105   | 71 | 8.81876   | 75 | 8.83246   | 67 | 8.86304   | 72 | 15 | 73 72 71          |
| 16 | .79175    | 71 | .81951    | 75 | .83313    | 67 | .86376    | 72 | 16 | 6 7.3 7.2 7.1     |
| 17 | .79246    | 71 | .82026    | 75 | .83380    | 67 | .86448    | 72 | 17 | 7 8.5 8.4 8.3     |
| 18 | .79317    | 70 | .82102    | 75 | .83447    | 67 | .86520    | 72 | 18 | 8 9.7 9.6 9.4     |
| 19 | .79387    | 70 | .82177    | 75 | .83515    | 67 | .86592    | 72 | 19 | 9 10.9 10.8 10.6  |
| 20 | 8.79458   | 70 | 8.82252   | 75 | 8.83582   | 67 | 8.86664   | 72 | 20 | 10 12.1 12.0 11.8 |
| 21 | .79528    | 70 | .82327    | 75 | .83649    | 67 | .86736    | 72 | 21 | 20 24.3 24.0 23.6 |
| 22 | .79598    | 70 | .82402    | 75 | .83716    | 67 | .86808    | 72 | 22 | 30 36.5 36.0 35.5 |
| 23 | .79669    | 70 | .82477    | 74 | .83783    | 67 | .86880    | 72 | 23 | 40 48.6 48.0 47.3 |
| 24 | .79739    | 70 | .82552    | 75 | .83850    | 67 | .86952    | 71 | 24 | 50 60.8 60.0 59.1 |
| 25 | 8.79809   | 70 | 8.82627   | 75 | 8.83916   | 66 | 8.87024   | 72 | 25 |                   |
| 26 | .79879    | 70 | .82702    | 74 | .83983    | 66 | .87095    | 72 | 26 | 70 69 68          |
| 27 | .79949    | 70 | .82776    | 74 | .84050    | 66 | .87167    | 72 | 27 | 6 7.0 6.9 6.8     |
| 28 | .80019    | 70 | .82851    | 75 | .84117    | 67 | .87239    | 71 | 28 | 7 8.1 8.0 7.9     |
| 29 | .80089    | 70 | .82926    | 74 | .84183    | 66 | .87310    | 71 | 29 | 8 9.3 9.2 9.0     |
| 30 | 8.80159   | 70 | 8.83000   | 74 | 8.84250   | 66 | 8.87382   | 71 | 30 | 9 10.5 10.3 10.2  |
| 31 | .80229    | 69 | .83075    | 74 | .84316    | 66 | .87453    | 71 | 31 | 10 11.6 11.5 11.3 |
| 32 | .80299    | 69 | .83149    | 74 | .84383    | 66 | .87525    | 71 | 32 | 20 23.3 23.0 22.6 |
| 33 | .80369    | 70 | .83224    | 74 | .84449    | 66 | .87596    | 71 | 33 | 30 35.0 34.5 34.0 |
| 34 | .80438    | 69 | .83298    | 74 | .84515    | 66 | .87668    | 71 | 34 | 40 46.6 46.0 45.3 |
| 35 | 8.80508   | 69 | 8.83373   | 74 | 8.84582   | 66 | 8.87739   | 71 | 35 | 50 58.3 57.5 56.6 |
| 36 | .80577    | 69 | .83447    | 74 | .84648    | 66 | .87810    | 71 | 36 |                   |
| 37 | .80647    | 69 | .83521    | 74 | .84714    | 66 | .87881    | 71 | 37 | 67 66 65          |
| 38 | .80716    | 69 | .83595    | 74 | .84780    | 66 | .87953    | 71 | 38 | 6 6.7 6.6 6.5     |
| 39 | .80786    | 69 | .83670    | 74 | .84846    | 66 | .88024    | 71 | 39 | 7 7.8 7.7 7.6     |
| 40 | 8.80855   | 69 | 8.83744   | 74 | 8.84912   | 66 | 8.88095   | 71 | 40 | 8 8.9 8.8 8.6     |
| 41 | .80924    | 69 | .83818    | 74 | .84978    | 66 | .88166    | 71 | 41 | 9 10.0 9.9 9.7    |
| 42 | .80993    | 69 | .83892    | 74 | .85044    | 66 | .88237    | 71 | 42 | 10 11.1 11.0 10.8 |
| 43 | .81063    | 69 | .83966    | 73 | .85110    | 66 | .88308    | 70 | 43 | 20 22.3 22.0 21.6 |
| 44 | .81132    | 69 | .84039    | 73 | .85176    | 66 | .88378    | 71 | 44 | 30 33.5 33.0 32.6 |
| 45 | 8.81201   | 69 | 8.84113   | 73 | 8.85242   | 65 | 8.88449   | 71 | 45 | 40 44.0 44.0 43.3 |
| 46 | .81270    | 69 | .84187    | 73 | .85308    | 65 | .88520    | 70 | 46 | 50 55.8 55.0 54.1 |
| 47 | .81339    | 69 | .84261    | 73 | .85373    | 65 | .88591    | 70 | 47 |                   |
| 48 | .81407    | 69 | .84334    | 73 | .85439    | 65 | .88661    | 71 | 48 |                   |
| 49 | .81476    | 69 | .84408    | 73 | .85505    | 65 | .88732    | 70 | 49 | 0                 |
| 50 | 8.81545   | 69 | 8.84481   | 73 | 8.85570   | 65 | 8.83003   | 70 | 50 | 6 0.0 0.0         |
| 51 | .81614    | 68 | .84555    | 73 | .85626    | 65 | .88873    | 70 | 51 | 7 0.0 0.0         |
| 52 | .81682    | 68 | .84628    | 73 | .85701    | 65 | .88944    | 70 | 52 | 8 0.0 0.0         |
| 53 | .81751    | 68 | .84702    | 73 | .85766    | 65 | .89014    | 70 | 53 | 9 0.1 0.1         |
| 54 | .81819    | 68 | .84775    | 73 | .85832    | 65 | .89085    | 70 | 54 | 10 0.1 0.1        |
| 55 | 8.81888   | 68 | 8.84848   | 73 | 8.85897   | 65 | 8.89155   | 70 | 55 | 20 0.1 0.2        |
| 56 | .81956    | 68 | .84922    | 73 | .85962    | 65 | .89225    | 70 | 56 | 30 0.2 0.3        |
| 57 | .82025    | 68 | .84995    | 73 | .86027    | 65 | .89295    | 70 | 57 | 40 0.3 0.4        |
| 58 | .82093    | 68 | .85068    | 73 | .86092    | 65 | .89366    | 70 | 58 | 50 0.4 0.4        |
| 59 | .82161    | 68 | .85141    | 73 | .86158    | 65 | .89436    | 70 | 59 |                   |
| 60 | 8.82229   | 68 | 8.85214   | 73 | 8.86223   | 65 | 8.89506   | 70 | 60 |                   |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |



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P. P.



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|
| 0  | 9.00520   | 55 | 9.05154   | 61 | 9.03740   | 52 | 9.08752   | 59 | 0  |       |
| 1  | .00575    | 54 | .05215    | 61 | .03792    | 52 | .08811    | 59 | 1  |       |
| 2  | .00630    | 54 | .05276    | 60 | .03845    | 53 | .08870    | 59 | 2  |       |
| 3  | .00684    | 54 | .05337    | 61 | .03898    | 53 | .08929    | 59 | 3  |       |
| 4  | .00739    | 54 | .05398    | 60 | .03950    | 52 | .08988    | 59 | 4  |       |
| 5  | 9.00794   | 55 | 9.05458   | 60 | 9.04002   | 52 | 9.09047   | 59 | 5  |       |
| 6  | .00848    | 54 | .05519    | 60 | .04055    | 52 | .09106    | 59 | 6  |       |
| 7  | .00903    | 54 | .05580    | 60 | .04107    | 52 | .09164    | 58 | 7  |       |
| 8  | .00957    | 54 | .05640    | 60 | .04160    | 52 | .09223    | 59 | 8  |       |
| 9  | .01011    | 54 | .05701    | 60 | .04212    | 52 | .09282    | 59 | 9  |       |
| 10 | 9.01066   | 54 | 9.05762   | 60 | 9.04264   | 52 | 9.09341   | 58 | 10 |       |
| 11 | .01120    | 54 | .05822    | 60 | .04317    | 52 | .09400    | 59 | 11 |       |
| 12 | .01174    | 54 | .05883    | 60 | .04369    | 52 | .09458    | 58 | 12 |       |
| 13 | .01229    | 54 | .05943    | 60 | .04421    | 52 | .09517    | 59 | 13 |       |
| 14 | .01283    | 54 | .06004    | 60 | .04473    | 52 | .09576    | 58 | 14 |       |
| 15 | 9.01337   | 54 | 9.06064   | 60 | 9.04525   | 52 | 9.09634   | 58 | 15 |       |
| 16 | .01391    | 54 | .06124    | 60 | .04577    | 52 | .09693    | 58 | 16 |       |
| 17 | .01445    | 54 | .06185    | 60 | .04630    | 52 | .09752    | 59 | 17 |       |
| 18 | .01499    | 54 | .06245    | 60 | .04682    | 52 | .09810    | 58 | 18 |       |
| 19 | .01554    | 54 | .06305    | 60 | .04734    | 52 | .09869    | 58 | 19 |       |
| 20 | 9.01608   | 54 | 9.06366   | 60 | 9.04786   | 51 | 9.09927   | 58 | 20 |       |
| 21 | .01662    | 53 | .06426    | 60 | .04837    | 52 | .09986    | 58 | 21 |       |
| 22 | .01715    | 54 | .06486    | 60 | .04889    | 52 | .10044    | 58 | 22 |       |
| 23 | .01769    | 54 | .06546    | 60 | .04941    | 52 | .10102    | 58 | 23 |       |
| 24 | .01823    | 54 | .06606    | 60 | .04993    | 52 | .10161    | 58 | 24 |       |
| 25 | 9.01877   | 54 | 9.06667   | 60 | 9.05045   | 51 | 9.10219   | 58 | 25 |       |
| 26 | .01931    | 53 | .06727    | 60 | .05097    | 52 | .10278    | 58 | 26 |       |
| 27 | .01985    | 53 | .06787    | 60 | .05148    | 52 | .10336    | 58 | 27 |       |
| 28 | .02038    | 54 | .06847    | 60 | .05200    | 52 | .10394    | 58 | 28 |       |
| 29 | .02092    | 53 | .06907    | 60 | .05252    | 52 | .10452    | 58 | 29 |       |
| 30 | 9.02146   | 53 | 9.06967   | 60 | 9.05303   | 52 | 9.10511   | 58 | 30 |       |
| 31 | .02199    | 54 | .07027    | 60 | .05355    | 51 | .10569    | 58 | 31 |       |
| 32 | .02253    | 53 | .07087    | 59 | .05407    | 51 | .10627    | 58 | 32 |       |
| 33 | .02307    | 53 | .07146    | 60 | .05458    | 51 | .10685    | 58 | 33 |       |
| 34 | .02360    | 53 | .07206    | 60 | .05510    | 51 | .10743    | 58 | 34 |       |
| 35 | 9.02414   | 53 | 9.07266   | 59 | 9.05561   | 51 | 9.10801   | 58 | 35 |       |
| 36 | .02467    | 53 | .07326    | 59 | .05613    | 51 | .10859    | 58 | 36 |       |
| 37 | .02521    | 53 | .07386    | 59 | .05664    | 51 | .10917    | 58 | 37 |       |
| 38 | .02574    | 53 | .07445    | 60 | .05715    | 51 | .10975    | 58 | 38 |       |
| 39 | .02627    | 53 | .07505    | 59 | .05767    | 51 | .11033    | 58 | 39 |       |
| 40 | 9.02681   | 53 | 9.07565   | 59 | 9.05818   | 51 | 9.11091   | 58 | 40 |       |
| 41 | .02734    | 53 | .07624    | 59 | .05869    | 51 | .11149    | 58 | 41 |       |
| 42 | .02787    | 53 | .07684    | 59 | .05921    | 51 | .11207    | 58 | 42 |       |
| 43 | .02840    | 53 | .07743    | 60 | .05972    | 51 | .11265    | 58 | 43 |       |
| 44 | .02894    | 53 | .07803    | 60 | .06023    | 51 | .11323    | 58 | 44 |       |
| 45 | 9.02947   | 53 | 9.07863   | 59 | 9.06074   | 51 | 9.11380   | 57 | 45 |       |
| 46 | .03000    | 53 | .07922    | 59 | .06125    | 51 | .11438    | 58 | 46 |       |
| 47 | .03053    | 53 | .07981    | 59 | .06176    | 51 | .11496    | 57 | 47 |       |
| 48 | .03106    | 53 | .08041    | 59 | .06227    | 51 | .11554    | 57 | 48 |       |
| 49 | .03159    | 53 | .08100    | 59 | .06279    | 51 | .11611    | 57 | 49 |       |
| 50 | 9.03212   | 53 | 9.08160   | 59 | 9.06330   | 50 | 9.11669   | 58 | 50 |       |
| 51 | .03265    | 53 | .08219    | 59 | .06380    | 51 | .11727    | 57 | 51 |       |
| 52 | .03318    | 53 | .08278    | 59 | .06431    | 51 | .11784    | 57 | 52 |       |
| 53 | .03371    | 52 | .08338    | 59 | .06482    | 51 | .11842    | 57 | 53 |       |
| 54 | .03423    | 53 | .08397    | 59 | .06533    | 51 | .11899    | 57 | 54 |       |
| 55 | 9.03476   | 53 | 9.08456   | 59 | 9.06584   | 50 | 9.11957   | 58 | 55 |       |
| 56 | .03529    | 52 | .08515    | 59 | .06635    | 51 | .12015    | 57 | 56 |       |
| 57 | .03582    | 52 | .08574    | 59 | .06686    | 50 | .12072    | 57 | 57 |       |
| 58 | .03634    | 53 | .08634    | 59 | .06736    | 51 | .12129    | 57 | 58 |       |
| 59 | .03687    | 52 | .08693    | 59 | .06787    | 50 | .12187    | 57 | 59 |       |
| 60 | 9.03740   | 52 | 9.08752   | 59 | 9.06838   | 50 | 9.12244   | 57 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|    |           |    |           |    |           |    |           |    |    |       |
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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

28°

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|    |           |    |           |    |           |    |           |    |    | P. P. |       |      |      |  |  |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|-------|------|------|--|--|
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    |       |       |      |      |  |  |
| 0  | 9.06838   | 50 | 9.12244   | 57 | 9.09823   | 49 | 9.15641   | 56 | 0  |       | 57    | 57   | 56   |  |  |
| 1  | .06888    | 51 | .12302    | 57 | .09872    | 48 | .15697    | 55 | 1  | 6     | 5.7   | 5.7  | 5.6  |  |  |
| 2  | .06939    | 50 | .12359    | 57 | .09920    | 49 | .15752    | 56 | 2  | 7     | 6.7   | 6.6  | 6.6  |  |  |
| 3  | .06990    | 50 | .12416    | 57 | .09969    | 48 | .15808    | 55 | 3  | 8     | 7.6   | 7.6  | 7.5  |  |  |
| 4  | .07040    | 50 | .12474    | 57 | .10018    | 48 | .15864    | 55 | 4  | 9     | 8.6   | 8.5  | 8.5  |  |  |
| 5  | 9.07091   | 50 | 9.12531   | 57 | 9.10067   | 49 | 9.15920   | 56 | 5  | 10    | 9.6   | 9.5  | 9.4  |  |  |
| 6  | .07141    | 50 | .12588    | 57 | .10115    | 48 | .15975    | 55 | 6  | 20    | 19.1  | 19.0 | 18.8 |  |  |
| 7  | .07192    | 50 | .12645    | 57 | .10164    | 48 | .16031    | 56 | 7  | 30    | 28.7  | 28.5 | 28.2 |  |  |
| 8  | .07242    | 50 | .12703    | 57 | .10213    | 49 | .16087    | 55 | 8  | 40    | 38.3  | 38.0 | 37.6 |  |  |
| 9  | .07293    | 50 | .12760    | 57 | .10261    | 48 | .16142    | 55 | 9  | 50    | 47.9  | 47.5 | 47.1 |  |  |
| 10 | 9.07343   | 50 | 9.12817   | 57 | 9.10310   | 48 | 9.16198   | 56 | 10 |       | 56    | 55   | 55   |  |  |
| 11 | .97393    | 50 | .12874    | 57 | .10358    | 48 | .16254    | 55 | 11 | 6     | 5.6   | 5.5  | 5.5  |  |  |
| 12 | .07444    | 50 | .12931    | 57 | .10407    | 48 | .16309    | 55 | 12 | 7     | 6.5   | 6.5  | 6.4  |  |  |
| 13 | .07494    | 50 | .12988    | 57 | .10455    | 48 | .16365    | 55 | 13 | 8     | 7.4   | 7.4  | 7.3  |  |  |
| 14 | .07544    | 50 | .13045    | 57 | .10504    | 48 | .16420    | 55 | 14 | 9     | 8.4   | 8.3  | 8.2  |  |  |
| 15 | 9.07594   | 50 | 9.13102   | 57 | 9.10552   | 48 | 9.16476   | 55 | 15 | 10    | 9.3   | 9.2  | 9.1  |  |  |
| 16 | .07644    | 50 | .13159    | 57 | .10601    | 48 | .16531    | 55 | 16 | 20    | 18.6  | 18.5 | 18.3 |  |  |
| 17 | .07695    | 50 | .13216    | 57 | .10649    | 48 | .16587    | 55 | 17 | 30    | 28.0  | 27.7 | 27.5 |  |  |
| 18 | .07745    | 50 | .13273    | 57 | .10697    | 48 | .16642    | 55 | 18 | 40    | 37.3  | 37.0 | 36.6 |  |  |
| 19 | .07795    | 50 | .13330    | 57 | .10746    | 48 | .16698    | 55 | 19 | 50    | 46.6  | 46.2 | 45.8 |  |  |
| 20 | 9.07845   | 50 | 9.13387   | 57 | 9.10794   | 48 | 9.16753   | 55 | 20 |       | 54    | 54   |      |  |  |
| 21 | .07895    | 50 | .13444    | 57 | .10842    | 48 | .16808    | 55 | 21 | 6     | 5.4   | 5.4  |      |  |  |
| 22 | .07945    | 50 | .13500    | 56 | .10890    | 48 | .16864    | 55 | 22 | 7     | 6.3   | 6.3  |      |  |  |
| 23 | .07995    | 50 | .13557    | 57 | .10939    | 48 | .16919    | 55 | 23 | 8     | 7.2   | 7.2  |      |  |  |
| 24 | .08045    | 50 | .13614    | 56 | .10987    | 48 | .16974    | 55 | 24 | 9     | 8.2   | 8.1  |      |  |  |
| 25 | 9.08095   | 50 | 9.13671   | 57 | 9.11035   | 48 | 9.17029   | 55 | 25 | 10    | 9.1   | 9.0  |      |  |  |
| 26 | .08145    | 50 | .13727    | 56 | .11083    | 48 | .17085    | 55 | 26 | 20    | 18.1  | 18.0 |      |  |  |
| 27 | .08195    | 49 | .13784    | 57 | .11131    | 48 | .17140    | 55 | 27 | 30    | 27.2  | 27.0 |      |  |  |
| 28 | .08244    | 50 | .13841    | 56 | .11179    | 48 | .17195    | 55 | 28 | 40    | 36.3  | 36.0 |      |  |  |
| 29 | .08294    | 49 | .13897    | 56 | .11227    | 48 | .17250    | 55 | 29 | 50    | 45.4  | 45.0 |      |  |  |
| 30 | 9.08344   | 49 | 9.13954   | 57 | 9.11275   | 48 | 9.17305   | 55 | 30 |       | 51    | 50   | 50   |  |  |
| 31 | .08394    | 50 | .14011    | 56 | .11323    | 48 | .17361    | 55 | 31 | 6     | 5.1   | 5.0  | 5.0  |  |  |
| 32 | .08443    | 49 | .14067    | 56 | .11371    | 48 | .17416    | 55 | 32 | 7     | 5.9   | 5.9  | 5.8  |  |  |
| 33 | .08493    | 49 | .14124    | 56 | .11419    | 47 | .17471    | 55 | 33 | 8     | 6.8   | 6.7  | 6.6  |  |  |
| 34 | .08543    | 50 | .14180    | 56 | .11467    | 48 | .17526    | 55 | 34 | 9     | 7.6   | 7.6  | 7.5  |  |  |
| 35 | 9.08592   | 49 | 9.14237   | 56 | 9.11515   | 48 | 9.17581   | 55 | 35 | 10    | 8.5   | 8.4  | 8.3  |  |  |
| 36 | .08642    | 49 | .14293    | 56 | .11562    | 47 | .17636    | 55 | 36 | 20    | 17.0  | 16.8 | 16.6 |  |  |
| 37 | .08691    | 49 | .14350    | 56 | .11610    | 48 | .17691    | 55 | 37 | 30    | 25.5  | 25.2 | 25.0 |  |  |
| 38 | .08741    | 49 | .14406    | 56 | .11658    | 47 | .17746    | 55 | 38 | 40    | 34.0  | 33.6 | 33.3 |  |  |
| 39 | .08790    | 49 | .14462    | 56 | .11706    | 47 | .17801    | 55 | 39 | 50    | 42.5  | 42.1 | 41.6 |  |  |
| 40 | 9.08840   | 49 | 9.14519   | 56 | 9.11754   | 48 | 9.17856   | 55 | 40 |       | 49    | 49   | 48   |  |  |
| 41 | .08889    | 49 | .14575    | 56 | .11801    | 47 | .17910    | 54 | 41 | 6     | 4.9   | 4.9  | 4.8  |  |  |
| 42 | .08939    | 49 | .14631    | 56 | .11849    | 47 | .17965    | 55 | 42 | 7     | 5.8   | 5.7  | 5.6  |  |  |
| 43 | .08988    | 49 | .14688    | 56 | .11897    | 48 | .18020    | 54 | 43 | 8     | 6.6   | 6.5  | 6.4  |  |  |
| 44 | .09087    | 49 | .14744    | 56 | .11944    | 47 | .18075    | 54 | 44 | 9     | 7.4   | 7.3  | 7.3  |  |  |
| 45 | 9.09087   | 49 | 9.14800   | 56 | 9.11992   | 47 | 9.18130   | 55 | 45 | 10    | 8.2   | 8.1  | 8.1  |  |  |
| 46 | .09136    | 49 | .14856    | 56 | .12039    | 47 | .18185    | 54 | 46 | 20    | 16.5  | 16.3 | 16.1 |  |  |
| 47 | .09185    | 49 | .14913    | 56 | .12087    | 47 | .18239    | 55 | 47 | 30    | 24.7  | 24.5 | 24.2 |  |  |
| 48 | .09234    | 49 | .14969    | 56 | .12134    | 47 | .18294    | 54 | 48 | 40    | 33.0  | 32.6 | 32.3 |  |  |
| 49 | .09284    | 49 | .15025    | 56 | .12182    | 47 | .18349    | 54 | 49 | 50    | 41.2  | 40.8 | 40.4 |  |  |
| 50 | 9.09333   | 49 | 9.15081   | 56 | 9.12229   | 47 | 9.18403   | 55 | 50 |       | 48    | 47   | 47   |  |  |
| 51 | .09382    | 49 | .15137    | 56 | .12277    | 47 | .18458    | 54 | 51 | 6     | 4.8   | 4.7  | 4.7  |  |  |
| 52 | .09431    | 49 | .15193    | 56 | .12324    | 47 | .18513    | 54 | 52 | 7     | 5.6   | 5.5  | 5.5  |  |  |
| 53 | .09480    | 49 | .15249    | 56 | .12371    | 47 | .18567    | 54 | 53 | 8     | 6.4   | 6.3  | 6.2  |  |  |
| 54 | .09529    | 49 | .15305    | 56 | .12419    | 47 | .18622    | 54 | 54 | 9     | 7.2   | 7.1  | 7.0  |  |  |
| 55 | 9.09578   | 49 | 9.15361   | 56 | 9.12466   | 47 | 9.18676   | 55 | 55 | 10    | 8.0   | 7.9  | 7.8  |  |  |
| 56 | .09627    | 49 | .15417    | 56 | .12513    | 47 | .18731    | 54 | 56 | 20    | 16.0  | 15.8 | 15.6 |  |  |
| 57 | .09676    | 49 | .15473    | 56 | .12560    | 47 | .18786    | 54 | 57 | 30    | 24.0  | 23.7 | 23.5 |  |  |
| 58 | .09725    | 48 | .15529    | 56 | .12608    | 47 | .18840    | 54 | 58 | 40    | 32.0  | 31.6 | 31.3 |  |  |
| 59 | .09774    | 49 | .15585    | 55 | .12655    | 47 | .18894    | 54 | 59 | 50    | 40.0  | 39.6 | 39.1 |  |  |
| 60 | 9.09823   | 49 | 9.15641   | 56 | 9.12702   | 47 | 9.18949   | 54 | 60 |       |       |      |      |  |  |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    |       | P. P. |      |      |  |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------------------|
| 0  | 9.12702   | 47 | 9.18949   | 54 | 9.15483   | 45 | 9.22176   | 53 | 0  |                   |
| 1  | .12749    | 47 | .19003    | 54 | .15528    | 45 | .22229    | 53 | 1  |                   |
| 2  | .12796    | 47 | .19058    | 54 | .15574    | 45 | .22282    | 53 | 2  |                   |
| 3  | .12843    | 47 | .19112    | 54 | .15619    | 45 | .22335    | 53 | 3  |                   |
| 4  | .12890    | 47 | .19167    | 54 | .15665    | 45 | .22388    | 53 | 4  | 54 54 53          |
| 5  | 9.12937   | 47 | 9.19221   | 54 | 9.15710   | 45 | 9.22441   | 53 | 5  | 6 5.4 5.4 5.3     |
| 6  | .12984    | 47 | .19275    | 54 | .15755    | 45 | .22494    | 53 | 6  | 7 6.3 6.3 6.2     |
| 7  | .13031    | 47 | .19329    | 54 | .15801    | 45 | .22547    | 53 | 7  | 8 7.2 7.2 7.1     |
| 8  | .13078    | 47 | .19384    | 54 | .15846    | 45 | .22600    | 53 | 8  | 9 8.2 8.1 8.0     |
| 9  | .13125    | 47 | .19438    | 54 | .15891    | 45 | .22653    | 53 | 9  | 10 9.1 9.0 8.9    |
| 10 | 9.13172   | 46 | 9.19492   | 54 | 9.15937   | 45 | 9.22706   | 53 | 10 | 20 18.1 18.0 17.8 |
| 11 | .13219    | 46 | .19546    | 54 | .15982    | 45 | .22759    | 53 | 11 | 30 27.2 27.0 26.7 |
| 12 | .13266    | 46 | .19601    | 54 | .16027    | 45 | .22812    | 53 | 12 | 40 36.3 36.0 35.6 |
| 13 | .13313    | 46 | .19655    | 54 | .16073    | 45 | .22865    | 53 | 13 | 50 45.4 45.0 44.6 |
| 14 | .13359    | 46 | .19709    | 54 | .16118    | 45 | .22918    | 53 | 14 |                   |
| 15 | 9.13406   | 46 | 9.19763   | 54 | 9.16163   | 45 | 9.22971   | 53 | 15 | 53 52 52          |
| 16 | .13453    | 46 | .19817    | 54 | .16208    | 45 | .23024    | 53 | 16 | 6 5.3 5.2 5.2     |
| 17 | .13500    | 46 | .19871    | 54 | .16253    | 45 | .23076    | 53 | 17 | 7 6.2 6.1 6.0     |
| 18 | .13546    | 46 | .19925    | 54 | .16298    | 45 | .23129    | 53 | 18 | 8 7.0 7.0 6.9     |
| 19 | .13593    | 46 | .19979    | 54 | .16343    | 45 | .23182    | 53 | 19 | 9 7.9 7.9 7.8     |
| 20 | 9.13639   | 46 | 9.20033   | 54 | 9.16388   | 45 | 9.23235   | 53 | 20 | 10 8.8 8.7 8.6    |
| 21 | .13686    | 46 | .20087    | 54 | .16434    | 45 | .23287    | 53 | 21 | 20 17.6 17.5 17.3 |
| 22 | .13733    | 46 | .20141    | 54 | .16479    | 44 | .23340    | 53 | 22 | 30 26.5 26.2 26.0 |
| 23 | .13779    | 46 | .20195    | 54 | .16523    | 45 | .23393    | 53 | 23 | 40 35.3 35.0 34.6 |
| 24 | .13826    | 46 | .20249    | 54 | .16568    | 45 | .23446    | 53 | 24 | 50 44.1 43.7 43.3 |
| 25 | 9.13872   | 46 | 9.20303   | 53 | 9.16613   | 45 | 9.23498   | 52 | 25 |                   |
| 26 | .13919    | 46 | .20357    | 54 | .16658    | 45 | .23551    | 52 | 26 |                   |
| 27 | .13965    | 46 | .20411    | 54 | .16703    | 45 | .23603    | 52 | 27 | 6 4.7 4.7 4.6     |
| 28 | .14011    | 46 | .20465    | 53 | .16748    | 44 | .23656    | 52 | 28 | 7 5.5 5.5 5.4     |
| 29 | .14058    | 46 | .20518    | 53 | .16793    | 44 | .23709    | 52 | 29 | 8 6.3 6.2 6.2     |
| 30 | 9.14104   | 46 | 9.20572   | 54 | 9.16838   | 45 | 9.23761   | 52 | 30 | 9 7.1 7.0 7.0     |
| 31 | .14151    | 46 | .20626    | 53 | .16882    | 45 | .23814    | 52 | 31 | 10 7.9 7.8 7.7    |
| 32 | .14197    | 46 | .20680    | 54 | .16927    | 45 | .23866    | 52 | 32 | 20 15.8 15.6 15.5 |
| 33 | .14243    | 46 | .20733    | 54 | .16972    | 44 | .23919    | 52 | 33 | 30 23.7 23.5 23.3 |
| 34 | .14289    | 46 | .20787    | 54 | .17017    | 44 | .23971    | 52 | 34 | 40 31.6 31.3 31.0 |
| 35 | 9.14336   | 46 | 9.20841   | 53 | 9.17061   | 44 | 9.24024   | 53 | 35 | 50 39.6 39.1 38.7 |
| 36 | .14382    | 46 | .20894    | 53 | .17106    | 44 | .24076    | 52 | 36 |                   |
| 37 | .14428    | 46 | .20948    | 54 | .17151    | 45 | .24128    | 52 | 37 |                   |
| 38 | .14474    | 46 | .21002    | 53 | .17195    | 44 | .24181    | 52 | 38 | 46 45 45          |
| 39 | .14520    | 46 | .21055    | 53 | .17240    | 44 | .24233    | 52 | 39 | 6 4.6 4.5 4.5     |
| 40 | 9.14566   | 46 | 9.21109   | 53 | 9.17284   | 44 | 9.24285   | 52 | 40 | 7 5.3 5.3 5.2     |
| 41 | .14612    | 46 | .21162    | 53 | .17329    | 44 | .24338    | 52 | 41 | 8 6.1 6.0 6.0     |
| 42 | .14658    | 46 | .21216    | 53 | .17373    | 44 | .24390    | 52 | 42 | 9 6.9 6.8 6.7     |
| 43 | .14704    | 46 | .21269    | 53 | .17418    | 44 | .24442    | 52 | 43 | 10 7.6 7.6 7.5    |
| 44 | .14750    | 46 | .21323    | 53 | .17462    | 44 | .24495    | 52 | 44 | 20 15.3 15.1 15.0 |
| 45 | 9.14796   | 46 | 9.21376   | 53 | 9.17507   | 44 | 9.24547   | 52 | 45 | 30 23.0 22.7 22.5 |
| 46 | .14842    | 46 | .21430    | 53 | .17551    | 44 | .24599    | 52 | 46 | 40 30.6 30.3 30.0 |
| 47 | .14888    | 46 | .21483    | 53 | .17596    | 44 | .24651    | 52 | 47 | 50 38.3 37.9 37.5 |
| 48 | .14934    | 45 | .21537    | 53 | .17640    | 44 | .24704    | 52 | 48 |                   |
| 49 | .14980    | 45 | .21590    | 53 | .17684    | 44 | .24756    | 52 | 49 |                   |
| 50 | 9.15026   | 45 | 9.21643   | 53 | 9.17729   | 44 | 9.24808   | 52 | 50 | 44 44             |
| 51 | .15071    | 46 | .21697    | 53 | .17773    | 44 | .24860    | 52 | 51 | 6 4.4 4.4 4.4     |
| 52 | .15117    | 46 | .21750    | 53 | .17817    | 44 | .24912    | 52 | 52 | 7 5.2 5.1 5.1     |
| 53 | .15163    | 45 | .21803    | 53 | .17861    | 44 | .24964    | 52 | 53 | 8 5.9 5.8 5.8     |
| 54 | .15209    | 45 | .21857    | 53 | .17906    | 44 | .25016    | 52 | 54 | 9 6.7 6.6 6.6     |
| 55 | 9.15254   | 45 | 9.21910   | 53 | 9.17950   | 44 | 9.25068   | 52 | 55 | 10 7.4 7.3 7.3    |
| 56 | .15300    | 45 | .21963    | 53 | .17994    | 44 | .25120    | 52 | 56 | 20 14.8 14.6      |
| 57 | .15346    | 45 | .22016    | 53 | .18038    | 44 | .25172    | 52 | 57 | 30 22.2 22.0      |
| 58 | .15391    | 45 | .22070    | 53 | .18082    | 44 | .25224    | 52 | 58 | 40 29.6 29.3      |
| 59 | .15437    | 45 | .22123    | 53 | .18126    | 44 | .25276    | 52 | 59 | 50 37.1 36.6      |
| 60 | 9.15483   | 46 | 9.22176   | 53 | 9.18170   | 44 | 9.25328   | 52 | 60 |                   |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |

## TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------------------|
| 0  | 9.18170   | 44 | 9.25328   | 52 | 9.20771   | 42 | 9.28412   | 51 | 0  |                   |
| 1  | .18214    | 44 | .25380    | 52 | .20814    | 42 | .28463    | 51 | 1  |                   |
| 2  | .18258    | 44 | .25432    | 52 | .20856    | 42 | .28514    | 50 | 2  |                   |
| 3  | .18302    | 44 | .25484    | 51 | .20899    | 43 | .28564    | 50 | 3  |                   |
| 4  | .18346    | 44 | .25536    | 52 | .20942    | 42 | .28615    | 51 | 4  | 52 51 51          |
| 5  | 9.18390   | 44 | 9.25588   | 52 | 9.20984   | 42 | 9.28665   | 51 | 5  | 6 5.2 5.1 5.1     |
| 6  | .18434    | 44 | .25640    | 52 | .21027    | 42 | .28717    | 50 | 6  | 7 6.0 6.0 5.9     |
| 7  | .18478    | 44 | .25692    | 52 | .21069    | 42 | .28768    | 51 | 7  | 8 6.9 6.8 6.8     |
| 8  | .18522    | 43 | .25743    | 52 | .21112    | 42 | .28818    | 50 | 8  | 9 7.8 7.7 7.6     |
| 9  | .18566    | 44 | .25795    | 51 | .21154    | 42 | .28869    | 50 | 9  | 10 8.6 8.6 8.5    |
| 10 | 9.18610   | 44 | 9.25847   | 52 | 9.21196   | 42 | 9.28920   | 51 | 10 | 20 17.3 17.1 17.0 |
| 11 | .18654    | 43 | .25899    | 51 | .21239    | 42 | .28970    | 50 | 11 | 30 26.0 25.7 25.5 |
| 12 | .18697    | 44 | .25950    | 52 | .21281    | 42 | .29021    | 51 | 12 | 40 34.6 34.3 34.0 |
| 13 | .18741    | 43 | .26002    | 51 | .21324    | 42 | .29072    | 50 | 13 | 50 43.3 42.9 42.5 |
| 14 | .18785    | 44 | .26054    | 51 | .21366    | 42 | .29122    | 50 | 14 |                   |
| 15 | 9.18829   | 43 | 9.26105   | 52 | 9.21408   | 42 | 9.29173   | 51 | 15 | 50 50 50 49       |
| 16 | .18872    | 43 | .26157    | 51 | .21451    | 42 | .29223    | 50 | 16 | 6 5.0 5.0 4.9     |
| 17 | .18916    | 43 | .26209    | 51 | .21493    | 42 | .29274    | 50 | 17 | 7 5.9 5.8 5.8     |
| 18 | .18959    | 44 | .26260    | 51 | .21535    | 42 | .29324    | 50 | 18 | 8 6.7 6.6 6.6     |
| 19 | .19003    | 43 | .26312    | 51 | .21577    | 42 | .29375    | 51 | 19 | 9 7.6 7.5 7.4     |
| 20 | 9.19047   | 43 | 9.26364   | 52 | 9.21620   | 42 | 9.29426   | 50 | 20 | 10 8.4 8.3 8.2    |
| 21 | .19090    | 43 | .26415    | 51 | .21662    | 42 | .29476    | 50 | 21 | 20 16.8 16.6 16.5 |
| 22 | .19134    | 43 | .26467    | 51 | .21704    | 42 | .29527    | 50 | 22 | 30 25.2 25.0 24.7 |
| 23 | .19177    | 43 | .26518    | 51 | .21746    | 42 | .29577    | 50 | 23 | 40 33.6 33.3 33.0 |
| 24 | .19221    | 43 | .26570    | 51 | .21788    | 42 | .29627    | 50 | 24 | 50 42.1 41.6 41.2 |
| 25 | 9.19264   | 43 | 9.26621   | 51 | 9.21830   | 42 | 9.29678   | 50 | 25 |                   |
| 26 | .19308    | 43 | .26673    | 51 | .21872    | 42 | .29728    | 50 | 26 | 44 43 43          |
| 27 | .19351    | 43 | .26724    | 51 | .21914    | 42 | .29779    | 50 | 27 | 6 4.4 4.3 4.3     |
| 28 | .19395    | 43 | .26776    | 51 | .21956    | 42 | .29829    | 50 | 28 | 7 5.1 5.1 5.0     |
| 29 | .19438    | 43 | .26827    | 51 | .21998    | 42 | .29879    | 50 | 29 | 8 5.8 5.8 5.7     |
| 30 | 9.19481   | 43 | 9.26878   | 51 | 9.22040   | 42 | 9.29930   | 50 | 30 | 9 6.6 6.5 6.4     |
| 31 | .19525    | 43 | .26930    | 51 | .22082    | 42 | .29980    | 50 | 31 | 10 7.3 7.2 7.1    |
| 32 | .19568    | 43 | .26981    | 51 | .22124    | 42 | .30030    | 50 | 32 | 20 14.6 14.5 14.3 |
| 33 | .19611    | 43 | .27032    | 51 | .22166    | 42 | .30081    | 50 | 33 | 30 22.0 21.7 21.5 |
| 34 | .19654    | 43 | .27084    | 51 | .22208    | 42 | .30131    | 50 | 34 | 40 29.3 29.0 28.6 |
| 35 | 9.19698   | 43 | 9.27135   | 51 | 9.22250   | 42 | 9.30181   | 50 | 35 | 50 36.6 36.2 35.8 |
| 36 | .19741    | 43 | .27186    | 51 | .22292    | 41 | .30231    | 50 | 36 |                   |
| 37 | .19784    | 43 | .27238    | 51 | .22334    | 42 | .30282    | 50 | 37 | 42 42 41          |
| 38 | .19827    | 43 | .27289    | 51 | .22376    | 41 | .30332    | 50 | 38 | 6 4.2 4.2 4.1     |
| 39 | .19870    | 43 | .27340    | 51 | .22417    | 41 | .30382    | 50 | 39 | 7 4.9 4.9 4.8     |
| 40 | 9.19914   | 43 | 9.27391   | 51 | 9.22459   | 42 | 9.30432   | 50 | 40 | 8 5.6 5.6 5.5     |
| 41 | .19957    | 43 | .27443    | 51 | .22501    | 42 | .30482    | 50 | 41 | 9 6.4 6.3 6.2     |
| 42 | .20000    | 43 | .27494    | 51 | .22543    | 41 | .30533    | 50 | 42 | 10 7.1 7.0 6.9    |
| 43 | .20043    | 43 | .27545    | 51 | .22584    | 41 | .30583    | 50 | 43 | 20 14.1 14.0 13.8 |
| 44 | .20086    | 43 | .27596    | 51 | .22626    | 41 | .30633    | 50 | 44 | 30 21.2 21.0 20.7 |
| 45 | 9.20129   | 43 | 9.27647   | 51 | 9.22668   | 42 | 9.30683   | 50 | 45 | 40 28.3 28.0 27.6 |
| 46 | .20172    | 43 | .27698    | 51 | .22709    | 41 | .30733    | 50 | 46 | 50 35.4 35.0 34.6 |
| 47 | .20215    | 43 | .27749    | 51 | .22751    | 41 | .30783    | 50 | 47 |                   |
| 48 | .20258    | 43 | .27800    | 51 | .22792    | 41 | .30833    | 50 | 48 |                   |
| 49 | .20301    | 42 | .27852    | 51 | .22834    | 41 | .30883    | 50 | 49 | 41                |
| 50 | 9.20343   | 43 | 9.27903   | 51 | 9.22876   | 41 | 9.30933   | 50 | 50 | 6 4.1             |
| 51 | .20386    | 43 | .27954    | 51 | .22917    | 41 | .30983    | 50 | 51 | 7 4.8             |
| 52 | .20429    | 43 | .28005    | 51 | .22959    | 41 | .31033    | 50 | 52 | 8 5.4             |
| 53 | .20472    | 43 | .28056    | 51 | .23000    | 41 | .31083    | 50 | 53 | 9 6.1             |
| 54 | .20515    | 42 | .28107    | 50 | .23042    | 41 | .31133    | 50 | 54 | 10 6.8            |
| 55 | 9.20558   | 43 | 9.28157   | 51 | 9.23083   | 41 | 9.31183   | 49 | 55 | 20 13.6           |
| 56 | .20600    | 43 | .28208    | 51 | .23124    | 41 | .31233    | 50 | 56 | 30 20.5           |
| 57 | .20643    | 43 | .28259    | 51 | .23166    | 41 | .31283    | 50 | 57 | 40 27.8           |
| 58 | .20686    | 42 | .28310    | 51 | .23207    | 41 | .31333    | 50 | 58 | 50 34.1           |
| 59 | .20728    | 43 | .28361    | 50 | .23248    | 41 | .31383    | 50 | 59 |                   |
| 60 | 9.20771   | 43 | 9.28412   | 50 | 9.23290   | 41 | 9.31432   | 49 | 60 |                   |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.             |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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| Lg. Vers. |         |    | Log. Exs. |    |         | Lg. Vers. |         |    | Log. Exs. |    |      | P. P. |      |  |
|-----------|---------|----|-----------|----|---------|-----------|---------|----|-----------|----|------|-------|------|--|
|           | D       |    |           | D  |         |           | D       |    | D         |    | D    |       | D    |  |
| 0         | 9.23290 | 41 | 9.31432   | 50 | 9.25731 | 40        | 9.34395 | 49 | 0         | 50 | 49   | 49    |      |  |
| 1         | .23331  | 41 | .31482    | 50 | .25777  | 40        | .34444  | 48 | 1         | 6  | 5.0  | 4.9   | 4.9  |  |
| 2         | .23372  | 41 | .31532    | 50 | .25811  | 40        | .34492  | 48 | 2         | 7  | 5.8  | 5.8   | 5.7  |  |
| 3         | .23414  | 41 | .31582    | 50 | .25851  | 40        | .34541  | 49 | 3         | 8  | 6.6  | 6.6   | 6.5  |  |
| 4         | .23455  | 41 | .31632    | 49 | .25891  | 40        | .34590  | 49 | 4         | 9  | 7.5  | 7.4   | 7.3  |  |
| 5         | 9.23496 | 41 | 9.31681   | 49 | 9.25931 | 40        | 9.34639 | 49 | 5         | 10 | 8.3  | 8.2   | 8.1  |  |
| 6         | .23537  | 41 | .31731    | 50 | .25977  | 40        | .34688  | 48 | 6         | 20 | 16.6 | 16.5  | 16.3 |  |
| 7         | .23579  | 41 | .31781    | 50 | .26011  | 39        | .34737  | 49 | 7         | 30 | 25.0 | 24.7  | 24.5 |  |
| 8         | .23620  | 41 | .31831    | 49 | .26051  | 40        | .34785  | 48 | 8         | 40 | 33.3 | 33.0  | 32.6 |  |
| 9         | .23661  | 41 | .31880    | 49 | .26091  | 40        | .34834  | 49 | 9         | 50 | 41.6 | 41.2  | 40.8 |  |
| 10        | 9.23702 | 41 | 9.31930   | 50 | 9.26131 | 40        | 9.34883 | 49 | 10        |    | 48   | 48    |      |  |
| 11        | .23743  | 41 | .31980    | 49 | .26171  | 39        | .34932  | 48 | 11        | 6  | 4.8  | 4.8   |      |  |
| 12        | .23784  | 41 | .32029    | 49 | .26210  | 40        | .34980  | 48 | 12        | 7  | 5.6  | 5.6   |      |  |
| 13        | .23825  | 41 | .32079    | 50 | .26250  | 40        | .35029  | 48 | 13        | 8  | 6.4  | 6.4   |      |  |
| 14        | .23866  | 41 | .32129    | 49 | .26290  | 39        | .35078  | 49 | 14        | 9  | 7.3  | 7.2   |      |  |
| 15        | 9.23907 | 41 | 9.32178   | 49 | 9.26330 | 40        | 9.35127 | 48 | 15        | 10 | 8.1  | 8.0   |      |  |
| 16        | .23948  | 41 | .32228    | 49 | .26370  | 39        | .35175  | 49 | 16        | 20 | 16.1 | 16.0  |      |  |
| 17        | .23989  | 41 | .32277    | 49 | .26409  | 40        | .35224  | 48 | 17        | 30 | 24.2 | 24.0  |      |  |
| 18        | .24030  | 41 | .32327    | 50 | .26449  | 39        | .35273  | 48 | 18        | 40 | 32.3 | 32.0  |      |  |
| 19        | .24071  | 40 | .32377    | 49 | .26489  | 39        | .35321  | 48 | 19        | 50 | 40.4 | 40.0  |      |  |
| 20        | 9.24112 | 41 | 9.32426   | 49 | 9.26528 | 40        | 9.35370 | 49 | 20        |    | 41   | 41    |      |  |
| 21        | .24153  | 41 | .32476    | 49 | .26568  | 39        | .35419  | 48 | 21        | 6  | 4.1  | 4.1   |      |  |
| 22        | .24194  | 41 | .32525    | 49 | .26608  | 39        | .35467  | 48 | 22        | 7  | 4.8  | 4.8   |      |  |
| 23        | .24235  | 40 | .32575    | 49 | .26647  | 39        | .35516  | 48 | 23        | 8  | 5.5  | 5.4   |      |  |
| 24        | .24275  | 41 | .32624    | 49 | .26687  | 39        | .35564  | 48 | 24        | 9  | 6.2  | 6.1   |      |  |
| 25        | 9.24316 | 40 | 9.32673   | 49 | 9.26726 | 40        | 9.35613 | 48 | 25        | 10 | 6.9  | 6.8   |      |  |
| 26        | .24357  | 40 | .32723    | 49 | .26766  | 39        | .35661  | 48 | 26        | 20 | 13.8 | 13.6  |      |  |
| 27        | .24398  | 40 | .32772    | 49 | .26806  | 39        | .35710  | 48 | 27        | 30 | 20.7 | 20.5  |      |  |
| 28        | .24438  | 41 | .32822    | 49 | .26845  | 39        | .35758  | 48 | 28        | 40 | 27.6 | 27.3  |      |  |
| 29        | .24479  | 40 | .32871    | 49 | .26885  | 39        | .35807  | 48 | 29        | 50 | 34.6 | 34.1  |      |  |
| 30        | 9.24520 | 41 | 9.32920   | 49 | 9.26924 | 39        | 9.35855 | 48 | 30        |    | 40   | 40    |      |  |
| 31        | .24561  | 40 | .32970    | 49 | .26964  | 39        | .35904  | 48 | 31        | 6  | 4.0  | 4.0   |      |  |
| 32        | .24601  | 40 | .33019    | 49 | .27003  | 39        | .35952  | 48 | 32        | 7  | 4.7  | 4.6   |      |  |
| 33        | .24642  | 40 | .33069    | 49 | .27042  | 39        | .36001  | 48 | 33        | 8  | 5.4  | 5.3   |      |  |
| 34        | .24682  | 41 | .33118    | 49 | .27082  | 39        | .36049  | 48 | 34        | 9  | 6.1  | 6.0   |      |  |
| 35        | 9.24723 | 40 | 9.33167   | 49 | 9.27121 | 39        | 9.36098 | 48 | 35        | 10 | 6.7  | 6.6   |      |  |
| 36        | .24764  | 40 | .33216    | 49 | .27161  | 39        | .36146  | 48 | 36        | 20 | 13.5 | 13.3  |      |  |
| 37        | .24804  | 40 | .33266    | 49 | .27200  | 39        | .36194  | 48 | 37        | 30 | 20.2 | 20.0  |      |  |
| 38        | .24845  | 40 | .33315    | 49 | .27239  | 39        | .36243  | 48 | 38        | 40 | 27.0 | 26.6  |      |  |
| 39        | .24885  | 40 | .33364    | 49 | .27278  | 39        | .36291  | 48 | 39        | 50 | 33.7 | 33.3  |      |  |
| 40        | 9.24926 | 40 | 9.33413   | 49 | 9.27318 | 39        | 9.36340 | 48 | 40        |    | 39   | 39    |      |  |
| 41        | .24966  | 40 | .33463    | 49 | .27357  | 39        | .36388  | 48 | 41        | 6  | 3.9  | 3.9   |      |  |
| 42        | .25007  | 40 | .33512    | 49 | .27396  | 39        | .36436  | 48 | 42        | 7  | 4.6  | 4.5   |      |  |
| 43        | .25047  | 40 | .33561    | 49 | .27435  | 39        | .36484  | 48 | 43        | 8  | 5.2  | 5.2   |      |  |
| 44        | .25087  | 40 | .33610    | 49 | .27475  | 39        | .36533  | 48 | 44        | 9  | 5.9  | 5.8   |      |  |
| 45        | 9.25128 | 40 | 9.33659   | 49 | 9.27514 | 39        | 9.36581 | 48 | 45        | 10 | 6.6  | 6.5   |      |  |
| 46        | .25168  | 40 | .33708    | 49 | .27553  | 39        | .36629  | 48 | 46        | 20 | 13.1 | 13.0  |      |  |
| 47        | .25209  | 40 | .33758    | 49 | .27592  | 39        | .36678  | 48 | 47        | 30 | 19.7 | 19.5  |      |  |
| 48        | .25249  | 40 | .33807    | 49 | .27631  | 39        | .36726  | 48 | 48        | 40 | 26.3 | 26.0  |      |  |
| 49        | .25289  | 40 | .33856    | 49 | .27670  | 39        | .36774  | 48 | 49        | 50 | 32.9 | 32.5  |      |  |
| 50        | 9.25329 | 40 | 9.33905   | 49 | 9.27709 | 39        | 9.36822 | 48 | 50        |    | 38   | 38    |      |  |
| 51        | .25370  | 40 | .33954    | 49 | .27749  | 39        | .36870  | 48 | 51        | 6  | 3.8  | 3.8   |      |  |
| 52        | .25410  | 40 | .34003    | 49 | .27788  | 39        | .36919  | 48 | 52        | 7  | 4.5  | 4.5   |      |  |
| 53        | .25450  | 40 | .34052    | 49 | .27827  | 39        | .36967  | 48 | 53        | 8  | 5.1  | 5.1   |      |  |
| 54        | .25490  | 40 | .34101    | 49 | .27866  | 39        | .37015  | 48 | 54        | 9  | 5.8  | 5.8   |      |  |
| 55        | 9.25531 | 40 | 9.34150   | 49 | 9.27905 | 39        | 9.37063 | 48 | 55        | 10 | 6.4  | 6.4   |      |  |
| 56        | .25571  | 40 | .34199    | 49 | .27944  | 38        | .37111  | 48 | 56        | 20 | 12.8 | 12.8  |      |  |
| 57        | .25611  | 40 | .34248    | 49 | .27983  | 39        | .37159  | 48 | 57        | 30 | 19.2 | 19.2  |      |  |
| 58        | .25651  | 40 | .34297    | 49 | .28021  | 39        | .37207  | 48 | 58        | 40 | 25.6 | 25.6  |      |  |
| 59        | .25691  | 40 | .34346    | 49 | .28060  | 39        | .37255  | 48 | 59        | 50 | 32.1 | 32.1  |      |  |
| 60        | 9.25731 | 40 | 9.34395   | 49 | 9.28099 | 39        | 9.37303 | 48 | 60        |    |      |       |      |  |
| Lg. Vers. |         |    | Log. Exs. |    |         | Lg. Vers. |         |    | Log. Exs. |    |      | P. P. |      |  |
|           | D       |    |           | D  |         |           | D       |    |           |    |      |       |      |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.28099   | 39 | 9.37303   | 48 | 9.30398   | 37 | 9.40163   | 47 | 0  | 48 48        |
| 1  | .28138    | 38 | .37352    | 48 | .30436    | 38 | .40210    | 47 | 1  | 6 4.8 4.8    |
| 2  | .28177    | 39 | .37400    | 48 | .30474    | 37 | .40258    | 47 | 2  | 7 5.6 5.6    |
| 3  | .28816    | 39 | .37448    | 48 | .30511    | 37 | .40305    | 47 | 3  | 8 6.4 6.4    |
| 4  | .28255    | 38 | .37496    | 48 | .30549    | 37 | .40352    | 47 | 4  | 9 7.3 7.2    |
| 5  | 9.28293   | 39 | 9.37544   | 48 | 9.30587   | 38 | 9.40399   | 47 | 5  | 10 8.1 8.0   |
| 6  | .28332    | 38 | .37592    | 48 | .30624    | 37 | .40447    | 47 | 6  | 20 16.1 16.0 |
| 7  | .28371    | 39 | .37640    | 47 | .30662    | 37 | .40494    | 47 | 7  | 30 24.2 24.0 |
| 8  | .28410    | 39 | .37687    | 48 | .30700    | 38 | .40541    | 47 | 8  | 40 32.3 32.0 |
| 9  | .28448    | 38 | .37735    | 48 | .30737    | 37 | .40588    | 47 | 9  | 50 40.4 40.0 |
| 10 | 9.28487   | 39 | 9.37783   | 48 | 9.30775   | 37 | 9.40635   | 47 | 10 | 47 47        |
| 11 | .28526    | 38 | .37831    | 48 | .30812    | 37 | .40682    | 47 | 11 | 6 4.7 4.7    |
| 12 | .28564    | 39 | .37879    | 48 | .30850    | 37 | .40730    | 47 | 12 | 7 5.5 5.5    |
| 13 | .28603    | 38 | .37927    | 47 | .30887    | 37 | .40777    | 47 | 13 | 8 6.3 6.2    |
| 14 | .28642    | 38 | .37975    | 48 | .30925    | 37 | .40824    | 47 | 14 | 9 7.1 7.0    |
| 15 | 9.28680   | 39 | 9.38023   | 48 | 9.30962   | 37 | 9.40871   | 47 | 15 | 10 7.9 7.8   |
| 16 | .28719    | 38 | .38071    | 48 | .31000    | 37 | .40918    | 47 | 16 | 20 15.8 15.6 |
| 17 | .28757    | 38 | .38119    | 48 | .31037    | 37 | .40965    | 47 | 17 | 30 23.7 23.5 |
| 18 | .28796    | 39 | .38166    | 47 | .31075    | 37 | .41012    | 47 | 18 | 40 31.6 31.3 |
| 19 | .28835    | 38 | .38214    | 47 | .31112    | 37 | .41059    | 47 | 19 | 50 39.6 39.1 |
| 20 | 9.28873   | 39 | 9.38262   | 48 | 9.31150   | 37 | 9.41106   | 47 | 20 | 46 46        |
| 21 | .28912    | 38 | .38310    | 47 | .31187    | 37 | .41153    | 47 | 21 | 6 4.6 4.6    |
| 22 | .28950    | 38 | .38357    | 48 | .31224    | 37 | .41200    | 47 | 22 | 7 5.4 5.4    |
| 23 | .28988    | 38 | .38405    | 47 | .31262    | 37 | .41247    | 47 | 23 | 8 6.2 6.2    |
| 24 | .29027    | 38 | .38453    | 48 | .31299    | 37 | .41294    | 47 | 24 | 9 7.0 7.0    |
| 25 | 9.29065   | 39 | 9.38501   | 47 | 9.31336   | 37 | 9.41341   | 47 | 25 | 10 7.7 7.7   |
| 26 | .29104    | 38 | .38548    | 48 | .31374    | 37 | .41388    | 47 | 26 | 20 15.5 15.5 |
| 27 | .29142    | 38 | .38596    | 47 | .31411    | 37 | .41435    | 47 | 27 | 30 23.2 23.2 |
| 28 | .29180    | 38 | .38644    | 48 | .31448    | 37 | .41482    | 47 | 28 | 40 31.0 31.0 |
| 29 | .29219    | 38 | .38692    | 47 | .31485    | 37 | .41529    | 47 | 29 | 50 38.7 38.7 |
| 30 | 9.29257   | 39 | 9.38739   | 47 | 9.31523   | 37 | 9.41576   | 46 | 30 | 39 38        |
| 31 | .29295    | 38 | .38787    | 47 | .31560    | 37 | .41623    | 47 | 31 | 6 3.9 3.8    |
| 32 | .29334    | 38 | .38834    | 48 | .31597    | 37 | .41670    | 47 | 32 | 7 4.5 4.5    |
| 33 | .29372    | 38 | .38882    | 47 | .31634    | 37 | .41717    | 47 | 33 | 8 5.2 5.1    |
| 34 | .29410    | 38 | .38930    | 47 | .31671    | 37 | .41763    | 46 | 34 | 9 5.8 5.8    |
| 35 | 9.29448   | 39 | 9.38977   | 47 | 9.31708   | 37 | 9.41810   | 47 | 35 | 10 6.5 6.4   |
| 36 | .29487    | 38 | .39025    | 47 | .31746    | 37 | .41857    | 47 | 36 | 20 13.0 12.8 |
| 37 | .29525    | 38 | .39072    | 48 | .31783    | 37 | .41904    | 46 | 37 | 30 10.5 10.2 |
| 38 | .29563    | 38 | .39120    | 47 | .31820    | 37 | .41951    | 47 | 38 | 40 26.0 25.6 |
| 39 | .29601    | 38 | .39168    | 47 | .31857    | 37 | .41998    | 47 | 39 | 50 32.5 32.1 |
| 40 | 9.29639   | 39 | 9.39215   | 48 | 9.31894   | 37 | 9.42044   | 46 | 40 | 38 37        |
| 41 | .29677    | 38 | .39263    | 47 | .31931    | 37 | .42091    | 47 | 41 | 6 3.8 3.7    |
| 42 | .29715    | 38 | .39310    | 47 | .31968    | 37 | .42138    | 46 | 42 | 7 4.4 4.4    |
| 43 | .29754    | 38 | .39358    | 47 | .32005    | 37 | .42185    | 47 | 43 | 8 5.0 5.0    |
| 44 | .29792    | 38 | .39405    | 47 | .32042    | 37 | .42231    | 46 | 44 | 9 5.7 5.6    |
| 45 | 9.29830   | 39 | 9.39453   | 47 | 9.32079   | 37 | 9.42278   | 47 | 45 | 10 6.3 6.2   |
| 46 | .29868    | 38 | .39500    | 47 | .32116    | 37 | .42325    | 46 | 46 | 20 12.6 12.5 |
| 47 | .29906    | 38 | .39548    | 47 | .32153    | 37 | .42372    | 47 | 47 | 30 19.0 18.7 |
| 48 | .29944    | 38 | .39595    | 47 | .32190    | 37 | .42418    | 46 | 48 | 40 25.3 25.0 |
| 49 | .29982    | 38 | .39642    | 47 | .32227    | 37 | .42465    | 47 | 49 | 50 31.6 31.2 |
| 50 | 9.30020   | 39 | 9.39690   | 47 | 9.32263   | 36 | 9.42512   | 46 | 50 | 37 36        |
| 51 | .30057    | 38 | .39737    | 47 | .32300    | 37 | .42558    | 46 | 51 | 6 3.7 3.6    |
| 52 | .30095    | 38 | .39785    | 47 | .32337    | 37 | .42605    | 47 | 52 | 7 4.3 4.2    |
| 53 | .30133    | 38 | .39832    | 47 | .32374    | 36 | .42652    | 46 | 53 | 8 4.9 4.8    |
| 54 | .30171    | 38 | .39879    | 47 | .32411    | 37 | .42698    | 46 | 54 | 9 5.5 5.5    |
| 55 | 9.30209   | 39 | 9.39927   | 47 | 9.32447   | 36 | 9.42745   | 47 | 55 | 10 6.1 6.1   |
| 56 | .30247    | 37 | .39974    | 47 | .32484    | 37 | .42792    | 46 | 56 | 20 12.3 12.1 |
| 57 | .30285    | 37 | .40021    | 47 | .32521    | 37 | .42838    | 46 | 57 | 30 18.5 18.2 |
| 58 | .30322    | 38 | .40069    | 47 | .32558    | 36 | .42885    | 46 | 58 | 40 24.6 24.3 |
| 59 | .30360    | 38 | .40116    | 47 | .32594    | 37 | .42931    | 46 | 59 | 50 30.8 30.4 |
| 60 | 9.30398   | 39 | 9.40163   | 47 | 9.32631   | 37 | 9.42978   | 46 | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.32831   |    | 9.42978   |    | 9.34802   |    | 9.45752   |    | 0  | 47 46        |
| 1  | .32868    | 36 | .43024    | 46 | .34837    | 35 | .45797    | 45 | 1  | 6 4.7 4.6    |
| 2  | .32704    | 37 | .43071    | 46 | .34873    | 35 | .45843    | 46 | 2  | 7 5.5 5.4    |
| 3  | .32741    | 36 | .43118    | 46 | .34909    | 35 | .45889    | 46 | 3  | 8 6.2 6.2    |
| 4  | .32778    | 36 | .43164    | 46 | .34944    | 35 | .45935    | 46 | 4  | 9 7.0 7.0    |
| 5  | 9.32814   | 36 | 9.43211   | 46 | 9.34980   | 35 | 9.45981   | 45 | 5  | 10 7.8 7.7   |
| 6  | .32851    | 37 | .43257    | 46 | .35016    | 35 | .46027    | 46 | 6  | 20 15.6 15.5 |
| 7  | .32888    | 36 | .43304    | 46 | .35051    | 35 | .46073    | 46 | 7  | 30 23.5 23.2 |
| 8  | .32924    | 36 | .43350    | 46 | .35087    | 35 | .46118    | 45 | 8  | 40 31.3 31.0 |
| 9  | .32961    | 36 | .43396    | 46 | .35122    | 35 | .46164    | 46 | 9  | 50 39.1 38.7 |
| 10 | 9.32997   | 36 | 9.43443   | 46 | 9.35158   | 35 | 9.46210   | 46 | 10 | 46 45        |
| 11 | .33034    | 36 | .43489    | 46 | .35193    | 35 | .46256    | 45 | 11 | 6 4.6 4.5    |
| 12 | .33070    | 36 | .43536    | 46 | .35229    | 35 | .46302    | 46 | 12 | 7 5.3 5.3    |
| 13 | .33107    | 36 | .43582    | 46 | .35264    | 35 | .46347    | 45 | 13 | 8 6.1 6.0    |
| 14 | .33143    | 36 | .43629    | 46 | .35300    | 35 | .46393    | 46 | 14 | 9 6.9 6.8    |
| 15 | 9.33180   | 36 | 9.43675   | 46 | 9.35335   | 35 | 9.46439   | 45 | 15 | 10 7.6 7.6   |
| 16 | .33216    | 36 | .43721    | 46 | .35370    | 35 | .46485    | 46 | 16 | 20 15.3 15.1 |
| 17 | .33252    | 36 | .43768    | 46 | .35406    | 35 | .46530    | 45 | 17 | 30 23.0 22.7 |
| 18 | .33289    | 36 | .43814    | 46 | .35441    | 35 | .46576    | 46 | 18 | 40 30.6 30.3 |
| 19 | .33325    | 36 | .43861    | 46 | .35477    | 35 | .46622    | 45 | 19 | 50 38.3 37.9 |
| 20 | 9.33361   | 36 | 9.43907   | 46 | 9.35512   | 35 | 9.46668   | 46 | 20 | 45           |
| 21 | .33398    | 36 | .43953    | 46 | .35547    | 35 | .46713    | 45 | 21 | 6 4.5        |
| 22 | .33434    | 36 | .43999    | 46 | .35583    | 35 | .46759    | 46 | 22 | 7 5.2 5.2    |
| 23 | .33470    | 36 | .44046    | 46 | .35618    | 35 | .46805    | 45 | 23 | 8 6.0 6.0    |
| 24 | .33507    | 36 | .44092    | 46 | .35653    | 35 | .46850    | 46 | 24 | 9 6.7 6.7    |
| 25 | 9.33543   | 36 | 9.44138   | 46 | 9.35689   | 35 | 9.46896   | 45 | 25 | 10 7.5 7.5   |
| 26 | .33579    | 36 | .44185    | 46 | .35724    | 35 | .46942    | 46 | 26 | 20 15.0 15.0 |
| 27 | .33615    | 36 | .44231    | 46 | .35759    | 35 | .46987    | 45 | 27 | 30 22.5 22.5 |
| 28 | .33652    | 36 | .44277    | 46 | .35794    | 35 | .47033    | 46 | 28 | 40 30.0 30.0 |
| 29 | .33688    | 36 | .44323    | 46 | .35829    | 35 | .47078    | 45 | 29 | 50 37.5 37.5 |
| 30 | 9.33724   | 36 | 9.44370   | 46 | 9.35865   | 35 | 9.47124   | 46 | 30 | 37 36        |
| 31 | .33760    | 36 | .44416    | 46 | .35900    | 35 | .47170    | 45 | 31 | 6 3.7 3.6    |
| 32 | .33796    | 36 | .44462    | 46 | .35935    | 35 | .47215    | 46 | 32 | 7 4.3 4.2    |
| 33 | .33833    | 36 | .44508    | 46 | .35970    | 35 | .47261    | 45 | 33 | 8 4.9 4.8    |
| 34 | .33869    | 36 | .44554    | 46 | .36005    | 35 | .47306    | 46 | 34 | 9 5.5 5.5    |
| 35 | 9.33905   | 36 | 9.44601   | 46 | 9.36040   | 35 | 9.47352   | 45 | 35 | 10 6.1 6.1   |
| 36 | .33941    | 36 | .44647    | 46 | .36076    | 35 | .47398    | 46 | 36 | 20 12.3 12.1 |
| 37 | .33977    | 36 | .44693    | 46 | .36111    | 35 | .47443    | 45 | 37 | 30 18.5 18.2 |
| 38 | .34013    | 36 | .44739    | 46 | .36146    | 35 | .47489    | 46 | 38 | 40 24.6 24.3 |
| 39 | .34049    | 36 | .44785    | 46 | .36181    | 35 | .47534    | 45 | 39 | 50 30.8 30.4 |
| 40 | 9.34085   | 36 | 9.44831   | 46 | 9.36216   | 35 | 9.47580   | 46 | 40 | 36 35        |
| 41 | .34121    | 36 | .44877    | 46 | .36251    | 35 | .47625    | 45 | 41 | 6 3.6 3.5    |
| 42 | .34157    | 36 | .44924    | 46 | .36286    | 35 | .47671    | 46 | 42 | 7 4.2 4.1    |
| 43 | .34193    | 36 | .44970    | 46 | .36321    | 35 | .47716    | 45 | 43 | 8 4.8 4.7    |
| 44 | .34229    | 36 | .45016    | 46 | .36356    | 35 | .47762    | 46 | 44 | 9 5.4 5.3    |
| 45 | 9.34265   | 36 | 9.45062   | 46 | 9.36391   | 35 | 9.47807   | 45 | 45 | 10 6.0 6.0   |
| 46 | .34301    | 36 | .45108    | 46 | .36426    | 35 | .47852    | 46 | 46 | 20 12.0 11.8 |
| 47 | .34337    | 36 | .45154    | 46 | .36461    | 34 | .47898    | 45 | 47 | 30 18.0 17.7 |
| 48 | .34373    | 35 | .45200    | 46 | .36495    | 35 | .47943    | 46 | 48 | 40 24.0 23.6 |
| 49 | .34408    | 36 | .45246    | 46 | .36530    | 35 | .47989    | 45 | 49 | 50 30.0 29.6 |
| 50 | 9.34444   | 36 | 9.45292   | 46 | 9.36565   | 35 | 9.48034   | 46 | 50 | 35 34        |
| 51 | .34480    | 35 | .45338    | 46 | .36600    | 34 | .48080    | 45 | 51 | 6 3.5 3.4    |
| 52 | .34516    | 36 | .45384    | 46 | .36635    | 35 | .48125    | 46 | 52 | 7 4.1 4.0    |
| 53 | .34552    | 36 | .45430    | 46 | .36670    | 35 | .48170    | 45 | 53 | 8 4.6 4.6    |
| 54 | .34587    | 36 | .45476    | 46 | .36705    | 34 | .48216    | 46 | 54 | 9 5.2 5.2    |
| 55 | 9.34623   | 35 | 9.45522   | 46 | 9.36739   | 35 | 9.48261   | 45 | 55 | 10 5.8 5.7   |
| 56 | .34659    | 35 | .45568    | 46 | .36774    | 34 | .48306    | 46 | 56 | 20 11.6 11.5 |
| 57 | .34695    | 35 | .45614    | 46 | .36809    | 35 | .48352    | 45 | 57 | 30 17.5 17.2 |
| 58 | .34730    | 36 | .45660    | 46 | .36844    | 34 | .48397    | 46 | 58 | 40 23.3 23.0 |
| 59 | .34766    | 35 | .45706    | 46 | .36878    | 35 | .48442    | 45 | 59 | 50 29.1 28.7 |
| 60 | 9.34802   |    | 9.45752   |    | 9.36913   |    | 9.48488   |    | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANT

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.36913   | 34 | 9.48488   | 45 | 9.38968   | 34 | 9.51190   | 45 | 0  |              |
| 1  | .36948    | 34 | .48533    | 45 | .39002    | 34 | .51235    | 44 | 1  |              |
| 2  | .36982    | 35 | .48578    | 45 | .39036    | 34 | .51279    | 45 | 2  |              |
| 3  | .37017    | 34 | .48624    | 45 | .39069    | 34 | .51324    | 44 | 3  |              |
| 4  | .37052    | 34 | .48669    | 45 | .39103    | 33 | .51369    | 44 | 4  | 45 45        |
| 5  | 9.37086   | 34 | 9.48714   | 45 | 9.39137   | 34 | 9.51414   | 45 | 5  | 6 4.5 4.5    |
| 6  | .37121    | 34 | .48759    | 45 | .39170    | 33 | .51458    | 44 | 6  | 7 5.3 5.2    |
| 7  | .37156    | 34 | .48805    | 45 | .39204    | 34 | .51503    | 45 | 7  | 8 6.0 6.0    |
| 8  | .37190    | 34 | .48850    | 45 | .39238    | 34 | .51548    | 44 | 8  | 9 6.8 6.7    |
| 9  | .37225    | 34 | .48895    | 45 | .39271    | 33 | .51592    | 44 | 9  | 10 7.6 7.5   |
| 10 | 9.37259   | 34 | 9.48940   | 45 | 9.39305   | 33 | 9.51637   | 45 | 10 | 20 15.1 15.0 |
| 11 | .37294    | 34 | .48986    | 45 | .39339    | 34 | .51682    | 44 | 11 | 30 22.7 22.5 |
| 12 | .37328    | 34 | .49031    | 45 | .39372    | 33 | .51726    | 44 | 12 | 40 30.3 30.0 |
| 13 | .37363    | 34 | .49076    | 45 | .39406    | 33 | .51771    | 45 | 13 | 50 37.9 37.5 |
| 14 | .37397    | 34 | .49121    | 45 | .39439    | 33 | .51816    | 44 | 14 |              |
| 15 | 9.37432   | 34 | 9.49166   | 45 | 9.39473   | 33 | 9.51860   | 44 | 15 | 44 44        |
| 16 | .37466    | 34 | .49211    | 45 | .39507    | 34 | .51905    | 45 | 16 | 6 4.4 4.4    |
| 17 | .37501    | 34 | .49257    | 45 | .39540    | 33 | .51950    | 44 | 17 | 7 5.2 5.1    |
| 18 | .37535    | 34 | .49302    | 45 | .39574    | 33 | .51994    | 44 | 18 | 8 5.9 5.8    |
| 19 | .37570    | 34 | .49347    | 45 | .39607    | 33 | .52039    | 44 | 19 | 9 6.7 6.6    |
| 20 | 9.37604   | 34 | 9.49392   | 45 | 9.39641   | 33 | 9.52084   | 45 | 20 | 10 7.4 7.3   |
| 21 | .37639    | 34 | .49437    | 45 | .39674    | 33 | .52128    | 44 | 21 | 20 14.8 14.6 |
| 22 | .37673    | 34 | .49482    | 45 | .39708    | 33 | .52173    | 44 | 22 | 30 22.2 22.0 |
| 23 | .37707    | 34 | .49527    | 45 | .39741    | 33 | .52217    | 44 | 23 | 40 29.6 29.3 |
| 24 | .37742    | 34 | .49572    | 45 | .39774    | 33 | .52262    | 44 | 24 | 50 37.1 36.6 |
| 25 | 9.37776   | 34 | 9.49618   | 45 | 9.39808   | 33 | 9.52306   | 44 | 25 |              |
| 26 | .37810    | 34 | .49663    | 45 | .39841    | 33 | .52351    | 45 | 26 | 35 35        |
| 27 | .37845    | 34 | .49708    | 45 | .39875    | 33 | .52396    | 44 | 27 | 6 3.5 3.4    |
| 28 | .37879    | 34 | .49753    | 45 | .39908    | 33 | .52440    | 44 | 28 | 7 4.1 4.0    |
| 29 | .37913    | 34 | .49798    | 45 | .39941    | 33 | .52485    | 44 | 29 | 8 4.8 4.6    |
| 30 | 9.37947   | 34 | 9.49843   | 45 | 9.39975   | 33 | 9.52529   | 44 | 30 | 9 5.2 5.2    |
| 31 | .37982    | 34 | .49888    | 45 | .40008    | 33 | .52574    | 44 | 31 | 10 5.8 5.7   |
| 32 | .38016    | 34 | .49933    | 45 | .40041    | 33 | .52618    | 44 | 32 | 20 11.6 11.5 |
| 33 | .38050    | 34 | .49978    | 45 | .40075    | 33 | .52663    | 44 | 33 | 30 17.5 17.2 |
| 34 | .38084    | 34 | .50023    | 45 | .40108    | 33 | .52707    | 44 | 34 | 40 23.3 23.0 |
| 35 | 9.38118   | 34 | 9.50068   | 45 | 9.40141   | 33 | 9.52752   | 44 | 35 | 50 29.1 28.7 |
| 36 | .38153    | 34 | .50113    | 45 | .40175    | 33 | .52796    | 44 | 36 |              |
| 37 | .38187    | 34 | .50158    | 45 | .40208    | 33 | .52841    | 44 | 37 | 34 33        |
| 38 | .38221    | 34 | .50203    | 45 | .40241    | 33 | .52885    | 44 | 38 | 6 3.4 3.3    |
| 39 | .38255    | 34 | .50248    | 45 | .40274    | 33 | .52930    | 44 | 39 | 7 3.9 3.9    |
| 40 | 9.38289   | 34 | 9.50293   | 45 | 9.40307   | 33 | 9.52974   | 44 | 40 | 8 4.5 4.4    |
| 41 | .38323    | 34 | .50338    | 45 | .40341    | 33 | .53018    | 44 | 41 | 9 5.1 5.0    |
| 42 | .38357    | 34 | .50383    | 45 | .40374    | 33 | .53063    | 44 | 42 | 10 5.8 5.6   |
| 43 | .38391    | 34 | .50427    | 44 | .40407    | 33 | .53107    | 44 | 43 | 20 11.3 11.1 |
| 44 | .38425    | 34 | .50472    | 44 | .40440    | 33 | .53152    | 44 | 44 | 30 17.0 16.7 |
| 45 | 9.38459   | 34 | 9.50517   | 45 | 9.40473   | 33 | 9.53196   | 44 | 45 | 40 22.6 22.3 |
| 46 | .38493    | 34 | .50562    | 45 | .40506    | 33 | .53240    | 44 | 46 | 50 28.3 27.9 |
| 47 | .38527    | 34 | .50607    | 45 | .40540    | 33 | .53285    | 44 | 47 |              |
| 48 | .38561    | 34 | .50652    | 44 | .40573    | 33 | .53329    | 44 | 48 |              |
| 49 | .38595    | 34 | .50697    | 45 | .40606    | 33 | .53374    | 44 | 49 |              |
| 50 | 9.38629   | 34 | 9.50742   | 45 | 9.40639   | 33 | 9.53418   | 44 | 50 | 33 33        |
| 51 | .38663    | 34 | .50787    | 45 | .40672    | 33 | .53462    | 44 | 51 | 6 3.3 3.3    |
| 52 | .38697    | 33 | .50831    | 44 | .40705    | 33 | .53507    | 44 | 52 | 7 3.8 3.8    |
| 53 | .38731    | 34 | .50876    | 45 | .40738    | 33 | .53551    | 44 | 53 | 8 4.4 4.4    |
| 54 | .38765    | 34 | .50921    | 44 | .40771    | 33 | .53595    | 44 | 54 | 9 4.9 4.9    |
| 55 | 9.38799   | 34 | 9.50966   | 45 | 9.40804   | 33 | 9.53640   | 44 | 55 | 10 5.5 5.5   |
| 56 | .38833    | 34 | .51011    | 44 | .40837    | 33 | .53684    | 44 | 56 | 20 11.0 11.0 |
| 57 | .38867    | 33 | .51055    | 45 | .40870    | 33 | .53728    | 44 | 57 | 30 16.5 16.5 |
| 58 | .38900    | 33 | .51100    | 45 | .40903    | 33 | .53773    | 44 | 58 | 40 22.0 22.0 |
| 59 | .38934    | 34 | .51145    | 44 | .40936    | 33 | .53817    | 44 | 59 | 50 27.5 27.5 |
| 60 | 9.38968   | 34 | 9.51190   | 44 | 9.40969   | 33 | 9.53861   | 44 | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|
| 0  | 9.40969   | 32 | 9.53661   | 44 | 9.42918   | 32 | 9.56505   | 43 | 0  |       |
| 1  | .41001    | 33 | .53906    | 44 | .42950    | 32 | .56549    | 44 | 1  |       |
| 2  | .41034    | 33 | .53950    | 44 | .42982    | 32 | .56593    | 44 | 2  |       |
| 3  | .41067    | 33 | .53994    | 44 | .43014    | 32 | .56637    | 44 | 3  |       |
| 4  | .41100    | 32 | .54038    | 44 | .43046    | 32 | .56680    | 43 | 4  |       |
| 5  | 9.41133   | 32 | 9.54083   | 44 | 9.43078   | 32 | 9.56724   | 44 | 5  |       |
| 6  | .41166    | 33 | .54127    | 44 | .43110    | 32 | .56768    | 44 | 6  |       |
| 7  | .41199    | 33 | .54171    | 44 | .43142    | 31 | .56812    | 43 | 7  |       |
| 8  | .41231    | 32 | .54215    | 44 | .43174    | 32 | .56856    | 44 | 8  |       |
| 9  | .41264    | 33 | .54259    | 44 | .43206    | 32 | .56899    | 43 | 9  |       |
| 10 | 9.41297   | 32 | 9.54304   | 44 | 9.43238   | 32 | 9.56943   | 44 | 10 |       |
| 11 | .41330    | 33 | .54348    | 44 | .43270    | 32 | .56987    | 44 | 11 |       |
| 12 | .41362    | 32 | .54392    | 44 | .43302    | 32 | .57031    | 43 | 12 |       |
| 13 | .41395    | 33 | .54436    | 44 | .43334    | 32 | .57075    | 44 | 13 |       |
| 14 | .41428    | 32 | .54480    | 44 | .43365    | 31 | .57118    | 43 | 14 |       |
| 15 | 9.41461   | 33 | 9.54525   | 44 | 9.43397   | 32 | 9.57162   | 44 | 15 |       |
| 16 | .41493    | 33 | .54569    | 44 | .43429    | 32 | .57206    | 43 | 16 |       |
| 17 | .41526    | 33 | .54613    | 44 | .43461    | 32 | .57250    | 44 | 17 |       |
| 18 | .41559    | 32 | .54657    | 44 | .43493    | 31 | .57293    | 43 | 18 |       |
| 19 | .41591    | 33 | .54701    | 44 | .43525    | 32 | .57337    | 44 | 19 |       |
| 20 | 9.41624   | 32 | 9.54745   | 44 | 9.43557   | 32 | 9.57381   | 43 | 20 |       |
| 21 | .41657    | 33 | .54790    | 44 | .43588    | 31 | .57424    | 43 | 21 |       |
| 22 | .41689    | 32 | .54834    | 44 | .43620    | 32 | .57468    | 44 | 22 |       |
| 23 | .41722    | 33 | .54878    | 44 | .43652    | 31 | .57512    | 43 | 23 |       |
| 24 | .41754    | 32 | .54922    | 44 | .43684    | 32 | .57556    | 44 | 24 |       |
| 25 | 9.41787   | 33 | 9.54966   | 44 | 9.43715   | 31 | 9.57599   | 43 | 25 |       |
| 26 | .41819    | 32 | .55010    | 44 | .43747    | 32 | .57643    | 43 | 26 |       |
| 27 | .41852    | 33 | .55054    | 44 | .43779    | 31 | .57687    | 44 | 27 |       |
| 28 | .41885    | 32 | .55098    | 44 | .43810    | 32 | .57730    | 43 | 28 |       |
| 29 | .41917    | 33 | .55142    | 44 | .43842    | 31 | .57774    | 43 | 29 |       |
| 30 | 9.41950   | 32 | 9.55186   | 44 | 9.43874   | 32 | 9.57818   | 44 | 30 |       |
| 31 | .41982    | 33 | .55230    | 44 | .43906    | 31 | .57861    | 43 | 31 |       |
| 32 | .42014    | 32 | .55275    | 44 | .43937    | 32 | .57905    | 44 | 32 |       |
| 33 | .42047    | 33 | .55319    | 44 | .43969    | 31 | .57949    | 43 | 33 |       |
| 34 | .42079    | 32 | .55363    | 44 | .44000    | 32 | .57992    | 44 | 34 |       |
| 35 | 9.42112   | 33 | 9.55407   | 44 | 9.44032   | 31 | 9.58036   | 43 | 35 |       |
| 36 | .42144    | 32 | .55451    | 44 | .44064    | 32 | .58079    | 43 | 36 |       |
| 37 | .42177    | 33 | .55495    | 44 | .44095    | 31 | .58123    | 44 | 37 |       |
| 38 | .42209    | 32 | .55539    | 44 | .44127    | 32 | .58167    | 43 | 38 |       |
| 39 | .42241    | 33 | .55583    | 44 | .44158    | 31 | .58210    | 43 | 39 |       |
| 40 | 9.42274   | 32 | 9.55627   | 44 | 9.44190   | 32 | 9.58254   | 44 | 40 |       |
| 41 | .42306    | 33 | .55671    | 44 | .44221    | 31 | .58297    | 43 | 41 |       |
| 42 | .42338    | 32 | .55715    | 44 | .44253    | 32 | .58341    | 44 | 42 |       |
| 43 | .42371    | 33 | .55759    | 44 | .44284    | 31 | .58385    | 43 | 43 |       |
| 44 | .42403    | 32 | .55803    | 44 | .44316    | 32 | .58428    | 44 | 44 |       |
| 45 | 9.42435   | 33 | 9.55847   | 44 | 9.44347   | 31 | 9.58472   | 43 | 45 |       |
| 46 | .42467    | 32 | .55890    | 44 | .44379    | 32 | .58515    | 44 | 46 |       |
| 47 | .42500    | 33 | .55934    | 44 | .44410    | 31 | .58559    | 43 | 47 |       |
| 48 | .42532    | 32 | .55978    | 44 | .44442    | 32 | .58602    | 44 | 48 |       |
| 49 | .42564    | 33 | .56022    | 44 | .44473    | 31 | .58646    | 43 | 49 |       |
| 50 | 9.42596   | 32 | 9.56066   | 44 | 9.44504   | 32 | 9.58689   | 44 | 50 |       |
| 51 | .42629    | 33 | .56110    | 44 | .44536    | 31 | .58733    | 43 | 51 |       |
| 52 | .42661    | 32 | .56154    | 44 | .44567    | 32 | .58776    | 44 | 52 |       |
| 53 | .42693    | 33 | .56198    | 44 | .44599    | 31 | .58820    | 43 | 53 |       |
| 54 | .42725    | 32 | .56242    | 44 | .44630    | 32 | .58864    | 44 | 54 |       |
| 55 | 9.42757   | 33 | 9.56286   | 44 | 9.44661   | 31 | 9.58907   | 43 | 55 |       |
| 56 | .42789    | 32 | .56330    | 44 | .44693    | 32 | .58951    | 44 | 56 |       |
| 57 | .42822    | 33 | .56374    | 44 | .44724    | 31 | .58994    | 43 | 57 |       |
| 58 | .42854    | 32 | .56417    | 44 | .44755    | 32 | .59037    | 44 | 58 |       |
| 59 | .42886    | 33 | .56461    | 44 | .44787    | 31 | .59081    | 43 | 59 |       |
| 60 | 9.42918   | 32 | 9.56505   | 43 | 9.44818   | 31 | 9.59124   | 43 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.44818   | 31 | 9.59124   | 43 | 9.46671   | 30 | 9.61722   | 43 | 0  |              |
| 1  | .44849    | 31 | .59168    | 43 | .46701    | 30 | .61765    | 43 | 1  |              |
| 2  | .44880    | 31 | .59211    | 43 | .46732    | 30 | .61808    | 43 | 2  |              |
| 3  | .44912    | 31 | .59255    | 43 | .46762    | 30 | .61852    | 43 | 3  |              |
| 4  | .44943    | 31 | .59298    | 43 | .46793    | 30 | .61895    | 43 | 4  | 43 43        |
| 5  | 9.44974   | 31 | 9.59342   | 43 | 9.46823   | 30 | 9.61938   | 43 | 5  | 6 4.3 4.3    |
| 6  | .45005    | 31 | .59385    | 43 | .46853    | 30 | .61981    | 43 | 6  | 7 5.1 5.0    |
| 7  | .45036    | 31 | .59429    | 43 | .46884    | 30 | .62024    | 43 | 7  | 8 5.8 5.7    |
| 8  | .45068    | 31 | .59472    | 43 | .46914    | 30 | .62067    | 43 | 8  | 9 6.5 6.4    |
| 9  | .45099    | 31 | .59515    | 43 | .46945    | 30 | .62110    | 43 | 9  | 10 7.2 7.1   |
| 10 | 9.45130   | 31 | 9.59559   | 43 | 9.46975   | 30 | 9.62153   | 43 | 10 | 20 14.5 14.3 |
| 11 | .45161    | 31 | .59602    | 43 | .47005    | 30 | .62196    | 43 | 11 | 30 21.7 21.5 |
| 12 | .45192    | 31 | .59646    | 43 | .47036    | 30 | .62239    | 43 | 12 | 40 29.0 28.6 |
| 13 | .45223    | 31 | .59689    | 43 | .47066    | 30 | .62282    | 43 | 13 | 50 36.2 35.8 |
| 14 | .45254    | 31 | .59732    | 43 | .47096    | 30 | .62326    | 43 | 14 |              |
| 15 | 9.45285   | 31 | 9.59776   | 43 | 9.47127   | 30 | 9.62369   | 43 | 15 |              |
| 16 | .45316    | 31 | .59819    | 43 | .47157    | 30 | .62412    | 43 | 16 | 6 4.2 4.2    |
| 17 | .45348    | 31 | .59863    | 43 | .47187    | 30 | .62455    | 43 | 17 | 7 4.9 4.9    |
| 18 | .45379    | 31 | .59906    | 43 | .47218    | 30 | .62498    | 43 | 18 | 8 5.6 5.6    |
| 19 | .45410    | 31 | .59949    | 43 | .47248    | 30 | .62541    | 43 | 19 | 9 6.4 6.4    |
| 20 | 9.45441   | 31 | 9.59993   | 43 | 9.47278   | 30 | 9.62584   | 43 | 20 | 10 7.1 7.1   |
| 21 | .45472    | 31 | .60036    | 43 | .47308    | 30 | .62627    | 43 | 21 | 20 14.1 14.1 |
| 22 | .45503    | 31 | .60079    | 43 | .47339    | 30 | .62670    | 43 | 22 | 30 21.2 21.2 |
| 23 | .45534    | 31 | .60123    | 43 | .47369    | 30 | .62713    | 43 | 23 | 40 28.3 28.3 |
| 24 | .45565    | 30 | .60166    | 43 | .47399    | 30 | .62756    | 43 | 24 | 50 35.4 35.4 |
| 25 | 9.45595   | 31 | 9.60209   | 43 | 9.47429   | 30 | 9.62799   | 43 | 25 |              |
| 26 | .45626    | 31 | .60253    | 43 | .47459    | 30 | .62842    | 43 | 26 |              |
| 27 | .45657    | 31 | .60296    | 43 | .47490    | 30 | .62885    | 43 | 27 | 31 31        |
| 28 | .45688    | 31 | .60339    | 43 | .47520    | 30 | .62928    | 43 | 28 | 6 3.1 3.1    |
| 29 | .45719    | 31 | .60383    | 43 | .47550    | 30 | .62971    | 43 | 29 | 7 3.7 3.6    |
| 30 | 9.45750   | 30 | 9.60426   | 43 | 9.47580   | 30 | 9.63014   | 43 | 30 | 8 4.2 4.1    |
| 31 | .45781    | 31 | .60469    | 43 | .47610    | 30 | .63057    | 43 | 31 | 9 4.7 4.6    |
| 32 | .45812    | 31 | .60512    | 43 | .47640    | 30 | .63100    | 43 | 32 | 10 5.2 5.1   |
| 33 | .45843    | 30 | .60556    | 43 | .47670    | 30 | .63143    | 43 | 33 | 20 10.5 10.5 |
| 34 | .45873    | 31 | .60599    | 43 | .47700    | 30 | .63186    | 43 | 34 | 30 15.7 15.3 |
| 35 | 9.45904   | 31 | 9.60642   | 43 | 9.47731   | 30 | 9.63229   | 43 | 35 | 40 21.0 20.6 |
| 36 | .45935    | 30 | .60685    | 43 | .47761    | 30 | .63272    | 43 | 36 | 50 26.2 25.8 |
| 37 | .45966    | 31 | .60729    | 43 | .47791    | 30 | .63315    | 43 | 37 |              |
| 38 | .45997    | 30 | .60772    | 43 | .47821    | 30 | .63358    | 43 | 38 | 30 30        |
| 39 | .46027    | 31 | .60815    | 43 | .47851    | 30 | .63401    | 43 | 39 | 6 3.0 3.0    |
| 40 | 9.46058   | 30 | 9.60858   | 43 | 9.47881   | 30 | 9.63443   | 43 | 40 | 7 3.5 3.5    |
| 41 | .46089    | 31 | .60902    | 43 | .47911    | 30 | .63486    | 43 | 41 | 8 4.0 4.0    |
| 42 | .46120    | 30 | .60945    | 43 | .47941    | 30 | .63529    | 43 | 42 | 9 4.6 4.5    |
| 43 | .46150    | 31 | .60988    | 43 | .47971    | 30 | .63572    | 43 | 43 | 10 5.1 5.0   |
| 44 | .46181    | 30 | .61031    | 43 | .48001    | 30 | .63615    | 43 | 44 | 20 10.1 10.1 |
| 45 | 9.46212   | 30 | 9.61075   | 43 | 9.48031   | 30 | 9.63658   | 43 | 45 | 30 15.2 15.0 |
| 46 | .46242    | 31 | .61118    | 43 | .48061    | 29 | .63701    | 43 | 46 | 40 20.3 20.0 |
| 47 | .46273    | 30 | .61161    | 43 | .48090    | 30 | .63744    | 43 | 47 | 50 25.4 25.0 |
| 48 | .46304    | 30 | .61204    | 43 | .48120    | 30 | .63787    | 43 | 48 |              |
| 49 | .46334    | 31 | .61247    | 43 | .48150    | 30 | .63830    | 43 | 49 |              |
| 50 | 9.46365   | 30 | 9.61291   | 43 | 9.48180   | 30 | 9.63873   | 43 | 50 | 29 29        |
| 51 | .46396    | 30 | .61334    | 43 | .48210    | 30 | .63915    | 43 | 51 | 6 2.9 2.9    |
| 52 | .46426    | 30 | .61377    | 43 | .48240    | 29 | .63956    | 43 | 52 | 7 3.4 3.4    |
| 53 | .46457    | 30 | .61420    | 43 | .48270    | 30 | .64001    | 43 | 53 | 8 3.9 3.9    |
| 54 | .46487    | 30 | .61463    | 43 | .48300    | 30 | .64044    | 43 | 54 | 9 4.4 4.4    |
| 55 | 9.46518   | 31 | 9.61506   | 43 | 9.48329   | 29 | 9.64087   | 43 | 55 | 10 4.9 4.9   |
| 56 | .46549    | 30 | .61550    | 43 | .48359    | 30 | .64130    | 43 | 56 | 20 9.8 9.8   |
| 57 | .46579    | 30 | .61593    | 43 | .48389    | 30 | .64173    | 43 | 57 | 30 14.7 14.7 |
| 58 | .46610    | 30 | .61636    | 43 | .48419    | 29 | .64216    | 43 | 58 | 40 19.6 19.6 |
| 59 | .46640    | 30 | .61679    | 43 | .48449    | 30 | .64258    | 43 | 59 | 50 24.6 24.6 |
| 60 | 9.46671   | 30 | 9.61722   | 43 | 9.48479   | 29 | 9.64301   | 43 | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |



TABLE VII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    |           |    |           |    |           |    |           |    |    |  |  |  |  | P. P. |       |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--|--|--|--|-------|-------|
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    |  |  |  |  |       |       |
| 0  | 9.48478   | 30 | 9.64301   | 43 | 9.50243   | 29 | 9.66864   | 42 | 0  |  |  |  |  |       |       |
| 1  | .48508    | 29 | .64344    | 42 | .50272    | 29 | .66907    | 43 | 1  |  |  |  |  |       |       |
| 2  | .48538    | 30 | .64387    | 43 | .50301    | 29 | .66950    | 42 | 2  |  |  |  |  |       |       |
| 3  | .48568    | 29 | .64430    | 43 | .50330    | 29 | .66992    | 42 | 3  |  |  |  |  |       |       |
| 4  | .48597    | 29 | .64473    | 43 | .50359    | 29 | .67035    | 42 | 4  |  |  |  |  |       |       |
| 5  | 3.48627   | 30 | 9.64515   | 42 | 9.50388   | 29 | 9.67077   | 42 | 5  |  |  |  |  |       |       |
| 6  | .48657    | 29 | .64558    | 43 | .50417    | 29 | .67120    | 42 | 6  |  |  |  |  |       |       |
| 7  | .48686    | 29 | .64601    | 43 | .50446    | 29 | .67162    | 42 | 7  |  |  |  |  |       |       |
| 8  | .48716    | 30 | .64644    | 42 | .50475    | 29 | .67205    | 42 | 8  |  |  |  |  |       |       |
| 9  | .48746    | 29 | .64687    | 43 | .50504    | 29 | .67248    | 42 | 9  |  |  |  |  |       |       |
| 10 | 3.48775   | 29 | 9.64729   | 42 | 9.50533   | 29 | 9.67290   | 42 | 10 |  |  |  |  |       |       |
| 11 | .48805    | 30 | .64772    | 43 | .50562    | 29 | .67333    | 42 | 11 |  |  |  |  |       |       |
| 12 | .48835    | 29 | .64815    | 43 | .50591    | 29 | .67375    | 42 | 12 |  |  |  |  |       |       |
| 13 | .48864    | 29 | .64858    | 43 | .50619    | 29 | .67418    | 42 | 13 |  |  |  |  |       |       |
| 14 | .48894    | 29 | .64901    | 42 | .50648    | 29 | .67460    | 42 | 14 |  |  |  |  |       |       |
| 15 | 3.48923   | 29 | 9.64943   | 43 | 9.50677   | 29 | 9.67503   | 42 | 15 |  |  |  |  |       |       |
| 16 | .48953    | 30 | .64986    | 42 | .50706    | 29 | .67546    | 42 | 16 |  |  |  |  |       |       |
| 17 | .48983    | 29 | .65029    | 43 | .50735    | 29 | .67588    | 42 | 17 |  |  |  |  |       |       |
| 18 | .49012    | 29 | .65072    | 42 | .50764    | 29 | .67631    | 42 | 18 |  |  |  |  |       |       |
| 19 | .49042    | 29 | .65114    | 43 | .50793    | 29 | .67673    | 42 | 19 |  |  |  |  |       |       |
| 20 | 3.49071   | 29 | 9.65157   | 42 | 9.50821   | 29 | 9.67716   | 42 | 20 |  |  |  |  |       |       |
| 21 | .49101    | 29 | .65200    | 43 | .50850    | 29 | .67758    | 42 | 21 |  |  |  |  |       |       |
| 22 | .49130    | 29 | .65243    | 42 | .50879    | 29 | .67801    | 42 | 22 |  |  |  |  |       |       |
| 23 | .49160    | 29 | .65285    | 43 | .50908    | 29 | .67843    | 42 | 23 |  |  |  |  |       |       |
| 24 | .49189    | 29 | .65328    | 42 | .50937    | 29 | .67886    | 42 | 24 |  |  |  |  |       |       |
| 25 | 9.49219   | 29 | 9.65371   | 43 | 9.50966   | 29 | 9.67928   | 42 | 25 |  |  |  |  |       |       |
| 26 | .49248    | 29 | .65414    | 42 | .50994    | 29 | .67971    | 42 | 26 |  |  |  |  |       |       |
| 27 | .49278    | 29 | .65456    | 43 | .51023    | 29 | .68013    | 42 | 27 |  |  |  |  |       |       |
| 28 | .49307    | 29 | .65499    | 42 | .51052    | 29 | .68056    | 42 | 28 |  |  |  |  |       |       |
| 29 | .49337    | 29 | .65542    | 43 | .51080    | 29 | .68098    | 42 | 29 |  |  |  |  |       |       |
| 30 | 3.49367   | 29 | 9.65585   | 42 | 9.51109   | 29 | 9.68141   | 42 | 30 |  |  |  |  |       |       |
| 31 | .49395    | 29 | .65627    | 43 | .51138    | 29 | .68183    | 42 | 31 |  |  |  |  |       |       |
| 32 | .49425    | 29 | .65670    | 42 | .51167    | 29 | .68226    | 42 | 32 |  |  |  |  |       |       |
| 33 | .49454    | 29 | .65713    | 42 | .51195    | 29 | .68268    | 42 | 33 |  |  |  |  |       |       |
| 34 | .49483    | 29 | .65755    | 43 | .51224    | 29 | .68311    | 42 | 34 |  |  |  |  |       |       |
| 35 | 9.49513   | 29 | 9.65798   | 42 | 9.51253   | 29 | 9.68353   | 42 | 35 |  |  |  |  |       |       |
| 36 | .49542    | 29 | .65841    | 43 | .51281    | 29 | .68396    | 42 | 36 |  |  |  |  |       |       |
| 37 | .49571    | 29 | .65884    | 42 | .51310    | 29 | .68438    | 42 | 37 |  |  |  |  |       |       |
| 38 | .49601    | 29 | .65926    | 43 | .51337    | 29 | .68481    | 42 | 38 |  |  |  |  |       |       |
| 39 | .49630    | 29 | .65969    | 42 | .51367    | 29 | .68523    | 42 | 39 |  |  |  |  |       |       |
| 40 | 3.49659   | 29 | 9.66012   | 43 | 9.51396   | 29 | 9.68566   | 42 | 40 |  |  |  |  |       |       |
| 41 | .49689    | 29 | .66054    | 42 | .51424    | 29 | .68608    | 42 | 41 |  |  |  |  |       |       |
| 42 | .49718    | 29 | .66097    | 43 | .51452    | 29 | .68651    | 42 | 42 |  |  |  |  |       |       |
| 43 | .49747    | 29 | .66140    | 42 | .51481    | 29 | .68693    | 42 | 43 |  |  |  |  |       |       |
| 44 | .49776    | 29 | .66182    | 43 | .51510    | 29 | .68735    | 42 | 44 |  |  |  |  |       |       |
| 45 | 9.49806   | 29 | 9.66225   | 42 | 9.51539   | 29 | 9.68778   | 42 | 45 |  |  |  |  |       |       |
| 46 | .49835    | 29 | .66268    | 43 | .51567    | 29 | .68820    | 42 | 46 |  |  |  |  |       |       |
| 47 | .49864    | 29 | .66310    | 42 | .51596    | 29 | .68863    | 42 | 47 |  |  |  |  |       |       |
| 48 | .49893    | 29 | .66353    | 43 | .51624    | 29 | .68905    | 42 | 48 |  |  |  |  |       |       |
| 49 | .49922    | 29 | .66396    | 42 | .51652    | 29 | .68948    | 42 | 49 |  |  |  |  |       |       |
| 50 | 3.49952   | 29 | 9.66438   | 43 | 9.51681   | 29 | 9.68990   | 42 | 50 |  |  |  |  |       |       |
| 51 | .49981    | 29 | .66481    | 42 | .51710    | 29 | .69033    | 42 | 51 |  |  |  |  |       |       |
| 52 | .50010    | 29 | .66523    | 43 | .51738    | 29 | .69075    | 42 | 52 |  |  |  |  |       |       |
| 53 | .50039    | 29 | .66566    | 42 | .51767    | 29 | .69117    | 42 | 53 |  |  |  |  |       |       |
| 54 | .50068    | 29 | .66609    | 43 | .51795    | 29 | .69160    | 42 | 54 |  |  |  |  |       |       |
| 55 | 9.50097   | 29 | 9.66651   | 42 | 9.51823   | 29 | 9.69202   | 42 | 55 |  |  |  |  |       |       |
| 56 | .50126    | 29 | .66694    | 43 | .51852    | 29 | .69245    | 42 | 56 |  |  |  |  |       |       |
| 57 | .50155    | 29 | .66737    | 42 | .51880    | 29 | .69287    | 42 | 57 |  |  |  |  |       |       |
| 58 | .50185    | 29 | .66779    | 43 | .51909    | 29 | .69330    | 42 | 58 |  |  |  |  |       |       |
| 59 | .50214    | 29 | .66822    | 42 | .51937    | 29 | .69372    | 42 | 59 |  |  |  |  |       |       |
| 60 | 9.50243   | 29 | 9.66864   | 43 | 9.51965   | 29 | 9.69414   | 42 | 60 |  |  |  |  |       |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    |  |  |  |  |       | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|
| 0  | 9.51965   | 28 | 9.69414   | 42 | 9.53648   | 27 | 9.71954   | 42 | 0  |       |
| 1  | .51994    | 28 | .69457    | 42 | .53676    | 27 | .71996    | 42 | 1  |       |
| 2  | .52022    | 28 | .69499    | 42 | .53704    | 28 | .72036    | 42 | 2  |       |
| 3  | .52050    | 28 | .69542    | 42 | .53731    | 27 | .72081    | 42 | 3  |       |
| 4  | .52079    | 28 | .69584    | 42 | .53759    | 27 | .72123    | 42 | 4  |       |
| 5  | 9.52107   | 28 | 9.69626   | 42 | 9.53787   | 28 | 9.72165   | 42 | 5  |       |
| 6  | .52135    | 28 | .69669    | 42 | .53814    | 27 | .72207    | 42 | 6  |       |
| 7  | .52164    | 28 | .69711    | 42 | .53842    | 28 | .72250    | 42 | 7  |       |
| 8  | .52192    | 28 | .69753    | 42 | .53870    | 28 | .72292    | 42 | 8  |       |
| 9  | .52220    | 28 | .69796    | 42 | .53897    | 27 | .72334    | 42 | 9  |       |
| 10 | 9.52249   | 28 | 9.69838   | 42 | 9.53925   | 27 | 9.72376   | 42 | 10 |       |
| 11 | .52277    | 28 | .69881    | 42 | .53952    | 27 | .72419    | 42 | 11 |       |
| 12 | .52305    | 28 | .69923    | 42 | .53980    | 28 | .72461    | 42 | 12 |       |
| 13 | .52333    | 28 | .69965    | 42 | .54008    | 27 | .72503    | 42 | 13 |       |
| 14 | .52362    | 28 | .70008    | 42 | .54035    | 27 | .72545    | 42 | 14 |       |
| 15 | 9.52390   | 28 | 9.70050   | 42 | 9.54063   | 27 | 9.72587   | 42 | 15 |       |
| 16 | .52418    | 28 | .70092    | 42 | .54090    | 27 | .72630    | 42 | 16 |       |
| 17 | .52446    | 28 | .70135    | 42 | .54118    | 27 | .72672    | 42 | 17 |       |
| 18 | .52474    | 28 | .70177    | 42 | .54145    | 27 | .72714    | 42 | 18 |       |
| 19 | .52503    | 28 | .70220    | 42 | .54173    | 27 | .72756    | 42 | 19 |       |
| 20 | 9.52531   | 28 | 9.70262   | 42 | 9.54200   | 27 | 9.72799   | 42 | 20 |       |
| 21 | .52559    | 28 | .70304    | 42 | .54228    | 27 | .72841    | 42 | 21 |       |
| 22 | .52587    | 28 | .70347    | 42 | .54255    | 27 | .72883    | 42 | 22 |       |
| 23 | .52615    | 28 | .70389    | 42 | .54283    | 27 | .72925    | 42 | 23 |       |
| 24 | .52643    | 28 | .70431    | 42 | .54310    | 27 | .72967    | 42 | 24 |       |
| 25 | 9.52671   | 28 | 9.70474   | 42 | 9.54338   | 27 | 9.73010   | 42 | 25 |       |
| 26 | .52699    | 28 | .70516    | 42 | .54365    | 27 | .73052    | 42 | 26 |       |
| 27 | .52727    | 28 | .70558    | 42 | .54393    | 27 | .73094    | 42 | 27 |       |
| 28 | .52756    | 28 | .70601    | 42 | .54420    | 27 | .73136    | 42 | 28 |       |
| 29 | .52784    | 28 | .70643    | 42 | .54448    | 27 | .73178    | 42 | 29 |       |
| 30 | 9.52812   | 28 | 9.70685   | 42 | 9.54475   | 27 | 9.73221   | 42 | 30 |       |
| 31 | .52840    | 28 | .70728    | 42 | .54502    | 27 | .73263    | 42 | 31 |       |
| 32 | .52868    | 28 | .70770    | 42 | .54530    | 27 | .73305    | 42 | 32 |       |
| 33 | .52896    | 28 | .70812    | 42 | .54557    | 27 | .73347    | 42 | 33 |       |
| 34 | .52924    | 28 | .70854    | 42 | .54585    | 27 | .73389    | 42 | 34 |       |
| 35 | 9.52952   | 28 | 9.70897   | 42 | 9.54612   | 27 | 9.73431   | 42 | 35 |       |
| 36 | .52980    | 28 | .70939    | 42 | .54639    | 27 | .73474    | 42 | 36 |       |
| 37 | .53008    | 28 | .70981    | 42 | .54667    | 27 | .73516    | 42 | 37 |       |
| 38 | .53036    | 28 | .71024    | 42 | .54694    | 27 | .73558    | 42 | 38 |       |
| 39 | .53064    | 28 | .71066    | 42 | .54721    | 27 | .73600    | 42 | 39 |       |
| 40 | 9.53092   | 28 | 9.71108   | 42 | 9.54748   | 27 | 9.73642   | 42 | 40 |       |
| 41 | .53120    | 27 | .71151    | 42 | .54776    | 27 | .73685    | 42 | 41 |       |
| 42 | .53147    | 28 | .71193    | 42 | .54803    | 27 | .73727    | 42 | 42 |       |
| 43 | .53175    | 28 | .71235    | 42 | .54830    | 27 | .73769    | 42 | 43 |       |
| 44 | .53203    | 28 | .71278    | 42 | .54858    | 27 | .73811    | 42 | 44 |       |
| 45 | 9.53231   | 28 | 9.71320   | 42 | 9.54885   | 27 | 9.73853   | 42 | 45 |       |
| 46 | .53259    | 27 | .71362    | 42 | .54912    | 27 | .73895    | 42 | 46 |       |
| 47 | .53287    | 28 | .71404    | 42 | .54939    | 27 | .73938    | 42 | 47 |       |
| 48 | .53315    | 28 | .71447    | 42 | .54967    | 27 | .73980    | 42 | 48 |       |
| 49 | .53343    | 28 | .71489    | 42 | .54994    | 27 | .74022    | 42 | 49 |       |
| 50 | 9.53370   | 27 | 9.71531   | 42 | 9.55021   | 27 | 9.74064   | 42 | 50 |       |
| 51 | .53398    | 28 | .71573    | 42 | .55048    | 27 | .74106    | 42 | 51 |       |
| 52 | .53426    | 28 | .71616    | 42 | .55075    | 27 | .74148    | 42 | 52 |       |
| 53 | .53454    | 27 | .71658    | 42 | .55103    | 27 | .74191    | 42 | 53 |       |
| 54 | .53482    | 28 | .71700    | 42 | .55130    | 27 | .74233    | 42 | 54 |       |
| 55 | 9.53509   | 27 | 9.71743   | 42 | 9.55157   | 27 | 9.74275   | 42 | 55 |       |
| 56 | .53537    | 27 | .71785    | 42 | .55184    | 27 | .74317    | 42 | 56 |       |
| 57 | .53565    | 28 | .71827    | 42 | .55211    | 27 | .74359    | 42 | 57 |       |
| 58 | .53593    | 28 | .71869    | 42 | .55238    | 27 | .74401    | 42 | 58 |       |
| 59 | .53620    | 27 | .71912    | 42 | .55265    | 27 | .74444    | 42 | 59 |       |
| 60 | 9.53648   | 28 | 9.71954   | 42 | 9.55292   | 27 | 9.74486   | 42 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |

|    |      |      |
|----|------|------|
|    | 42   | 42   |
| 6  | 4.2  | 4.2  |
| 7  | 4.9  | 4.9  |
| 8  | 5.6  | 5.6  |
| 9  | 6.4  | 6.3  |
| 10 | 7.1  | 7.0  |
| 20 | 14.1 | 14.0 |
| 30 | 21.2 | 21.0 |
| 40 | 28.3 | 28.0 |
| 50 | 35.4 | 35.0 |

|    |      |      |
|----|------|------|
|    | 28   | 28   |
| 6  | 2.8  | 2.8  |
| 7  | 3.3  | 3.2  |
| 8  | 3.8  | 3.7  |
| 9  | 4.3  | 4.2  |
| 10 | 4.7  | 4.6  |
| 20 | 9.5  | 9.3  |
| 30 | 14.2 | 14.0 |
| 40 | 19.0 | 18.6 |
| 50 | 23.7 | 23.3 |

|    |      |      |
|----|------|------|
|    | 27   | 27   |
| 6  | 2.7  | 2.7  |
| 7  | 3.2  | 3.1  |
| 8  | 3.6  | 3.6  |
| 9  | 4.1  | 4.0  |
| 10 | 4.6  | 4.5  |
| 20 | 9.1  | 9.0  |
| 30 | 13.7 | 13.5 |
| 40 | 18.3 | 18.0 |
| 50 | 22.9 | 22.5 |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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55°

|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|
| 0  | 9.55292   | 27 | 9.74486   | 42 | 9.56900   | 26 | 9.77012   | 42 | 0  |       |
| 1  | .55319    | 27 | .74528    | 42 | .56926    | 26 | .77055    | 42 | 1  |       |
| 2  | .55347    | 27 | .74570    | 42 | .56953    | 26 | .77097    | 42 | 2  |       |
| 3  | .55374    | 27 | .74612    | 42 | .56979    | 26 | .77139    | 42 | 3  |       |
| 4  | .55401    | 27 | .74654    | 42 | .57005    | 26 | .77181    | 42 | 4  |       |
| 5  | 9.55428   | 27 | 9.74696   | 42 | 9.57032   | 26 | 9.77223   | 42 | 5  | 42    |
| 6  | .55455    | 27 | .74739    | 42 | .57058    | 26 | .77265    | 42 | 6  | 4.2   |
| 7  | .55482    | 27 | .74781    | 42 | .57085    | 26 | .77307    | 42 | 7  | 4.9   |
| 8  | .55509    | 27 | .74823    | 42 | .57111    | 26 | .77349    | 42 | 8  | 5.6   |
| 9  | .55536    | 27 | .74865    | 42 | .57138    | 26 | .77391    | 42 | 9  | 6.4   |
| 10 | 9.55563   | 27 | 9.74907   | 42 | 9.57164   | 26 | 9.77433   | 42 | 10 | 7.1   |
| 11 | .55590    | 27 | .74949    | 42 | .57190    | 26 | .77475    | 42 | 11 | 14.1  |
| 12 | .55617    | 27 | .74991    | 42 | .57217    | 26 | .77517    | 42 | 12 | 21.0  |
| 13 | .55644    | 27 | .75033    | 42 | .57243    | 26 | .77560    | 42 | 13 | 28.0  |
| 14 | .55671    | 27 | .75076    | 42 | .57269    | 26 | .77602    | 42 | 14 | 35.0  |
| 15 | 9.55698   | 27 | 9.75118   | 42 | 9.57296   | 26 | 9.77644   | 42 | 15 |       |
| 16 | .55725    | 26 | .75160    | 42 | .57322    | 26 | .77686    | 42 | 16 |       |
| 17 | .55751    | 27 | .75202    | 42 | .57348    | 26 | .77728    | 42 | 17 |       |
| 18 | .55778    | 27 | .75244    | 42 | .57375    | 26 | .77770    | 42 | 18 |       |
| 19 | .55805    | 27 | .75286    | 42 | .57401    | 26 | .77812    | 42 | 19 |       |
| 20 | 9.55832   | 27 | 9.75328   | 42 | 9.57427   | 26 | 9.77854   | 42 | 20 | 27    |
| 21 | .55859    | 26 | .75370    | 42 | .57454    | 26 | .77896    | 42 | 21 | 2.7   |
| 22 | .55886    | 27 | .75413    | 42 | .57480    | 26 | .77938    | 42 | 22 | 3.1   |
| 23 | .55913    | 27 | .75455    | 42 | .57506    | 26 | .77980    | 42 | 23 | 3.6   |
| 24 | .55940    | 26 | .75497    | 42 | .57532    | 26 | .78022    | 42 | 24 | 4.0   |
| 25 | 9.55966   | 27 | 9.75539   | 42 | 9.57559   | 26 | 9.78064   | 42 | 25 | 4.5   |
| 26 | .55993    | 27 | .75581    | 42 | .57585    | 26 | .78107    | 42 | 26 | 9.0   |
| 27 | .56020    | 26 | .75623    | 42 | .57611    | 26 | .78149    | 42 | 27 | 13.5  |
| 28 | .56047    | 27 | .75665    | 42 | .57637    | 26 | .78191    | 42 | 28 | 18.0  |
| 29 | .56074    | 27 | .75707    | 42 | .57664    | 26 | .78233    | 42 | 29 | 22.5  |
| 30 | 9.56101   | 26 | 9.75750   | 42 | 9.57690   | 26 | 9.78275   | 42 | 30 |       |
| 31 | .56127    | 27 | .75792    | 42 | .57716    | 26 | .78317    | 42 | 31 |       |
| 32 | .56154    | 26 | .75834    | 42 | .57742    | 26 | .78359    | 42 | 32 |       |
| 33 | .56181    | 27 | .75876    | 42 | .57768    | 26 | .78401    | 42 | 33 | 26    |
| 34 | .56208    | 26 | .75918    | 42 | .57794    | 26 | .78443    | 42 | 34 | 2.6   |
| 35 | 9.56234   | 27 | 9.75960   | 42 | 9.57821   | 26 | 9.78485   | 42 | 35 | 3.1   |
| 36 | .56261    | 26 | .76002    | 42 | .57847    | 26 | .78527    | 42 | 36 | 3.4   |
| 37 | .56288    | 27 | .76044    | 42 | .57873    | 26 | .78569    | 42 | 37 | 3.9   |
| 38 | .56315    | 26 | .76086    | 42 | .57899    | 26 | .78611    | 42 | 38 | 4.3   |
| 39 | .56341    | 26 | .76128    | 42 | .57925    | 26 | .78653    | 42 | 39 | 8.6   |
| 40 | 9.56368   | 27 | 9.76171   | 42 | 9.57951   | 26 | 9.78696   | 42 | 40 | 13.0  |
| 41 | .56395    | 26 | .76213    | 42 | .57977    | 26 | .78738    | 42 | 41 | 17.3  |
| 42 | .56421    | 26 | .76255    | 42 | .58003    | 26 | .78780    | 42 | 42 | 21.6  |
| 43 | .56448    | 27 | .76297    | 42 | .58029    | 26 | .78822    | 42 | 43 |       |
| 44 | .56475    | 26 | .76339    | 42 | .58055    | 26 | .78864    | 42 | 44 |       |
| 45 | 9.56501   | 26 | 9.76381   | 42 | 9.58082   | 26 | 9.78906   | 42 | 45 |       |
| 46 | .56528    | 26 | .76423    | 42 | .58108    | 26 | .78948    | 42 | 46 |       |
| 47 | .56554    | 27 | .76465    | 42 | .58134    | 26 | .78990    | 42 | 47 | 25    |
| 48 | .56581    | 26 | .76507    | 42 | .58160    | 26 | .79032    | 42 | 48 | 2.5   |
| 49 | .56608    | 26 | .76549    | 42 | .58186    | 26 | .79074    | 42 | 49 | 3.0   |
| 50 | 9.56634   | 26 | 9.76592   | 42 | 9.58212   | 26 | 9.79116   | 42 | 50 | 3.4   |
| 51 | .56661    | 26 | .76634    | 42 | .58238    | 26 | .79158    | 42 | 51 | 3.8   |
| 52 | .56687    | 26 | .76676    | 42 | .58264    | 26 | .79200    | 42 | 52 | 4.2   |
| 53 | .56714    | 27 | .76718    | 42 | .58290    | 26 | .79242    | 42 | 53 | 8.5   |
| 54 | .56741    | 26 | .76760    | 42 | .58316    | 26 | .79285    | 42 | 54 | 12.7  |
| 55 | 9.56767   | 26 | 9.76802   | 42 | 9.58342   | 26 | 9.79327   | 42 | 55 | 17.0  |
| 56 | .56794    | 26 | .76844    | 42 | .58367    | 26 | .79369    | 42 | 56 | 21.2  |
| 57 | .56820    | 26 | .76886    | 42 | .58393    | 26 | .79411    | 42 | 57 |       |
| 58 | .56847    | 26 | .76928    | 42 | .58419    | 26 | .79453    | 42 | 58 |       |
| 59 | .56873    | 26 | .76970    | 42 | .58445    | 26 | .79495    | 42 | 59 |       |
| 60 | 9.56900   | 26 | 9.77012   | 42 | 9.58471   | 26 | 9.79537   | 42 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|
| 0  | 9.58471   | 26 | 9.79537   | 42 | 9.60008   | 25 | 9.82062   | 42 | 0  |       |
| 1  | .58497    | 25 | .79579    | 42 | .60034    | 25 | .82104    | 42 | 1  |       |
| 2  | .58523    | 26 | .79621    | 42 | .60059    | 25 | .82146    | 42 | 2  |       |
| 3  | .58549    | 26 | .79663    | 42 | .60084    | 25 | .82188    | 42 | 3  |       |
| 4  | .58575    | 26 | .79705    | 42 | .60110    | 25 | .82230    | 42 | 4  |       |
| 5  | 9.58601   | 26 | 9.79747   | 42 | 9.60135   | 25 | 9.82272   | 42 | 5  |       |
| 6  | .58626    | 25 | .79789    | 42 | .60160    | 25 | .82315    | 42 | 6  |       |
| 7  | .58652    | 26 | .79831    | 42 | .60185    | 25 | .82357    | 42 | 7  |       |
| 8  | .58678    | 25 | .79874    | 42 | .60211    | 25 | .82399    | 42 | 8  |       |
| 9  | .58704    | 25 | .79916    | 42 | .60236    | 25 | .82441    | 42 | 9  |       |
| 10 | 9.58730   | 26 | 9.79958   | 42 | 9.60261   | 25 | 9.82483   | 42 | 10 |       |
| 11 | .58755    | 25 | .80000    | 42 | .60286    | 25 | .82525    | 42 | 11 |       |
| 12 | .58781    | 26 | .80042    | 42 | .60312    | 25 | .82567    | 42 | 12 |       |
| 13 | .58807    | 26 | .80084    | 42 | .60337    | 25 | .82609    | 42 | 13 |       |
| 14 | .58833    | 25 | .80126    | 42 | .60362    | 25 | .82651    | 42 | 14 |       |
| 15 | 9.58859   | 26 | 9.80168   | 42 | 9.60387   | 25 | 9.82694   | 42 | 15 |       |
| 16 | .58884    | 25 | .80210    | 42 | .60412    | 25 | .82736    | 42 | 16 |       |
| 17 | .58910    | 26 | .80252    | 42 | .60438    | 25 | .82778    | 42 | 17 |       |
| 18 | .58936    | 26 | .80294    | 42 | .60463    | 25 | .82820    | 42 | 18 |       |
| 19 | .58962    | 25 | .80336    | 42 | .60488    | 25 | .82862    | 42 | 19 |       |
| 20 | 9.58987   | 25 | 9.80378   | 42 | 9.60513   | 25 | 9.82904   | 42 | 20 |       |
| 21 | .59013    | 26 | .80420    | 42 | .60538    | 25 | .82946    | 42 | 21 |       |
| 22 | .59039    | 25 | .80463    | 42 | .60563    | 25 | .82988    | 42 | 22 |       |
| 23 | .59064    | 26 | .80505    | 42 | .60589    | 25 | .83031    | 42 | 23 |       |
| 24 | .59090    | 25 | .80547    | 42 | .60614    | 25 | .83073    | 42 | 24 |       |
| 25 | 9.59116   | 25 | 9.80589   | 42 | 9.60639   | 25 | 9.83115   | 42 | 25 |       |
| 26 | .59141    | 26 | .80631    | 42 | .60664    | 25 | .83157    | 42 | 26 |       |
| 27 | .59167    | 25 | .80673    | 42 | .60689    | 25 | .83199    | 42 | 27 |       |
| 28 | .59193    | 25 | .80715    | 42 | .60714    | 25 | .83241    | 42 | 28 |       |
| 29 | .59218    | 25 | .80757    | 42 | .60739    | 25 | .83283    | 42 | 29 |       |
| 30 | 9.59244   | 26 | 9.80799   | 42 | 9.60764   | 25 | 9.83325   | 42 | 30 |       |
| 31 | .59270    | 25 | .80841    | 42 | .60789    | 25 | .83368    | 42 | 31 |       |
| 32 | .59295    | 26 | .80883    | 42 | .60814    | 25 | .83410    | 42 | 32 |       |
| 33 | .59321    | 25 | .80925    | 42 | .60839    | 25 | .83452    | 42 | 33 |       |
| 34 | .59346    | 25 | .80968    | 42 | .60864    | 25 | .83494    | 42 | 34 |       |
| 35 | 9.59372   | 25 | 9.81010   | 42 | 9.60889   | 25 | 9.83536   | 42 | 35 |       |
| 36 | .59397    | 26 | .81052    | 42 | .60914    | 25 | .83578    | 42 | 36 |       |
| 37 | .59423    | 26 | .81094    | 42 | .60939    | 25 | .83620    | 42 | 37 |       |
| 38 | .59449    | 25 | .81136    | 42 | .60964    | 25 | .83663    | 42 | 38 |       |
| 39 | .59474    | 25 | .81178    | 42 | .60989    | 25 | .83705    | 42 | 39 |       |
| 40 | 9.59500   | 25 | 9.81220   | 42 | 9.61014   | 25 | 9.83747   | 42 | 40 |       |
| 41 | .59525    | 25 | .81262    | 42 | .61039    | 25 | .83789    | 42 | 41 |       |
| 42 | .59551    | 25 | .81304    | 42 | .61064    | 25 | .83831    | 42 | 42 |       |
| 43 | .59576    | 25 | .81346    | 42 | .61089    | 25 | .83873    | 42 | 43 |       |
| 44 | .59602    | 25 | .81388    | 42 | .61114    | 25 | .83916    | 42 | 44 |       |
| 45 | 9.59627   | 25 | 9.81430   | 42 | 9.61139   | 25 | 9.83958   | 42 | 45 |       |
| 46 | .59653    | 26 | .81473    | 42 | .61164    | 25 | .84000    | 42 | 46 |       |
| 47 | .59678    | 25 | .81515    | 42 | .61189    | 24 | .84042    | 42 | 47 |       |
| 48 | .59704    | 25 | .81557    | 42 | .61214    | 25 | .84084    | 42 | 48 |       |
| 49 | .59729    | 25 | .81599    | 42 | .61239    | 25 | .84126    | 42 | 49 |       |
| 50 | 9.59754   | 25 | 9.81641   | 42 | 9.61264   | 25 | 9.84168   | 42 | 50 |       |
| 51 | .59780    | 25 | .81683    | 42 | .61289    | 24 | .84211    | 42 | 51 |       |
| 52 | .59805    | 25 | .81725    | 42 | .61314    | 25 | .84253    | 42 | 52 |       |
| 53 | .59831    | 25 | .81767    | 42 | .61339    | 25 | .84295    | 42 | 53 |       |
| 54 | .59856    | 25 | .81809    | 42 | .61363    | 25 | .84337    | 42 | 54 |       |
| 55 | 9.59881   | 25 | 9.81851   | 42 | 9.61388   | 25 | 9.84379   | 42 | 55 |       |
| 56 | .59907    | 25 | .81894    | 42 | .61413    | 25 | .84422    | 42 | 56 |       |
| 57 | .59932    | 25 | .81936    | 42 | .61438    | 24 | .84464    | 42 | 57 |       |
| 58 | .59958    | 25 | .81978    | 42 | .61462    | 25 | .84506    | 42 | 58 |       |
| 59 | .59983    | 25 | .82020    | 42 | .61487    | 25 | .84548    | 42 | 59 |       |
| 60 | 9.60008   | 25 | 9.82062   | 42 | 9.61512   | 25 | 9.84590   | 42 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |

6 4.2 4.2  
 7 4.4 4.9  
 8 5.6 5.6  
 9 6.4 6.3  
 10 7.1 7.0  
 20 14.1 14.0  
 30 21.2 21.0  
 40 28.3 28.0  
 50 35.4 35.0

26 25  
 6 2.6 2.5  
 7 3.0 3.0  
 8 3.4 3.4  
 9 3.9 3.8  
 10 4.3 4.2  
 20 8.6 8.5  
 30 13.0 12.7  
 40 17.3 17.0  
 50 21.6 21.2

25 24  
 6 2.5 2.4  
 7 2.9 2.8  
 8 3.3 3.2  
 9 3.7 3.7  
 10 4.1 4.1  
 20 8.3 8.1  
 30 12.5 12.2  
 40 16.6 16.3  
 50 20.8 20.4

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.61512   | 24 | 9.84590   | 42 | 9.62984   | 24 | 9.87125   | 42 | 0  |              |
| 1  | .61537    | 25 | .84632    | 42 | .63008    | 24 | .87167    | 42 | 1  |              |
| 2  | .61562    | 24 | .84675    | 42 | .63032    | 24 | .87209    | 42 | 2  |              |
| 3  | .61586    | 25 | .84717    | 42 | .63057    | 24 | .87252    | 42 | 3  |              |
| 4  | .61611    | 24 | .84759    | 42 | .63081    | 24 | .87294    | 42 | 4  |              |
| 5  | 9.61636   | 24 | 9.84801   | 42 | 9.63105   | 24 | 9.87336   | 42 | 5  |              |
| 6  | .61661    | 24 | .84843    | 42 | .63129    | 24 | .87379    | 42 | 6  |              |
| 7  | .61685    | 25 | .84886    | 42 | .63154    | 24 | .87421    | 42 | 7  |              |
| 8  | .61710    | 24 | .84928    | 42 | .63178    | 24 | .87463    | 42 | 8  | 42 42        |
| 9  | .61733    | 25 | .84970    | 42 | .63202    | 24 | .87506    | 42 | 9  | 6 4.2 4.2    |
| 10 | 9.61769   | 24 | 9.85012   | 42 | 9.63226   | 24 | 9.87548   | 42 | 10 | 7 4.9 4.9    |
| 11 | .61784    | 24 | .85054    | 42 | .63250    | 24 | .87590    | 42 | 11 | 8 5.6 5.6    |
| 12 | .61809    | 25 | .85097    | 42 | .63274    | 24 | .87633    | 42 | 12 | 9 6.4 6.3    |
| 13 | .61834    | 24 | .85139    | 42 | .63299    | 24 | .87675    | 42 | 13 | 10 7.1 7.0   |
| 14 | .61858    | 25 | .85181    | 42 | .63323    | 24 | .87717    | 42 | 14 | 20 14.1 14.0 |
| 15 | 9.61883   | 24 | 9.85223   | 42 | 9.63347   | 24 | 9.87760   | 42 | 15 | 30 21.2 21.0 |
| 16 | .61908    | 24 | .85265    | 42 | .63371    | 24 | .87802    | 42 | 16 | 40 28.3 28.0 |
| 17 | .61932    | 24 | .85308    | 42 | .63395    | 24 | .87844    | 42 | 17 | 50 35.4 35.0 |
| 18 | .61957    | 25 | .85350    | 42 | .63419    | 24 | .87887    | 42 | 18 |              |
| 19 | .61982    | 24 | .85392    | 42 | .63443    | 24 | .87929    | 42 | 19 |              |
| 20 | 9.62006   | 24 | 9.85434   | 42 | 9.63468   | 24 | 9.87971   | 42 | 20 |              |
| 21 | .62031    | 24 | .85476    | 42 | .63492    | 24 | .88014    | 42 | 21 |              |
| 22 | .62055    | 25 | .85519    | 42 | .63516    | 24 | .88056    | 42 | 22 |              |
| 23 | .62080    | 24 | .85561    | 42 | .63540    | 24 | .88099    | 42 | 23 |              |
| 24 | .62105    | 24 | .85603    | 42 | .63564    | 24 | .88141    | 42 | 24 |              |
| 25 | 9.62129   | 24 | 9.85645   | 42 | 9.63588   | 24 | 9.88183   | 42 | 25 | 25 24        |
| 26 | .62154    | 24 | .85688    | 42 | .63612    | 24 | .88226    | 42 | 26 | 6 2.5 2.4    |
| 27 | .62178    | 24 | .85730    | 42 | .63636    | 24 | .88268    | 42 | 27 | 7 2.9 2.8    |
| 28 | .62203    | 24 | .85772    | 42 | .63660    | 24 | .88310    | 42 | 28 | 8 3.3 3.2    |
| 29 | .62227    | 24 | .85814    | 42 | .63684    | 24 | .88353    | 42 | 29 | 9 3.7 3.7    |
| 30 | 9.62252   | 24 | 9.85857   | 42 | 9.63708   | 24 | 9.88395   | 42 | 30 | 10 4.1 4.1   |
| 31 | .62276    | 24 | .85899    | 42 | .63732    | 24 | .88438    | 42 | 31 | 20 8.3 8.1   |
| 32 | .62301    | 24 | .85941    | 42 | .63756    | 24 | .88480    | 42 | 32 | 30 12.5 12.2 |
| 33 | .62325    | 24 | .85983    | 42 | .63780    | 24 | .88522    | 42 | 33 | 40 16.6 16.3 |
| 34 | .62350    | 24 | .86026    | 42 | .63804    | 24 | .88565    | 42 | 34 | 50 20.8 20.4 |
| 35 | 9.62374   | 24 | 9.86068   | 42 | 9.63828   | 24 | 9.88607   | 42 | 35 |              |
| 36 | .62399    | 24 | .86110    | 42 | .63852    | 24 | .88650    | 42 | 36 |              |
| 37 | .62423    | 24 | .86152    | 42 | .63876    | 24 | .88692    | 42 | 37 |              |
| 38 | .62448    | 24 | .86195    | 42 | .63900    | 24 | .88734    | 42 | 38 |              |
| 39 | .62472    | 24 | .86237    | 42 | .63924    | 24 | .88777    | 42 | 39 |              |
| 40 | 9.62497   | 24 | 9.86279   | 42 | 9.63948   | 24 | 9.88819   | 42 | 40 |              |
| 41 | .62521    | 24 | .86321    | 42 | .63972    | 24 | .88862    | 42 | 41 |              |
| 42 | .62546    | 24 | .86364    | 42 | .63996    | 23 | .88904    | 42 | 42 |              |
| 43 | .62570    | 24 | .86406    | 42 | .64019    | 24 | .88947    | 42 | 43 |              |
| 44 | .62594    | 24 | .86448    | 42 | .64043    | 24 | .88989    | 42 | 44 | 24 23        |
| 45 | 9.62619   | 24 | 9.86490   | 42 | 9.64067   | 24 | 9.89031   | 42 | 45 | 6 2.4 2.3    |
| 46 | .62643    | 24 | .86533    | 42 | .64091    | 24 | .89074    | 42 | 46 | 7 2.8 2.7    |
| 47 | .62668    | 24 | .86575    | 42 | .64115    | 23 | .89116    | 42 | 47 | 8 3.2 3.1    |
| 48 | .62692    | 24 | .86617    | 42 | .64139    | 24 | .89159    | 42 | 48 | 9 3.6 3.5    |
| 49 | .62716    | 24 | .86659    | 42 | .64163    | 24 | .89201    | 42 | 49 | 10 4.0 3.9   |
| 50 | 9.62741   | 24 | 9.86702   | 42 | 9.64187   | 24 | 9.89244   | 42 | 50 | 20 8.0 7.8   |
| 51 | .62765    | 24 | .86744    | 42 | .64210    | 24 | .89286    | 42 | 51 | 30 12.0 11.7 |
| 52 | .62789    | 24 | .86786    | 42 | .64234    | 24 | .89329    | 42 | 52 | 40 16.0 15.6 |
| 53 | .62814    | 24 | .86829    | 42 | .64258    | 23 | .89371    | 42 | 53 | 50 20.0 19.6 |
| 54 | .62838    | 24 | .86871    | 42 | .64282    | 24 | .89414    | 42 | 54 |              |
| 55 | 9.62862   | 24 | 9.86913   | 42 | 9.64306   | 24 | 9.89456   | 42 | 55 |              |
| 56 | .62887    | 24 | .86956    | 42 | .64330    | 23 | .89499    | 42 | 56 |              |
| 57 | .62911    | 24 | .86998    | 42 | .64353    | 24 | .89541    | 42 | 57 |              |
| 58 | .62935    | 24 | .87040    | 42 | .64377    | 23 | .89583    | 42 | 58 |              |
| 59 | .62960    | 24 | .87082    | 42 | .64401    | 24 | .89626    | 42 | 59 |              |
| 60 | 9.62984   | 24 | 9.87125   | 42 | 9.64425   | 24 | 9.89668   | 42 | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    |           |    |           |    |           |    |           |    |    |  |  | P. P. |  |  |  |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--|--|-------|--|--|--|
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    |  |  |       |  |  |  |
| 0  | 9.64425   |    | 9.89668   |    | 9.65835   |    | 9.92224   |    | 0  |  |  |       |  |  |  |
| 1  | 64448     | 23 | .89711    | 42 | .65859    | 23 | .92267    | 43 | 1  |  |  |       |  |  |  |
| 2  | 64472     | 24 | .89753    | 42 | .65882    | 23 | .92310    | 42 | 2  |  |  |       |  |  |  |
| 3  | 64496     | 23 | .89796    | 42 | .65905    | 23 | .92353    | 43 | 3  |  |  |       |  |  |  |
| 4  | 64520     | 24 | .89838    | 42 | .65928    | 23 | .92395    | 42 | 4  |  |  |       |  |  |  |
| 5  | 9.64543   | 23 | 9.89881   | 42 | 9.65952   | 23 | 9.92438   | 43 | 5  |  |  |       |  |  |  |
| 6  | 64567     | 24 | .89923    | 42 | .65975    | 23 | .92481    | 42 | 6  |  |  |       |  |  |  |
| 7  | 64591     | 23 | .89966    | 42 | .65998    | 23 | .92524    | 43 | 7  |  |  |       |  |  |  |
| 8  | 64614     | 23 | .90008    | 42 | .66021    | 23 | .92566    | 42 | 8  |  |  |       |  |  |  |
| 9  | 64638     | 24 | .90051    | 42 | .66044    | 23 | .92609    | 43 | 9  |  |  |       |  |  |  |
| 10 | 9.64662   | 23 | 9.90094   | 43 | 9.66068   | 23 | 9.92652   | 42 | 10 |  |  |       |  |  |  |
| 11 | 64685     | 24 | .90136    | 42 | .66091    | 23 | .92695    | 43 | 11 |  |  |       |  |  |  |
| 12 | 64709     | 23 | .90179    | 42 | .66114    | 23 | .92737    | 42 | 12 |  |  |       |  |  |  |
| 13 | 64733     | 23 | .90221    | 42 | .66137    | 23 | .92780    | 43 | 13 |  |  |       |  |  |  |
| 14 | 64756     | 24 | .90264    | 42 | .66160    | 23 | .92823    | 42 | 14 |  |  |       |  |  |  |
| 15 | 9.64780   | 23 | 9.90306   | 42 | 9.66183   | 23 | 9.92866   | 43 | 15 |  |  |       |  |  |  |
| 16 | 64804     | 24 | .90349    | 42 | .66207    | 23 | .92909    | 42 | 16 |  |  |       |  |  |  |
| 17 | 64827     | 23 | .90391    | 42 | .66230    | 23 | .92951    | 43 | 17 |  |  |       |  |  |  |
| 18 | 64851     | 24 | .90434    | 42 | .66253    | 23 | .92994    | 42 | 18 |  |  |       |  |  |  |
| 19 | 64875     | 23 | .90476    | 42 | .66276    | 23 | .93037    | 43 | 19 |  |  |       |  |  |  |
| 20 | 9.64898   | 23 | 9.90519   | 42 | 9.66299   | 23 | 9.93080   | 43 | 20 |  |  |       |  |  |  |
| 21 | 64922     | 24 | .90561    | 42 | .66322    | 23 | .93123    | 42 | 21 |  |  |       |  |  |  |
| 22 | 64945     | 23 | .90604    | 42 | .66345    | 23 | .93165    | 43 | 22 |  |  |       |  |  |  |
| 23 | 64969     | 23 | .90647    | 42 | .66368    | 23 | .93208    | 42 | 23 |  |  |       |  |  |  |
| 24 | 64992     | 24 | .90689    | 42 | .66391    | 23 | .93251    | 43 | 24 |  |  |       |  |  |  |
| 25 | 9.65016   | 23 | 9.90732   | 42 | 9.66415   | 23 | 9.93294   | 42 | 25 |  |  |       |  |  |  |
| 26 | 65040     | 24 | .90774    | 42 | .66438    | 23 | .93337    | 43 | 26 |  |  |       |  |  |  |
| 27 | 65063     | 23 | .90817    | 42 | .66461    | 23 | .93380    | 42 | 27 |  |  |       |  |  |  |
| 28 | 65087     | 23 | .90860    | 42 | .66484    | 23 | .93422    | 43 | 28 |  |  |       |  |  |  |
| 29 | 65110     | 24 | .90902    | 42 | .66507    | 23 | .93465    | 42 | 29 |  |  |       |  |  |  |
| 30 | 9.65134   | 23 | 9.90945   | 42 | 9.66530   | 23 | 9.93508   | 43 | 30 |  |  |       |  |  |  |
| 31 | 65157     | 24 | .90987    | 42 | .66553    | 23 | .93551    | 42 | 31 |  |  |       |  |  |  |
| 32 | 65181     | 23 | .91030    | 42 | .66576    | 23 | .93594    | 43 | 32 |  |  |       |  |  |  |
| 33 | 65204     | 23 | .91073    | 42 | .66599    | 23 | .93637    | 42 | 33 |  |  |       |  |  |  |
| 34 | 65228     | 24 | .91115    | 42 | .66622    | 23 | .93680    | 43 | 34 |  |  |       |  |  |  |
| 35 | 9.65251   | 23 | 9.91158   | 42 | 9.66645   | 23 | 9.93722   | 42 | 35 |  |  |       |  |  |  |
| 36 | 65275     | 24 | .91200    | 42 | .66668    | 23 | .93765    | 43 | 36 |  |  |       |  |  |  |
| 37 | 65298     | 23 | .91243    | 42 | .66691    | 23 | .93808    | 42 | 37 |  |  |       |  |  |  |
| 38 | 65321     | 23 | .91286    | 42 | .66714    | 23 | .93851    | 43 | 38 |  |  |       |  |  |  |
| 39 | 65345     | 24 | .91329    | 42 | .66737    | 23 | .93894    | 42 | 39 |  |  |       |  |  |  |
| 40 | 9.65368   | 23 | 9.91371   | 42 | 9.66760   | 23 | 9.93937   | 43 | 40 |  |  |       |  |  |  |
| 41 | 65392     | 24 | .91414    | 42 | .66783    | 23 | .93980    | 42 | 41 |  |  |       |  |  |  |
| 42 | 65415     | 23 | .91456    | 42 | .66805    | 23 | .94023    | 43 | 42 |  |  |       |  |  |  |
| 43 | 65439     | 23 | .91499    | 42 | .66828    | 23 | .94066    | 42 | 43 |  |  |       |  |  |  |
| 44 | 65462     | 24 | .91541    | 42 | .66851    | 23 | .94109    | 43 | 44 |  |  |       |  |  |  |
| 45 | 9.65485   | 23 | 9.91584   | 42 | 9.66874   | 23 | 9.94151   | 42 | 45 |  |  |       |  |  |  |
| 46 | 65509     | 24 | .91627    | 42 | .66897    | 23 | .94194    | 43 | 46 |  |  |       |  |  |  |
| 47 | 65532     | 23 | .91669    | 42 | .66920    | 22 | .94237    | 42 | 47 |  |  |       |  |  |  |
| 48 | 65556     | 23 | .91712    | 42 | .66943    | 23 | .94280    | 43 | 48 |  |  |       |  |  |  |
| 49 | 65579     | 24 | .91755    | 42 | .66966    | 23 | .94323    | 42 | 49 |  |  |       |  |  |  |
| 50 | 9.65602   | 23 | 9.91797   | 42 | 9.66989   | 23 | 9.94366   | 43 | 50 |  |  |       |  |  |  |
| 51 | 65626     | 24 | .91840    | 42 | .67012    | 22 | .94409    | 42 | 51 |  |  |       |  |  |  |
| 52 | 65649     | 23 | .91883    | 42 | .67034    | 23 | .94452    | 43 | 52 |  |  |       |  |  |  |
| 53 | 65672     | 23 | .91926    | 42 | .67057    | 23 | .94495    | 42 | 53 |  |  |       |  |  |  |
| 54 | 65696     | 24 | .91968    | 42 | .67080    | 22 | .94538    | 43 | 54 |  |  |       |  |  |  |
| 55 | 9.65719   | 23 | 9.92011   | 42 | 9.67103   | 22 | 9.94581   | 42 | 55 |  |  |       |  |  |  |
| 56 | 65742     | 24 | .92054    | 42 | .67126    | 23 | .94624    | 43 | 56 |  |  |       |  |  |  |
| 57 | 65765     | 23 | .92096    | 42 | .67149    | 22 | .94667    | 42 | 57 |  |  |       |  |  |  |
| 58 | 65789     | 23 | .92139    | 42 | .67171    | 23 | .94710    | 43 | 58 |  |  |       |  |  |  |
| 59 | 65812     | 24 | .92182    | 42 | .67194    | 22 | .94753    | 42 | 59 |  |  |       |  |  |  |
| 60 | 9.65835   | 23 | 9.92224   | 42 | 9.67217   | 22 | 9.94796   | 43 | 60 |  |  |       |  |  |  |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    |  |  |       |  |  |  |

43 42  
6 4.3 4.2  
7 5.0 4.9  
8 5.7 5.6  
9 6.4 6.4  
10 7.1 7.1  
11 7.8 7.8  
12 8.5 8.5  
13 9.2 9.2  
14 9.9 9.9  
15 10.6 10.6  
16 11.3 11.3  
17 12.0 12.0  
18 12.7 12.7  
19 13.4 13.4

24 23  
6 2.4 2.3  
7 2.8 2.7  
8 3.2 3.1  
9 3.6 3.5  
10 4.0 3.9  
11 4.4 4.3  
12 4.8 4.7  
13 5.2 5.1  
14 5.6 5.5  
15 6.0 5.9  
16 6.4 6.3  
17 6.8 6.7  
18 7.2 7.1  
19 7.6 7.5  
20 8.0 7.9  
21 8.4 8.3  
22 8.8 8.7  
23 9.2 9.1  
24 9.6 9.5  
25 10.0 9.9  
26 10.4 10.3  
27 10.8 10.7  
28 11.2 11.1  
29 11.6 11.5  
30 12.0 11.9  
31 12.4 12.3  
32 12.8 12.7  
33 13.2 13.1  
34 13.6 13.5  
35 14.0 13.9  
36 14.4 14.3  
37 14.8 14.7  
38 15.2 15.1  
39 15.6 15.5  
40 16.0 15.9  
41 16.4 16.3  
42 16.8 16.7  
43 17.2 17.1  
44 17.6 17.5  
45 18.0 17.9  
46 18.4 18.3  
47 18.8 18.7  
48 19.2 19.1  
49 19.6 19.5  
50 20.0 19.9

23 22  
6 2.3 2.2  
7 2.7 2.6  
8 3.0 3.0  
9 3.4 3.4  
10 3.8 3.7  
11 4.2 4.1  
12 4.6 4.5  
13 5.0 4.9  
14 5.4 5.3  
15 5.8 5.7  
16 6.2 6.1  
17 6.6 6.5  
18 7.0 6.9  
19 7.4 7.3  
20 7.8 7.7  
21 8.2 8.1  
22 8.6 8.5  
23 9.0 8.9  
24 9.4 9.3  
25 9.8 9.7  
26 10.2 10.1  
27 10.6 10.5  
28 11.0 10.9  
29 11.4 11.3  
30 11.8 11.7  
31 12.2 12.1  
32 12.6 12.5  
33 13.0 12.9  
34 13.4 13.3  
35 13.8 13.7  
36 14.2 14.1  
37 14.6 14.5  
38 15.0 14.9  
39 15.4 15.3  
40 15.8 15.7  
41 16.2 16.1  
42 16.6 16.5  
43 17.0 16.9  
44 17.4 17.3  
45 17.8 17.7  
46 18.2 18.1  
47 18.6 18.5  
48 19.0 18.9  
49 19.4 19.3  
50 19.8 19.7  
51 20.2 20.1  
52 20.6 20.5  
53 21.0 20.9  
54 21.4 21.3  
55 21.8 21.7  
56 22.2 22.1  
57 22.6 22.5  
58 23.0 22.9  
59 23.4 23.3  
60 23.8 23.7

P. P.



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    |         | Lg. Vers. | D       |    |         | Log. Exs. | D        |    |    | Lg. Vers. | D |  |  | Log. Exs. | D |  |  | P. P. |  |
|----|---------|-----------|---------|----|---------|-----------|----------|----|----|-----------|---|--|--|-----------|---|--|--|-------|--|
| 0  | 9.67217 | 23        | 9.94796 | 43 | 9.68571 | 22        | 9.97387  | 43 | 0  |           |   |  |  |           |   |  |  |       |  |
| 1  | .67240  | 23        | .94839  | 43 | .68593  | 22        | .97430   | 43 | 1  |           |   |  |  |           |   |  |  |       |  |
| 2  | .67263  | 22        | .94882  | 43 | .68615  | 22        | .97473   | 43 | 2  |           |   |  |  |           |   |  |  |       |  |
| 3  | .67285  | 22        | .94925  | 43 | .68637  | 22        | .97517   | 43 | 3  |           |   |  |  |           |   |  |  |       |  |
| 4  | .67308  | 22        | .94968  | 43 | .68660  | 22        | .97560   | 43 | 4  |           |   |  |  |           |   |  |  |       |  |
| 5  | 9.67331 | 22        | 9.95011 | 43 | 9.68682 | 22        | 9.97603  | 43 | 5  |           |   |  |  |           |   |  |  |       |  |
| 6  | .67354  | 22        | .95054  | 43 | .68704  | 22        | .97647   | 43 | 6  |           |   |  |  |           |   |  |  |       |  |
| 7  | .67376  | 22        | .95097  | 43 | .68727  | 22        | .97690   | 43 | 7  |           |   |  |  |           |   |  |  |       |  |
| 8  | .67399  | 22        | .95140  | 43 | .68749  | 22        | .97734   | 43 | 8  |           |   |  |  |           |   |  |  |       |  |
| 9  | .67422  | 22        | .95183  | 43 | .68771  | 22        | .97777   | 43 | 9  |           |   |  |  |           |   |  |  |       |  |
| 10 | 9.67445 | 23        | 9.95226 | 43 | 9.68793 | 22        | 9.97820  | 43 | 10 |           |   |  |  |           |   |  |  |       |  |
| 11 | .67467  | 22        | .95269  | 43 | .68816  | 22        | .97864   | 43 | 11 |           |   |  |  |           |   |  |  |       |  |
| 12 | .67490  | 23        | .95313  | 43 | .68838  | 22        | .97907   | 43 | 12 |           |   |  |  |           |   |  |  |       |  |
| 13 | .67513  | 22        | .95356  | 43 | .68860  | 22        | .97951   | 43 | 13 |           |   |  |  |           |   |  |  |       |  |
| 14 | .67535  | 22        | .95399  | 43 | .68882  | 22        | .97994   | 43 | 14 |           |   |  |  |           |   |  |  |       |  |
| 15 | 9.67558 | 23        | 9.95442 | 43 | 9.68905 | 22        | 9.98038  | 43 | 15 |           |   |  |  |           |   |  |  |       |  |
| 16 | .67581  | 22        | .95485  | 43 | .68927  | 22        | .98081   | 43 | 16 |           |   |  |  |           |   |  |  |       |  |
| 17 | .67603  | 22        | .95528  | 43 | .68949  | 22        | .98125   | 43 | 17 |           |   |  |  |           |   |  |  |       |  |
| 18 | .67626  | 23        | .95571  | 43 | .68971  | 22        | .98168   | 43 | 18 |           |   |  |  |           |   |  |  |       |  |
| 19 | .67649  | 22        | .95614  | 43 | .68993  | 22        | .98211   | 43 | 19 |           |   |  |  |           |   |  |  |       |  |
| 20 | 9.67671 | 22        | 9.95657 | 43 | 9.69016 | 22        | 9.98255  | 43 | 20 |           |   |  |  |           |   |  |  |       |  |
| 21 | .67694  | 22        | .95700  | 43 | .69038  | 22        | .98298   | 43 | 21 |           |   |  |  |           |   |  |  |       |  |
| 22 | .67717  | 23        | .95744  | 43 | .69060  | 22        | .98342   | 43 | 22 |           |   |  |  |           |   |  |  |       |  |
| 23 | .67739  | 22        | .95787  | 43 | .69082  | 22        | .98385   | 43 | 23 |           |   |  |  |           |   |  |  |       |  |
| 24 | .67762  | 22        | .95830  | 43 | .69104  | 22        | .98429   | 43 | 24 |           |   |  |  |           |   |  |  |       |  |
| 25 | 9.67784 | 22        | 9.95873 | 43 | 9.69126 | 22        | 9.98472  | 43 | 25 |           |   |  |  |           |   |  |  |       |  |
| 26 | .67807  | 23        | .95916  | 43 | .69149  | 22        | .98516   | 43 | 26 |           |   |  |  |           |   |  |  |       |  |
| 27 | .67830  | 22        | .95959  | 43 | .69171  | 22        | .98559   | 43 | 27 |           |   |  |  |           |   |  |  |       |  |
| 28 | .67852  | 22        | .96002  | 43 | .69193  | 22        | .98603   | 43 | 28 |           |   |  |  |           |   |  |  |       |  |
| 29 | .67875  | 22        | .96046  | 43 | .69215  | 22        | .98647   | 43 | 29 |           |   |  |  |           |   |  |  |       |  |
| 30 | 9.67897 | 22        | 9.96089 | 43 | 9.69237 | 22        | 9.98690  | 43 | 30 |           |   |  |  |           |   |  |  |       |  |
| 31 | .67920  | 22        | .96132  | 43 | .69259  | 22        | .98734   | 43 | 31 |           |   |  |  |           |   |  |  |       |  |
| 32 | .67942  | 22        | .96175  | 43 | .69281  | 22        | .98777   | 43 | 32 |           |   |  |  |           |   |  |  |       |  |
| 33 | .67965  | 22        | .96218  | 43 | .69303  | 22        | .98821   | 43 | 33 |           |   |  |  |           |   |  |  |       |  |
| 34 | .67987  | 22        | .96261  | 43 | .69325  | 22        | .98864   | 43 | 34 |           |   |  |  |           |   |  |  |       |  |
| 35 | 9.68010 | 22        | 9.96305 | 43 | 9.69347 | 22        | 9.98908  | 43 | 35 |           |   |  |  |           |   |  |  |       |  |
| 36 | .68032  | 22        | .96348  | 43 | .69369  | 22        | .98952   | 43 | 36 |           |   |  |  |           |   |  |  |       |  |
| 37 | .68055  | 22        | .96391  | 43 | .69392  | 22        | .98995   | 43 | 37 |           |   |  |  |           |   |  |  |       |  |
| 38 | .68077  | 22        | .96434  | 43 | .69414  | 22        | .99039   | 43 | 38 |           |   |  |  |           |   |  |  |       |  |
| 39 | .68100  | 22        | .96478  | 43 | .69436  | 22        | .99082   | 43 | 39 |           |   |  |  |           |   |  |  |       |  |
| 40 | 9.68122 | 22        | 9.96521 | 43 | 9.69458 | 22        | 9.99126  | 43 | 40 |           |   |  |  |           |   |  |  |       |  |
| 41 | .68145  | 22        | .96564  | 43 | .69480  | 22        | .99170   | 43 | 41 |           |   |  |  |           |   |  |  |       |  |
| 42 | .68167  | 22        | .96607  | 43 | .69502  | 22        | .99213   | 43 | 42 |           |   |  |  |           |   |  |  |       |  |
| 43 | .68190  | 22        | .96650  | 43 | .69524  | 22        | .99257   | 43 | 43 |           |   |  |  |           |   |  |  |       |  |
| 44 | .68212  | 22        | .96694  | 43 | .69546  | 22        | .99300   | 43 | 44 |           |   |  |  |           |   |  |  |       |  |
| 45 | 9.68235 | 22        | 9.96737 | 43 | 9.69568 | 22        | 9.99344  | 43 | 45 |           |   |  |  |           |   |  |  |       |  |
| 46 | .68257  | 22        | .96780  | 43 | .69590  | 22        | .99388   | 43 | 46 |           |   |  |  |           |   |  |  |       |  |
| 47 | .68280  | 22        | .96824  | 43 | .69612  | 22        | .99431   | 43 | 47 |           |   |  |  |           |   |  |  |       |  |
| 48 | .68302  | 22        | .96867  | 43 | .69634  | 22        | .99475   | 43 | 48 |           |   |  |  |           |   |  |  |       |  |
| 49 | .68324  | 22        | .96910  | 43 | .69656  | 22        | .99519   | 43 | 49 |           |   |  |  |           |   |  |  |       |  |
| 50 | 9.68347 | 22        | 9.96953 | 43 | 9.69678 | 22        | 9.99562  | 43 | 50 |           |   |  |  |           |   |  |  |       |  |
| 51 | .68369  | 22        | .96997  | 43 | .69700  | 21        | .99606   | 43 | 51 |           |   |  |  |           |   |  |  |       |  |
| 52 | .68392  | 22        | .97040  | 43 | .69721  | 22        | .99650   | 43 | 52 |           |   |  |  |           |   |  |  |       |  |
| 53 | .68414  | 22        | .97083  | 43 | .69743  | 22        | .99694   | 43 | 53 |           |   |  |  |           |   |  |  |       |  |
| 54 | .68436  | 22        | .97127  | 43 | .69765  | 22        | .99737   | 43 | 54 |           |   |  |  |           |   |  |  |       |  |
| 55 | 9.68459 | 22        | 9.97170 | 43 | 9.69787 | 22        | 9.99781  | 43 | 55 |           |   |  |  |           |   |  |  |       |  |
| 56 | .68481  | 22        | .97213  | 43 | .69809  | 22        | .99825   | 43 | 56 |           |   |  |  |           |   |  |  |       |  |
| 57 | .68503  | 22        | .97257  | 43 | .69831  | 21        | .99868   | 43 | 57 |           |   |  |  |           |   |  |  |       |  |
| 58 | .68526  | 22        | .97300  | 43 | .69853  | 22        | .99912   | 43 | 58 |           |   |  |  |           |   |  |  |       |  |
| 59 | .68548  | 22        | .97343  | 43 | .69875  | 22        | .99956   | 43 | 59 |           |   |  |  |           |   |  |  |       |  |
| 60 | 9.68571 | 23        | 9.97387 | 43 | 9.69897 | 22        | 10.00000 | 43 | 60 |           |   |  |  |           |   |  |  |       |  |
|    |         | Lg. Vers. | D       |    |         | Log. Exs. | D        |    |    | Lg. Vers. | D |  |  | Log. Exs. | D |  |  | P. P. |  |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.69897   | 22 | 10.00000  | 44 | 9.71197   | 21 | 10.02639  | 44 | 0  |              |
| 1  | .69919    | 21 | .00044    | 43 | .71213    | 21 | .02684    | 44 | 1  |              |
| 2  | .69940    | 22 | .00087    | 44 | .71233    | 21 | .02728    | 44 | 2  |              |
| 3  | .69962    | 22 | .00131    | 43 | .71261    | 21 | .02772    | 44 | 3  |              |
| 4  | .69984    | 22 | .00175    | 43 | .71282    | 21 | .02816    | 44 | 4  |              |
| 5  | 9.70006   | 22 | 10.00219  | 44 | 9.71304   | 21 | 10.02861  | 44 | 5  | 45 44        |
| 6  | .70028    | 21 | .00262    | 43 | .71325    | 21 | .02905    | 44 | 6  | 6 4.5 4.4    |
| 7  | .70050    | 22 | .00306    | 44 | .71346    | 21 | .02949    | 44 | 7  | 7 5.2 5.2    |
| 8  | .70072    | 21 | .00350    | 43 | .71368    | 21 | .02994    | 44 | 8  | 8 6.0 5.9    |
| 9  | .70093    | 21 | .00394    | 43 | .71389    | 21 | .03038    | 44 | 9  | 9 6.7 6.7    |
| 10 | 9.70115   | 22 | 10.00438  | 44 | 9.71411   | 21 | 10.03082  | 44 | 10 | 10 7.5 7.4   |
| 11 | .70137    | 22 | .00482    | 43 | .71432    | 21 | .03127    | 44 | 11 | 20 15.0 14.8 |
| 12 | .70159    | 22 | .00525    | 44 | .71453    | 21 | .03171    | 44 | 12 | 30 22.5 22.2 |
| 13 | .70181    | 21 | .00569    | 44 | .71475    | 21 | .03215    | 44 | 13 | 40 30.0 29.6 |
| 14 | .70202    | 22 | .00613    | 44 | .71496    | 21 | .03260    | 44 | 14 | 50 37.5 37.1 |
| 15 | 9.70224   | 22 | 10.00657  | 44 | 9.71517   | 21 | 10.03304  | 44 | 15 |              |
| 16 | .70246    | 21 | .00701    | 43 | .71539    | 21 | .03348    | 44 | 16 |              |
| 17 | .70268    | 22 | .00745    | 44 | .71560    | 21 | .03393    | 44 | 17 |              |
| 18 | .70289    | 21 | .00789    | 44 | .71581    | 21 | .03437    | 44 | 18 |              |
| 19 | .70311    | 22 | .00833    | 44 | .71603    | 21 | .03481    | 44 | 19 |              |
| 20 | 9.70333   | 22 | 10.00876  | 43 | 9.71624   | 21 | 10.03526  | 44 | 20 | 44 43        |
| 21 | .70355    | 21 | .00920    | 44 | .71645    | 21 | .03570    | 44 | 21 | 6 4.4 4.3    |
| 22 | .70376    | 22 | .00964    | 44 | .71667    | 21 | .03615    | 44 | 22 | 7 5.1 5.1    |
| 23 | .70398    | 21 | .01008    | 44 | .71688    | 21 | .03659    | 44 | 23 | 8 5.8 5.8    |
| 24 | .70420    | 21 | .01052    | 44 | .71709    | 21 | .03704    | 44 | 24 | 9 6.6 6.5    |
| 25 | 9.70441   | 22 | 10.01096  | 44 | 9.71730   | 21 | 10.03748  | 44 | 25 | 10 7.3 7.2   |
| 26 | .70463    | 21 | .01140    | 44 | .71752    | 21 | .03793    | 44 | 26 | 20 14.6 14.5 |
| 27 | .70485    | 22 | .01184    | 44 | .71773    | 21 | .03837    | 44 | 27 | 30 22.0 21.7 |
| 28 | .70507    | 21 | .01228    | 44 | .71794    | 21 | .03881    | 44 | 28 | 40 29.3 29.0 |
| 29 | .70528    | 21 | .01272    | 44 | .71815    | 21 | .03926    | 44 | 29 | 50 36.6 36.2 |
| 30 | 9.70550   | 22 | 10.01316  | 44 | 9.71837   | 21 | 10.03970  | 44 | 30 |              |
| 31 | .70572    | 21 | .01360    | 44 | .71858    | 21 | .04015    | 44 | 31 |              |
| 32 | .70593    | 21 | .01404    | 44 | .71879    | 21 | .04059    | 44 | 32 |              |
| 33 | .70615    | 21 | .01448    | 44 | .71900    | 21 | .04104    | 44 | 33 | 22 21        |
| 34 | .70636    | 22 | .01492    | 44 | .71922    | 21 | .04149    | 44 | 34 | 6 2.2 2.1    |
| 35 | 9.70658   | 22 | 10.01536  | 44 | 9.71943   | 21 | 10.04193  | 44 | 35 | 7 2.5 2.5    |
| 36 | .70680    | 21 | .01580    | 44 | .71964    | 21 | .04238    | 44 | 36 | 8 2.9 2.8    |
| 37 | .70701    | 21 | .01624    | 44 | .71985    | 21 | .04282    | 44 | 37 | 9 3.3 3.2    |
| 38 | .70723    | 22 | .01668    | 44 | .72006    | 21 | .04327    | 44 | 38 | 10 3.6 3.6   |
| 39 | .70745    | 22 | .01712    | 44 | .72028    | 21 | .04371    | 44 | 39 | 20 7.3 7.1   |
| 40 | 9.70766   | 21 | 10.01756  | 44 | 9.72049   | 21 | 10.04416  | 44 | 40 | 30 11.0 10.7 |
| 41 | .70788    | 21 | .01800    | 44 | .72070    | 21 | .04461    | 44 | 41 | 40 14.6 14.3 |
| 42 | .70809    | 21 | .01844    | 44 | .72091    | 21 | .04505    | 44 | 42 | 50 18.3 17.9 |
| 43 | .70831    | 21 | .01889    | 44 | .72112    | 21 | .04550    | 44 | 43 |              |
| 44 | .70852    | 22 | .01933    | 44 | .72133    | 21 | .04594    | 44 | 44 |              |
| 45 | 9.70874   | 21 | 10.01977  | 44 | 9.72154   | 21 | 10.04639  | 44 | 45 |              |
| 46 | .70896    | 21 | .02021    | 44 | .72176    | 21 | .04684    | 44 | 46 |              |
| 47 | .70917    | 21 | .02065    | 44 | .72197    | 21 | .04728    | 44 | 47 | 21           |
| 48 | .70939    | 21 | .02109    | 44 | .72218    | 21 | .04773    | 44 | 48 | 6 2.1 2.1    |
| 49 | .70960    | 21 | .02153    | 44 | .72239    | 21 | .04818    | 44 | 49 | 7 2.4 2.4    |
| 50 | 9.70982   | 21 | 10.02197  | 44 | 9.72260   | 21 | 10.04862  | 44 | 50 | 8 2.8 2.8    |
| 51 | .71003    | 21 | .02242    | 44 | .72281    | 21 | .04907    | 44 | 51 | 9 3.1 3.1    |
| 52 | .71025    | 21 | .02286    | 44 | .72302    | 21 | .04952    | 44 | 52 | 10 3.5 3.5   |
| 53 | .71046    | 21 | .02330    | 44 | .72323    | 21 | .04996    | 44 | 53 | 20 7.0 7.0   |
| 54 | .71068    | 21 | .02374    | 44 | .72344    | 21 | .05041    | 44 | 54 | 30 10.5 10.5 |
| 55 | 9.71089   | 21 | 10.02418  | 44 | 9.72365   | 21 | 10.05086  | 44 | 55 | 40 14.0 14.0 |
| 56 | .71111    | 21 | .02463    | 44 | .72386    | 21 | .05131    | 44 | 56 | 50 17.5 17.5 |
| 57 | .71132    | 21 | .02507    | 44 | .72408    | 21 | .05175    | 44 | 57 |              |
| 58 | .71154    | 21 | .02551    | 44 | .72429    | 21 | .05220    | 44 | 58 |              |
| 59 | .71175    | 21 | .02595    | 44 | .72450    | 21 | .05265    | 44 | 59 |              |
| 60 | 9.71197   | 21 | 10.02639  | 44 | 9.72471   | 21 | 10.05310  | 44 | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    |         | Lg. Vers. | D        |    |         | Log. Exs. | D        |    |    | Lg. Vers. | D |  |  | Log. Exs. | D |
|----|---------|-----------|----------|----|---------|-----------|----------|----|----|-----------|---|--|--|-----------|---|
|    |         |           |          |    |         |           |          |    |    |           |   |  |  |           |   |
| 0  | 9.72471 | 21        | 10.05310 | 44 | 9.73720 | 20        | 10.08015 | 45 | 0  |           |   |  |  |           |   |
| 1  | .72492  | 21        | .05354   | 45 | .73740  | 20        | .08061   | 45 | 1  |           |   |  |  |           |   |
| 2  | .72513  | 21        | .05399   | 45 | .73761  | 21        | .08106   | 45 | 2  |           |   |  |  |           |   |
| 3  | .72534  | 21        | .05444   | 44 | .73782  | 20        | .08151   | 45 | 3  |           |   |  |  |           |   |
| 4  | .72555  | 21        | .05489   | 45 | .73802  | 20        | .08197   | 45 | 4  |           |   |  |  |           |   |
| 5  | 9.72576 | 21        | 10.05534 | 45 | 9.73823 | 20        | 10.08242 | 45 | 5  |           |   |  |  |           |   |
| 6  | .72597  | 21        | .05579   | 44 | .73843  | 20        | .08288   | 45 | 6  |           |   |  |  |           |   |
| 7  | .72618  | 21        | .05623   | 45 | .73864  | 20        | .08333   | 45 | 7  |           |   |  |  |           |   |
| 8  | .72639  | 21        | .05668   | 45 | .73884  | 21        | .08379   | 45 | 8  |           |   |  |  |           |   |
| 9  | .72660  | 21        | .05713   | 45 | .73905  | 20        | .08424   | 45 | 9  |           |   |  |  |           |   |
| 10 | 9.72681 | 20        | 10.05758 | 44 | 9.73926 | 20        | 10.08470 | 45 | 10 |           |   |  |  |           |   |
| 11 | .72701  | 21        | .05803   | 45 | .73946  | 20        | .08515   | 45 | 11 |           |   |  |  |           |   |
| 12 | .72722  | 21        | .05848   | 45 | .73967  | 20        | .08561   | 45 | 12 |           |   |  |  |           |   |
| 13 | .72743  | 21        | .05893   | 45 | .73987  | 20        | .08606   | 45 | 13 |           |   |  |  |           |   |
| 14 | .72764  | 21        | .05938   | 45 | .74008  | 20        | .08652   | 45 | 14 |           |   |  |  |           |   |
| 15 | 9.72785 | 21        | 10.05983 | 45 | 9.74028 | 20        | 10.08697 | 46 | 15 |           |   |  |  |           |   |
| 16 | .72806  | 21        | .06028   | 44 | .74049  | 20        | .08743   | 45 | 16 |           |   |  |  |           |   |
| 17 | .72827  | 20        | .06072   | 45 | .74069  | 20        | .08789   | 45 | 17 |           |   |  |  |           |   |
| 18 | .72848  | 21        | .06117   | 45 | .74090  | 20        | .08834   | 45 | 18 |           |   |  |  |           |   |
| 19 | .72869  | 21        | .06162   | 45 | .74110  | 20        | .08880   | 46 | 19 |           |   |  |  |           |   |
| 20 | 9.72890 | 21        | 10.06207 | 45 | 9.74131 | 20        | 10.08926 | 45 | 20 |           |   |  |  |           |   |
| 21 | .72911  | 20        | .06252   | 45 | .74151  | 20        | .08971   | 45 | 21 |           |   |  |  |           |   |
| 22 | .72931  | 20        | .06297   | 45 | .74172  | 20        | .09017   | 45 | 22 |           |   |  |  |           |   |
| 23 | .72952  | 21        | .06342   | 45 | .74192  | 20        | .09062   | 46 | 23 |           |   |  |  |           |   |
| 24 | .72973  | 21        | .06387   | 45 | .74213  | 20        | .09108   | 45 | 24 |           |   |  |  |           |   |
| 25 | 9.72994 | 21        | 10.06432 | 45 | 9.74233 | 20        | 10.09154 | 46 | 25 |           |   |  |  |           |   |
| 26 | .73015  | 20        | .06477   | 45 | .74254  | 20        | .09200   | 45 | 26 |           |   |  |  |           |   |
| 27 | .73036  | 21        | .06522   | 45 | .74274  | 20        | .09245   | 45 | 27 |           |   |  |  |           |   |
| 28 | .73057  | 20        | .06568   | 45 | .74294  | 20        | .09291   | 46 | 28 |           |   |  |  |           |   |
| 29 | .73077  | 20        | .06613   | 45 | .74315  | 20        | .09337   | 45 | 29 |           |   |  |  |           |   |
| 30 | 9.73098 | 21        | 10.06658 | 45 | 9.74335 | 20        | 10.09382 | 46 | 30 |           |   |  |  |           |   |
| 31 | .73119  | 20        | .06703   | 45 | .74356  | 20        | .09428   | 46 | 31 |           |   |  |  |           |   |
| 32 | .73140  | 20        | .06748   | 45 | .74376  | 20        | .09474   | 45 | 32 |           |   |  |  |           |   |
| 33 | .73161  | 20        | .06793   | 45 | .74396  | 20        | .09520   | 46 | 33 |           |   |  |  |           |   |
| 34 | .73181  | 21        | .06838   | 45 | .74417  | 20        | .09566   | 45 | 34 |           |   |  |  |           |   |
| 35 | 9.73202 | 21        | 10.06883 | 45 | 9.74437 | 20        | 10.09611 | 46 | 35 |           |   |  |  |           |   |
| 36 | .73223  | 20        | .06928   | 45 | .74458  | 20        | .09657   | 46 | 36 |           |   |  |  |           |   |
| 37 | .73244  | 21        | .06974   | 45 | .74478  | 20        | .09703   | 46 | 37 |           |   |  |  |           |   |
| 38 | .73265  | 20        | .07019   | 45 | .74498  | 20        | .09749   | 46 | 38 |           |   |  |  |           |   |
| 39 | .73285  | 21        | .07064   | 45 | .74519  | 20        | .09795   | 46 | 39 |           |   |  |  |           |   |
| 40 | 9.73306 | 21        | 10.07109 | 45 | 9.74539 | 20        | 10.09841 | 45 | 40 |           |   |  |  |           |   |
| 41 | .73327  | 20        | .07154   | 45 | .74559  | 20        | .09886   | 46 | 41 |           |   |  |  |           |   |
| 42 | .73348  | 20        | .07200   | 45 | .74580  | 20        | .09932   | 46 | 42 |           |   |  |  |           |   |
| 43 | .73368  | 21        | .07245   | 45 | .74600  | 20        | .09978   | 46 | 43 |           |   |  |  |           |   |
| 44 | .73389  | 20        | .07290   | 45 | .74620  | 20        | .10024   | 46 | 44 |           |   |  |  |           |   |
| 45 | 9.73410 | 20        | 10.07335 | 45 | 9.74641 | 20        | 10.10070 | 46 | 45 |           |   |  |  |           |   |
| 46 | .73430  | 21        | .07380   | 45 | .74661  | 20        | .10116   | 46 | 46 |           |   |  |  |           |   |
| 47 | .73451  | 20        | .07426   | 45 | .74681  | 20        | .10162   | 45 | 47 |           |   |  |  |           |   |
| 48 | .73472  | 21        | .07471   | 45 | .74702  | 20        | .10208   | 46 | 48 |           |   |  |  |           |   |
| 49 | .73493  | 20        | .07516   | 45 | .74722  | 20        | .10254   | 46 | 49 |           |   |  |  |           |   |
| 50 | 9.73513 | 20        | 10.07562 | 45 | 9.74742 | 20        | 10.10300 | 46 | 50 |           |   |  |  |           |   |
| 51 | .73534  | 21        | .07607   | 45 | .74762  | 20        | .10346   | 46 | 51 |           |   |  |  |           |   |
| 52 | .73555  | 20        | .07652   | 45 | .74783  | 20        | .10392   | 46 | 52 |           |   |  |  |           |   |
| 53 | .73575  | 20        | .07697   | 45 | .74803  | 20        | .10438   | 46 | 53 |           |   |  |  |           |   |
| 54 | .73596  | 21        | .07743   | 45 | .74823  | 20        | .10484   | 46 | 54 |           |   |  |  |           |   |
| 55 | 9.73617 | 20        | 10.07788 | 45 | 9.74844 | 20        | 10.10530 | 46 | 55 |           |   |  |  |           |   |
| 56 | .73637  | 20        | .07834   | 45 | .74864  | 20        | .10576   | 46 | 56 |           |   |  |  |           |   |
| 57 | .73658  | 21        | .07879   | 45 | .74884  | 20        | .10622   | 46 | 57 |           |   |  |  |           |   |
| 58 | .73679  | 20        | .07924   | 45 | .74904  | 20        | .10668   | 46 | 58 |           |   |  |  |           |   |
| 59 | .73699  | 20        | .07970   | 45 | .74924  | 20        | .10714   | 46 | 59 |           |   |  |  |           |   |
| 60 | 9.73720 | 20        | 10.08015 | 45 | 9.74945 | 20        | 10.10760 | 46 | 60 |           |   |  |  |           |   |
|    |         | Lg. Vers. | D        |    |         | Log. Exs. | D        |    |    |           |   |  |  |           |   |

P. P.

46 46

6 4.6 4.6  
7 5.4 5.3  
8 6.2 6.1  
9 7.0 6.9  
10 7.7 7.6  
20 15.5 15.3  
30 23.2 23.0  
40 31.0 30.6  
50 38.7 38.3

45 45

6 4.5 4.5  
7 5.3 5.2  
8 6.0 6.0  
9 6.8 6.7  
10 7.6 7.5  
20 15.1 15.0  
30 22.7 22.5  
40 30.3 30.0  
50 37.9 37.5

41 41

6 4.4 4.4  
7 5.2 5.2  
8 5.9 5.9  
9 6.7 6.7  
10 7.4 7.4  
20 14.8 14.8  
30 22.2 22.2  
40 29.6 29.6  
50 37.1 37.1

21 20

6 2.1 2.0  
7 2.4 2.4  
8 2.8 2.7  
9 3.1 3.1  
10 3.5 3.4  
20 7.0 6.8  
30 10.5 10.2  
40 14.0 13.6  
50 17.5 17.1

20 20

6 2.0 2.0  
7 2.3 2.3  
8 2.6 2.6  
9 3.0 3.0  
10 3.3 3.3  
20 6.6 6.6  
30 10.0 10.0  
40 13.3 13.3  
50 16.6 16.6

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.74945   | 20 | 10.10760  | 46 | 9.76146   | 19 | 10.13551  | 47 | 0  |              |
| 1  | .74965    | 20 | .10807    | 46 | .76168    | 20 | .13598    | 47 | 1  |              |
| 2  | .74985    | 20 | .10853    | 46 | .76186    | 20 | .13645    | 47 | 2  |              |
| 3  | .75005    | 20 | .10899    | 46 | .76206    | 19 | .13692    | 47 | 3  |              |
| 4  | .75028    | 20 | .10945    | 46 | .76225    | 20 | .13739    | 47 | 4  | 48 47        |
| 5  | 9.75046   | 20 | 10.10991  | 46 | 9.76245   | 19 | 10.13786  | 47 | 5  | 6 4.8 4.7    |
| 6  | .75066    | 20 | .11037    | 46 | .76265    | 20 | .13833    | 47 | 6  | 7 5.6 5.5    |
| 7  | .75086    | 20 | .11084    | 46 | .76285    | 19 | .13880    | 47 | 7  | 8 6.4 6.3    |
| 8  | .75106    | 20 | .11130    | 46 | .76304    | 20 | .13927    | 47 | 8  | 9 7.2 7.1    |
| 9  | .75126    | 20 | .11176    | 46 | .76324    | 19 | .13974    | 47 | 9  | 10 8.0 7.9   |
| 10 | 9.75147   | 20 | 10.11222  | 46 | 9.76344   | 20 | 10.14021  | 47 | 10 | 20 16.0 15.8 |
| 11 | .75167    | 20 | .11289    | 46 | .76364    | 19 | .14068    | 47 | 11 | 30 24.0 23.7 |
| 12 | .75187    | 20 | .11315    | 46 | .76384    | 20 | .14115    | 47 | 12 | 40 32.0 31.6 |
| 13 | .75207    | 20 | .11361    | 46 | .76403    | 19 | .14162    | 47 | 13 | 50 40.0 39.6 |
| 14 | .75227    | 20 | .11407    | 46 | .76423    | 20 | .14210    | 47 | 14 |              |
| 15 | 9.75247   | 20 | 10.11454  | 46 | 9.76443   | 19 | 10.14257  | 47 | 15 | 47 46        |
| 16 | .75267    | 20 | .11500    | 46 | .76463    | 20 | .14304    | 47 | 16 | 6 4.7 4.6    |
| 17 | .75287    | 20 | .11546    | 46 | .76482    | 19 | .14351    | 47 | 17 | 7 5.5 5.4    |
| 18 | .75308    | 20 | .11593    | 46 | .76502    | 20 | .14398    | 47 | 18 | 8 6.2 6.2    |
| 19 | .75328    | 20 | .11639    | 46 | .76522    | 19 | .14445    | 47 | 19 | 9 7.0 7.0    |
| 20 | 9.75348   | 20 | 10.11685  | 46 | 9.76541   | 20 | 10.14493  | 47 | 20 | 10 7.8 7.7   |
| 21 | .75368    | 20 | .11732    | 46 | .76561    | 19 | .14540    | 47 | 21 | 20 15.6 15.5 |
| 22 | .75388    | 20 | .11778    | 46 | .76581    | 20 | .14587    | 47 | 22 | 30 23.5 23.2 |
| 23 | .75408    | 20 | .11825    | 46 | .76600    | 19 | .14634    | 47 | 23 | 40 31.3 31.0 |
| 24 | .75428    | 20 | .11871    | 46 | .76620    | 20 | .14682    | 47 | 24 | 50 39.1 38.7 |
| 25 | 9.75448   | 20 | 10.11917  | 46 | 9.76640   | 19 | 10.14729  | 47 | 25 |              |
| 26 | .75468    | 20 | .11964    | 46 | .76659    | 20 | .14776    | 47 | 26 | 46           |
| 27 | .75488    | 20 | .12010    | 46 | .76679    | 19 | .14823    | 47 | 27 | 6 4.6        |
| 28 | .75508    | 20 | .12057    | 46 | .76699    | 20 | .14871    | 47 | 28 | 7 5.3        |
| 29 | .75528    | 20 | .12103    | 46 | .76718    | 19 | .14918    | 47 | 29 | 8 6.1        |
| 30 | 9.75548   | 20 | 10.12150  | 46 | 9.76738   | 20 | 10.14965  | 47 | 30 | 9 6.9        |
| 31 | .75568    | 20 | .12196    | 46 | .76758    | 19 | .15013    | 47 | 31 | 10 7.6       |
| 32 | .75588    | 20 | .12243    | 46 | .76777    | 20 | .15060    | 47 | 32 | 20 15.3      |
| 33 | .75608    | 20 | .12289    | 46 | .76797    | 19 | .15108    | 47 | 33 | 30 23.0      |
| 34 | .75628    | 20 | .12336    | 46 | .76817    | 20 | .15155    | 47 | 34 | 40 30.6      |
| 35 | 9.75648   | 20 | 10.12383  | 47 | 9.76836   | 19 | 10.15202  | 47 | 35 | 50 38.3      |
| 36 | .75668    | 20 | .12429    | 46 | .76856    | 20 | .15250    | 47 | 36 |              |
| 37 | .75688    | 20 | .12476    | 46 | .76875    | 19 | .15297    | 47 | 37 | 20           |
| 38 | .75708    | 20 | .12522    | 46 | .76895    | 20 | .15345    | 47 | 38 | 6 2.0 2.0    |
| 39 | .75728    | 20 | .12569    | 46 | .76915    | 19 | .15392    | 47 | 39 | 7 2.4 2.3    |
| 40 | 9.75748   | 20 | 10.12616  | 47 | 9.76934   | 20 | 10.15440  | 47 | 40 | 8 2.7 2.6    |
| 41 | .75768    | 20 | .12662    | 46 | .76954    | 19 | .15487    | 47 | 41 | 9 3.1 3.0    |
| 42 | .75788    | 20 | .12709    | 46 | .76973    | 20 | .15535    | 47 | 42 | 10 3.4 3.3   |
| 43 | .75808    | 20 | .12756    | 46 | .76993    | 19 | .15582    | 47 | 43 | 20 6.8 6.6   |
| 44 | .75828    | 19 | .12802    | 46 | .77012    | 20 | .15630    | 47 | 44 | 30 10.2 10.0 |
| 45 | 9.75848   | 20 | 10.12849  | 46 | 9.77032   | 19 | 10.15678  | 47 | 45 | 40 13.6 13.3 |
| 46 | .75868    | 20 | .12896    | 46 | .77052    | 20 | .15725    | 47 | 46 | 50 17.1 16.6 |
| 47 | .75888    | 20 | .12942    | 47 | .77071    | 19 | .15773    | 47 | 47 |              |
| 48 | .75908    | 20 | .12989    | 46 | .77091    | 20 | .15820    | 47 | 48 | 19           |
| 49 | .75928    | 20 | .13036    | 46 | .77110    | 19 | .15868    | 47 | 49 | 6 1.9        |
| 50 | 9.75947   | 19 | 10.13083  | 47 | 9.77130   | 20 | 10.15916  | 47 | 50 | 7 2.3        |
| 51 | .75967    | 20 | .13130    | 46 | .77149    | 19 | .15963    | 47 | 51 | 8 2.6        |
| 52 | .75987    | 20 | .13176    | 46 | .77169    | 20 | .16011    | 47 | 52 | 9 2.9        |
| 53 | .76007    | 20 | .13223    | 46 | .77188    | 19 | .16059    | 47 | 53 | 10 3.2       |
| 54 | .76027    | 19 | .13270    | 46 | .77208    | 20 | .16106    | 47 | 54 | 20 6.5       |
| 55 | 9.76047   | 20 | 10.13317  | 47 | 9.77227   | 19 | 10.16154  | 47 | 55 | 30 9.7       |
| 56 | .76067    | 20 | .13364    | 47 | .77247    | 20 | .16202    | 47 | 56 | 40 13.0      |
| 57 | .76087    | 19 | .13411    | 46 | .77266    | 19 | .16250    | 47 | 57 | 50 16.2      |
| 58 | .76106    | 20 | .13457    | 46 | .77286    | 20 | .16298    | 47 | 58 |              |
| 59 | .76126    | 20 | .13504    | 47 | .77305    | 19 | .16345    | 47 | 59 |              |
| 60 | 9.76146   | 20 | 10.13551  | 47 | 9.77325   | 19 | 10.16393  | 47 | 60 |              |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------|
| 0  | 9.77325   |    | 10.16393  |    | 9.78481   |    | 10.19293  |    | 0  |       |
| 1  | .77344    | 19 | .16441    | 48 | .78500    | 19 | .19342    | 49 | 1  |       |
| 2  | .77363    | 19 | .16489    | 47 | .78519    | 19 | .19391    | 48 | 2  |       |
| 3  | .77383    | 19 | .16537    | 48 | .78538    | 19 | .19439    | 49 | 3  |       |
| 4  | .77402    | 19 | .16585    | 48 | .78557    | 19 | .19488    | 49 | 4  |       |
| 5  | 9.77422   | 19 | 10.16635  | 48 | 9.78576   | 19 | 10.19537  | 49 | 5  |       |
| 6  | .77441    | 19 | .16680    | 47 | .78595    | 19 | .19586    | 49 | 6  |       |
| 7  | .77461    | 19 | .16728    | 48 | .78614    | 19 | .19635    | 49 | 7  |       |
| 8  | .77480    | 19 | .16776    | 48 | .78633    | 19 | .19684    | 49 | 8  |       |
| 9  | .77499    | 19 | .16824    | 48 | .78652    | 19 | .19733    | 49 | 9  |       |
| 10 | 9.77519   | 19 | 10.16872  | 48 | 9.78671   | 19 | 10.19782  | 49 | 10 |       |
| 11 | .77538    | 19 | .16920    | 48 | .78690    | 19 | .19831    | 49 | 11 |       |
| 12 | .77557    | 19 | .16968    | 48 | .78709    | 19 | .19880    | 49 | 12 |       |
| 13 | .77577    | 19 | .17016    | 48 | .78728    | 19 | .19929    | 49 | 13 |       |
| 14 | .77596    | 19 | .17064    | 48 | .78747    | 19 | .19979    | 49 | 14 |       |
| 15 | 9.77616   | 19 | 10.17112  | 48 | 9.78766   | 19 | 10.20028  | 49 | 15 |       |
| 16 | .77635    | 19 | .17160    | 48 | .78785    | 19 | .20077    | 49 | 16 |       |
| 17 | .77654    | 19 | .17209    | 48 | .78804    | 19 | .20126    | 49 | 17 |       |
| 18 | .77674    | 19 | .17257    | 48 | .78823    | 19 | .20175    | 49 | 18 |       |
| 19 | .77693    | 19 | .17305    | 48 | .78842    | 19 | .20224    | 49 | 19 |       |
| 20 | 9.77712   | 19 | 10.17353  | 48 | 9.78861   | 19 | 10.20273  | 49 | 20 |       |
| 21 | .77732    | 19 | .17401    | 48 | .78880    | 19 | .20323    | 49 | 21 |       |
| 22 | .77751    | 19 | .17449    | 48 | .78899    | 19 | .20372    | 49 | 22 |       |
| 23 | .77770    | 19 | .17498    | 48 | .78918    | 19 | .20421    | 49 | 23 |       |
| 24 | .77790    | 19 | .17546    | 48 | .78937    | 19 | .20470    | 49 | 24 |       |
| 25 | 9.77809   | 19 | 10.17594  | 48 | 9.78956   | 19 | 10.20520  | 49 | 25 |       |
| 26 | .77828    | 19 | .17642    | 48 | .78975    | 19 | .20569    | 49 | 26 |       |
| 27 | .77847    | 19 | .17690    | 48 | .78994    | 19 | .20618    | 49 | 27 |       |
| 28 | .77867    | 19 | .17739    | 48 | .79013    | 19 | .20668    | 49 | 28 |       |
| 29 | .77886    | 19 | .17787    | 48 | .79032    | 19 | .20717    | 49 | 29 |       |
| 30 | 9.77905   | 19 | 10.17835  | 48 | 9.79051   | 19 | 10.20767  | 49 | 30 |       |
| 31 | .77925    | 19 | .17884    | 48 | .79069    | 19 | .20816    | 49 | 31 |       |
| 32 | .77944    | 19 | .17932    | 48 | .79088    | 19 | .20865    | 49 | 32 |       |
| 33 | .77963    | 19 | .17980    | 48 | .79107    | 19 | .20915    | 49 | 33 |       |
| 34 | .77982    | 19 | .18029    | 48 | .79126    | 19 | .20964    | 49 | 34 |       |
| 35 | 9.78002   | 19 | 10.18077  | 48 | 9.79145   | 19 | 10.21014  | 49 | 35 |       |
| 36 | .78021    | 19 | .18126    | 48 | .79164    | 19 | .21063    | 49 | 36 |       |
| 37 | .78040    | 19 | .18174    | 48 | .79183    | 19 | .21113    | 49 | 37 |       |
| 38 | .78059    | 19 | .18222    | 48 | .79202    | 19 | .21162    | 50 | 38 |       |
| 39 | .78078    | 19 | .18271    | 48 | .79220    | 19 | .21212    | 50 | 39 |       |
| 40 | 9.78098   | 19 | 10.18319  | 48 | 9.79239   | 19 | 10.21262  | 49 | 40 |       |
| 41 | .78117    | 19 | .18368    | 48 | .79258    | 19 | .21311    | 49 | 41 |       |
| 42 | .78136    | 19 | .18416    | 49 | .79277    | 19 | .21361    | 49 | 42 |       |
| 43 | .78155    | 19 | .18465    | 48 | .79296    | 19 | .21410    | 50 | 43 |       |
| 44 | .78174    | 19 | .18514    | 48 | .79315    | 19 | .21460    | 50 | 44 |       |
| 45 | 9.78194   | 19 | 10.18562  | 48 | 9.79333   | 19 | 10.21510  | 50 | 45 |       |
| 46 | .78213    | 19 | .18611    | 48 | .79352    | 19 | .21560    | 49 | 46 |       |
| 47 | .78232    | 19 | .18659    | 48 | .79371    | 19 | .21609    | 50 | 47 |       |
| 48 | .78251    | 19 | .18708    | 49 | .79390    | 19 | .21659    | 49 | 48 |       |
| 49 | .78270    | 19 | .18757    | 48 | .79409    | 19 | .21709    | 49 | 49 |       |
| 50 | 9.78289   | 19 | 10.18805  | 48 | 9.79427   | 19 | 10.21759  | 50 | 50 |       |
| 51 | .78309    | 19 | .18854    | 49 | .79446    | 19 | .21808    | 50 | 51 |       |
| 52 | .78328    | 19 | .18903    | 48 | .79465    | 19 | .21858    | 50 | 52 |       |
| 53 | .78347    | 19 | .18951    | 49 | .79484    | 19 | .21908    | 49 | 53 |       |
| 54 | .78366    | 19 | .19000    | 48 | .79503    | 19 | .21958    | 50 | 54 |       |
| 55 | 9.78385   | 19 | 10.19049  | 49 | 9.79521   | 19 | 10.22008  | 50 | 55 |       |
| 56 | .78404    | 19 | .19098    | 48 | .79540    | 19 | .22058    | 50 | 56 |       |
| 57 | .78423    | 19 | .19146    | 48 | .79559    | 19 | .22108    | 50 | 57 |       |
| 58 | .78442    | 19 | .19195    | 49 | .79578    | 19 | .22158    | 50 | 58 |       |
| 59 | .78462    | 19 | .19244    | 48 | .79596    | 19 | .22208    | 50 | 59 |       |
| 60 | 9.78481   | 19 | 10.19293  | 48 | 9.79615   | 19 | 10.22258  | 50 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P. |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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| 68° |           |    |           | 69° |           |    |           | P. P. |    |       |      |
|-----|-----------|----|-----------|-----|-----------|----|-----------|-------|----|-------|------|
| '   | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D     | '  | P. P. |      |
| 0   | 9.79615   | 18 | 10.22258  | 50  | 9.80728   | 18 | 10.25295  | 51    | 0  | 53    | 52   |
| 1   | .79634    | 19 | .22308    | 50  | .80747    | 18 | .25347    | 51    | 1  | 6     | 5.3  |
| 2   | .79653    | 19 | .22358    | 50  | .80765    | 18 | .25398    | 51    | 2  | 7     | 6.2  |
| 3   | .79671    | 18 | .22408    | 50  | .80783    | 18 | .25449    | 51    | 3  | 8     | 7.0  |
| 4   | .79690    | 18 | .22458    | 50  | .80802    | 18 | .25501    | 51    | 4  | 9     | 7.9  |
| 5   | 9.79709   | 19 | 10.22508  | 50  | 9.80820   | 18 | 10.25552  | 51    | 5  | 10    | 8.8  |
| 6   | .79727    | 18 | .22558    | 50  | .80839    | 18 | .25604    | 51    | 6  | 20    | 17.6 |
| 7   | .79746    | 19 | .22608    | 50  | .80857    | 18 | .25655    | 51    | 7  | 30    | 26.5 |
| 8   | .79765    | 18 | .22658    | 50  | .80875    | 18 | .25707    | 51    | 8  | 40    | 35.3 |
| 9   | .79783    | 18 | .22708    | 50  | .80894    | 18 | .25758    | 51    | 9  | 50    | 44.1 |
| 10  | 9.79802   | 19 | 10.22759  | 50  | 9.80912   | 18 | 10.25810  | 51    | 10 | 52    | 51   |
| 11  | .79821    | 18 | .22809    | 50  | .80930    | 18 | .25861    | 51    | 11 | 6     | 5.2  |
| 12  | .79839    | 18 | .22859    | 50  | .80949    | 18 | .25913    | 51    | 12 | 7     | 6.0  |
| 13  | .79858    | 18 | .22909    | 50  | .80967    | 18 | .25964    | 51    | 13 | 8     | 6.9  |
| 14  | .79877    | 18 | .22960    | 50  | .80985    | 18 | .26016    | 51    | 14 | 9     | 7.8  |
| 15  | 9.79895   | 18 | 10.23010  | 50  | 9.81003   | 18 | 10.26067  | 51    | 15 | 10    | 8.6  |
| 16  | .79914    | 19 | .23060    | 50  | .81022    | 18 | .26119    | 52    | 16 | 20    | 17.3 |
| 17  | .79933    | 18 | .23110    | 50  | .81040    | 18 | .26171    | 51    | 17 | 30    | 26.0 |
| 18  | .79951    | 18 | .23161    | 50  | .81058    | 18 | .26222    | 51    | 18 | 40    | 34.6 |
| 19  | .79970    | 18 | .23211    | 50  | .81077    | 18 | .26274    | 52    | 19 | 50    | 43.3 |
| 20  | 9.79988   | 19 | 10.23262  | 50  | 9.81095   | 18 | 10.26326  | 51    | 20 | 51    | 50   |
| 21  | .80007    | 18 | .23312    | 50  | .81113    | 18 | .26378    | 52    | 21 | 6     | 5.1  |
| 22  | .80026    | 18 | .23362    | 50  | .81131    | 18 | .26429    | 52    | 22 | 7     | 5.9  |
| 23  | .80044    | 18 | .23413    | 50  | .81150    | 18 | .26481    | 52    | 23 | 8     | 6.8  |
| 24  | .80063    | 18 | .23463    | 50  | .81168    | 18 | .26533    | 52    | 24 | 9     | 7.6  |
| 25  | 9.80081   | 19 | 10.23514  | 50  | 9.81186   | 18 | 10.26585  | 51    | 25 | 10    | 8.5  |
| 26  | .80100    | 18 | .23564    | 50  | .81204    | 18 | .26637    | 52    | 26 | 20    | 17.0 |
| 27  | .80119    | 18 | .23615    | 50  | .81223    | 18 | .26689    | 52    | 27 | 30    | 25.5 |
| 28  | .80137    | 18 | .23666    | 50  | .81241    | 18 | .26741    | 52    | 28 | 40    | 34.0 |
| 29  | .80156    | 18 | .23716    | 50  | .81259    | 18 | .26793    | 52    | 29 | 50    | 42.5 |
| 30  | 9.80174   | 18 | 10.23767  | 50  | 9.81277   | 18 | 10.26845  | 52    | 30 | 52    | 50   |
| 31  | .80193    | 18 | .23817    | 51  | .81295    | 18 | .26897    | 52    | 31 | 6     | 5.0  |
| 32  | .80211    | 18 | .23868    | 50  | .81314    | 18 | .26949    | 52    | 32 | 7     | 5.8  |
| 33  | .80230    | 18 | .23919    | 50  | .81332    | 18 | .27001    | 52    | 33 | 8     | 6.6  |
| 34  | .80248    | 18 | .23969    | 51  | .81350    | 18 | .27053    | 52    | 34 | 9     | 7.5  |
| 35  | 9.80267   | 19 | 10.24020  | 50  | 9.81368   | 18 | 10.27105  | 52    | 35 | 10    | 8.3  |
| 36  | .80286    | 18 | .24071    | 51  | .81386    | 18 | .27157    | 52    | 36 | 20    | 16.6 |
| 37  | .80304    | 18 | .24122    | 50  | .81405    | 18 | .27209    | 52    | 37 | 30    | 25.0 |
| 38  | .80323    | 18 | .24172    | 51  | .81423    | 18 | .27261    | 52    | 38 | 40    | 33.3 |
| 39  | .80341    | 18 | .24223    | 51  | .81441    | 18 | .27314    | 52    | 39 | 50    | 41.6 |
| 40  | 9.80360   | 18 | 10.24274  | 50  | 9.81459   | 18 | 10.27366  | 52    | 40 | 19    | 18   |
| 41  | .80378    | 18 | .24325    | 51  | .81477    | 18 | .27418    | 52    | 41 | 6     | 1.9  |
| 42  | .80397    | 18 | .24376    | 51  | .81495    | 18 | .27470    | 52    | 42 | 7     | 2.2  |
| 43  | .80415    | 18 | .24427    | 51  | .81513    | 18 | .27523    | 52    | 43 | 8     | 2.5  |
| 44  | .80434    | 18 | .24478    | 51  | .81532    | 18 | .27575    | 52    | 44 | 9     | 2.8  |
| 45  | 9.80452   | 18 | 10.24529  | 51  | 9.81550   | 18 | 10.22627  | 52    | 45 | 10    | 3.1  |
| 46  | .80470    | 18 | .24580    | 51  | .81568    | 18 | .27680    | 52    | 46 | 20    | 6.3  |
| 47  | .80489    | 18 | .24631    | 51  | .81586    | 18 | .27732    | 52    | 47 | 30    | 9.5  |
| 48  | .80507    | 18 | .24682    | 51  | .81604    | 18 | .27785    | 52    | 48 | 40    | 12.6 |
| 49  | .80526    | 18 | .24733    | 51  | .81622    | 18 | .27837    | 52    | 49 | 50    | 15.8 |
| 50  | 9.80544   | 18 | 10.24784  | 51  | 9.81640   | 18 | 10.27890  | 52    | 50 | 18    | 18   |
| 51  | .80563    | 18 | .24835    | 51  | .81658    | 18 | .27942    | 52    | 51 | 6     | 1.8  |
| 52  | .80587    | 18 | .24886    | 51  | .81676    | 18 | .27995    | 52    | 52 | 7     | 2.1  |
| 53  | .80600    | 18 | .24937    | 51  | .81695    | 18 | .28047    | 52    | 53 | 8     | 2.4  |
| 54  | .80618    | 18 | .24988    | 51  | .81713    | 18 | .28100    | 52    | 54 | 9     | 2.7  |
| 55  | 9.80636   | 18 | 10.25039  | 51  | 9.81731   | 18 | 10.28152  | 52    | 55 | 10    | 3.0  |
| 56  | .80655    | 18 | .25090    | 51  | .81749    | 18 | .28205    | 52    | 56 | 20    | 6.0  |
| 57  | .80673    | 18 | .25142    | 51  | .81767    | 18 | .28258    | 52    | 57 | 30    | 9.0  |
| 58  | .80692    | 18 | .25193    | 51  | .81785    | 18 | .28310    | 53    | 58 | 40    | 12.0 |
| 59  | .80710    | 18 | .25244    | 51  | .81803    | 18 | .28363    | 52    | 59 | 50    | 15.0 |
| 60  | 9.80728   | 18 | 10.25295  | 51  | 9.81821   | 18 | 10.28416  | 52    | 60 | P. P. |      |
| '   | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D     | '  | P. P. |      |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.81821   | 18 | 10.28416  | 53 | 9.82894   | 17 | 10.31629  | 54 | 0  | 56 56        |
| 1  | .81839    | 18 | .28469    | 52 | .82911    | 17 | .31684    | 54 | 1  | 6 5.6 5.6    |
| 2  | .81857    | 18 | .28521    | 53 | .82929    | 17 | .31738    | 54 | 2  | 7 6.6 6.5    |
| 3  | .81875    | 18 | .28574    | 53 | .82947    | 18 | .31793    | 54 | 3  | 8 7.5 7.4    |
| 4  | .81893    | 18 | .28627    | 53 | .82964    | 17 | .31847    | 54 | 4  | 9 8.5 8.4    |
| 5  | 9.81911   | 18 | 10.28680  | 52 | 9.82982   | 18 | 10.31902  | 54 | 5  | 10 9.4 9.3   |
| 6  | .81929    | 18 | .28733    | 53 | .83000    | 17 | .31956    | 54 | 6  | 20 18.6 18.6 |
| 7  | .81947    | 18 | .28786    | 53 | .83017    | 17 | .32011    | 54 | 7  | 30 28.2 28.0 |
| 8  | .81965    | 18 | .28839    | 53 | .83035    | 18 | .32066    | 54 | 8  | 40 37.6 37.3 |
| 9  | .81983    | 18 | .28892    | 53 | .83053    | 17 | .32120    | 54 | 9  | 50 47.1 46.6 |
| 10 | 9.82001   | 18 | 10.28945  | 53 | 9.83070   | 17 | 10.32175  | 54 | 10 | 55 55        |
| 11 | .82019    | 18 | .28998    | 53 | .83088    | 18 | .32230    | 54 | 11 | 6 5.5 5.5    |
| 12 | .82037    | 18 | .29051    | 53 | .83106    | 17 | .32284    | 54 | 12 | 7 6.5 6.4    |
| 13 | .82055    | 18 | .29104    | 53 | .83123    | 17 | .32339    | 54 | 13 | 8 7.4 7.3    |
| 14 | .82073    | 18 | .29157    | 53 | .83141    | 17 | .32394    | 54 | 14 | 9 8.3 8.2    |
| 15 | 9.82091   | 18 | 10.29210  | 53 | 9.83159   | 18 | 10.32449  | 55 | 15 | 10 9.2 9.1   |
| 16 | .82109    | 17 | .29263    | 53 | .83176    | 17 | .32504    | 54 | 16 | 20 18.5 18.3 |
| 17 | .82127    | 18 | .29316    | 53 | .83194    | 17 | .32558    | 54 | 17 | 30 27.7 27.5 |
| 18 | .82145    | 18 | .29370    | 53 | .83211    | 17 | .32613    | 55 | 18 | 40 37.0 36.6 |
| 19 | .82163    | 18 | .29423    | 53 | .83229    | 18 | .32668    | 55 | 19 | 50 46.2 45.8 |
| 20 | 9.82181   | 18 | 10.29476  | 53 | 9.83247   | 17 | 10.32723  | 55 | 20 | 54 54        |
| 21 | .82199    | 18 | .29529    | 53 | .83264    | 17 | .32778    | 55 | 21 | 6 5.4 5.4    |
| 22 | .82217    | 18 | .29583    | 53 | .83282    | 17 | .32833    | 55 | 22 | 7 6.3 6.3    |
| 23 | .82235    | 17 | .29636    | 53 | .83299    | 17 | .32888    | 55 | 23 | 8 7.2 7.2    |
| 24 | .82252    | 17 | .29689    | 53 | .83317    | 18 | .32944    | 55 | 24 | 9 8.2 8.1    |
| 25 | 9.82270   | 18 | 10.29743  | 53 | 9.83335   | 17 | 10.32999  | 55 | 25 | 10 9.1 9.0   |
| 26 | .82288    | 18 | .29796    | 53 | .83352    | 17 | .33054    | 55 | 26 | 20 18.1 18.0 |
| 27 | .82306    | 18 | .29850    | 53 | .83370    | 17 | .33109    | 55 | 27 | 30 27.2 27.0 |
| 28 | .82324    | 17 | .29903    | 53 | .83387    | 17 | .33164    | 55 | 28 | 40 36.3 36.0 |
| 29 | .82342    | 17 | .29957    | 53 | .83405    | 17 | .33220    | 55 | 29 | 50 45.4 45.0 |
| 30 | 9.82360   | 18 | 10.30010  | 53 | 9.83422   | 17 | 10.33275  | 55 | 30 | 53 53        |
| 31 | .82378    | 18 | .30064    | 53 | .83440    | 18 | .33330    | 55 | 31 | 6 5.3 5.3    |
| 32 | .82396    | 17 | .30117    | 54 | .83458    | 17 | .33385    | 55 | 32 | 7 6.2 6.2    |
| 33 | .82413    | 17 | .30171    | 53 | .83475    | 17 | .33441    | 55 | 33 | 8 7.1 7.0    |
| 34 | .82431    | 18 | .30225    | 53 | .83493    | 17 | .33496    | 55 | 34 | 9 8.0 7.9    |
| 35 | 9.82449   | 18 | 10.30278  | 53 | 9.83510   | 17 | 10.33552  | 55 | 35 | 10 8.9 8.8   |
| 36 | .82467    | 17 | .30332    | 54 | .83528    | 17 | .33607    | 55 | 36 | 20 17.8 17.6 |
| 37 | .82485    | 18 | .30386    | 54 | .83545    | 17 | .33663    | 55 | 37 | 30 26.7 26.5 |
| 38 | .82503    | 17 | .30440    | 54 | .83563    | 17 | .33718    | 55 | 38 | 40 35.6 35.3 |
| 39 | .82520    | 17 | .30493    | 54 | .83580    | 17 | .33774    | 55 | 39 | 50 44.6 44.1 |
| 40 | 9.82538   | 18 | 10.30547  | 53 | 9.83598   | 17 | 10.33829  | 56 | 40 | 52           |
| 41 | .82556    | 17 | .30601    | 54 | .83615    | 17 | .33885    | 55 | 41 | 6 5.2        |
| 42 | .82574    | 18 | .30655    | 54 | .83633    | 17 | .33941    | 55 | 42 | 7 6.1        |
| 43 | .82592    | 17 | .30709    | 54 | .83650    | 17 | .33996    | 56 | 43 | 8 7.0        |
| 44 | .82609    | 18 | .30763    | 54 | .83668    | 17 | .34052    | 56 | 44 | 9 7.9        |
| 45 | 9.82627   | 18 | 10.30817  | 54 | 9.83685   | 17 | 10.34108  | 56 | 45 | 10 8.7       |
| 46 | .82645    | 17 | .30871    | 54 | .83703    | 17 | .34164    | 56 | 46 | 20 17.5      |
| 47 | .82663    | 18 | .30925    | 54 | .83720    | 17 | .34220    | 55 | 47 | 30 26.2      |
| 48 | .82681    | 17 | .30979    | 54 | .83737    | 17 | .34275    | 56 | 48 | 40 35.0      |
| 49 | .82698    | 17 | .31033    | 54 | .83755    | 17 | .34331    | 56 | 49 | 50 43.7      |
| 50 | 9.82716   | 18 | 10.31087  | 54 | 9.83772   | 17 | 10.34387  | 56 | 50 | 18 17 17     |
| 51 | .82734    | 18 | .31141    | 54 | .83790    | 17 | .34443    | 56 | 51 | 6 1.8 1.7    |
| 52 | .82752    | 17 | .31195    | 54 | .83807    | 17 | .34499    | 56 | 52 | 7 2.1 2.0    |
| 53 | .82769    | 18 | .31249    | 54 | .83825    | 17 | .34555    | 56 | 53 | 8 2.4 2.3    |
| 54 | .82787    | 17 | .31303    | 54 | .83842    | 17 | .34611    | 56 | 54 | 9 2.7 2.6    |
| 55 | 9.82805   | 18 | 10.31358  | 54 | 9.83859   | 17 | 10.34667  | 56 | 55 | 10 3.0 2.9   |
| 56 | .82823    | 17 | .31412    | 54 | .83877    | 17 | .34723    | 56 | 56 | 20 6.0 5.8   |
| 57 | .82840    | 18 | .31466    | 54 | .83894    | 17 | .34780    | 56 | 57 | 30 9.0 8.7   |
| 58 | .82858    | 17 | .31521    | 54 | .83912    | 17 | .34836    | 56 | 58 | 40 12.0 11.6 |
| 59 | .82876    | 18 | .31575    | 54 | .83929    | 17 | .34892    | 56 | 59 | 50 15.0 14.6 |
| 60 | 9.82894   | 18 | 10.31629  | 54 | 9.83946   | 17 | 10.34948  | 56 | 60 | 14.1         |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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| '  | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  | '  | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.83946   | 17 | 10.34948  | 56 | 9.84980   | 17 | 10.38387  | 58 | 0  | 61 60        |
| 1  | .83964    | 17 | .35005    | 56 | .84997    | 17 | .38445    | 58 | 1  | 6 6.1 6.0    |
| 2  | .83981    | 17 | .35061    | 56 | .85014    | 17 | .38504    | 58 | 2  | 7 7.1 7.0    |
| 3  | .83999    | 17 | .35117    | 56 | .85031    | 17 | .38562    | 58 | 3  | 8 8.1 8.0    |
| 4  | .84016    | 17 | .35174    | 56 | .85049    | 17 | .38621    | 58 | 4  | 9 9.1 9.1    |
| 5  | 9.84033   | 17 | 10.35230  | 56 | 9.85066   | 17 | 10.38679  | 58 | 5  | 10 10.1 10.1 |
| 6  | .84051    | 17 | .35286    | 56 | .85083    | 17 | .38738    | 58 | 6  | 20 20.3 20.1 |
| 7  | .84068    | 17 | .35343    | 56 | .85100    | 17 | .38796    | 59 | 7  | 30 30.5 30.2 |
| 8  | .84085    | 17 | .35399    | 57 | .85117    | 17 | .38855    | 59 | 8  | 40 40.6 40.3 |
| 9  | .84103    | 17 | .35456    | 57 | .85134    | 17 | .38914    | 58 | 9  | 50 50.8 50.4 |
| 10 | 9.84120   | 17 | 10.35513  | 56 | 9.85151   | 17 | 10.38973  | 59 | 10 | 60 59        |
| 11 | .84137    | 17 | .35569    | 56 | .85168    | 17 | .39031    | 59 | 11 | 6 6.0 5.9    |
| 12 | .84155    | 17 | .35626    | 56 | .85185    | 17 | .39090    | 59 | 12 | 7 7.0 6.9    |
| 13 | .84172    | 17 | .35683    | 57 | .85202    | 17 | .39149    | 59 | 13 | 8 8.0 7.9    |
| 14 | .84189    | 17 | .35739    | 56 | .85219    | 17 | .39208    | 58 | 14 | 9 9.0 8.9    |
| 15 | 9.84207   | 17 | 10.35796  | 57 | 9.85236   | 17 | 10.39267  | 59 | 15 | 10 10.0 9.9  |
| 16 | .84224    | 17 | .35853    | 56 | .85253    | 17 | .39326    | 59 | 16 | 20 20.0 19.8 |
| 17 | .84241    | 17 | .35910    | 57 | .85270    | 17 | .39385    | 59 | 17 | 30 30.0 29.7 |
| 18 | .84259    | 17 | .35967    | 56 | .85287    | 17 | .39444    | 59 | 18 | 40 40.0 39.6 |
| 19 | .84276    | 17 | .36023    | 57 | .85304    | 17 | .39503    | 59 | 19 | 50 50.0 49.6 |
| 20 | 9.84293   | 17 | 10.36080  | 57 | 9.85321   | 17 | 10.39562  | 59 | 20 | 59 58        |
| 21 | .84310    | 17 | .36137    | 57 | .85338    | 17 | .39621    | 59 | 21 | 6 5.9 5.8    |
| 22 | .84328    | 17 | .36194    | 57 | .85355    | 17 | .39681    | 59 | 22 | 7 6.9 6.8    |
| 23 | .84345    | 17 | .36251    | 57 | .85372    | 17 | .39740    | 59 | 23 | 8 7.8 7.8    |
| 24 | .84362    | 17 | .36308    | 57 | .85389    | 17 | .39799    | 59 | 24 | 9 8.8 8.8    |
| 25 | 9.84380   | 17 | 10.36366  | 57 | 9.85405   | 16 | 10.39859  | 59 | 25 | 10 9.8 9.7   |
| 26 | .84397    | 17 | .36423    | 57 | .85422    | 17 | .39918    | 59 | 26 | 20 19.6 19.5 |
| 27 | .84414    | 17 | .36480    | 57 | .85439    | 17 | .39977    | 59 | 27 | 30 29.5 29.2 |
| 28 | .84431    | 17 | .36537    | 57 | .85456    | 17 | .40037    | 59 | 28 | 40 39.3 39.0 |
| 29 | .84449    | 17 | .36594    | 57 | .85473    | 17 | .40096    | 59 | 29 | 50 49.1 48.7 |
| 30 | 9.84466   | 17 | 10.36652  | 57 | 9.85490   | 17 | 10.40156  | 59 | 30 | 58 57        |
| 31 | .84483    | 17 | .36709    | 57 | .85507    | 17 | .40216    | 60 | 31 | 6 5.8 5.7    |
| 32 | .84500    | 17 | .36766    | 57 | .85524    | 17 | .40275    | 59 | 32 | 7 6.7 6.7    |
| 33 | .84517    | 17 | .36824    | 57 | .85541    | 17 | .40335    | 59 | 33 | 8 7.7 7.6    |
| 34 | .84535    | 17 | .36881    | 57 | .85558    | 17 | .40395    | 60 | 34 | 9 8.7 8.6    |
| 35 | 9.84552   | 17 | 10.36938  | 57 | 9.85575   | 17 | 10.40454  | 59 | 35 | 10 9.6 9.6   |
| 36 | .84569    | 17 | .36996    | 57 | .85592    | 16 | .40514    | 60 | 36 | 20 19.3 19.1 |
| 37 | .84586    | 17 | .37054    | 57 | .85609    | 17 | .40574    | 59 | 37 | 30 29.0 28.7 |
| 38 | .84603    | 17 | .37111    | 57 | .85625    | 17 | .40634    | 60 | 38 | 40 38.6 38.3 |
| 39 | .84620    | 17 | .37169    | 57 | .85642    | 17 | .40694    | 60 | 39 | 50 48.3 47.9 |
| 40 | 9.84638   | 17 | 10.37226  | 57 | 9.85659   | 16 | 10.40754  | 60 | 40 | 57 56        |
| 41 | .84655    | 17 | .37284    | 57 | .85676    | 17 | .40814    | 60 | 41 | 6 5.7 5.6    |
| 42 | .84672    | 17 | .37342    | 57 | .85693    | 17 | .40874    | 60 | 42 | 7 6.6 6.6    |
| 43 | .84689    | 17 | .37399    | 58 | .85710    | 16 | .40934    | 60 | 43 | 8 7.6 7.5    |
| 44 | .84706    | 17 | .37457    | 58 | .85726    | 16 | .40994    | 60 | 44 | 9 8.5 8.5    |
| 45 | 9.84724   | 17 | 10.37515  | 58 | 9.85743   | 17 | 10.41054  | 60 | 45 | 10 9.5 9.4   |
| 46 | .84741    | 17 | .37573    | 57 | .85760    | 16 | .41114    | 60 | 46 | 20 19.0 18.8 |
| 47 | .84758    | 17 | .37631    | 58 | .85777    | 17 | .41174    | 60 | 47 | 30 28.5 28.2 |
| 48 | .84775    | 17 | .37689    | 58 | .85794    | 17 | .41235    | 60 | 48 | 40 38.0 37.6 |
| 49 | .84792    | 17 | .37747    | 58 | .85811    | 17 | .41295    | 60 | 49 | 50 47.5 47.1 |
| 50 | 9.84809   | 17 | 10.37805  | 58 | 9.85827   | 16 | 10.41355  | 60 | 50 | 17 17 16     |
| 51 | .84826    | 17 | .37863    | 58 | .85844    | 17 | .41416    | 60 | 51 | 6 1.7 1.7    |
| 52 | .84844    | 17 | .37921    | 58 | .85861    | 16 | .41476    | 60 | 52 | 7 2.0 2.0    |
| 53 | .84861    | 17 | .37979    | 58 | .85878    | 17 | .41537    | 60 | 53 | 8 2.3 2.2    |
| 54 | .84878    | 17 | .38037    | 58 | .85895    | 16 | .41597    | 60 | 54 | 9 2.6 2.5    |
| 55 | 9.84895   | 17 | 10.38095  | 58 | 9.85911   | 17 | 10.41658  | 60 | 55 | 10 2.9 2.8   |
| 56 | .84912    | 17 | .38153    | 58 | .85928    | 17 | .41719    | 61 | 56 | 20 5.8 5.6   |
| 57 | .84929    | 17 | .38212    | 58 | .85945    | 16 | .41779    | 60 | 57 | 30 8.7 8.5   |
| 58 | .84946    | 17 | .38270    | 58 | .85962    | 17 | .41840    | 61 | 58 | 40 11.6 11.3 |
| 59 | .84963    | 17 | .38328    | 58 | .85979    | 16 | .41901    | 61 | 59 | 50 14.6 14.1 |
| 60 | 9.84980   | 17 | 10.38387  | 58 | 9.85995   | 16 | 10.41962  | 61 | 60 | 13.7         |
| '  | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  | '  | P. P.        |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|--------------|
| 0  | 9.85995   | 17 | 10.41962  | 60 | 9.86992   | 16 | 10.45693  | 63 | 0  | 67 66 66     |
| 1  | .86012    | 16 | .42022    | 61 | .87009    | 16 | .45756    | 63 | 1  | 6 6.7 6.6    |
| 2  | .86029    | 17 | .42083    | 61 | .87025    | 16 | .45820    | 64 | 2  | 7 7.8 7.7    |
| 3  | .86046    | 16 | .42144    | 61 | .87042    | 16 | .45884    | 63 | 3  | 8 8.9 8.8    |
| 4  | .86062    | 17 | .42205    | 61 | .87058    | 16 | .45947    | 63 | 4  | 9 10.0 10.0  |
| 5  | 9.86079   | 16 | 10.42266  | 61 | 9.87074   | 16 | 10.46011  | 64 | 5  | 10 11.1 11.1 |
| 6  | .86096    | 17 | .42327    | 61 | .87091    | 16 | .46075    | 64 | 6  | 20 22.3 22.1 |
| 7  | .86113    | 16 | .42388    | 61 | .87107    | 16 | .46139    | 64 | 7  | 30 33.5 33.2 |
| 8  | .86129    | 17 | .42450    | 61 | .87124    | 16 | .46203    | 64 | 8  | 40 44.6 44.3 |
| 9  | .86146    | 16 | .42511    | 61 | .87140    | 16 | .46267    | 64 | 9  | 50 55.8 55.4 |
| 10 | 9.86163   | 17 | 10.42572  | 61 | 9.87157   | 16 | 10.46331  | 64 | 10 | 65 65 64     |
| 11 | .86179    | 16 | .42633    | 61 | .87173    | 16 | .46395    | 64 | 11 | 6 6.5 6.5    |
| 12 | .86196    | 17 | .42695    | 61 | .87189    | 16 | .46460    | 64 | 12 | 7 7.6 7.6    |
| 13 | .86213    | 16 | .42756    | 61 | .87206    | 16 | .46524    | 64 | 13 | 8 8.7 8.6    |
| 14 | .86230    | 17 | .42817    | 61 | .87222    | 16 | .46588    | 64 | 14 | 9 9.8 9.7    |
| 15 | 9.86246   | 16 | 10.42879  | 61 | 9.87239   | 16 | 10.46652  | 64 | 15 | 10 10.9 10.8 |
| 16 | .86263    | 17 | .42940    | 61 | .87255    | 16 | .46717    | 64 | 16 | 20 21.8 21.5 |
| 17 | .86280    | 16 | .43002    | 61 | .87271    | 16 | .46781    | 64 | 17 | 30 32.7 32.5 |
| 18 | .86296    | 17 | .43063    | 62 | .87288    | 16 | .46846    | 64 | 18 | 40 43.6 43.3 |
| 19 | .86313    | 16 | .43125    | 61 | .87304    | 16 | .46910    | 64 | 19 | 50 54.6 54.1 |
| 20 | 9.86330   | 17 | 10.43187  | 62 | 9.87320   | 16 | 10.46975  | 65 | 20 | 64 63 63     |
| 21 | .86346    | 16 | .43249    | 61 | .87337    | 16 | .47040    | 64 | 21 | 6 6.4 6.3    |
| 22 | .86363    | 17 | .43310    | 61 | .87353    | 16 | .47104    | 64 | 22 | 7 7.4 7.4    |
| 23 | .86380    | 16 | .43372    | 61 | .87370    | 16 | .47169    | 64 | 23 | 8 8.5 8.4    |
| 24 | .86396    | 17 | .43434    | 62 | .87386    | 16 | .47234    | 64 | 24 | 9 9.6 9.5    |
| 25 | 9.86413   | 16 | 10.43496  | 62 | 9.87402   | 16 | 10.47299  | 65 | 25 | 10 10.6 10.6 |
| 26 | .86430    | 17 | .43558    | 62 | .87419    | 16 | .47364    | 65 | 26 | 20 21.3 21.1 |
| 27 | .86446    | 16 | .43620    | 62 | .87435    | 16 | .47429    | 65 | 27 | 30 32.0 31.7 |
| 28 | .86463    | 17 | .43682    | 62 | .87451    | 16 | .47494    | 65 | 28 | 40 42.6 42.3 |
| 29 | .86479    | 16 | .43744    | 62 | .87468    | 16 | .47559    | 65 | 29 | 50 53.3 52.9 |
| 30 | 9.86496   | 17 | 10.43806  | 62 | 9.87484   | 16 | 10.47624  | 65 | 30 | 62 62 61     |
| 31 | .86513    | 16 | .43868    | 62 | .87500    | 16 | .47689    | 65 | 31 | 6 6.2 6.2    |
| 32 | .86529    | 17 | .43931    | 62 | .87516    | 16 | .47754    | 65 | 32 | 7 7.3 7.2    |
| 33 | .86546    | 16 | .43993    | 62 | .87533    | 16 | .47820    | 65 | 33 | 8 8.3 8.2    |
| 34 | .86562    | 17 | .44055    | 62 | .87549    | 16 | .47885    | 65 | 34 | 9 9.4 9.3    |
| 35 | 9.86579   | 16 | 10.44118  | 62 | 9.87565   | 16 | 10.47950  | 65 | 35 | 10 10.4 10.3 |
| 36 | .86596    | 17 | .44180    | 62 | .87582    | 16 | .48016    | 65 | 36 | 20 20.8 20.6 |
| 37 | .86612    | 16 | .44242    | 62 | .87598    | 16 | .48081    | 65 | 37 | 30 31.2 31.0 |
| 38 | .86629    | 17 | .44305    | 62 | .87614    | 16 | .48147    | 65 | 38 | 40 41.6 41.3 |
| 39 | .86645    | 16 | .44368    | 62 | .87631    | 16 | .48213    | 65 | 39 | 50 52.1 51.6 |
| 40 | 9.86662   | 17 | 10.44430  | 62 | 9.87647   | 16 | 10.48278  | 65 | 40 | 61 60 60     |
| 41 | .86678    | 16 | .44493    | 63 | .87663    | 16 | .48344    | 66 | 41 | 6 6.1 6.0    |
| 42 | .86695    | 17 | .44556    | 62 | .87679    | 16 | .48410    | 66 | 42 | 7 7.1 7.0    |
| 43 | .86712    | 16 | .44618    | 63 | .87696    | 16 | .48476    | 66 | 43 | 8 8.1 8.0    |
| 44 | .86728    | 17 | .44681    | 62 | .87712    | 16 | .48542    | 66 | 44 | 9 9.1 9.1    |
| 45 | 9.86745   | 16 | 10.44744  | 63 | 9.87728   | 16 | 10.48607  | 66 | 45 | 10 10.1 10.1 |
| 46 | .86761    | 17 | .44807    | 63 | .87744    | 16 | .48674    | 66 | 46 | 20 20.3 20.1 |
| 47 | .86778    | 16 | .44870    | 63 | .87761    | 16 | .48740    | 66 | 47 | 30 30.5 30.2 |
| 48 | .86794    | 17 | .44933    | 63 | .87777    | 16 | .48806    | 66 | 48 | 40 40.6 40.3 |
| 49 | .86811    | 16 | .44996    | 63 | .87793    | 16 | .48872    | 66 | 49 | 50 50.8 50.4 |
| 50 | 9.86827   | 17 | 10.45059  | 63 | 9.87809   | 16 | 10.48938  | 66 | 50 | 17 16 16     |
| 51 | .86844    | 16 | .45122    | 63 | .87825    | 16 | .49004    | 66 | 51 | 6 1.7 1.6    |
| 52 | .86860    | 17 | .45185    | 63 | .87842    | 16 | .49071    | 66 | 52 | 7 2.0 1.9    |
| 53 | .86877    | 16 | .45248    | 63 | .87858    | 16 | .49137    | 66 | 53 | 8 2.2 2.2    |
| 54 | .86893    | 17 | .45312    | 63 | .87874    | 16 | .49204    | 66 | 54 | 9 2.5 2.4    |
| 55 | 9.86910   | 16 | 10.45375  | 63 | 9.87890   | 16 | 10.49270  | 66 | 55 | 10 2.8 2.7   |
| 56 | .86926    | 17 | .45439    | 63 | .87906    | 16 | .49337    | 66 | 56 | 20 5.6 5.5   |
| 57 | .86943    | 16 | .45502    | 63 | .87923    | 16 | .49403    | 67 | 57 | 30 8.5 8.2   |
| 58 | .86959    | 17 | .45565    | 63 | .87939    | 16 | .49470    | 67 | 58 | 40 11.3 11.0 |
| 59 | .86976    | 16 | .45629    | 63 | .87955    | 16 | .49537    | 67 | 59 | 50 14.1 13.7 |
| 60 | 9.86992   | 17 | 10.45693  | 64 | 9.87971   | 16 | 10.49604  | 67 | 60 | 13.7 13.3    |
|    | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  |    | P. P.        |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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| '  | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  | '  | P. P.             |
|----|-----------|----|-----------|----|-----------|----|-----------|----|----|-------------------|
| 0  | 9.87971   | 16 | 10.49604  | 66 | 9.88933   | 16 | 10.53724  | 70 | 0  |                   |
| 1  | .87987    | 16 | .49670    | 67 | .88949    | 15 | .53794    | 71 | 1  |                   |
| 2  | .88003    | 16 | .49737    | 67 | .88964    | 15 | .53865    | 71 | 2  |                   |
| 3  | .88020    | 16 | .49804    | 67 | .88980    | 16 | .53936    | 71 | 3  |                   |
| 4  | .88036    | 16 | .49871    | 67 | .88996    | 16 | .54007    | 71 | 4  | 75 74 73          |
| 5  | 9.88052   | 16 | 10.49939  | 67 | 9.89012   | 16 | 10.54078  | 71 | 5  | 6 7.5 7.4 7.3     |
| 6  | .88068    | 16 | .50006    | 67 | .89028    | 15 | .54149    | 71 | 6  | 7 8.7 8.6 8.5     |
| 7  | .88084    | 16 | .50073    | 67 | .89044    | 16 | .54220    | 71 | 7  | 8 10.0 9.8 9.7    |
| 8  | .88100    | 16 | .50140    | 67 | .89060    | 15 | .54291    | 71 | 8  | 9 11.2 11.1 10.9  |
| 9  | .88116    | 16 | .50208    | 67 | .89075    | 15 | .54362    | 71 | 9  | 10 12.5 12.3 12.1 |
| 10 | 9.88133   | 16 | 10.50275  | 67 | 9.89091   | 16 | 10.54433  | 71 | 10 | 20 25.0 24.6 24.3 |
| 11 | .88149    | 16 | .50342    | 67 | .89107    | 15 | .54505    | 71 | 11 | 30 37.5 37.0 36.5 |
| 12 | .88165    | 16 | .50410    | 67 | .89123    | 16 | .54576    | 71 | 12 | 40 50.0 49.3 48.6 |
| 13 | .88181    | 16 | .50477    | 67 | .89139    | 16 | .54647    | 71 | 13 | 50 62.5 61.6 60.8 |
| 14 | .88197    | 16 | .50545    | 68 | .89155    | 16 | .54719    | 72 | 14 |                   |
| 15 | 9.88213   | 16 | 10.50613  | 67 | 9.89170   | 15 | 10.54791  | 71 | 15 | 72 71 70          |
| 16 | .88229    | 16 | .50681    | 67 | .89186    | 15 | .54862    | 71 | 16 | 6 7.2 7.1 7.0     |
| 17 | .88245    | 16 | .50748    | 67 | .89202    | 15 | .54934    | 71 | 17 | 7 8.4 8.3 8.2     |
| 18 | .88261    | 16 | .50816    | 68 | .89218    | 16 | .55006    | 71 | 18 | 8 9.6 9.4 9.4     |
| 19 | .88277    | 16 | .50884    | 68 | .89234    | 15 | .55078    | 72 | 19 | 9 10.8 10.6 10.6  |
| 20 | 9.88294   | 16 | 10.50952  | 68 | 9.89249   | 16 | 10.55150  | 72 | 20 | 10 12.0 11.8 11.7 |
| 21 | .88310    | 16 | .51020    | 68 | .89265    | 15 | .55222    | 72 | 21 | 20 24.0 23.6 23.3 |
| 22 | .88326    | 16 | .51088    | 68 | .89281    | 15 | .55294    | 72 | 22 | 30 36.0 35.5 35.2 |
| 23 | .88342    | 16 | .51157    | 68 | .89297    | 15 | .55366    | 72 | 23 | 40 48.0 47.3 47.0 |
| 24 | .88358    | 16 | .51225    | 68 | .89312    | 15 | .55438    | 72 | 24 | 50 60.0 59.1 58.7 |
| 25 | 9.88374   | 16 | 10.51293  | 68 | 9.89328   | 16 | 10.55511  | 72 | 25 |                   |
| 26 | .88390    | 16 | .51361    | 68 | .89344    | 15 | .55583    | 72 | 26 | 69 68 67          |
| 27 | .88406    | 16 | .51430    | 68 | .89360    | 16 | .55655    | 72 | 27 | 6 6.9 6.8 6.7     |
| 28 | .88422    | 16 | .51498    | 68 | .89376    | 15 | .55728    | 72 | 28 | 7 8.0 7.9 7.8     |
| 29 | .88438    | 16 | .51567    | 69 | .89391    | 15 | .55801    | 72 | 29 | 8 9.2 9.0 8.9     |
| 30 | 9.88454   | 16 | 10.51636  | 68 | 9.89407   | 16 | 10.55873  | 72 | 30 | 9 10.3 10.2 10.0  |
| 31 | .88470    | 16 | .51704    | 68 | .89423    | 15 | .55946    | 72 | 31 | 10 11.5 11.3 11.1 |
| 32 | .88486    | 16 | .51773    | 68 | .89438    | 16 | .56019    | 72 | 32 | 20 23.0 22.6 22.3 |
| 33 | .88502    | 16 | .51842    | 69 | .89454    | 15 | .56092    | 72 | 33 | 30 34.5 34.0 33.5 |
| 34 | .88518    | 16 | .51911    | 69 | .89470    | 15 | .56165    | 72 | 34 | 40 46.0 45.3 44.6 |
| 35 | 9.88534   | 16 | 10.51980  | 69 | 9.89486   | 16 | 10.56238  | 72 | 35 | 50 57.5 56.6 55.8 |
| 36 | .88550    | 16 | .52049    | 69 | .89501    | 15 | .56311    | 72 | 36 |                   |
| 37 | .88566    | 16 | .52118    | 69 | .89517    | 15 | .56384    | 72 | 37 | 66 65 64          |
| 38 | .88582    | 16 | .52187    | 69 | .89533    | 15 | .56457    | 72 | 38 | 6 6.6 6.5 6.4     |
| 39 | .88598    | 16 | .52256    | 69 | .89548    | 15 | .56531    | 72 | 39 | 7 7.7 7.6 7.5     |
| 40 | 9.88614   | 16 | 10.52325  | 69 | 9.89564   | 16 | 10.56604  | 72 | 40 | 8 8.8 8.7 8.6     |
| 41 | .88630    | 16 | .52394    | 69 | .89580    | 15 | .56678    | 72 | 41 | 9 9.9 9.8 9.7     |
| 42 | .88646    | 15 | .52464    | 69 | .89596    | 15 | .56751    | 72 | 42 | 10 11.0 10.9 10.8 |
| 43 | .88662    | 16 | .52533    | 69 | .89611    | 15 | .56825    | 72 | 43 | 20 22.0 21.8 21.6 |
| 44 | .88678    | 16 | .52603    | 69 | .89627    | 15 | .56899    | 72 | 44 | 30 33.0 32.7 32.5 |
| 45 | 9.88694   | 16 | 10.52672  | 69 | 9.89643   | 16 | 10.56973  | 72 | 45 | 40 44.0 43.6 43.3 |
| 46 | .88710    | 16 | .52742    | 70 | .89658    | 15 | .57047    | 72 | 46 | 50 55.0 54.4 53.8 |
| 47 | .88726    | 16 | .52812    | 69 | .89674    | 16 | .57120    | 72 | 47 |                   |
| 48 | .88742    | 16 | .52881    | 69 | .89690    | 15 | .57195    | 72 | 48 | 16 15 14          |
| 49 | .88758    | 16 | .52951    | 70 | .89705    | 15 | .57269    | 72 | 49 | 6 1.6 1.6 1.5     |
| 50 | 9.88774   | 16 | 10.53021  | 70 | 9.89721   | 16 | 10.57343  | 72 | 50 | 7 1.9 1.8 1.8     |
| 51 | .88790    | 15 | .53091    | 70 | .89737    | 15 | .57417    | 72 | 51 | 8 2.2 2.1 2.0     |
| 52 | .88805    | 16 | .53161    | 70 | .89752    | 15 | .57491    | 72 | 52 | 9 2.5 2.4 2.3     |
| 53 | .88821    | 16 | .53231    | 70 | .89768    | 15 | .57566    | 72 | 53 | 10 2.7 2.6 2.6    |
| 54 | .88837    | 16 | .53301    | 70 | .89783    | 15 | .57640    | 72 | 54 | 20 5.5 5.3 5.1    |
| 55 | 9.88853   | 16 | 10.53372  | 70 | 9.89799   | 16 | 10.57715  | 72 | 55 | 30 8.2 8.0 7.7    |
| 56 | .88869    | 15 | .53442    | 70 | .89815    | 15 | .57790    | 72 | 56 | 40 11.0 10.8 10.3 |
| 57 | .88885    | 15 | .53512    | 70 | .89830    | 15 | .57864    | 72 | 57 | 50 13.7 13.3 12.9 |
| 58 | .88901    | 16 | .53583    | 70 | .89846    | 16 | .57939    | 72 | 58 |                   |
| 59 | .88917    | 16 | .53653    | 70 | .89862    | 16 | .58014    | 72 | 59 |                   |
| 60 | 9.88933   | 16 | 10.53724  | 70 | 9.89877   | 15 | 10.58089  | 72 | 60 |                   |
| '  | Lg. Vers. | D  | Log. Exs. | D  | Lg. Vers. | D  | Log. Exs. | D  | '  | P. P.             |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    |         | Lg. Vers. | D        |    |         | Log. Exs. | D        |    |    | Lg. Vers. | D    |      |      | Log. Exs. | D |
|----|---------|-----------|----------|----|---------|-----------|----------|----|----|-----------|------|------|------|-----------|---|
| 0  | 9.89877 | 15        | 10.58089 | 75 | 9.90805 | 15        | 10.62745 | 80 | 0  | 6         | 8.6  | 8.5  | 8.4  |           |   |
| 1  | .89893  | 15        | .58164   | 75 | .90820  | 15        | .62825   | 80 | 1  | 7         | 10.0 | 9.9  | 9.8  |           |   |
| 2  | .89908  | 15        | .58239   | 75 | .90835  | 15        | .62906   | 80 | 2  | 8         | 11.4 | 11.3 | 11.2 |           |   |
| 3  | .89924  | 15        | .58315   | 75 | .90851  | 15        | .62986   | 81 | 3  | 9         | 12.9 | 12.7 | 12.6 |           |   |
| 4  | .89939  | 15        | .58390   | 75 | .90866  | 15        | .63067   | 81 | 4  | 10        | 14.3 | 14.1 | 14.0 |           |   |
| 5  | 9.89955 | 15        | 10.58465 | 75 | 9.90881 | 15        | 10.63148 | 81 | 5  | 20        | 28.6 | 28.3 | 28.0 |           |   |
| 6  | .89971  | 15        | .58541   | 75 | .90897  | 15        | .63229   | 81 | 6  | 30        | 43.0 | 42.5 | 42.0 |           |   |
| 7  | .89986  | 15        | .58616   | 76 | .90912  | 15        | .63310   | 81 | 7  | 40        | 57.3 | 56.6 | 56.0 |           |   |
| 8  | .90002  | 15        | .58692   | 75 | .90927  | 15        | .63391   | 81 | 8  | 50        | 71.6 | 70.8 | 70.0 |           |   |
| 9  | .90017  | 15        | .58768   | 76 | .90943  | 15        | .63472   | 81 | 9  |           |      |      |      |           |   |
| 10 | 9.90033 | 15        | 10.58844 | 76 | 9.90958 | 15        | 10.63553 | 81 | 10 | 6         | 8.3  | 8.2  | 8.1  |           |   |
| 11 | .90048  | 15        | .58920   | 75 | .90973  | 15        | .63634   | 81 | 11 | 7         | 9.7  | 9.5  | 9.4  |           |   |
| 12 | .90064  | 15        | .58995   | 75 | .90988  | 15        | .63716   | 81 | 12 | 8         | 11.0 | 10.9 | 10.8 |           |   |
| 13 | .90080  | 15        | .59072   | 76 | .91004  | 15        | .63797   | 81 | 13 | 9         | 12.4 | 12.3 | 12.1 |           |   |
| 14 | .90095  | 15        | .59148   | 76 | .91019  | 15        | .63879   | 82 | 14 | 10        | 13.8 | 13.6 | 13.5 |           |   |
| 15 | 9.90111 | 15        | 10.59224 | 76 | 9.91034 | 15        | 10.63961 | 82 | 15 | 20        | 27.6 | 27.3 | 27.0 |           |   |
| 16 | .90126  | 15        | .59300   | 76 | .91049  | 15        | .64043   | 82 | 16 | 30        | 41.5 | 41.0 | 40.5 |           |   |
| 17 | .90142  | 15        | .59377   | 76 | .91065  | 15        | .64125   | 82 | 17 | 40        | 55.3 | 54.6 | 54.0 |           |   |
| 18 | .90157  | 15        | .59453   | 76 | .91080  | 15        | .64207   | 82 | 18 | 50        | 69.1 | 68.3 | 67.5 |           |   |
| 19 | .90173  | 15        | .59530   | 76 | .91095  | 15        | .64289   | 82 | 19 |           |      |      |      |           |   |
| 20 | 9.90188 | 15        | 10.59606 | 77 | 9.91110 | 15        | 10.64371 | 82 | 20 | 6         | 8.0  | 7.9  | 7.8  |           |   |
| 21 | .90204  | 15        | .59683   | 77 | .91126  | 15        | .64453   | 82 | 21 | 7         | 9.3  | 9.2  | 9.1  |           |   |
| 22 | .90219  | 15        | .59760   | 77 | .91141  | 15        | .64536   | 82 | 22 | 8         | 10.6 | 10.5 | 10.4 |           |   |
| 23 | .90235  | 15        | .59837   | 77 | .91156  | 15        | .64618   | 83 | 23 | 9         | 12.0 | 11.8 | 11.7 |           |   |
| 24 | .90250  | 15        | .59914   | 77 | .91171  | 15        | .64701   | 83 | 24 | 10        | 13.3 | 13.1 | 13.0 |           |   |
| 25 | 9.90266 | 15        | 10.59991 | 77 | 9.91187 | 15        | 10.64784 | 83 | 25 | 20        | 26.6 | 26.3 | 26.0 |           |   |
| 26 | .90281  | 15        | .60038   | 77 | .91202  | 15        | .64867   | 83 | 26 | 30        | 40.0 | 39.5 | 39.0 |           |   |
| 27 | .90297  | 15        | .60145   | 77 | .91217  | 15        | .64950   | 83 | 27 | 40        | 53.3 | 52.6 | 52.0 |           |   |
| 28 | .90312  | 15        | .60223   | 77 | .91232  | 15        | .65033   | 83 | 28 | 50        | 66.6 | 65.8 | 65.0 |           |   |
| 29 | .90328  | 15        | .60300   | 77 | .91247  | 15        | .65116   | 83 | 29 |           |      |      |      |           |   |
| 30 | 9.90343 | 15        | 10.60378 | 77 | 9.91263 | 15        | 10.65199 | 83 | 30 | 6         | 7.7  | 7.6  | 7.5  |           |   |
| 31 | .90359  | 15        | .60455   | 77 | .91278  | 15        | .65283   | 83 | 31 | 7         | 9.0  | 8.8  | 8.7  |           |   |
| 32 | .90374  | 15        | .60533   | 78 | .91293  | 15        | .65366   | 83 | 32 | 8         | 10.3 | 10.1 | 10.0 |           |   |
| 33 | .90389  | 15        | .60611   | 78 | .91308  | 15        | .65450   | 84 | 33 | 9         | 11.5 | 11.4 | 11.2 |           |   |
| 34 | .90405  | 15        | .60688   | 78 | .91323  | 15        | .65534   | 83 | 34 | 10        | 12.8 | 12.6 | 12.5 |           |   |
| 35 | 9.90420 | 15        | 10.60766 | 78 | 9.91338 | 15        | 10.65617 | 84 | 35 | 20        | 25.6 | 25.3 | 25.0 |           |   |
| 36 | .90436  | 15        | .60844   | 78 | .91354  | 15        | .65701   | 84 | 36 | 30        | 38.5 | 38.0 | 37.5 |           |   |
| 37 | .90451  | 15        | .60923   | 78 | .91369  | 15        | .65785   | 84 | 37 | 40        | 51.3 | 50.6 | 50.0 |           |   |
| 38 | .90467  | 15        | .61001   | 78 | .91384  | 15        | .65870   | 84 | 38 | 50        | 64.1 | 63.3 | 62.5 |           |   |
| 39 | .90482  | 15        | .61079   | 78 | .91399  | 15        | .65954   | 84 | 39 |           |      |      |      |           |   |
| 40 | 9.90497 | 15        | 10.61158 | 78 | 9.91414 | 15        | 10.66038 | 84 | 40 | 6         | 7.0  | 7.0  | 7.0  |           |   |
| 41 | .90513  | 15        | .61236   | 78 | .91429  | 15        | .66123   | 84 | 41 | 7         | 8.0  | 8.0  | 8.0  |           |   |
| 42 | .90528  | 15        | .61315   | 78 | .91445  | 15        | .66207   | 84 | 42 | 8         | 9.0  | 9.0  | 9.0  |           |   |
| 43 | .90544  | 15        | .61393   | 79 | .91460  | 15        | .66292   | 85 | 43 | 9         | 10.1 | 10.1 | 10.1 |           |   |
| 44 | .90559  | 15        | .61472   | 79 | .91475  | 15        | .66377   | 85 | 44 | 10        | 11.1 | 11.1 | 11.1 |           |   |
| 45 | 9.90574 | 15        | 10.61551 | 79 | 9.91490 | 15        | 10.66462 | 85 | 45 | 20        | 24.4 | 24.3 | 24.2 |           |   |
| 46 | .90590  | 15        | .61630   | 79 | .91505  | 15        | .66547   | 85 | 46 | 30        | 37.5 | 37.0 | 36.5 |           |   |
| 47 | .90605  | 15        | .61709   | 79 | .91520  | 15        | .66632   | 85 | 47 | 40        | 50.6 | 50.0 | 49.4 |           |   |
| 48 | .90621  | 15        | .61788   | 79 | .91535  | 15        | .66717   | 85 | 48 |           |      |      |      |           |   |
| 49 | .90636  | 15        | .61867   | 79 | .91550  | 15        | .66803   | 85 | 49 |           |      |      |      |           |   |
| 50 | 9.90651 | 15        | 10.61947 | 79 | 9.91565 | 15        | 10.66888 | 85 | 50 | 6         | 1.6  | 1.5  | 1.5  |           |   |
| 51 | .90667  | 15        | .62026   | 79 | .91581  | 15        | .66974   | 85 | 51 | 7         | 1.8  | 1.8  | 1.7  |           |   |
| 52 | .90682  | 15        | .62105   | 80 | .91596  | 15        | .67059   | 86 | 52 | 8         | 2.1  | 2.0  | 2.0  |           |   |
| 53 | .90697  | 15        | .62185   | 79 | .91611  | 15        | .67145   | 86 | 53 | 9         | 2.4  | 2.3  | 2.2  |           |   |
| 54 | .90713  | 15        | .62265   | 80 | .91626  | 15        | .67231   | 86 | 54 | 10        | 2.6  | 2.6  | 2.5  |           |   |
| 55 | 9.90728 | 15        | 10.62345 | 79 | 9.91641 | 15        | 10.67317 | 86 | 55 | 20        | 5.3  | 5.1  | 5.0  |           |   |
| 56 | .90744  | 15        | .62424   | 80 | .91656  | 15        | .67403   | 86 | 56 | 30        | 8.0  | 7.7  | 7.5  |           |   |
| 57 | .90759  | 15        | .62504   | 80 | .91671  | 15        | .67490   | 86 | 57 | 40        | 10.6 | 10.3 | 10.0 |           |   |
| 58 | .90774  | 15        | .62585   | 80 | .91686  | 15        | .67576   | 86 | 58 | 50        | 13.3 | 12.9 | 12.5 |           |   |
| 59 | .90790  | 15        | .62665   | 80 | .91701  | 15        | .67663   | 86 | 59 |           |      |      |      |           |   |
| 60 | 9.90805 | 15        | 10.62745 | 80 | 9.91716 | 15        | 10.67749 | 86 | 60 |           |      |      |      |           |   |
|    |         | Lg. Vers. | D        |    |         | Log. Exs. | D        |    |    | Lg. Vers. | D    |      |      | Log. Exs. | D |

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TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    |         |           |          |    |         |           |          |     |    |           |      | P. P. |  |           |   |
|----|---------|-----------|----------|----|---------|-----------|----------|-----|----|-----------|------|-------|--|-----------|---|
|    |         | Lg. Vers. | D        |    |         | Log. Exs. | D        |     |    | Lg. Vers. | D    |       |  | Log. Exs. | D |
| 0  | 9.91716 | 15        | 10.67749 | 86 | 9.92612 | 14        | 10.73178 | 95  | 0  | 90        | 80   |       |  |           |   |
| 1  | .91731  | 15        | .67836   | 87 | .92626  | 15        | .73273   | 94  | 1  | 6         | 9.0  | 8.0   |  |           |   |
| 2  | .91746  | 15        | .67923   | 87 | .92641  | 14        | .73368   | 95  | 2  | 7         | 10.5 | 9.3   |  |           |   |
| 3  | .91761  | 15        | .68010   | 87 | .92656  | 15        | .73463   | 95  | 3  | 8         | 12.0 | 10.6  |  |           |   |
| 4  | .91776  | 15        | .68097   | 87 | .92671  | 15        | .73558   | 95  | 4  | 9         | 13.5 | 12.0  |  |           |   |
| 5  | 9.91791 | 15        | 10.68184 | 87 | 9.92686 | 15        | 10.73653 | 95  | 5  | 10        | 15.0 | 13.3  |  |           |   |
| 6  | .91807  | 15        | .68272   | 87 | .92700  | 14        | .73748   | 95  | 6  | 20        | 30.0 | 26.6  |  |           |   |
| 7  | .91822  | 15        | .68359   | 87 | .92715  | 15        | .73844   | 95  | 7  | 30        | 45.0 | 40.0  |  |           |   |
| 8  | .91837  | 15        | .68447   | 87 | .92730  | 14        | .73940   | 96  | 8  | 40        | 60.0 | 53.3  |  |           |   |
| 9  | .91852  | 15        | .68534   | 87 | .92745  | 15        | .74035   | 95  | 9  | 50        | 75.0 | 66.6  |  |           |   |
| 10 | 9.91867 | 15        | 10.68622 | 88 | 9.92759 | 14        | 10.74131 | 96  | 10 | 9         | 8    |       |  |           |   |
| 11 | .91882  | 15        | .68710   | 88 | .92774  | 15        | .74227   | 96  | 11 | 6         | 0.9  | 0.8   |  |           |   |
| 12 | .91897  | 15        | .68798   | 88 | .92789  | 14        | .74324   | 96  | 12 | 7         | 1.0  | 0.9   |  |           |   |
| 13 | .91912  | 15        | .68886   | 88 | .92804  | 15        | .74420   | 96  | 13 | 8         | 1.2  | 1.0   |  |           |   |
| 14 | .91927  | 15        | .68975   | 88 | .92818  | 14        | .74517   | 96  | 14 | 9         | 1.3  | 1.2   |  |           |   |
| 15 | 9.91942 | 15        | 10.69063 | 88 | 9.92833 | 15        | 10.74613 | 96  | 15 | 10        | 1.5  | 1.3   |  |           |   |
| 16 | .91957  | 15        | .69152   | 88 | .92848  | 14        | .74710   | 97  | 16 | 20        | 3.0  | 2.6   |  |           |   |
| 17 | .91972  | 15        | .69240   | 88 | .92862  | 15        | .74807   | 97  | 17 | 30        | 4.5  | 4.0   |  |           |   |
| 18 | .91987  | 15        | .69329   | 89 | .92877  | 14        | .74905   | 97  | 18 | 40        | 6.0  | 5.3   |  |           |   |
| 19 | .92002  | 15        | .69418   | 89 | .92892  | 15        | .75002   | 97  | 19 | 50        | 7.5  | 6.6   |  |           |   |
| 20 | 9.92016 | 14        | 10.69507 | 89 | 9.92907 | 14        | 10.75099 | 97  | 20 | 7         | 6    |       |  |           |   |
| 21 | .92031  | 15        | .69596   | 89 | .92921  | 15        | .75197   | 98  | 21 | 6         | 0.7  | 0.6   |  |           |   |
| 22 | .92046  | 15        | .69686   | 89 | .92936  | 14        | .75295   | 97  | 22 | 7         | 0.8  | 0.7   |  |           |   |
| 23 | .92061  | 15        | .69775   | 89 | .92951  | 15        | .75393   | 98  | 23 | 8         | 0.9  | 0.8   |  |           |   |
| 24 | .92076  | 15        | .69865   | 89 | .92965  | 14        | .75491   | 98  | 24 | 9         | 1.0  | 0.9   |  |           |   |
| 25 | 9.92091 | 15        | 10.69955 | 89 | 9.92980 | 15        | 10.75589 | 98  | 25 | 10        | 1.1  | 1.0   |  |           |   |
| 26 | .92106  | 15        | .70044   | 89 | .92995  | 14        | .75688   | 98  | 26 | 20        | 2.3  | 2.0   |  |           |   |
| 27 | .92121  | 15        | .70134   | 90 | .93009  | 15        | .75786   | 98  | 27 | 30        | 3.5  | 3.0   |  |           |   |
| 28 | .92136  | 14        | .70224   | 90 | .93024  | 14        | .75885   | 99  | 28 | 40        | 4.6  | 4.0   |  |           |   |
| 29 | .92151  | 15        | .70315   | 90 | .93039  | 15        | .75984   | 99  | 29 | 50        | 5.8  | 5.0   |  |           |   |
| 30 | 9.92166 | 15        | 10.70405 | 90 | 9.93053 | 14        | 10.76083 | 99  | 30 | 5         | 4    |       |  |           |   |
| 31 | .92181  | 15        | .70495   | 91 | .93068  | 15        | .76182   | 99  | 31 | 6         | 0.5  | 0.4   |  |           |   |
| 32 | .92196  | 15        | .70586   | 91 | .93083  | 14        | .76282   | 100 | 32 | 7         | 0.6  | 0.4   |  |           |   |
| 33 | .92211  | 15        | .70677   | 91 | .93097  | 15        | .76382   | 99  | 33 | 8         | 0.6  | 0.5   |  |           |   |
| 34 | .92226  | 14        | .70768   | 91 | .93112  | 14        | .76481   | 100 | 34 | 9         | 0.7  | 0.6   |  |           |   |
| 35 | 9.92240 | 15        | 10.70859 | 91 | 9.93127 | 15        | 10.76581 | 100 | 35 | 10        | 0.8  | 0.6   |  |           |   |
| 36 | .92255  | 15        | .70950   | 91 | .93141  | 14        | .76681   | 100 | 36 | 20        | 1.6  | 1.3   |  |           |   |
| 37 | .92270  | 15        | .71041   | 91 | .93156  | 15        | .76782   | 100 | 37 | 30        | 2.5  | 2.0   |  |           |   |
| 38 | .92285  | 15        | .71133   | 91 | .93171  | 14        | .76882   | 100 | 38 | 40        | 3.3  | 2.6   |  |           |   |
| 39 | .92300  | 15        | .71224   | 91 | .93185  | 14        | .76983   | 100 | 39 | 50        | 4.1  | 3.3   |  |           |   |
| 40 | 9.92315 | 14        | 10.71316 | 92 | 9.93200 | 15        | 10.77083 | 101 | 40 | 15        | 15   |       |  |           |   |
| 41 | .92330  | 15        | .71408   | 92 | .93214  | 14        | .77184   | 101 | 41 | 6         | 1.5  | 1.5   |  |           |   |
| 42 | .92345  | 15        | .71500   | 92 | .93229  | 15        | .77286   | 101 | 42 | 7         | 1.8  | 1.7   |  |           |   |
| 43 | .92360  | 14        | .71592   | 92 | .93244  | 14        | .77387   | 101 | 43 | 8         | 2.0  | 2.0   |  |           |   |
| 44 | .92374  | 15        | .71684   | 92 | .93258  | 15        | .77488   | 101 | 44 | 9         | 2.3  | 2.2   |  |           |   |
| 45 | 9.92389 | 15        | 10.71776 | 92 | 9.93273 | 14        | 10.77590 | 101 | 45 | 10        | 2.6  | 2.5   |  |           |   |
| 46 | .92404  | 14        | .71869   | 92 | .93287  | 15        | .77692   | 102 | 46 | 20        | 5.1  | 5.0   |  |           |   |
| 47 | .92419  | 15        | .71961   | 93 | .93302  | 14        | .77794   | 102 | 47 | 30        | 7.7  | 7.5   |  |           |   |
| 48 | .92434  | 15        | .72054   | 92 | .93317  | 15        | .77896   | 102 | 48 | 40        | 10.3 | 10.0  |  |           |   |
| 49 | .92449  | 14        | .72147   | 92 | .93331  | 14        | .77998   | 102 | 49 | 50        | 12.9 | 12.5  |  |           |   |
| 50 | 9.92463 | 15        | 10.72240 | 93 | 9.93346 | 14        | 10.78101 | 102 | 50 | 6         | 1.4  |       |  |           |   |
| 51 | .92478  | 15        | .72333   | 93 | .93360  | 15        | .78203   | 103 | 51 | 7         | 1.7  |       |  |           |   |
| 52 | .92493  | 15        | .72427   | 93 | .93375  | 14        | .78306   | 103 | 52 | 8         | 1.9  |       |  |           |   |
| 53 | .92508  | 14        | .72520   | 93 | .93389  | 15        | .78409   | 103 | 53 | 9         | 2.2  |       |  |           |   |
| 54 | .92523  | 15        | .72614   | 93 | .93404  | 14        | .78513   | 103 | 54 | 10        | 2.4  |       |  |           |   |
| 55 | 9.92538 | 14        | 10.72707 | 93 | 9.93419 | 15        | 10.78616 | 103 | 55 | 20        | 4.8  |       |  |           |   |
| 56 | .92552  | 15        | .72801   | 94 | .93433  | 14        | .78720   | 104 | 56 | 30        | 7.2  |       |  |           |   |
| 57 | .92567  | 15        | .72895   | 94 | .93448  | 15        | .78823   | 104 | 57 | 40        | 9.6  |       |  |           |   |
| 58 | .92582  | 14        | .72990   | 94 | .93462  | 14        | .78927   | 104 | 58 | 50        | 12.1 |       |  |           |   |
| 59 | .92597  | 15        | .73084   | 94 | .93477  | 14        | .79031   | 104 | 59 | 50        | 12.1 |       |  |           |   |
| 60 | 9.92612 | 15        | 10.73178 | 94 | 9.93491 | 15        | 10.79136 | 104 | 60 |           |      |       |  |           |   |
|    |         | Lg. Vers. | D        |    |         | Log. Exs. | D        |     |    |           |      | P. P. |  |           |   |



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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|    | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D   |    | P. P. |
|----|-----------|----|-----------|-----|-----------|----|-----------|-----|----|-------|
| 0  | 9.93191   | 14 | 10.79136  | 104 | 9.94356   | 14 | 10.85766  | 117 | 0  |       |
| 1  | .93506    | 14 | .79240    | 105 | .94370    | 14 | .85884    | 117 | 1  |       |
| 2  | .93520    | 14 | .79345    | 104 | .94384    | 14 | .86001    | 117 | 2  |       |
| 3  | .93535    | 14 | .79450    | 105 | .94396    | 14 | .86119    | 118 | 3  |       |
| 4  | .93549    | 14 | .79555    | 105 | .94413    | 14 | .86237    | 118 | 4  |       |
| 5  | 9.93564   | 14 | 10.79660  | 105 | 9.94427   | 14 | 10.86355  | 118 | 5  |       |
| 6  | .93578    | 14 | .79766    | 105 | .94441    | 14 | .86474    | 118 | 6  |       |
| 7  | .93593    | 14 | .79871    | 106 | .94456    | 14 | .86592    | 119 | 7  |       |
| 8  | .93607    | 14 | .79977    | 106 | .94470    | 14 | .86711    | 119 | 8  |       |
| 9  | .93622    | 14 | .80083    | 106 | .94484    | 14 | .86831    | 119 | 9  |       |
| 10 | 9.93636   | 14 | 10.80189  | 106 | 9.94498   | 14 | 10.86950  | 119 | 10 |       |
| 11 | .93651    | 14 | .80296    | 106 | .94512    | 14 | .87070    | 120 | 11 |       |
| 12 | .93665    | 14 | .80402    | 107 | .94527    | 14 | .87190    | 120 | 12 |       |
| 13 | .93680    | 14 | .80509    | 107 | .94541    | 14 | .87310    | 120 | 13 |       |
| 14 | .93694    | 14 | .80616    | 107 | .94555    | 14 | .87431    | 120 | 14 |       |
| 15 | 9.93709   | 14 | 10.80723  | 107 | 9.94569   | 14 | 10.87552  | 121 | 15 |       |
| 16 | .93723    | 14 | .80831    | 107 | .94584    | 14 | .87673    | 121 | 16 |       |
| 17 | .93738    | 14 | .80938    | 108 | .94598    | 14 | .87794    | 121 | 17 |       |
| 18 | .93752    | 14 | .81046    | 108 | .94612    | 14 | .87916    | 122 | 18 |       |
| 19 | .93767    | 14 | .81154    | 108 | .94626    | 14 | .88038    | 122 | 19 |       |
| 20 | 9.93781   | 14 | 10.81262  | 108 | 9.94640   | 14 | 10.88160  | 122 | 20 |       |
| 21 | .93796    | 14 | .81371    | 108 | .94655    | 14 | .88282    | 122 | 21 |       |
| 22 | .93810    | 14 | .81479    | 109 | .94669    | 14 | .88405    | 123 | 22 |       |
| 23 | .93824    | 14 | .81588    | 109 | .94683    | 14 | .88528    | 123 | 23 |       |
| 24 | .93839    | 14 | .81697    | 109 | .94697    | 14 | .88651    | 123 | 24 |       |
| 25 | 9.93853   | 14 | 10.81806  | 109 | 9.94711   | 14 | 10.88775  | 124 | 25 |       |
| 26 | .93868    | 14 | .81916    | 109 | .94726    | 14 | .88898    | 124 | 26 |       |
| 27 | .93882    | 14 | .82025    | 110 | .94740    | 14 | .89022    | 124 | 27 |       |
| 28 | .93897    | 14 | .82135    | 110 | .94754    | 14 | .89147    | 124 | 28 |       |
| 29 | .93911    | 14 | .82245    | 110 | .94768    | 14 | .89271    | 125 | 29 |       |
| 30 | 9.93925   | 14 | 10.82356  | 110 | 9.94782   | 14 | 10.89396  | 125 | 30 |       |
| 31 | .93940    | 14 | .82466    | 110 | .94796    | 14 | .89521    | 125 | 31 |       |
| 32 | .93954    | 14 | .82577    | 111 | .94810    | 14 | .89647    | 126 | 32 |       |
| 33 | .93969    | 14 | .82688    | 111 | .94825    | 14 | .89773    | 126 | 33 |       |
| 34 | .93983    | 14 | .82799    | 111 | .94839    | 14 | .89899    | 126 | 34 |       |
| 35 | 9.93997   | 14 | 10.82910  | 111 | 9.94853   | 14 | 10.90025  | 126 | 35 |       |
| 36 | .94012    | 14 | .83022    | 111 | .94867    | 14 | .90152    | 127 | 36 |       |
| 37 | .94026    | 14 | .83133    | 112 | .94881    | 14 | .90279    | 127 | 37 |       |
| 38 | .94041    | 14 | .83245    | 112 | .94895    | 14 | .90406    | 127 | 38 |       |
| 39 | .94055    | 14 | .83358    | 112 | .94909    | 14 | .90533    | 127 | 39 |       |
| 40 | 9.94069   | 14 | 10.83470  | 112 | 9.94923   | 14 | 10.90661  | 128 | 40 |       |
| 41 | .94084    | 14 | .83583    | 112 | .94938    | 14 | .90789    | 128 | 41 |       |
| 42 | .94098    | 14 | .83695    | 113 | .94952    | 14 | .90917    | 129 | 42 |       |
| 43 | .94112    | 14 | .83809    | 113 | .94966    | 14 | .91046    | 129 | 43 |       |
| 44 | .94127    | 14 | .83922    | 113 | .94980    | 14 | .91175    | 129 | 44 |       |
| 45 | 9.94141   | 14 | 10.84035  | 113 | 9.94994   | 14 | 10.91304  | 130 | 45 |       |
| 46 | .94155    | 14 | .84149    | 114 | .95008    | 14 | .91434    | 129 | 46 |       |
| 47 | .94170    | 14 | .84263    | 114 | .95022    | 14 | .91564    | 130 | 47 |       |
| 48 | .94184    | 14 | .84377    | 114 | .95036    | 14 | .91694    | 130 | 48 |       |
| 49 | .94198    | 14 | .84492    | 114 | .95050    | 14 | .91825    | 131 | 49 |       |
| 50 | 9.94213   | 14 | 10.84607  | 115 | 9.95064   | 14 | 10.91956  | 131 | 50 |       |
| 51 | .94227    | 14 | .84721    | 115 | .95078    | 14 | .92087    | 131 | 51 |       |
| 52 | .94241    | 14 | .84837    | 115 | .95093    | 14 | .92218    | 132 | 52 |       |
| 53 | .94256    | 14 | .84952    | 116 | .95107    | 14 | .92350    | 132 | 53 |       |
| 54 | .94270    | 14 | .85068    | 116 | .95121    | 14 | .92482    | 132 | 54 |       |
| 55 | 9.94284   | 14 | 10.85183  | 116 | 9.95135   | 14 | 10.92614  | 133 | 55 |       |
| 56 | .94299    | 14 | .85299    | 116 | .95149    | 14 | .92747    | 133 | 56 |       |
| 57 | .94313    | 14 | .85416    | 116 | .95163    | 14 | .92880    | 133 | 57 |       |
| 58 | .94327    | 14 | .85532    | 117 | .95177    | 14 | .93014    | 133 | 58 |       |
| 59 | .94341    | 14 | .85649    | 117 | .95191    | 14 | .93147    | 134 | 59 |       |
| 60 | 9.94356   | 14 | 10.85766  | 117 | 9.95205   | 14 | 10.93281  | 134 | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D   |    | P. P. |

|    |       |       |
|----|-------|-------|
| 6  | 130   | 120   |
| 7  | 15.0  | 14.0  |
| 8  | 17.3  | 16.0  |
| 9  | 19.5  | 18.0  |
| 10 | 21.6  | 20.0  |
| 20 | 43.3  | 40.0  |
| 30 | 65.0  | 60.0  |
| 40 | 86.6  | 80.0  |
| 50 | 108.3 | 100.0 |

|    |      |      |
|----|------|------|
| 6  | 110  | 100  |
| 7  | 11.0 | 10.0 |
| 8  | 12.8 | 11.6 |
| 9  | 14.6 | 13.3 |
| 10 | 16.5 | 15.0 |
| 20 | 36.6 | 33.3 |
| 30 | 55.0 | 50.0 |
| 40 | 73.3 | 66.6 |
| 50 | 91.6 | 83.3 |

|    |     |     |
|----|-----|-----|
| 6  | 3   | 2   |
| 7  | 0.3 | 0.2 |
| 8  | 0.4 | 0.2 |
| 9  | 0.4 | 0.3 |
| 10 | 0.5 | 0.3 |
| 20 | 1.0 | 0.6 |
| 30 | 1.5 | 1.0 |
| 40 | 2.0 | 1.3 |
| 50 | 2.5 | 1.6 |

|    |     |     |
|----|-----|-----|
| 6  | 1   | 0   |
| 7  | 0.1 | 0.0 |
| 8  | 0.1 | 0.0 |
| 9  | 0.1 | 0.1 |
| 10 | 0.1 | 0.1 |
| 20 | 0.3 | 0.1 |
| 30 | 0.5 | 0.2 |
| 40 | 0.6 | 0.3 |
| 50 | 0.8 | 0.4 |

|    |      |      |
|----|------|------|
| 6  | 14   | 14   |
| 7  | 1.4  | 1.4  |
| 8  | 1.7  | 1.6  |
| 9  | 1.9  | 1.8  |
| 10 | 2.2  | 2.1  |
| 20 | 4.8  | 4.6  |
| 30 | 7.2  | 7.0  |
| 40 | 9.6  | 9.3  |
| 50 | 12.1 | 11.6 |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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| 84°        |    |           |     | 85°       |    |           |     | P. P. |     |       |       |
|------------|----|-----------|-----|-----------|----|-----------|-----|-------|-----|-------|-------|
| Lg. Vers.  | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D   |       |     |       |       |
| 0 9.95205  | 14 | 10.93281  | 134 | 9.96039   | 14 | 11.02010  | 158 | 0     | 190 | 180   |       |
| 1 .95219   | 14 | .93416    | 135 | .96053    | 13 | .02168    | 159 | 1     | 6   | 19.0  | 18.0  |
| 2 .95233   | 14 | .93551    | 135 | .96067    | 13 | .02327    | 159 | 2     | 7   | 22.1  | 21.0  |
| 3 .95247   | 14 | .93686    | 135 | .96081    | 14 | .02487    | 159 | 3     | 8   | 25.3  | 24.0  |
| 4 .95261   | 14 | .93821    | 135 | .96095    | 14 | .02646    | 159 | 4     | 9   | 28.5  | 27.0  |
| 5 9.95275  | 14 | 10.93957  | 135 | 9.96108   | 13 | 11.02807  | 160 | 5     | 10  | 31.6  | 30.0  |
| 6 .95289   | 14 | .94093    | 136 | .96122    | 14 | .02968    | 161 | 6     | 20  | 63.3  | 60.0  |
| 7 .95303   | 14 | .94229    | 136 | .96136    | 13 | .03129    | 161 | 7     | 30  | 95.0  | 90.0  |
| 8 .95317   | 14 | .94366    | 137 | .96150    | 14 | .03291    | 161 | 8     | 40  | 126.6 | 120.0 |
| 9 .95331   | 14 | .94503    | 137 | .96163    | 13 | .03453    | 162 | 9     | 50  | 158.3 | 150.0 |
| 10 9.95345 | 14 | 10.94641  | 137 | 9.96177   | 14 | 11.03616  | 163 | 10    | 170 | 160   |       |
| 11 .95359  | 13 | .94778    | 137 | .96191    | 13 | .03780    | 163 | 11    | 6   | 17.0  | 16.0  |
| 12 .95373  | 14 | .94917    | 138 | .96205    | 14 | .03944    | 164 | 12    | 7   | 19.8  | 18.6  |
| 13 .95387  | 14 | .95055    | 138 | .96218    | 13 | .04108    | 164 | 13    | 8   | 22.6  | 21.3  |
| 14 .95401  | 14 | .95194    | 139 | .96232    | 14 | .04273    | 165 | 14    | 9   | 25.5  | 24.0  |
| 15 9.95415 | 14 | 10.95333  | 139 | 9.96246   | 13 | 11.04438  | 165 | 15    | 10  | 28.3  | 26.6  |
| 16 .95429  | 14 | .95473    | 139 | .96259    | 14 | .04604    | 166 | 16    | 20  | 56.6  | 53.3  |
| 17 .95443  | 14 | .95613    | 140 | .96273    | 14 | .04771    | 167 | 17    | 30  | 85.0  | 80.0  |
| 18 .95457  | 14 | .95753    | 140 | .96287    | 13 | .04938    | 167 | 18    | 40  | 113.3 | 106.6 |
| 19 .95471  | 14 | .95894    | 141 | .96301    | 14 | .05106    | 167 | 19    | 50  | 141.6 | 133.3 |
| 20 9.95485 | 14 | 10.96033  | 141 | 9.96314   | 13 | 11.05274  | 168 | 20    | 150 | 140   |       |
| 21 .95499  | 14 | .96176    | 141 | .96328    | 14 | .05443    | 169 | 21    | 6   | 15.0  | 14.0  |
| 22 .95513  | 14 | .96316    | 142 | .96342    | 13 | .05612    | 169 | 22    | 7   | 17.5  | 16.3  |
| 23 .95527  | 13 | .96461    | 142 | .96355    | 14 | .05782    | 169 | 23    | 8   | 20.0  | 18.6  |
| 24 .95540  | 14 | .96603    | 142 | .96369    | 13 | .05952    | 170 | 24    | 9   | 22.5  | 21.0  |
| 25 9.95554 | 14 | 10.96746  | 143 | 9.96383   | 14 | 11.06123  | 171 | 25    | 10  | 25.0  | 23.3  |
| 26 .95568  | 14 | .96889    | 143 | .96397    | 13 | .06295    | 171 | 26    | 20  | 50.0  | 46.6  |
| 27 .95582  | 14 | .97033    | 144 | .96410    | 14 | .06467    | 172 | 27    | 30  | 75.0  | 70.0  |
| 28 .95596  | 14 | .97177    | 144 | .96424    | 13 | .06640    | 173 | 28    | 40  | 100.0 | 93.3  |
| 29 .95610  | 14 | .97322    | 144 | .96438    | 14 | .06813    | 173 | 29    | 50  | 125.0 | 116.6 |
| 30 9.95624 | 13 | 10.97467  | 145 | 9.96451   | 13 | 11.06987  | 174 | 30    | 130 | 9     | 8     |
| 31 .95638  | 14 | .97612    | 145 | .96465    | 14 | .07161    | 174 | 31    | 6   | 13.0  | 0.9   |
| 32 .95652  | 14 | .97758    | 145 | .96479    | 13 | .07336    | 175 | 32    | 7   | 15.1  | 1.0   |
| 33 .95666  | 14 | .97904    | 146 | .96492    | 14 | .07512    | 176 | 33    | 8   | 17.3  | 1.2   |
| 34 .95680  | 13 | .98050    | 146 | .96506    | 13 | .07688    | 176 | 34    | 9   | 19.5  | 1.3   |
| 35 9.95693 | 14 | 10.98197  | 147 | 9.96519   | 14 | 11.07865  | 177 | 35    | 10  | 21.6  | 1.5   |
| 36 .95707  | 14 | .98345    | 147 | .96533    | 13 | .08043    | 177 | 36    | 20  | 43.3  | 3.0   |
| 37 .95721  | 14 | .98492    | 148 | .96547    | 14 | .08221    | 178 | 37    | 30  | 65.0  | 4.5   |
| 38 .95735  | 14 | .98640    | 148 | .96560    | 13 | .08400    | 179 | 38    | 40  | 86.6  | 6.0   |
| 39 .95749  | 13 | .98789    | 149 | .96574    | 14 | .08579    | 179 | 39    | 50  | 108.3 | 7.5   |
| 40 9.95763 | 14 | 10.98938  | 149 | 9.96588   | 13 | 11.08759  | 180 | 40    | 6   | 0.7   | 0.6   |
| 41 .95777  | 14 | .99087    | 149 | .96601    | 14 | .08940    | 180 | 41    | 7   | 0.8   | 0.7   |
| 42 .95791  | 13 | .99237    | 150 | .96615    | 13 | .09121    | 181 | 42    | 8   | 0.9   | 0.8   |
| 43 .95804  | 14 | .99387    | 150 | .96629    | 14 | .09303    | 182 | 43    | 9   | 1.0   | 0.9   |
| 44 .95818  | 14 | .99538    | 151 | .96642    | 13 | .09486    | 182 | 44    | 10  | 1.1   | 1.0   |
| 45 9.95832 | 14 | 10.99689  | 151 | 9.96656   | 13 | 11.09669  | 183 | 45    | 20  | 2.3   | 2.0   |
| 46 .95846  | 13 | .99841    | 152 | .96669    | 14 | .09853    | 184 | 46    | 30  | 3.5   | 3.0   |
| 47 .95860  | 14 | 10.99993  | 152 | .96683    | 13 | .10038    | 185 | 47    | 40  | 4.6   | 4.0   |
| 48 .95874  | 14 | 11.00145  | 152 | .96697    | 14 | .10223    | 186 | 48    | 50  | 5.8   | 5.0   |
| 49 .95888  | 13 | .00298    | 153 | .96710    | 13 | .10409    | 186 | 49    | 6   | 0.7   | 0.6   |
| 50 9.95901 | 14 | 11.00451  | 153 | 9.96724   | 13 | 11.10595  | 187 | 50    | 7   | 0.8   | 0.7   |
| 51 .95915  | 13 | .00605    | 154 | .96737    | 14 | .10783    | 187 | 51    | 8   | 0.9   | 0.8   |
| 52 .95929  | 14 | .00759    | 154 | .96751    | 13 | .10971    | 188 | 52    | 9   | 1.0   | 0.9   |
| 53 .95943  | 14 | .00914    | 155 | .96764    | 14 | .11160    | 189 | 53    | 10  | 1.1   | 1.0   |
| 54 .95957  | 13 | .01069    | 155 | .96778    | 13 | .11349    | 189 | 54    | 20  | 2.3   | 2.0   |
| 55 9.95970 | 14 | 11.01225  | 155 | 9.96792   | 13 | 11.11539  | 190 | 55    | 30  | 3.5   | 3.0   |
| 56 .95984  | 14 | .01381    | 156 | .96805    | 14 | .11730    | 191 | 56    | 40  | 4.6   | 4.0   |
| 57 .95998  | 13 | .01537    | 156 | .96819    | 13 | .11922    | 191 | 57    | 50  | 5.8   | 5.0   |
| 58 .96012  | 14 | .01694    | 157 | .96832    | 14 | .12114    | 192 | 58    | 6   | 0.7   | 0.6   |
| 59 .96026  | 13 | .01852    | 157 | .96846    | 13 | .12307    | 193 | 59    | 7   | 0.8   | 0.7   |
| 60 9.96039 | 13 | 11.02010  | 158 | 9.96859   | 13 | 11.12501  | 193 | 60    | 8   | 0.9   | 0.8   |
| Lg. Vers.  | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D   |       |     |       |       |

P. P.



TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

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| Lg. Vers. |         | D  | Log. Exs. |     | D       | Lg. Vers. |          | D   | Log. Exs. |    | D     | P. P. |       |
|-----------|---------|----|-----------|-----|---------|-----------|----------|-----|-----------|----|-------|-------|-------|
| 0         | 9.96859 | 13 | 11.12501  | 195 | 9.97665 | 13        | 11.25785 | 255 | 0         |    |       | 250   | 240   |
| 1         | .96837  | 14 | .12696    | 195 | .97679  | 13        | .26040   | 256 | 1         | 6  | 25.0  | 24.0  |       |
| 2         | .96837  | 13 | .12891    | 196 | .97692  | 13        | .26297   | 257 | 2         | 7  | 29.1  | 28.0  |       |
| 3         | .96900  | 13 | .13087    | 196 | .97705  | 13        | .26554   | 259 | 3         | 8  | 33.3  | 32.0  |       |
| 4         | .96914  | 13 | .13284    | 198 | .97718  | 13        | .26814   | 260 | 4         | 9  | 37.5  | 36.0  |       |
| 5         | 9.96927 | 13 | 11.13482  | 198 | 9.97732 | 13        | 11.27074 | 262 | 5         | 10 | 41.6  | 40.0  |       |
| 6         | .96941  | 13 | .13680    | 199 | .97745  | 13        | .27336   | 263 | 6         | 20 | 83.3  | 80.0  |       |
| 7         | .96954  | 13 | .13879    | 200 | .97758  | 13        | .27599   | 265 | 7         | 30 | 125.0 | 120.0 |       |
| 8         | .96968  | 13 | .14079    | 201 | .97772  | 13        | .27864   | 266 | 8         | 40 | 166.6 | 160.0 |       |
| 9         | .96981  | 13 | .14280    | 201 | .97785  | 13        | .28131   | 267 | 9         | 50 | 208.3 | 200.0 |       |
| 10        | 9.96995 | 13 | 11.14482  | 202 | 9.97799 | 13        | 11.28398 | 268 | 10        |    |       | 230   | 220   |
| 11        | .97008  | 13 | .14684    | 203 | .97811  | 13        | .28668   | 270 | 11        | 6  | 23.0  | 22.0  |       |
| 12        | .97022  | 13 | .14887    | 204 | .97825  | 13        | .28938   | 272 | 12        | 7  | 26.8  | 25.6  |       |
| 13        | .97035  | 13 | .15092    | 205 | .97838  | 13        | .29211   | 274 | 13        | 8  | 30.6  | 29.3  |       |
| 14        | .97049  | 13 | .15297    | 205 | .97851  | 13        | .29485   | 275 | 14        | 9  | 34.5  | 33.0  |       |
| 15        | 9.97062 | 13 | 11.15502  | 206 | 9.97864 | 13        | 11.29760 | 277 | 15        | 10 | 38.3  | 36.6  |       |
| 16        | .97076  | 13 | .15709    | 208 | .97878  | 13        | .30037   | 278 | 16        | 20 | 76.6  | 73.3  |       |
| 17        | .97089  | 13 | .15917    | 208 | .97891  | 13        | .30316   | 280 | 17        | 30 | 115.0 | 110.0 |       |
| 18        | .97103  | 13 | .16125    | 209 | .97904  | 13        | .30596   | 282 | 18        | 40 | 153.3 | 146.6 |       |
| 19        | .97116  | 13 | .16334    | 210 | .97917  | 13        | .30878   | 283 | 19        | 50 | 191.6 | 183.3 |       |
| 20        | 9.97130 | 13 | 11.16544  | 211 | 9.97931 | 13        | 11.31162 | 285 | 20        |    |       | 210   | 200   |
| 21        | .97143  | 13 | .16755    | 212 | .97944  | 13        | .31447   | 287 | 21        | 6  | 21.0  | 20.0  |       |
| 22        | .97157  | 13 | .16967    | 213 | .97957  | 13        | .31734   | 288 | 22        | 7  | 24.5  | 23.3  |       |
| 23        | .97170  | 13 | .17180    | 214 | .97970  | 13        | .32023   | 290 | 23        | 8  | 28.0  | 26.6  |       |
| 24        | .97183  | 13 | .17394    | 214 | .97984  | 13        | .32313   | 292 | 24        | 9  | 31.5  | 30.0  |       |
| 25        | 9.97197 | 13 | 11.17609  | 215 | 9.97997 | 13        | 11.32606 | 294 | 25        | 10 | 35.0  | 33.3  |       |
| 26        | .97210  | 13 | .17824    | 216 | .98010  | 13        | .32900   | 296 | 26        | 20 | 70.0  | 66.6  |       |
| 27        | .97224  | 13 | .18041    | 218 | .98023  | 13        | .33196   | 298 | 27        | 30 | 105.0 | 100.0 |       |
| 28        | .97237  | 13 | .18259    | 218 | .98036  | 13        | .33494   | 299 | 28        | 40 | 140.0 | 133.3 |       |
| 29        | .97251  | 13 | .18477    | 219 | .98050  | 13        | .33793   | 301 | 29        | 50 | 175.0 | 166.6 |       |
| 30        | 9.97264 | 13 | 11.18697  | 220 | 9.98063 | 13        | 11.34095 | 303 | 30        |    |       | 190   | 4 3   |
| 31        | .97277  | 13 | .18917    | 221 | .98076  | 13        | .34398   | 305 | 31        | 6  | 19.0  | 0.4   | 0.3   |
| 32        | .97291  | 13 | .19138    | 222 | .98089  | 13        | .34704   | 307 | 32        | 7  | 22.1  | 0.4   | 0.3   |
| 33        | .97304  | 13 | .19361    | 223 | .98102  | 13        | .35011   | 309 | 33        | 8  | 25.3  | 0.5   | 0.4   |
| 34        | .97318  | 13 | .19584    | 224 | .98116  | 13        | .35321   | 311 | 34        | 9  | 28.5  | 0.6   | 0.4   |
| 35        | 9.97331 | 13 | 11.19809  | 225 | 9.98129 | 13        | 11.35632 | 313 | 35        | 10 | 31.6  | 0.6   | 0.5   |
| 36        | .97345  | 13 | .20034    | 227 | .98142  | 13        | .35946   | 315 | 36        | 20 | 63.3  | 1.3   | 1.0   |
| 37        | .97358  | 13 | .20261    | 227 | .98155  | 13        | .36261   | 318 | 37        | 30 | 95.0  | 2.0   | 1.5   |
| 38        | .97371  | 33 | .20489    | 228 | .98168  | 13        | .36579   | 320 | 38        | 40 | 126.6 | 2.6   | 2.0   |
| 39        | .97385  | 13 | .20717    | 230 | .98181  | 13        | .36899   | 322 | 39        | 50 | 158.3 | 3.3   | 2.5   |
| 40        | 9.97398 | 13 | 11.20947  | 230 | 9.98195 | 13        | 11.37221 | 324 | 40        |    |       | 2     | 1 0   |
| 41        | .97412  | 13 | .21178    | 232 | .98208  | 13        | .37546   | 326 | 41        | 6  | 0.2   | 0.1   | 0.0   |
| 42        | .97425  | 13 | .21410    | 233 | .98221  | 13        | .37872   | 328 | 42        | 7  | 0.2   | 0.1   | 0.0   |
| 43        | .97438  | 13 | .21643    | 234 | .98234  | 13        | .38201   | 331 | 43        | 8  | 0.2   | 0.1   | 0.0   |
| 44        | .97452  | 13 | .21877    | 235 | .98247  | 13        | .38532   | 333 | 44        | 9  | 0.3   | 0.1   | 0.1   |
| 45        | 9.97465 | 13 | 11.22112  | 236 | 9.98260 | 13        | 11.38866 | 335 | 45        | 10 | 0.3   | 0.1   | 0.1   |
| 46        | .97478  | 13 | .22349    | 237 | .98273  | 13        | .39201   | 338 | 46        | 20 | 0.6   | 0.3   | 0.1   |
| 47        | .97492  | 13 | .22586    | 239 | .98287  | 13        | .39540   | 340 | 47        | 30 | 1.0   | 0.5   | 0.2   |
| 48        | .97505  | 13 | .22825    | 239 | .98300  | 13        | .39880   | 343 | 48        | 40 | 1.3   | 0.6   | 0.3   |
| 49        | .97519  | 13 | .23065    | 241 | .98313  | 13        | .40224   | 345 | 49        | 50 | 1.6   | 0.8   | 0.4   |
| 50        | 9.97532 | 13 | 11.23306  | 242 | 9.98326 | 13        | 11.40569 | 348 | 50        |    |       | 14    | 13 13 |
| 51        | .97545  | 13 | .23548    | 243 | .98339  | 13        | .40918   | 351 | 51        | 6  | 1.4   | 1.3   | 1.3   |
| 52        | .97559  | 13 | .23792    | 245 | .98352  | 13        | .41269   | 353 | 52        | 7  | 1.6   | 1.6   | 1.5   |
| 53        | .97572  | 13 | .24037    | 246 | .98365  | 13        | .41622   | 356 | 53        | 8  | 1.8   | 1.8   | 1.7   |
| 54        | .97585  | 13 | .24283    | 247 | .98378  | 13        | .41979   | 359 | 54        | 9  | 2.1   | 2.0   | 1.9   |
| 55        | 9.97599 | 13 | 11.24530  | 248 | 9.98392 | 13        | 11.42338 | 361 | 55        | 10 | 2.3   | 2.2   | 2.1   |
| 56        | .97612  | 13 | .24778    | 250 | .98405  | 13        | .42699   | 364 | 56        | 20 | 4.6   | 4.5   | 4.3   |
| 57        | .97625  | 13 | .25028    | 251 | .98418  | 13        | .43064   | 367 | 57        | 30 | 7.0   | 6.7   | 6.5   |
| 58        | .97639  | 13 | .25279    | 252 | .98431  | 13        | .43431   | 370 | 58        | 40 | 9.3   | 9.0   | 8.6   |
| 59        | .97652  | 13 | .25531    | 254 | .98444  | 13        | .43802   | 373 | 59        | 50 | 11.6  | 11.2  | 10.8  |
| 60        | 9.97665 | 13 | 11.25785  | 254 | 9.98457 | 13        | 11.44175 | 376 | 60        |    |       |       |       |
| Lg. Vers. |         | D  | Log. Exs. |     | D       | Lg. Vers. |          | D   | Log. Exs. |    | D     | P. P. |       |

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS

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|    | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D     |    | P. P. |
|----|-----------|----|-----------|-----|-----------|----|-----------|-------|----|-------|
| 0  | 9.98457   | 13 | 11.44175  | 376 | 9.99235   | 12 | 11.75050  | 742   | 0  |       |
| 1  | .98470    | 13 | .44551    | 379 | .99248    | 13 | .75792    | 755   | 1  |       |
| 2  | .98483    | 13 | .44931    | 382 | .99261    | 13 | .76547    | 768   | 2  |       |
| 3  | .98496    | 13 | .45313    | 386 | .99274    | 13 | .77316    | 781   | 3  |       |
| 4  | .98509    | 13 | .45699    | 389 | .99287    | 13 | .78097    | 795   | 4  |       |
| 5  | 9.98522   | 13 | 11.46088  | 392 | 9.99299   | 12 | 11.78892  | 809   | 5  |       |
| 6  | .98535    | 13 | .46480    | 395 | .99312    | 13 | .79702    | 825   | 6  |       |
| 7  | .98548    | 13 | .46876    | 399 | .99325    | 12 | .80527    | 840   | 7  |       |
| 8  | .98562    | 13 | .47275    | 402 | .99338    | 13 | .81367    | 856   | 8  |       |
| 9  | .98575    | 13 | .47677    | 406 | .99351    | 12 | .82223    | 872   | 9  |       |
| 10 | 9.98588   | 13 | 11.48083  | 409 | 9.99363   | 13 | 11.83095  | 890   | 10 |       |
| 11 | .98601    | 13 | .48493    | 413 | .99376    | 13 | .83986    | 908   | 11 |       |
| 12 | .98614    | 13 | .48906    | 417 | .99389    | 13 | .84894    | 927   | 12 |       |
| 13 | .98627    | 13 | .49323    | 420 | .99402    | 12 | .85821    | 947   | 13 |       |
| 14 | .98640    | 13 | .49743    | 425 | .99415    | 13 | .86768    | 967   | 14 |       |
| 15 | 9.98653   | 13 | 11.50168  | 428 | 9.99428   | 12 | 11.87735  | 989   | 15 |       |
| 16 | .98666    | 13 | .50597    | 432 | .99440    | 13 | .88724    | 1009  | 16 |       |
| 17 | .98679    | 13 | .51029    | 436 | .99453    | 12 | .89735    | 1034  | 17 |       |
| 18 | .98692    | 13 | .51466    | 440 | .99466    | 13 | .90769    | 1059  | 18 |       |
| 19 | .98705    | 13 | .51906    | 445 | .99479    | 12 | .91829    | 1085  | 19 |       |
| 20 | 9.98718   | 13 | 11.52351  | 449 | 9.99491   | 13 | 11.92914  | 1112  | 20 |       |
| 21 | .98731    | 13 | .52801    | 454 | .99504    | 13 | .94026    | 1140  | 21 |       |
| 22 | .98744    | 13 | .53255    | 458 | .99517    | 12 | .95167    | 1171  | 22 |       |
| 23 | .98757    | 13 | .53713    | 463 | .99530    | 13 | .96338    | 1203  | 23 |       |
| 24 | .98770    | 13 | .54176    | 467 | .99543    | 12 | .97541    | 1236  | 24 |       |
| 25 | 9.98783   | 13 | 11.54643  | 472 | 9.99555   | 13 | 11.98777  | 1271  | 25 |       |
| 26 | .98796    | 13 | .55116    | 477 | .99568    | 13 | 12.00048  | 1309  | 26 |       |
| 27 | .98809    | 13 | .55593    | 482 | .99581    | 12 | .01358    | 1349  | 27 |       |
| 28 | .98822    | 13 | .56076    | 487 | .99594    | 13 | .02707    | 1391  | 28 |       |
| 29 | .98835    | 13 | .56563    | 492 | .99606    | 12 | .04098    | 1436  | 29 |       |
| 30 | 9.98848   | 13 | 11.57058  | 498 | 9.99619   | 13 | 12.05535  | 1485  | 30 |       |
| 31 | .98861    | 13 | .57554    | 504 | .99632    | 12 | .07020    | 1537  | 31 |       |
| 32 | .98874    | 13 | .58058    | 509 | .99645    | 13 | .08557    | 1592  | 32 |       |
| 33 | .98887    | 13 | .58567    | 515 | .99657    | 12 | .10149    | 1652  | 33 |       |
| 34 | .98900    | 13 | .59082    | 520 | .99670    | 13 | .11801    | 1716  | 34 |       |
| 35 | 9.98913   | 12 | 11.59602  | 527 | 9.99683   | 12 | 12.13517  | 1785  | 35 |       |
| 36 | .98925    | 13 | .60129    | 533 | .99695    | 13 | .15302    | 1861  | 36 |       |
| 37 | .98938    | 13 | .60662    | 539 | .99708    | 12 | .17163    | 1943  | 37 |       |
| 38 | .98951    | 13 | .61202    | 545 | .99721    | 13 | .19106    | 2033  | 38 |       |
| 39 | .98964    | 13 | .61747    | 552 | .99734    | 12 | .21139    | 2131  | 39 |       |
| 40 | 9.98977   | 13 | 11.62300  | 559 | 9.99746   | 13 | 12.23271  | 2240  | 40 |       |
| 41 | .98990    | 13 | .62859    | 566 | .99759    | 12 | .25511    | 2361  | 41 |       |
| 42 | .99003    | 13 | .63425    | 573 | .99772    | 12 | .27872    | 2495  | 42 |       |
| 43 | .99016    | 12 | .63998    | 581 | .99784    | 13 | .30367    | 2645  | 43 |       |
| 44 | .99029    | 13 | .64579    | 588 | .99797    | 12 | .33013    | 2815  | 44 |       |
| 45 | 9.99042   | 13 | 11.65167  | 595 | 9.99810   | 13 | 12.35828  | 3009  | 45 |       |
| 46 | .99055    | 13 | .65762    | 604 | .99823    | 12 | .38837    | 3231  | 46 |       |
| 47 | .99068    | 13 | .66366    | 611 | .99835    | 12 | .42068    | 3489  | 47 |       |
| 48 | .99081    | 12 | .66978    | 620 | .99848    | 13 | .45557    | 3791  | 48 |       |
| 49 | .99093    | 13 | .67598    | 628 | .99861    | 12 | .49349    | 4152  | 49 |       |
| 50 | 9.99106   | 13 | 11.68227  | 638 | 9.99873   | 12 | 12.53501  | 4588  | 50 |       |
| 51 | .99119    | 13 | .68865    | 646 | .99886    | 13 | .58089    | 5127  | 51 |       |
| 52 | .99132    | 13 | .69511    | 655 | .99899    | 12 | .63217    | 5812  | 52 |       |
| 53 | .99145    | 12 | .70168    | 666 | .99911    | 12 | .69029    | 6707  | 53 |       |
| 54 | .99158    | 13 | .70834    | 675 | .99924    | 13 | .75736    | 7931  | 54 |       |
| 55 | 9.99171   | 13 | 11.71509  | 686 | 9.99937   | 12 | 12.83667  | 9704  | 55 |       |
| 56 | .99184    | 13 | .72196    | 696 | .99949    | 12 | .93371    | 12506 | 56 |       |
| 57 | .99197    | 12 | .72892    | 707 | .99962    | 13 | 13.05877  | 17621 | 57 |       |
| 58 | .99209    | 13 | .73600    | 719 | .99974    | 12 | .23499    | 30118 | 58 |       |
| 59 | .99222    | 13 | .74319    | 730 | .99987    | 13 | .53615    |       | 59 |       |
| 60 | 9.99235   | 18 | 11.75050  | 730 | 10.00000  | 12 | Infinity  |       | 60 |       |
|    | Lg. Vers. | D  | Log. Exs. | D   | Lg. Vers. | D  | Log. Exs. | D     |    | P. P. |

|    |      |      |
|----|------|------|
|    | 13   | 13   |
| 6  | 1.3  | 1.3  |
| 7  | 1.6  | 1.5  |
| 8  | 1.8  | 1.7  |
| 9  | 2.0  | 1.9  |
| 10 | 2.2  | 2.1  |
| 20 | 4.5  | 4.3  |
| 30 | 6.7  | 6.5  |
| 40 | 9.0  | 8.6  |
| 50 | 11.2 | 10.8 |

|    |      |
|----|------|
|    | 12   |
| 6  | 1.2  |
| 7  | 1.4  |
| 8  | 1.6  |
| 9  | 1.9  |
| 10 | 2.1  |
| 20 | 4.1  |
| 30 | 6.1  |
| 40 | 8.3  |
| 50 | 10.4 |

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

0°-10°

| °    | Sin.   | d. | Tan.   | d. | Cot.    | d.     | Cos.   | d. | P. P. |
|------|--------|----|--------|----|---------|--------|--------|----|-------|
| 0 0  | 0.0000 | 29 | 0.0000 | 29 | ∞       |        | 1.0000 | 0  | 0 90  |
| 10   | 0.0029 | 29 | 0.0029 | 29 | 343.773 |        | 1.0000 | 0  | 50    |
| 20   | 0.0058 | 29 | 0.0058 | 29 | 171.885 |        | 1.0000 | 0  | 40    |
| 30   | 0.0087 | 29 | 0.0087 | 29 | 114.588 |        | 0.9999 | 0  | 30    |
| 40   | 0.0116 | 29 | 0.0116 | 29 | 85.9398 |        | 0.9999 | 0  | 20    |
| 50   | 0.0145 | 29 | 0.0145 | 29 | 68.7501 |        | 0.9999 | 0  | 10    |
| 1 0  | 0.0174 | 29 | 0.0174 | 29 | 57.2899 | 8.1860 | 0.9998 | 0  | 0 89  |
| 10   | 0.0203 | 29 | 0.0203 | 29 | 49.1039 | 6.1398 | 0.9998 | 0  | 50    |
| 20   | 0.0232 | 29 | 0.0233 | 29 | 42.9641 | 4.7756 | 0.9997 | 0  | 40    |
| 30   | 0.0262 | 29 | 0.0262 | 29 | 38.1884 | 3.8207 | 0.9996 | 1  | 30    |
| 40   | 0.0291 | 29 | 0.0291 | 29 | 34.3677 | 3.1261 | 0.9996 | 1  | 20    |
| 50   | 0.0320 | 29 | 0.0320 | 29 | 31.2416 | 2.6053 | 0.9995 | 1  | 10    |
| 2 0  | 0.0349 | 29 | 0.0349 | 29 | 28.6362 | 2.2046 | 0.9994 | 1  | 0 88  |
| 10   | 0.0378 | 29 | 0.0378 | 29 | 26.4316 | 1.8898 | 0.9993 | 1  | 50    |
| 20   | 0.0407 | 29 | 0.0407 | 29 | 24.5417 | 1.6390 | 0.9991 | 1  | 40    |
| 30   | 0.0436 | 29 | 0.0436 | 29 | 22.9037 | 1.4333 | 0.9990 | 1  | 30    |
| 40   | 0.0465 | 29 | 0.0466 | 29 | 21.4704 | 1.2648 | 0.9989 | 1  | 20    |
| 50   | 0.0494 | 29 | 0.0495 | 29 | 20.2055 | 1.1244 | 0.9988 | 1  | 10    |
| 3 0  | 0.0523 | 29 | 0.0521 | 29 | 19.0811 | 1.0061 | 0.9986 | 2  | 0 87  |
| 10   | 0.0552 | 29 | 0.0553 | 29 | 18.0750 | 0.9056 | 0.9984 | 2  | 50    |
| 20   | 0.0581 | 29 | 0.0582 | 29 | 17.1693 | 0.8195 | 0.9983 | 2  | 40    |
| 30   | 0.0610 | 29 | 0.0611 | 29 | 16.3498 | 0.7456 | 0.9981 | 2  | 30    |
| 40   | 0.0639 | 29 | 0.0641 | 29 | 15.6048 | 0.6804 | 0.9979 | 2  | 20    |
| 50   | 0.0668 | 29 | 0.0670 | 29 | 14.9244 | 0.6237 | 0.9977 | 2  | 10    |
| 4 0  | 0.0697 | 29 | 0.0699 | 29 | 14.3006 | 0.5739 | 0.9975 | 2  | 0 86  |
| 10   | 0.0726 | 29 | 0.0728 | 29 | 13.7267 | 0.5298 | 0.9973 | 2  | 50    |
| 20   | 0.0755 | 29 | 0.0758 | 29 | 13.1969 | 0.4907 | 0.9971 | 2  | 40    |
| 30   | 0.0784 | 29 | 0.0787 | 29 | 12.7062 | 0.4557 | 0.9969 | 2  | 30    |
| 40   | 0.0813 | 29 | 0.0816 | 29 | 12.2505 | 0.4243 | 0.9967 | 2  | 20    |
| 50   | 0.0842 | 29 | 0.0845 | 29 | 11.8261 | 0.3961 | 0.9964 | 2  | 10    |
| 5 0  | 0.0871 | 29 | 0.0875 | 29 | 11.4300 | 0.3706 | 0.9962 | 2  | 0 85  |
| 10   | 0.0900 | 29 | 0.0904 | 29 | 11.0594 | 0.3475 | 0.9959 | 3  | 50    |
| 20   | 0.0929 | 29 | 0.0933 | 29 | 10.7119 | 0.3265 | 0.9956 | 3  | 40    |
| 30   | 0.0958 | 29 | 0.0963 | 29 | 10.3854 | 0.3073 | 0.9954 | 3  | 30    |
| 40   | 0.0987 | 29 | 0.0992 | 29 | 10.0780 | 0.2899 | 0.9951 | 3  | 20    |
| 50   | 0.1016 | 29 | 0.1021 | 29 | 9.7881  | 0.2738 | 0.9948 | 3  | 10    |
| 6 0  | 0.1045 | 28 | 0.1051 | 29 | 9.5143  | 0.2590 | 0.9945 | 3  | 0 84  |
| 10   | 0.1074 | 29 | 0.1080 | 29 | 9.2553  | 0.2454 | 0.9942 | 3  | 50    |
| 20   | 0.1103 | 29 | 0.1110 | 29 | 9.0098  | 0.2329 | 0.9939 | 3  | 40    |
| 30   | 0.1132 | 29 | 0.1139 | 29 | 8.7769  | 0.2213 | 0.9935 | 3  | 30    |
| 40   | 0.1161 | 29 | 0.1169 | 29 | 8.5555  | 0.2106 | 0.9932 | 3  | 20    |
| 50   | 0.1190 | 28 | 0.1198 | 29 | 8.3449  | 0.2006 | 0.9929 | 3  | 10    |
| 7 0  | 0.1218 | 29 | 0.1228 | 29 | 8.1443  | 0.1913 | 0.9925 | 3  | 0 83  |
| 10   | 0.1247 | 29 | 0.1257 | 29 | 7.9530  | 0.1826 | 0.9922 | 4  | 50    |
| 20   | 0.1276 | 29 | 0.1287 | 29 | 7.7703  | 0.1746 | 0.9918 | 4  | 40    |
| 30   | 0.1305 | 29 | 0.1316 | 29 | 7.5957  | 0.1670 | 0.9914 | 4  | 30    |
| 40   | 0.1334 | 28 | 0.1346 | 30 | 7.4287  | 0.1599 | 0.9910 | 4  | 20    |
| 50   | 0.1363 | 28 | 0.1376 | 29 | 7.2687  | 0.1534 | 0.9906 | 4  | 10    |
| 8 0  | 0.1391 | 29 | 0.1405 | 29 | 7.1153  | 0.1471 | 0.9902 | 4  | 0 82  |
| 10   | 0.1420 | 29 | 0.1435 | 30 | 6.9682  | 0.1413 | 0.9898 | 4  | 50    |
| 20   | 0.1449 | 28 | 0.1465 | 29 | 6.8269  | 0.1358 | 0.9894 | 4  | 40    |
| 30   | 0.1478 | 29 | 0.1494 | 30 | 6.6911  | 0.1306 | 0.9890 | 4  | 30    |
| 40   | 0.1507 | 28 | 0.1524 | 29 | 6.5605  | 0.1257 | 0.9886 | 4  | 20    |
| 50   | 0.1535 | 29 | 0.1554 | 30 | 6.4348  | 0.1211 | 0.9881 | 4  | 10    |
| 9 0  | 0.1561 | 29 | 0.1584 | 29 | 6.3137  | 0.1167 | 0.9877 | 4  | 0 81  |
| 10   | 0.1593 | 28 | 0.1613 | 30 | 6.1970  | 0.1126 | 0.9872 | 5  | 50    |
| 20   | 0.1622 | 28 | 0.1643 | 30 | 6.0844  | 0.1087 | 0.9867 | 5  | 40    |
| 30   | 0.1650 | 29 | 0.1673 | 30 | 5.9757  | 0.1049 | 0.9863 | 5  | 30    |
| 40   | 0.1679 | 28 | 0.1703 | 30 | 5.8708  | 0.1014 | 0.9858 | 5  | 20    |
| 50   | 0.1708 | 28 | 0.1733 | 30 | 5.7693  | 0.980  | 0.9853 | 5  | 10    |
| 10 0 | 0.1736 | 28 | 0.1763 | 30 | 5.6713  | 0.9848 | 0.9848 | 5  | 0 80  |
|      | Cos.   | d. | Cot.   | d. | Tan.    | d.     | Sin.   | d. | P. P. |

80°-90°

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

10°-20°

| °    | Sin.   | d. | Tan.   | d. | Cot.   | d.  | Cos.   | d.     | P. P. |
|------|--------|----|--------|----|--------|-----|--------|--------|-------|
| 10 0 | 0.1736 | 28 | 0.1763 | 30 | 5.6713 | 949 | 0.9848 | 5 80   |       |
| 10   | 0.1765 | 28 | 0.1793 | 30 | 5.5764 | 919 | 0.9843 | 5 50   |       |
| 20   | 0.1793 | 28 | 0.1823 | 30 | 5.4845 | 890 | 0.9838 | 5 40   |       |
| 30   | 0.1822 | 28 | 0.1853 | 30 | 5.3955 | 862 | 0.9832 | 5 30   |       |
| 40   | 0.1851 | 28 | 0.1883 | 30 | 5.3093 | 836 | 0.9827 | 5 20   |       |
| 50   | 0.1879 | 28 | 0.1913 | 30 | 5.2256 | 811 | 0.9822 | 5 10   |       |
| 11 0 | 0.1908 | 28 | 0.1944 | 30 | 5.1445 | 787 | 0.9816 | 5 0 79 |       |
| 10   | 0.1936 | 28 | 0.1974 | 30 | 5.0658 | 764 | 0.9810 | 6 50   |       |
| 20   | 0.1965 | 28 | 0.2004 | 30 | 4.9894 | 742 | 0.9805 | 6 40   |       |
| 30   | 0.1993 | 28 | 0.2034 | 30 | 4.9151 | 721 | 0.9799 | 6 30   |       |
| 40   | 0.2022 | 28 | 0.2065 | 30 | 4.8430 | 701 | 0.9793 | 6 20   |       |
| 50   | 0.2050 | 28 | 0.2095 | 30 | 4.7728 | 682 | 0.9787 | 6 10   |       |
| 12 0 | 0.2079 | 28 | 0.2125 | 30 | 4.7046 | 664 | 0.9781 | 6 0 78 |       |
| 10   | 0.2107 | 28 | 0.2156 | 30 | 4.6382 | 646 | 0.9775 | 6 50   |       |
| 20   | 0.2136 | 28 | 0.2186 | 30 | 4.5736 | 629 | 0.9769 | 6 40   |       |
| 30   | 0.2164 | 28 | 0.2217 | 30 | 4.5107 | 613 | 0.9763 | 6 30   |       |
| 40   | 0.2193 | 28 | 0.2247 | 30 | 4.4494 | 597 | 0.9756 | 6 20   |       |
| 50   | 0.2221 | 28 | 0.2278 | 30 | 4.3897 | 582 | 0.9750 | 6 10   |       |
| 13 0 | 0.2249 | 28 | 0.2308 | 31 | 4.3315 | 568 | 0.9743 | 6 0 77 |       |
| 10   | 0.2278 | 28 | 0.2339 | 30 | 4.2747 | 553 | 0.9737 | 6 50   |       |
| 20   | 0.2306 | 28 | 0.2370 | 31 | 4.2193 | 540 | 0.9730 | 6 40   |       |
| 30   | 0.2334 | 28 | 0.2401 | 30 | 4.1653 | 527 | 0.9723 | 6 30   |       |
| 40   | 0.2362 | 28 | 0.2431 | 31 | 4.1125 | 515 | 0.9717 | 6 20   |       |
| 50   | 0.2391 | 28 | 0.2462 | 31 | 4.0610 | 502 | 0.9710 | 6 10   |       |
| 14 0 | 0.2419 | 28 | 0.2493 | 30 | 4.0108 | 491 | 0.9703 | 6 0 76 |       |
| 10   | 0.2447 | 28 | 0.2524 | 31 | 3.9616 | 480 | 0.9696 | 7 50   |       |
| 20   | 0.2475 | 28 | 0.2555 | 31 | 3.9136 | 469 | 0.9688 | 7 40   |       |
| 30   | 0.2504 | 28 | 0.2586 | 31 | 3.8667 | 458 | 0.9681 | 7 30   |       |
| 40   | 0.2532 | 28 | 0.2617 | 31 | 3.8208 | 449 | 0.9674 | 7 20   |       |
| 50   | 0.2560 | 28 | 0.2648 | 31 | 3.7759 | 439 | 0.9666 | 7 10   |       |
| 15 0 | 0.2588 | 28 | 0.2679 | 31 | 3.7320 | 429 | 0.9659 | 7 0 75 |       |
| 10   | 0.2616 | 28 | 0.2710 | 31 | 3.6891 | 420 | 0.9651 | 8 50   |       |
| 20   | 0.2644 | 28 | 0.2742 | 31 | 3.6470 | 411 | 0.9644 | 8 40   |       |
| 30   | 0.2672 | 28 | 0.2773 | 31 | 3.6059 | 403 | 0.9636 | 8 30   |       |
| 40   | 0.2700 | 28 | 0.2804 | 31 | 3.5655 | 394 | 0.9628 | 8 20   |       |
| 50   | 0.2728 | 28 | 0.2836 | 31 | 3.5261 | 387 | 0.9620 | 8 10   |       |
| 16 0 | 0.2756 | 28 | 0.2867 | 31 | 3.4874 | 379 | 0.9612 | 8 0 74 |       |
| 10   | 0.2784 | 28 | 0.2899 | 31 | 3.4495 | 371 | 0.9604 | 8 50   |       |
| 20   | 0.2812 | 28 | 0.2930 | 31 | 3.4123 | 364 | 0.9596 | 8 40   |       |
| 30   | 0.2840 | 28 | 0.2962 | 32 | 3.3759 | 357 | 0.9588 | 8 30   |       |
| 40   | 0.2868 | 28 | 0.2994 | 31 | 3.3402 | 350 | 0.9580 | 8 20   |       |
| 50   | 0.2896 | 27 | 0.3025 | 32 | 3.3052 | 343 | 0.9571 | 8 10   |       |
| 17 0 | 0.2923 | 28 | 0.3057 | 31 | 3.2708 | 337 | 0.9563 | 8 0 73 |       |
| 10   | 0.2951 | 28 | 0.3089 | 32 | 3.2371 | 331 | 0.9554 | 8 50   |       |
| 20   | 0.2979 | 27 | 0.3121 | 32 | 3.2040 | 324 | 0.9546 | 8 40   |       |
| 30   | 0.3007 | 28 | 0.3153 | 32 | 3.1716 | 319 | 0.9537 | 8 30   |       |
| 40   | 0.3035 | 27 | 0.3185 | 32 | 3.1397 | 313 | 0.9528 | 8 20   |       |
| 50   | 0.3062 | 27 | 0.3217 | 32 | 3.1084 | 307 | 0.9519 | 8 10   |       |
| 18 0 | 0.3090 | 28 | 0.3249 | 32 | 3.0777 | 302 | 0.9510 | 8 0 72 |       |
| 10   | 0.3118 | 27 | 0.3281 | 32 | 3.0475 | 296 | 0.9501 | 9 50   |       |
| 20   | 0.3145 | 27 | 0.3313 | 32 | 3.0178 | 291 | 0.9492 | 9 40   |       |
| 30   | 0.3173 | 27 | 0.3346 | 32 | 2.9887 | 286 | 0.9483 | 9 30   |       |
| 40   | 0.3200 | 27 | 0.3378 | 32 | 2.9600 | 281 | 0.9474 | 9 20   |       |
| 50   | 0.3228 | 27 | 0.3411 | 32 | 2.9319 | 277 | 0.9464 | 9 10   |       |
| 19 0 | 0.3255 | 27 | 0.3443 | 32 | 2.9042 | 272 | 0.9455 | 9 0 71 |       |
| 10   | 0.3283 | 27 | 0.3476 | 32 | 2.8770 | 267 | 0.9445 | 9 50   |       |
| 20   | 0.3310 | 27 | 0.3508 | 32 | 2.8502 | 263 | 0.9436 | 9 40   |       |
| 30   | 0.3338 | 27 | 0.3541 | 33 | 2.8239 | 259 | 0.9426 | 9 30   |       |
| 40   | 0.3365 | 27 | 0.3574 | 33 | 2.7980 | 254 | 0.9416 | 9 20   |       |
| 50   | 0.3393 | 27 | 0.3607 | 32 | 2.7725 | 250 | 0.9407 | 9 10   |       |
| 20 0 | 0.3420 | 27 | 0.3639 | 32 | 2.7475 | 250 | 0.9397 | 9 0 70 |       |
|      | Cos.   | d. | Cot.   | d. | Tan.   | d.  | Sin.   | d.     | P. P. |



TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

20°-30°

| °    | Sin.   | d. | Tan.   | d. | Cot.   | d.  | Cos.   | d. | P. P. |
|------|--------|----|--------|----|--------|-----|--------|----|-------|
| 20 0 | 0.3420 | 27 | 0.3639 | 33 | 2.7475 | 247 | 0.9397 | 10 | 0 70  |
| 10   | 0.3447 | 27 | 0.3672 | 33 | 2.7228 | 242 | 0.9387 | 10 | 50    |
| 20   | 0.3475 | 27 | 0.3705 | 33 | 2.6985 | 239 | 0.9377 | 10 | 40    |
| 30   | 0.3502 | 27 | 0.3739 | 33 | 2.6746 | 235 | 0.9366 | 10 | 30    |
| 40   | 0.3529 | 27 | 0.3772 | 33 | 2.6511 | 232 | 0.9355 | 10 | 20    |
| 50   | 0.3556 | 27 | 0.3805 | 33 | 2.6279 | 228 | 0.9346 | 10 | 10    |
| 21 0 | 0.3583 | 27 | 0.3838 | 33 | 2.6051 | 225 | 0.9336 | 10 | 0 69  |
| 10   | 0.3611 | 27 | 0.3872 | 33 | 2.5826 | 221 | 0.9325 | 10 | 50    |
| 20   | 0.3638 | 27 | 0.3905 | 33 | 2.5604 | 218 | 0.9315 | 10 | 40    |
| 30   | 0.3665 | 27 | 0.3939 | 33 | 2.5386 | 215 | 0.9304 | 10 | 30    |
| 40   | 0.3692 | 27 | 0.3972 | 33 | 2.5171 | 212 | 0.9293 | 10 | 20    |
| 50   | 0.3719 | 27 | 0.4006 | 34 | 2.4959 | 208 | 0.9282 | 10 | 10    |
| 22 0 | 0.3746 | 27 | 0.4040 | 33 | 2.4751 | 206 | 0.9272 | 11 | 0 68  |
| 10   | 0.3773 | 27 | 0.4074 | 34 | 2.4545 | 203 | 0.9261 | 11 | 50    |
| 20   | 0.3800 | 27 | 0.4108 | 34 | 2.4342 | 200 | 0.9250 | 11 | 40    |
| 30   | 0.3827 | 26 | 0.4142 | 34 | 2.4142 | 197 | 0.9239 | 11 | 30    |
| 40   | 0.3853 | 27 | 0.4176 | 34 | 2.3945 | 194 | 0.9227 | 11 | 20    |
| 50   | 0.3880 | 27 | 0.4210 | 34 | 2.3750 | 192 | 0.9216 | 11 | 10    |
| 23 0 | 0.3907 | 26 | 0.4244 | 34 | 2.3558 | 189 | 0.9205 | 11 | 0 67  |
| 10   | 0.3934 | 27 | 0.4279 | 34 | 2.3369 | 187 | 0.9193 | 11 | 50    |
| 20   | 0.3961 | 26 | 0.4313 | 34 | 2.3182 | 184 | 0.9182 | 11 | 40    |
| 30   | 0.3987 | 26 | 0.4348 | 35 | 2.2998 | 182 | 0.9170 | 11 | 30    |
| 40   | 0.4014 | 27 | 0.4383 | 34 | 2.2816 | 179 | 0.9159 | 12 | 20    |
| 50   | 0.4041 | 26 | 0.4417 | 35 | 2.2637 | 177 | 0.9147 | 12 | 10    |
| 24 0 | 0.4067 | 26 | 0.4452 | 34 | 2.2460 | 175 | 0.9135 | 12 | 0 66  |
| 10   | 0.4094 | 26 | 0.4487 | 35 | 2.2285 | 172 | 0.9123 | 12 | 50    |
| 20   | 0.4120 | 26 | 0.4522 | 35 | 2.2113 | 170 | 0.9111 | 12 | 40    |
| 30   | 0.4147 | 26 | 0.4557 | 35 | 2.1943 | 168 | 0.9099 | 12 | 30    |
| 40   | 0.4173 | 26 | 0.4592 | 35 | 2.1775 | 166 | 0.9087 | 12 | 20    |
| 50   | 0.4200 | 26 | 0.4627 | 35 | 2.1609 | 164 | 0.9075 | 12 | 10    |
| 25 0 | 0.4226 | 26 | 0.4663 | 35 | 2.1445 | 162 | 0.9063 | 12 | 0 65  |
| 10   | 0.4252 | 26 | 0.4698 | 35 | 2.1283 | 159 | 0.9050 | 12 | 50    |
| 20   | 0.4279 | 26 | 0.4734 | 36 | 2.1123 | 158 | 0.9038 | 12 | 40    |
| 30   | 0.4305 | 26 | 0.4770 | 35 | 2.0965 | 156 | 0.9026 | 12 | 30    |
| 40   | 0.4331 | 26 | 0.4805 | 36 | 2.0809 | 154 | 0.9013 | 12 | 20    |
| 50   | 0.4357 | 26 | 0.4841 | 36 | 2.0655 | 152 | 0.9000 | 12 | 10    |
| 26 0 | 0.4383 | 26 | 0.4877 | 36 | 2.0503 | 150 | 0.8988 | 13 | 0 64  |
| 10   | 0.4410 | 26 | 0.4913 | 36 | 2.0352 | 148 | 0.8975 | 13 | 50    |
| 20   | 0.4436 | 26 | 0.4949 | 36 | 2.0204 | 147 | 0.8962 | 13 | 40    |
| 30   | 0.4462 | 26 | 0.4986 | 36 | 2.0057 | 145 | 0.8949 | 13 | 30    |
| 40   | 0.4488 | 26 | 0.5022 | 36 | 1.9911 | 143 | 0.8936 | 13 | 20    |
| 50   | 0.4514 | 26 | 0.5058 | 37 | 1.9768 | 142 | 0.8923 | 13 | 10    |
| 27 0 | 0.4540 | 26 | 0.5095 | 36 | 1.9626 | 140 | 0.8910 | 13 | 0 63  |
| 10   | 0.4566 | 25 | 0.5132 | 37 | 1.9486 | 139 | 0.8897 | 13 | 50    |
| 20   | 0.4591 | 26 | 0.5169 | 36 | 1.9347 | 137 | 0.8883 | 13 | 40    |
| 30   | 0.4617 | 26 | 0.5205 | 37 | 1.9210 | 136 | 0.8870 | 13 | 30    |
| 40   | 0.4643 | 25 | 0.5242 | 37 | 1.9074 | 134 | 0.8856 | 13 | 20    |
| 50   | 0.4669 | 25 | 0.5280 | 37 | 1.8940 | 132 | 0.8843 | 13 | 10    |
| 28 0 | 0.4694 | 26 | 0.5317 | 37 | 1.8807 | 131 | 0.8829 | 13 | 0 62  |
| 10   | 0.4720 | 25 | 0.5354 | 37 | 1.8676 | 130 | 0.8816 | 14 | 50    |
| 20   | 0.4746 | 25 | 0.5392 | 37 | 1.8546 | 128 | 0.8802 | 14 | 40    |
| 30   | 0.4771 | 25 | 0.5429 | 38 | 1.8417 | 127 | 0.8788 | 13 | 30    |
| 40   | 0.4797 | 25 | 0.5467 | 37 | 1.8290 | 125 | 0.8774 | 14 | 20    |
| 50   | 0.4822 | 25 | 0.5505 | 38 | 1.8165 | 124 | 0.8760 | 14 | 10    |
| 29 0 | 0.4848 | 25 | 0.5543 | 38 | 1.8040 | 123 | 0.8746 | 14 | 0 61  |
| 10   | 0.4873 | 25 | 0.5581 | 38 | 1.7917 | 122 | 0.8732 | 14 | 50    |
| 20   | 0.4899 | 25 | 0.5619 | 38 | 1.7795 | 120 | 0.8718 | 14 | 40    |
| 30   | 0.4924 | 25 | 0.5657 | 38 | 1.7675 | 119 | 0.8703 | 14 | 30    |
| 40   | 0.4949 | 25 | 0.5696 | 39 | 1.7555 | 118 | 0.8689 | 14 | 20    |
| 50   | 0.4975 | 25 | 0.5735 | 38 | 1.7437 | 117 | 0.8675 | 14 | 10    |
| 30 0 | 0.5000 | d. | 0.5773 | d. | 1.7320 | d.  | 0.8660 | d. | 0 60  |
|      | Cos.   | d. | Cot.   | d. | Tan.   | d.  | Sin.   | d. | P. P. |

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TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

30°-40°

| °    | Sin.   | d. | Tan.   | d. | Cot.   | d.  | Cos.   | d. | P. P.    |
|------|--------|----|--------|----|--------|-----|--------|----|----------|
| 30 0 | 0.5000 |    | 0.5773 |    | 1.7320 |     | 0.8660 |    | 0 60     |
| 10   | 0.5025 | 25 | 0.5812 | 39 | 1.7204 | 116 | 0.8645 | 15 | 49 48 47 |
| 20   | 0.5050 | 25 | 0.5851 | 39 | 1.7090 | 114 | 0.8631 | 14 | 49 48 47 |
| 30   | 0.5075 | 25 | 0.5890 | 39 | 1.6976 | 113 | 0.8616 | 14 | 49 48 47 |
| 40   | 0.5100 | 25 | 0.5929 | 39 | 1.6864 | 112 | 0.8601 | 15 | 49 48 47 |
| 50   | 0.5125 | 25 | 0.5969 | 39 | 1.6753 | 111 | 0.8586 | 15 | 49 48 47 |
| 31 0 | 0.5150 | 25 | 0.6008 | 40 | 1.6643 | 110 | 0.8571 | 15 | 0 59     |
| 10   | 0.5175 | 24 | 0.6048 | 39 | 1.6533 | 109 | 0.8556 | 15 | 0 59     |
| 20   | 0.5200 | 25 | 0.6088 | 40 | 1.6425 | 108 | 0.8541 | 15 | 0 59     |
| 30   | 0.5225 | 25 | 0.6128 | 40 | 1.6318 | 107 | 0.8526 | 15 | 0 59     |
| 40   | 0.5250 | 25 | 0.6168 | 40 | 1.6212 | 106 | 0.8511 | 15 | 0 59     |
| 50   | 0.5274 | 24 | 0.6208 | 40 | 1.6107 | 105 | 0.8496 | 15 | 0 59     |
| 32 0 | 0.5299 | 25 | 0.6248 | 40 | 1.6003 | 104 | 0.8480 | 15 | 0 58     |
| 10   | 0.5324 | 24 | 0.6289 | 41 | 1.5900 | 103 | 0.8465 | 15 | 0 58     |
| 20   | 0.5348 | 24 | 0.6330 | 41 | 1.5798 | 102 | 0.8449 | 15 | 0 58     |
| 30   | 0.5373 | 24 | 0.6370 | 41 | 1.5697 | 101 | 0.8434 | 15 | 0 58     |
| 40   | 0.5397 | 24 | 0.6411 | 41 | 1.5596 | 100 | 0.8418 | 16 | 0 58     |
| 50   | 0.5422 | 24 | 0.6453 | 41 | 1.5497 | 99  | 0.8402 | 16 | 0 58     |
| 33 0 | 0.5446 | 24 | 0.6494 | 41 | 1.5398 | 98  | 0.8386 | 16 | 0 57     |
| 10   | 0.5471 | 24 | 0.6535 | 41 | 1.5301 | 97  | 0.8371 | 16 | 0 57     |
| 20   | 0.5495 | 24 | 0.6577 | 42 | 1.5204 | 96  | 0.8355 | 16 | 0 57     |
| 30   | 0.5519 | 24 | 0.6619 | 42 | 1.5108 | 95  | 0.8339 | 16 | 0 57     |
| 40   | 0.5543 | 24 | 0.6661 | 42 | 1.5013 | 94  | 0.8323 | 16 | 0 57     |
| 50   | 0.5568 | 24 | 0.6703 | 42 | 1.4919 | 93  | 0.8306 | 16 | 0 57     |
| 34 0 | 0.5592 | 24 | 0.6745 | 42 | 1.4825 | 92  | 0.8290 | 16 | 0 56     |
| 10   | 0.5616 | 24 | 0.6787 | 42 | 1.4733 | 91  | 0.8274 | 16 | 0 56     |
| 20   | 0.5640 | 24 | 0.6830 | 43 | 1.4641 | 90  | 0.8257 | 16 | 0 56     |
| 30   | 0.5664 | 24 | 0.6873 | 42 | 1.4550 | 89  | 0.8241 | 16 | 0 56     |
| 40   | 0.5688 | 24 | 0.6915 | 43 | 1.4460 | 88  | 0.8225 | 17 | 0 56     |
| 50   | 0.5712 | 24 | 0.6959 | 43 | 1.4370 | 87  | 0.8208 | 17 | 0 56     |
| 35 0 | 0.5736 | 23 | 0.7002 | 43 | 1.4281 | 86  | 0.8191 | 17 | 0 55     |
| 10   | 0.5759 | 24 | 0.7045 | 43 | 1.4193 | 85  | 0.8175 | 17 | 0 55     |
| 20   | 0.5783 | 23 | 0.7089 | 44 | 1.4106 | 84  | 0.8158 | 17 | 0 55     |
| 30   | 0.5807 | 23 | 0.7133 | 44 | 1.4019 | 83  | 0.8141 | 17 | 0 55     |
| 40   | 0.5830 | 23 | 0.7177 | 44 | 1.3933 | 82  | 0.8124 | 17 | 0 55     |
| 50   | 0.5854 | 23 | 0.7221 | 44 | 1.3848 | 81  | 0.8107 | 17 | 0 55     |
| 36 0 | 0.5878 | 23 | 0.7265 | 44 | 1.3764 | 80  | 0.8090 | 17 | 0 54     |
| 10   | 0.5901 | 23 | 0.7310 | 44 | 1.3680 | 79  | 0.8073 | 17 | 0 54     |
| 20   | 0.5925 | 23 | 0.7354 | 45 | 1.3597 | 78  | 0.8056 | 17 | 0 54     |
| 30   | 0.5948 | 23 | 0.7399 | 45 | 1.3514 | 77  | 0.8038 | 17 | 0 54     |
| 40   | 0.5971 | 23 | 0.7444 | 45 | 1.3432 | 76  | 0.8021 | 17 | 0 54     |
| 50   | 0.5995 | 23 | 0.7490 | 45 | 1.3351 | 75  | 0.8004 | 17 | 0 54     |
| 37 0 | 0.6018 | 23 | 0.7535 | 45 | 1.3270 | 74  | 0.7986 | 17 | 0 53     |
| 10   | 0.6041 | 23 | 0.7581 | 46 | 1.3190 | 73  | 0.7969 | 18 | 0 53     |
| 20   | 0.6064 | 23 | 0.7627 | 46 | 1.3111 | 72  | 0.7951 | 18 | 0 53     |
| 30   | 0.6087 | 23 | 0.7673 | 46 | 1.3032 | 71  | 0.7933 | 18 | 0 53     |
| 40   | 0.6110 | 23 | 0.7719 | 46 | 1.2954 | 70  | 0.7916 | 18 | 0 53     |
| 50   | 0.6133 | 23 | 0.7766 | 47 | 1.2876 | 69  | 0.7898 | 18 | 0 53     |
| 38 0 | 0.6156 | 23 | 0.7813 | 47 | 1.2799 | 68  | 0.7880 | 18 | 0 52     |
| 10   | 0.6179 | 23 | 0.7860 | 47 | 1.2723 | 67  | 0.7862 | 18 | 0 52     |
| 20   | 0.6202 | 22 | 0.7907 | 47 | 1.2647 | 66  | 0.7844 | 18 | 0 52     |
| 30   | 0.6225 | 22 | 0.7954 | 47 | 1.2571 | 65  | 0.7826 | 18 | 0 52     |
| 40   | 0.6248 | 22 | 0.8002 | 48 | 1.2497 | 64  | 0.7808 | 18 | 0 52     |
| 50   | 0.6270 | 22 | 0.8050 | 48 | 1.2422 | 63  | 0.7789 | 18 | 0 52     |
| 39 0 | 0.6293 | 23 | 0.8098 | 48 | 1.2349 | 62  | 0.7771 | 18 | 0 51     |
| 10   | 0.6316 | 22 | 0.8146 | 48 | 1.2276 | 61  | 0.7753 | 18 | 0 51     |
| 20   | 0.6338 | 22 | 0.8194 | 49 | 1.2203 | 60  | 0.7734 | 18 | 0 51     |
| 30   | 0.6361 | 22 | 0.8243 | 49 | 1.2131 | 59  | 0.7716 | 18 | 0 51     |
| 40   | 0.6383 | 22 | 0.8292 | 49 | 1.2059 | 58  | 0.7697 | 18 | 0 51     |
| 50   | 0.6405 | 22 | 0.8341 | 49 | 1.1988 | 57  | 0.7679 | 18 | 0 51     |
| 40 0 | 0.6428 | 22 | 0.8391 | 49 | 1.1917 | 56  | 0.7660 | 18 | 0 50     |
|      | Cos.   | d. | Cot.   | d. | Tan.   | d.  | Sin.   | d. | P. P.    |



TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

40°-45°

| °    | Sin.   | d. | Tan.   | d. | Cot.   | d. | Cos.   | d. | P. P. |
|------|--------|----|--------|----|--------|----|--------|----|-------|
| 40 0 | 0.6428 | 22 | 0.8391 | 49 | 1.1917 | 70 | 0.7660 | 19 | 0 50  |
| 10   | 0.6450 | 22 | 0.8440 | 50 | 1.1847 | 70 | 0.7641 | 18 | 50    |
| 20   | 0.6472 | 22 | 0.8490 | 50 | 1.1777 | 69 | 0.7623 | 19 | 40    |
| 30   | 0.6494 | 22 | 0.8541 | 50 | 1.1708 | 68 | 0.7604 | 19 | 30    |
| 40   | 0.6516 | 22 | 0.8591 | 51 | 1.1640 | 68 | 0.7585 | 19 | 20    |
| 50   | 0.6538 | 22 | 0.8642 | 51 | 1.1571 | 68 | 0.7566 | 19 | 10    |
| 41 0 | 0.6560 | 22 | 0.8693 | 51 | 1.1503 | 67 | 0.7547 | 19 | 0 49  |
| 10   | 0.6582 | 22 | 0.8744 | 51 | 1.1436 | 67 | 0.7528 | 19 | 50    |
| 20   | 0.6604 | 22 | 0.8795 | 51 | 1.1369 | 66 | 0.7509 | 19 | 40    |
| 30   | 0.6626 | 22 | 0.8847 | 52 | 1.1303 | 66 | 0.7489 | 19 | 30    |
| 40   | 0.6648 | 22 | 0.8899 | 52 | 1.1237 | 65 | 0.7470 | 19 | 20    |
| 50   | 0.6669 | 22 | 0.8951 | 52 | 1.1171 | 65 | 0.7451 | 19 | 10    |
| 42 0 | 0.6691 | 22 | 0.9004 | 53 | 1.1106 | 64 | 0.7431 | 19 | 0 48  |
| 10   | 0.6713 | 21 | 0.9057 | 53 | 1.1041 | 64 | 0.7412 | 19 | 50    |
| 20   | 0.6734 | 21 | 0.9110 | 53 | 1.0977 | 64 | 0.7392 | 19 | 40    |
| 30   | 0.6756 | 21 | 0.9163 | 53 | 1.0913 | 63 | 0.7373 | 19 | 30    |
| 40   | 0.6777 | 21 | 0.9217 | 54 | 1.0849 | 63 | 0.7353 | 19 | 20    |
| 50   | 0.6798 | 21 | 0.9271 | 54 | 1.0786 | 63 | 0.7333 | 19 | 10    |
| 43 0 | 0.6820 | 21 | 0.9325 | 54 | 1.0723 | 62 | 0.7313 | 20 | 0 47  |
| 10   | 0.6841 | 21 | 0.9379 | 55 | 1.0661 | 62 | 0.7293 | 20 | 50    |
| 20   | 0.6862 | 21 | 0.9434 | 55 | 1.0599 | 61 | 0.7273 | 20 | 40    |
| 30   | 0.6883 | 21 | 0.9489 | 55 | 1.0538 | 61 | 0.7253 | 20 | 30    |
| 40   | 0.6904 | 21 | 0.9545 | 55 | 1.0476 | 60 | 0.7233 | 20 | 20    |
| 50   | 0.6925 | 21 | 0.9601 | 56 | 1.0416 | 60 | 0.7213 | 20 | 10    |
| 44 0 | 0.6946 | 21 | 0.9657 | 56 | 1.0355 | 60 | 0.7193 | 20 | 0 46  |
| 10   | 0.6967 | 21 | 0.9713 | 56 | 1.0295 | 59 | 0.7173 | 20 | 50    |
| 20   | 0.6988 | 21 | 0.9770 | 57 | 1.0235 | 59 | 0.7153 | 20 | 40    |
| 30   | 0.7009 | 21 | 0.9827 | 57 | 1.0176 | 59 | 0.7132 | 20 | 30    |
| 40   | 0.7030 | 21 | 0.9884 | 57 | 1.0117 | 58 | 0.7112 | 20 | 20    |
| 50   | 0.7050 | 20 | 0.9942 | 58 | 1.0058 | 58 | 0.7091 | 20 | 10    |
| 45 0 | 0.7071 | 20 | 1.0000 | 58 | 1.0000 | 58 | 0.7071 | 20 | 0 45  |
|      | Cos.   | d. | Cot.   | d. | Tan.   | d. | Sin.   | d. | P. P. |

| 68     | 67   | 66   | 65   | 64   | 64   | 63   | 62   | 61   | 60   | 59   | 59   | 58   | 58   | 57   |
|--------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 6.8  | 6.7  | 6.6  | 6.5  | 6.4  | 6.4  | 6.3  | 6.2  | 6.1  | 6.0  | 5.9  | 5.9  | 5.8  | 5.8  | 5.7  |
| 2 13.6 | 13.4 | 13.2 | 13.1 | 12.9 | 12.8 | 12.6 | 12.4 | 12.3 | 12.1 | 11.9 | 11.8 | 11.7 | 11.6 | 11.5 |
| 3 20.4 | 20.1 | 19.8 | 19.6 | 19.3 | 19.2 | 18.9 | 18.6 | 18.4 | 18.1 | 17.8 | 17.7 | 17.5 | 17.4 | 17.2 |
| 4 27.2 | 26.8 | 26.4 | 26.2 | 25.8 | 25.6 | 25.2 | 24.8 | 24.6 | 24.2 | 23.8 | 23.6 | 23.4 | 23.2 | 23.0 |
| 5 34.0 | 33.5 | 33.0 | 32.7 | 32.2 | 32.0 | 31.5 | 31.0 | 30.7 | 30.2 | 29.7 | 29.5 | 29.2 | 29.0 | 28.7 |
| 6 40.8 | 40.2 | 39.6 | 39.3 | 38.7 | 38.4 | 37.8 | 37.2 | 36.9 | 36.3 | 35.7 | 35.4 | 35.1 | 34.8 | 34.5 |
| 7 47.6 | 46.9 | 46.2 | 45.8 | 45.1 | 44.8 | 44.1 | 43.4 | 43.0 | 42.3 | 41.6 | 41.3 | 40.9 | 40.6 | 40.2 |
| 8 54.4 | 53.6 | 52.8 | 52.4 | 51.6 | 51.2 | 50.4 | 49.6 | 49.2 | 48.4 | 47.6 | 47.2 | 46.8 | 46.4 | 46.0 |
| 9 61.2 | 60.3 | 59.4 | 58.9 | 58.0 | 57.6 | 56.7 | 55.8 | 55.3 | 54.4 | 53.5 | 53.1 | 52.6 | 52.2 | 51.7 |

| 57     | 56   | 55   | 54   | 54   | 53   | 53   | 52   | 52   | 51   | 51   | 50   | 50   | 49   |
|--------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 5.7  | 5.6  | 5.6  | 5.5  | 5.4  | 5.4  | 5.3  | 5.3  | 5.2  | 5.2  | 5.1  | 5.1  | 5.0  | 4.9  |
| 2 11.4 | 11.3 | 11.2 | 11.0 | 10.9 | 10.8 | 10.7 | 10.6 | 10.5 | 10.4 | 10.3 | 10.2 | 10.1 | 9.9  |
| 3 17.1 | 16.9 | 16.8 | 16.6 | 16.3 | 16.2 | 16.0 | 15.9 | 15.7 | 15.6 | 15.4 | 15.3 | 15.1 | 14.8 |
| 4 22.8 | 22.6 | 22.4 | 22.0 | 21.8 | 21.6 | 21.4 | 21.2 | 21.0 | 20.8 | 20.6 | 20.4 | 20.2 | 19.8 |
| 5 28.5 | 28.2 | 28.0 | 27.5 | 27.2 | 27.0 | 26.7 | 26.5 | 26.2 | 26.0 | 25.7 | 25.5 | 25.2 | 24.7 |
| 6 34.2 | 33.9 | 33.6 | 33.0 | 32.7 | 32.4 | 32.1 | 31.8 | 31.5 | 31.2 | 30.9 | 30.6 | 30.3 | 29.7 |
| 7 39.9 | 39.5 | 39.2 | 38.5 | 38.1 | 37.8 | 37.4 | 37.1 | 36.7 | 36.4 | 36.0 | 35.7 | 35.3 | 34.6 |
| 8 45.6 | 45.2 | 44.8 | 44.0 | 43.6 | 43.2 | 42.8 | 42.4 | 42.0 | 41.6 | 41.2 | 40.8 | 40.4 | 39.6 |
| 9 51.3 | 50.8 | 50.4 | 49.5 | 49.0 | 48.6 | 48.1 | 47.7 | 47.2 | 46.8 | 46.3 | 45.9 | 45.4 | 44.5 |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANT

0°-10

10°-20°

| °    | Vers.  | d. | Exsec. | d. | °    | Vers.  | d. | Exsec. | d.  | P. P.               |
|------|--------|----|--------|----|------|--------|----|--------|-----|---------------------|
| 0 0  | .00000 | 0  | .00000 | 0  | 10 0 | .01519 | 51 | .01542 | 52  | 110 100 90 80       |
| 10   | .00000 | 1  | .00000 | 1  | 10   | .01570 | 51 | .01595 | 53  | 11 10 9 8           |
| 20   | .00001 | 2  | .00001 | 2  | 20   | .01622 | 52 | .01648 | 54  | 22 20 18 16         |
| 30   | .00004 | 3  | .00004 | 3  | 30   | .01674 | 53 | .01703 | 55  | 33 30 27 24         |
| 40   | .00007 | 4  | .00007 | 4  | 40   | .01728 | 54 | .01758 | 56  | 44 40 36 32         |
| 50   | .00010 | 5  | .00010 | 5  | 50   | .01782 | 55 | .01814 | 57  | 55 50 45 40         |
| 1 0  | .00015 | 6  | .00015 | 6  | 11 0 | .01837 | 55 | .01871 | 58  | 66 60 54 48         |
| 10   | .00020 | 7  | .00020 | 7  | 10   | .01893 | 57 | .01929 | 59  | 77 70 63 56         |
| 20   | .00027 | 8  | .00027 | 8  | 20   | .01950 | 57 | .01988 | 60  | 88 80 72 64         |
| 30   | .00034 | 9  | .00034 | 9  | 30   | .02007 | 58 | .02048 | 61  | 99 90 81 72         |
| 40   | .00042 | 10 | .00042 | 10 | 40   | .02066 | 59 | .02109 | 62  |                     |
| 50   | .00051 | 11 | .00051 | 11 | 50   | .02125 | 60 | .02171 | 63  | 60 50 40 30         |
| 2 0  | .00061 | 12 | .00061 | 12 | 12 0 | .02185 | 61 | .02234 | 64  | 1 6 5 4             |
| 10   | .00071 | 13 | .00071 | 13 | 10   | .02246 | 62 | .02297 | 65  | 12 10 8 6           |
| 20   | .00083 | 14 | .00083 | 14 | 20   | .02308 | 62 | .02362 | 66  | 18 15 12 9          |
| 30   | .00095 | 15 | .00095 | 15 | 30   | .02370 | 63 | .02428 | 67  | 24 20 16 12         |
| 40   | .00108 | 16 | .00108 | 16 | 40   | .02434 | 64 | .02494 | 68  | 30 25 20 15         |
| 50   | .00122 | 17 | .00122 | 17 | 50   | .02498 | 65 | .02562 | 69  | 36 30 24 18         |
| 3 0  | .00137 | 18 | .00137 | 18 | 13 0 | .02563 | 66 | .02630 | 70  | 42 35 28 21         |
| 10   | .00152 | 19 | .00152 | 19 | 10   | .02629 | 67 | .02700 | 71  | 48 40 32 24         |
| 20   | .00169 | 20 | .00169 | 20 | 20   | .02695 | 67 | .02770 | 72  | 54 45 36 27         |
| 30   | .00186 | 21 | .00186 | 21 | 30   | .02763 | 68 | .02841 | 73  |                     |
| 40   | .00204 | 22 | .00204 | 22 | 40   | .02831 | 69 | .02914 | 74  | 10 9 9 8            |
| 50   | .00223 | 23 | .00223 | 23 | 50   | .02900 | 70 | .02987 | 75  | 1 0.9 0.9 0.8       |
| 4 0  | .00243 | 24 | .00243 | 24 | 14 0 | .02970 | 70 | .03061 | 76  | 2 1.9 1.8 1.7       |
| 10   | .00264 | 25 | .00264 | 25 | 10   | .03041 | 72 | .03136 | 77  | 3 2.8 2.7 2.5       |
| 20   | .00286 | 26 | .00286 | 26 | 20   | .03113 | 72 | .03213 | 78  | 4 3.8 3.6 3.4       |
| 30   | .00308 | 27 | .00308 | 27 | 30   | .03185 | 72 | .03290 | 79  | 5 4.7 4.5 4.2       |
| 40   | .00331 | 28 | .00331 | 28 | 40   | .03258 | 74 | .03368 | 80  | 6 5.7 5.4 5.1       |
| 50   | .00355 | 29 | .00355 | 29 | 50   | .03332 | 75 | .03447 | 81  | 7 6.6 6.3 5.9       |
| 5 0  | .00380 | 30 | .00380 | 30 | 15 0 | .03407 | 75 | .03527 | 82  | 8 7.6 7.2 6.8       |
| 10   | .00406 | 31 | .00406 | 31 | 10   | .03483 | 76 | .03609 | 83  | 9 8.5 8.1 7.6       |
| 20   | .00433 | 32 | .00433 | 32 | 20   | .03559 | 77 | .03691 | 84  |                     |
| 30   | .00460 | 33 | .00460 | 33 | 30   | .03637 | 78 | .03774 | 85  | 7 7 6 6             |
| 40   | .00488 | 34 | .00488 | 34 | 40   | .03715 | 79 | .03858 | 86  | 0.7 0.7 0.6 0.6     |
| 50   | .00518 | 35 | .00518 | 35 | 50   | .03794 | 80 | .03943 | 87  | 1.5 1.4 1.3 1.2     |
| 6 0  | .00548 | 36 | .00548 | 36 | 16 0 | .03874 | 80 | .04030 | 88  | 2.2 2.1 1.9 1.8     |
| 10   | .00578 | 37 | .00578 | 37 | 10   | .03954 | 81 | .04117 | 89  | 3.0 2.8 2.6 2.4     |
| 20   | .00610 | 38 | .00610 | 38 | 20   | .04036 | 82 | .04205 | 90  | 4.0 3.7 3.5 3.2     |
| 30   | .00643 | 39 | .00643 | 39 | 30   | .04118 | 83 | .04295 | 91  | 5.0 4.5 4.2 3.9     |
| 40   | .00676 | 40 | .00676 | 40 | 40   | .04201 | 84 | .04385 | 92  | 6.0 5.4 5.1 4.8     |
| 50   | .00710 | 41 | .00710 | 41 | 50   | .04285 | 84 | .04476 | 93  | 7.0 6.3 5.8 5.4     |
| 7 0  | .00745 | 42 | .00745 | 42 | 17 0 | .04369 | 85 | .04569 | 94  |                     |
| 10   | .00781 | 43 | .00781 | 43 | 10   | .04455 | 86 | .04662 | 95  | 5 4 4 3             |
| 20   | .00818 | 44 | .00818 | 44 | 20   | .04541 | 87 | .04757 | 96  | 10.0 9.4 8.8 8.2    |
| 30   | .00855 | 45 | .00855 | 45 | 30   | .04628 | 87 | .04853 | 97  | 21.0 20.0 19.0 18.0 |
| 40   | .00894 | 46 | .00894 | 46 | 40   | .04716 | 89 | .04949 | 98  | 31.5 31.3 31.2 31.0 |
| 50   | .00933 | 47 | .00933 | 47 | 50   | .04805 | 89 | .05047 | 99  | 42.0 42.0 42.0 42.0 |
| 8 0  | .00973 | 48 | .00973 | 48 | 18 0 | .04894 | 90 | .05146 | 100 | 52.5 52.2 52.0 51.7 |
| 10   | .01014 | 49 | .01014 | 49 | 10   | .04984 | 91 | .05246 | 101 | 63.0 62.7 62.4 62.1 |
| 20   | .01056 | 50 | .01056 | 50 | 20   | .05075 | 92 | .05347 | 102 | 73.5 73.1 72.8 72.4 |
| 30   | .01098 | 51 | .01098 | 51 | 30   | .05167 | 93 | .05449 | 103 | 84.0 83.6 83.2 82.8 |
| 40   | .01142 | 52 | .01142 | 52 | 40   | .05260 | 93 | .05552 | 104 | 94.5 94.0 93.6 93.1 |
| 50   | .01186 | 53 | .01186 | 53 | 50   | .05354 | 94 | .05656 | 105 |                     |
| 9 0  | .01231 | 54 | .01231 | 54 | 19 0 | .05448 | 95 | .05762 | 106 | 2 2 1 1             |
| 10   | .01277 | 55 | .01277 | 55 | 10   | .05543 | 96 | .05868 | 107 | 0.2 0.2 0.1 0.1     |
| 20   | .01324 | 56 | .01324 | 56 | 20   | .05639 | 97 | .05976 | 108 | 0.5 0.4 0.3 0.2     |
| 30   | .01371 | 57 | .01371 | 57 | 30   | .05736 | 97 | .06085 | 109 | 0.7 0.6 0.4 0.3     |
| 40   | .01420 | 58 | .01420 | 58 | 40   | .05833 | 98 | .06194 | 110 | 1.0 0.8 0.6 0.4     |
| 50   | .01469 | 59 | .01469 | 59 | 50   | .05931 | 99 | .06305 | 111 | 1.2 1.0 0.7 0.5     |
| 10 0 | .01519 | 60 | .01519 | 60 | 20 0 | .06030 | 99 | .06418 | 112 | 1.5 1.2 0.9 0.6     |
| °    | Vers.  | d. | Exsec. | d. | °    | Vers.  | d. | Exsec. | d.  | P. P.               |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

20°-30°

30°-40°

| °  | ' | Vers. | d. | Exsec. | d. | °  | ' | Vers. | d. | Exsec. | d. | P. P. |      |      |      |      |
|----|---|-------|----|--------|----|----|---|-------|----|--------|----|-------|------|------|------|------|
| 20 | 0 | .0603 |    | .0642  |    | 30 | 0 | .1339 |    | .1547  |    | 19    | 31   | 30   | 29   | 28   |
| 10 |   | .0613 | 10 | .0653  | 10 | 10 |   | .1354 | 15 | .1566  | 15 | 1     | 3.1  | 3.0  | 2.9  | 2.8  |
| 20 |   | .0623 | 10 | .0664  | 11 | 20 |   | .1369 | 14 | .1586  | 14 | 2     | 6.2  | 6.0  | 5.8  | 5.6  |
| 30 |   | .0633 | 10 | .0676  | 11 | 30 |   | .1383 | 15 | .1606  | 15 | 3     | 9.3  | 9.0  | 8.7  | 8.4  |
| 40 |   | .0643 | 10 | .0688  | 12 | 40 |   | .1398 | 15 | .1626  | 15 | 4     | 12.4 | 12.0 | 11.6 | 11.2 |
| 50 |   | .0654 | 10 | .0699  | 12 | 50 |   | .1413 | 15 | .1646  | 15 | 5     | 15.5 | 15.0 | 14.5 | 14.0 |
| 21 | 0 | .0664 | 10 | .0711  | 12 | 31 | 0 | .1428 | 15 | .1666  | 15 | 6     | 18.6 | 18.0 | 17.4 | 16.8 |
| 10 |   | .0674 | 10 | .0723  | 12 | 10 |   | .1443 | 15 | .1687  | 15 | 7     | 21.7 | 21.0 | 20.3 | 19.6 |
| 20 |   | .0685 | 10 | .0735  | 12 | 20 |   | .1458 | 15 | .1707  | 15 | 8     | 24.8 | 24.0 | 23.2 | 22.4 |
| 30 |   | .0696 | 10 | .0748  | 12 | 30 |   | .1473 | 15 | .1728  | 15 | 9     | 27.9 | 27.0 | 26.1 | 25.2 |
| 40 |   | .0706 | 10 | .0762  | 12 | 40 |   | .1489 | 15 | .1749  | 15 |       |      |      |      |      |
| 50 |   | .0717 | 11 | .0772  | 13 | 50 |   | .1504 | 15 | .1770  | 15 |       |      |      |      |      |
| 22 | 0 | .0728 | 10 | .0785  | 12 | 32 | 0 | .1519 | 15 | .1792  | 15 | 1     | 2.7  | 2.6  | 2.5  | 2.4  |
| 10 |   | .0739 | 11 | .0798  | 13 | 10 |   | .1535 | 15 | .1813  | 15 | 2     | 5.4  | 5.2  | 5.0  | 4.8  |
| 20 |   | .0750 | 11 | .0811  | 13 | 20 |   | .1550 | 15 | .1835  | 15 | 3     | 8.1  | 7.8  | 7.5  | 7.2  |
| 30 |   | .0761 | 11 | .0824  | 13 | 30 |   | .1566 | 16 | .1857  | 16 | 4     | 10.8 | 10.4 | 10.0 | 9.6  |
| 40 |   | .0772 | 11 | .0837  | 13 | 40 |   | .1582 | 16 | .1879  | 16 | 5     | 13.5 | 13.0 | 12.5 | 12.0 |
| 50 |   | .0783 | 11 | .0850  | 13 | 50 |   | .1597 | 16 | .1901  | 16 | 6     | 16.2 | 15.6 | 15.0 | 14.4 |
| 23 | 0 | .0795 | 11 | .0863  | 13 | 33 | 0 | .1613 | 15 | .1923  | 15 | 7     | 18.9 | 18.2 | 17.5 | 16.8 |
| 10 |   | .0806 | 11 | .0877  | 13 | 10 |   | .1629 | 16 | .1946  | 16 | 8     | 21.6 | 20.8 | 20.0 | 19.2 |
| 20 |   | .0818 | 11 | .0890  | 14 | 20 |   | .1645 | 16 | .1969  | 16 | 9     | 24.3 | 23.4 | 22.5 | 21.6 |
| 30 |   | .0829 | 11 | .0904  | 14 | 30 |   | .1661 | 16 | .1992  | 16 |       |      |      |      |      |
| 40 |   | .0841 | 12 | .0918  | 14 | 40 |   | .1677 | 16 | .2015  | 16 | 1     | 2.3  | 2.2  | 2.1  | 2.0  |
| 50 |   | .0853 | 12 | .0932  | 14 | 50 |   | .1693 | 16 | .2038  | 16 | 2     | 4.6  | 4.4  | 4.2  | 4.0  |
| 24 | 0 | .0864 | 11 | .0946  | 14 | 34 | 0 | .1709 | 16 | .2062  | 16 | 3     | 6.9  | 6.6  | 6.3  | 6.0  |
| 10 |   | .0876 | 12 | .0960  | 14 | 10 |   | .1726 | 16 | .2086  | 16 | 4     | 9.2  | 8.8  | 8.4  | 8.0  |
| 20 |   | .0888 | 12 | .0975  | 14 | 20 |   | .1742 | 16 | .2110  | 16 | 5     | 11.5 | 11.0 | 10.5 | 10.0 |
| 30 |   | .0900 | 12 | .0989  | 14 | 30 |   | .1758 | 16 | .2134  | 16 | 6     | 13.8 | 13.2 | 12.6 | 12.0 |
| 40 |   | .0912 | 12 | .1004  | 15 | 40 |   | .1775 | 17 | .2158  | 17 | 7     | 16.1 | 15.4 | 14.7 | 14.0 |
| 50 |   | .0924 | 12 | .1019  | 15 | 50 |   | .1792 | 16 | .2183  | 16 | 8     | 18.4 | 17.6 | 16.8 | 16.0 |
| 25 | 0 | .0937 | 12 | .1034  | 15 | 35 | 0 | .1808 | 16 | .2207  | 16 | 9     | 20.7 | 19.8 | 18.9 | 18.0 |
| 10 |   | .0949 | 12 | .1049  | 15 | 10 |   | .1825 | 17 | .2232  | 17 |       |      |      |      |      |
| 20 |   | .0961 | 12 | .1064  | 15 | 20 |   | .1842 | 17 | .2258  | 17 | 1     | 1.9  | 1.8  | 1.7  | 1.6  |
| 30 |   | .0974 | 12 | .1079  | 15 | 30 |   | .1859 | 17 | .2283  | 17 | 2     | 3.8  | 3.6  | 3.4  | 3.2  |
| 40 |   | .0986 | 13 | .1094  | 16 | 40 |   | .1876 | 17 | .2309  | 17 | 3     | 5.7  | 5.4  | 5.1  | 4.8  |
| 50 |   | .0999 | 13 | .1110  | 16 | 50 |   | .1893 | 17 | .2334  | 17 | 4     | 7.6  | 7.2  | 6.8  | 6.4  |
| 26 | 0 | .1012 | 12 | .1126  | 16 | 36 | 0 | .1910 | 17 | .2360  | 17 | 5     | 9.5  | 9.0  | 8.5  | 8.0  |
| 10 |   | .1025 | 13 | .1142  | 16 | 10 |   | .1927 | 17 | .2387  | 17 | 6     | 11.4 | 10.8 | 10.2 | 9.6  |
| 20 |   | .1037 | 13 | .1158  | 16 | 20 |   | .1944 | 17 | .2413  | 17 | 7     | 13.3 | 12.6 | 11.9 | 11.2 |
| 30 |   | .1050 | 13 | .1174  | 16 | 30 |   | .1961 | 17 | .2440  | 17 | 8     | 15.2 | 14.4 | 13.6 | 12.8 |
| 40 |   | .1063 | 13 | .1190  | 16 | 40 |   | .1979 | 17 | .2467  | 17 | 9     | 17.1 | 16.2 | 15.3 | 14.4 |
| 50 |   | .1077 | 13 | .1206  | 17 | 50 |   | .1996 | 17 | .2494  | 17 |       |      |      |      |      |
| 27 | 0 | .1090 | 13 | .1223  | 16 | 37 | 0 | .2013 | 17 | .2521  | 17 | 1     | 1.5  | 1.4  | 1.3  | 1.2  |
| 10 |   | .1103 | 13 | .1240  | 17 | 10 |   | .2031 | 18 | .2549  | 18 | 2     | 3.0  | 2.8  | 2.6  | 2.4  |
| 20 |   | .1116 | 13 | .1257  | 17 | 20 |   | .2049 | 17 | .2576  | 17 | 3     | 4.5  | 4.2  | 3.9  | 3.6  |
| 30 |   | .1130 | 13 | .1274  | 17 | 30 |   | .2066 | 17 | .2604  | 17 | 4     | 6.0  | 5.6  | 5.2  | 4.8  |
| 40 |   | .1143 | 13 | .1291  | 17 | 40 |   | .2084 | 18 | .2633  | 18 | 5     | 7.5  | 7.0  | 6.5  | 6.0  |
| 50 |   | .1157 | 13 | .1308  | 17 | 50 |   | .2102 | 18 | .2661  | 18 | 6     | 9.0  | 8.4  | 7.8  | 7.2  |
| 28 | 0 | .1170 | 13 | .1325  | 18 | 38 | 0 | .2120 | 18 | .2690  | 18 | 7     | 10.5 | 9.8  | 9.1  | 8.4  |
| 10 |   | .1184 | 14 | .1343  | 17 | 10 |   | .2138 | 18 | .2719  | 18 | 8     | 12.0 | 11.2 | 10.4 | 9.6  |
| 20 |   | .1198 | 14 | .1361  | 18 | 20 |   | .2156 | 18 | .2748  | 18 | 9     | 13.5 | 12.6 | 11.7 | 10.8 |
| 30 |   | .1212 | 14 | .1379  | 18 | 30 |   | .2174 | 18 | .2778  | 18 |       |      |      |      |      |
| 40 |   | .1225 | 14 | .1397  | 18 | 40 |   | .2192 | 18 | .2807  | 18 | 1     | 1.1  | 1.0  | 0.9  | 0.8  |
| 50 |   | .1239 | 14 | .1415  | 18 | 50 |   | .2210 | 18 | .2837  | 18 | 2     | 2.2  | 2.0  | 1.9  | 1.7  |
| 29 | 0 | .1254 | 14 | .1433  | 18 | 39 | 0 | .2228 | 18 | .2867  | 18 | 3     | 3.3  | 3.0  | 2.8  | 2.6  |
| 10 |   | .1268 | 14 | .1452  | 18 | 10 |   | .2247 | 18 | .2898  | 18 | 4     | 4.4  | 4.0  | 3.7  | 3.4  |
| 20 |   | .1282 | 14 | .1470  | 19 | 20 |   | .2265 | 18 | .2928  | 18 | 5     | 5.5  | 5.0  | 4.6  | 4.2  |
| 30 |   | .1296 | 14 | .1489  | 19 | 30 |   | .2284 | 18 | .2959  | 18 | 6     | 6.6  | 6.0  | 5.5  | 5.0  |
| 40 |   | .1311 | 14 | .1508  | 19 | 40 |   | .2302 | 18 | .2991  | 18 | 7     | 7.7  | 7.0  | 6.4  | 5.8  |
| 50 |   | .1325 | 14 | .1527  | 19 | 50 |   | .2321 | 18 | .3022  | 18 | 8     | 8.8  | 8.0  | 7.3  | 6.6  |
| 30 | 0 | .1339 | 14 | .1547  | 19 | 40 | 0 | .2339 | 18 | .3054  | 18 | 9     | 9.9  | 9.0  | 8.2  | 7.4  |
| °  | ' | Vers. | d. | Exsec. | d. | °  | ' | Vers. | d. | Exsec. | d. | P. P. |      |      |      |      |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS

40°-50°

50°-60°

| °    | Vers. | d. | Exsec. | d. | °    | Vers. | d. | Exsec. | d. | P. P.                     |
|------|-------|----|--------|----|------|-------|----|--------|----|---------------------------|
| 40 0 | .2339 | 19 | .3054  | 32 | 50 0 | .3572 | 22 | .5557  | 53 | 9 8 7 6 5                 |
| 10   | .2358 | 18 | .3086  | 32 | 10   | .3594 | 22 | .5611  | 54 | 1 0 9 0 8 0 7 0 6 0 5 0 0 |
| 20   | .2377 | 18 | .3118  | 32 | 20   | .3617 | 22 | .5666  | 54 | 2 1 8 1 6 1 4 1 2 1 0 0 0 |
| 30   | .2396 | 19 | .3151  | 32 | 30   | .3639 | 22 | .5721  | 55 | 3 2 7 2 4 2 1 1 8 1 5 1 1 |
| 40   | .2415 | 19 | .3183  | 33 | 40   | .3661 | 22 | .5777  | 56 | 4 3 6 3 2 2 8 2 4 2 0 1 1 |
| 50   | .2434 | 19 | .3217  | 33 | 50   | .3684 | 23 | .5833  | 56 | 5 4 5 4 0 3 5 3 0 2 5 1 1 |
| 41 0 | .2453 | 19 | .3250  | 34 | 51 0 | .3707 | 22 | .5890  | 57 | 6 5 4 4 8 4 2 3 6 3 0 2 2 |
| 10   | .2472 | 19 | .3284  | 34 | 10   | .3729 | 22 | .5947  | 58 | 7 6 3 5 6 4 9 4 2 3 5 2 2 |
| 20   | .2491 | 19 | .3317  | 34 | 20   | .3752 | 23 | .6005  | 58 | 8 7 2 6 4 5 6 4 8 4 0 3 3 |
| 30   | .2510 | 19 | .3352  | 34 | 30   | .3775 | 22 | .6064  | 59 | 9 8 1 7 2 6 3 5 4 4 5 3 3 |
| 40   | .2529 | 19 | .3386  | 34 | 40   | .3797 | 23 | .6123  | 59 |                           |
| 50   | .2549 | 19 | .3421  | 35 | 50   | .3820 | 23 | .6182  | 59 |                           |
| 42 0 | .2568 | 19 | .3456  | 35 | 52 0 | .3843 | 23 | .6242  | 60 | 3 2 1 9 8                 |
| 10   | .2588 | 19 | .3491  | 36 | 10   | .3866 | 23 | .6303  | 61 | 1 0 3 0 2 0 1 0 9 0 8 0   |
| 20   | .2607 | 19 | .3527  | 36 | 20   | .3889 | 23 | .6365  | 61 | 2 0 6 0 4 0 2 1 9 1 7 1   |
| 30   | .2627 | 20 | .3563  | 36 | 30   | .3912 | 23 | .6427  | 62 | 3 0 9 0 6 0 3 2 8 2 5 2   |
| 40   | .2647 | 20 | .3599  | 37 | 40   | .3935 | 23 | .6489  | 62 | 4 1 2 0 8 0 4 3 8 3 4     |
| 50   | .2666 | 20 | .3636  | 37 | 50   | .3958 | 23 | .6552  | 63 | 5 1 5 1 0 0 5 4 7 4 2 3   |
| 43 0 | .2686 | 20 | .3673  | 37 | 53 0 | .3982 | 23 | .6616  | 64 | 6 1 8 1 2 0 6 5 7 5 1 4   |
| 10   | .2706 | 20 | .3710  | 37 | 10   | .4005 | 23 | .6681  | 64 | 7 2 1 1 4 0 7 6 6 5 9 5   |
| 20   | .2726 | 20 | .3748  | 38 | 20   | .4028 | 23 | .6746  | 65 | 8 2 4 1 6 0 8 7 6 6 8 6   |
| 30   | .2746 | 20 | .3786  | 38 | 30   | .4052 | 23 | .6811  | 65 | 9 2 7 1 8 0 9 8 5 7 6 6   |
| 40   | .2766 | 20 | .3824  | 39 | 40   | .4075 | 23 | .6878  | 66 |                           |
| 50   | .2786 | 20 | .3863  | 38 | 50   | .4098 | 23 | .6945  | 66 | 6 5 4 3 2 1               |
| 44 0 | .2806 | 20 | .3901  | 39 | 54 0 | .4122 | 23 | .7013  | 67 | 1 0 6 0 5 0 4 0 3 0 2 0   |
| 10   | .2827 | 20 | .3941  | 39 | 10   | .4145 | 24 | .7081  | 68 | 2 1 3 1 1 0 9 0 7 0 5 0   |
| 20   | .2847 | 20 | .3980  | 40 | 20   | .4169 | 24 | .7150  | 68 | 3 1 9 1 6 1 3 1 0 0 7 0   |
| 30   | .2867 | 20 | .4020  | 40 | 30   | .4193 | 23 | .7220  | 69 | 4 2 6 2 2 1 8 1 4 1 0 0   |
| 40   | .2888 | 20 | .4060  | 40 | 40   | .4216 | 23 | .7291  | 70 | 5 3 2 2 7 2 2 1 7 1 2 0   |
| 50   | .2908 | 20 | .4101  | 41 | 50   | .4240 | 24 | .7362  | 70 | 6 3 9 3 3 2 7 2 1 1 5 0   |
| 45 0 | .2929 | 20 | .4142  | 41 | 55 0 | .4261 | 24 | .7434  | 71 | 7 4 5 3 8 3 1 2 4 1 7 0   |
| 10   | .2949 | 20 | .4183  | 41 | 10   | .4288 | 24 | .7507  | 72 | 8 5 2 4 4 3 6 2 8 2 0 1   |
| 20   | .2970 | 21 | .4225  | 42 | 20   | .4312 | 24 | .7581  | 73 | 9 5 8 4 9 4 0 3 1 2 2 1   |
| 30   | .2991 | 20 | .4267  | 42 | 30   | .4336 | 24 | .7655  | 73 | 25 25 24 24 23            |
| 40   | .3011 | 21 | .4309  | 43 | 40   | .4360 | 24 | .7730  | 74 | 1 2 5 5 1 5 0 4 9 4 8     |
| 50   | .3032 | 21 | .4352  | 43 | 50   | .4384 | 24 | .7806  | 75 | 2 5 7 6 7 5 7 3 7 2 7     |
| 46 0 | .3053 | 21 | .4395  | 43 | 56 0 | .4404 | 24 | .7883  | 75 | 3 7 6 10 2 10 0 9 8 9 6   |
| 10   | .3074 | 21 | .4439  | 44 | 10   | .4432 | 24 | .7930  | 77 | 4 10 2 12 5 12 2 12 0 11  |
| 20   | .3095 | 21 | .4483  | 44 | 20   | .4456 | 24 | .8039  | 77 | 5 12 7 12 5 12 2 12 0 11  |
| 30   | .3116 | 21 | .4527  | 44 | 30   | .4480 | 24 | .8118  | 78 | 6 13 8 13 5 14 7 14 4 14  |
| 40   | .3137 | 21 | .4572  | 45 | 40   | .4505 | 24 | .8198  | 78 | 7 15 3 15 0 14 7 14 4 14  |
| 50   | .3159 | 21 | .4617  | 45 | 50   | .4529 | 24 | .8279  | 79 | 8 17 8 17 5 17 1 16 8 16  |
| 47 0 | .3180 | 21 | .4663  | 45 | 57 0 | .4553 | 24 | .8361  | 79 | 9 20 4 20 0 19 6 19 2 18  |
| 10   | .3201 | 21 | .4708  | 46 | 10   | .4578 | 24 | .8443  | 80 | 22 9 22 5 22 0 21 6 21    |
| 20   | .3222 | 21 | .4755  | 46 | 20   | .4602 | 24 | .8527  | 81 |                           |
| 30   | .3244 | 21 | .4802  | 47 | 30   | .4627 | 24 | .8611  | 82 | 23 22 22 21 21            |
| 40   | .3265 | 21 | .4849  | 47 | 40   | .4651 | 24 | .8697  | 82 | 1 2 3 2 2 2 2 1 2 1       |
| 50   | .3287 | 21 | .4896  | 48 | 50   | .4676 | 25 | .8783  | 83 | 2 4 6 4 5 4 4 4 3 4       |
| 48 0 | .3308 | 22 | .4945  | 48 | 58 0 | .4701 | 24 | .8871  | 84 | 3 6 9 6 7 6 6 6 4 6       |
| 10   | .3330 | 21 | .4993  | 48 | 10   | .4725 | 24 | .8959  | 85 | 4 9 2 9 0 8 8 8 6 8       |
| 20   | .3352 | 22 | .5042  | 49 | 20   | .4750 | 25 | .9048  | 85 | 5 11 5 11 2 11 0 10 7 10  |
| 30   | .3374 | 21 | .5091  | 50 | 30   | .4775 | 25 | .9139  | 87 | 6 13 8 13 5 13 2 12 9 12  |
| 40   | .3395 | 22 | .5141  | 50 | 40   | .4800 | 24 | .9230  | 87 | 7 16 1 15 7 15 4 15 0 14  |
| 50   | .3417 | 22 | .5192  | 50 | 50   | .4824 | 25 | .9322  | 88 | 8 18 4 18 0 17 6 17 2 16  |
| 49 0 | .3439 | 22 | .5242  | 51 | 59 0 | .4849 | 25 | .9416  | 89 | 9 20 7 20 2 19 8 19 3 18  |
| 10   | .3461 | 22 | .5294  | 51 | 10   | .4874 | 25 | .9510  | 90 |                           |
| 20   | .3483 | 22 | .5345  | 52 | 20   | .4899 | 25 | .9606  | 91 | 20 20 19 19 18            |
| 30   | .3505 | 22 | .5397  | 53 | 30   | .4924 | 25 | .9703  | 92 | 1 2 0 2 0 1 9 1 9 1 8     |
| 40   | .3527 | 22 | .5450  | 53 | 40   | .4949 | 25 | .9801  | 93 | 2 4 1 4 0 3 9 3 8 3 7     |
| 50   | .3550 | 22 | .5503  | 53 | 50   | .4975 | 25 | .9900  | 94 | 3 6 1 6 0 5 8 5 7 5 5     |
| 50 0 | .3572 | 22 | .5557  | 53 | 60 0 | .5000 | 25 | 1.0000 | 95 | 4 8 2 8 0 7 8 7 6 7 4     |
| °    | Vers. | d. | Exsec. | d. | °    | Vers. | d. | Exsec. | d. | P. P.                     |



TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

60°-70°

70°-80°

| °    | Vers. | d. | Exsec. | d.  | °    | Vers. | d. | Exsec. | d.  | P. P. |      |      |      |      |
|------|-------|----|--------|-----|------|-------|----|--------|-----|-------|------|------|------|------|
| 60 0 | .5000 | 25 | 1.0000 | 101 | 70 0 | .6580 | 27 | 1.9238 | 235 | 9     | 8    | 7    | 6    | 5    |
| 10   | .5025 | 25 | 1.0101 | 102 | 10   | .6607 | 27 | 1.9473 | 240 | 10.9  | 0.8  | 0.7  | 0.6  | 0.5  |
| 20   | .5050 | 25 | 1.0204 | 103 | 20   | .6634 | 27 | 1.9713 | 244 | 21.8  | 1.6  | 1.4  | 1.2  | 1.0  |
| 30   | .5076 | 25 | 1.0307 | 105 | 30   | .6662 | 27 | 1.9957 | 248 | 32.7  | 2.4  | 2.1  | 1.8  | 1.5  |
| 40   | .5101 | 25 | 1.0413 | 106 | 40   | .6689 | 27 | 2.0205 | 253 | 43.6  | 3.2  | 2.8  | 2.4  | 2.0  |
| 50   | .5126 | 25 | 1.0519 | 107 | 50   | .6717 | 27 | 2.0458 | 257 | 54.5  | 4.0  | 3.5  | 3.0  | 2.5  |
| 61 0 | .5152 | 25 | 1.0626 | 109 | 71 0 | .6744 | 27 | 2.0715 | 262 | 65.4  | 4.8  | 4.2  | 3.6  | 3.0  |
| 10   | .5177 | 25 | 1.0735 | 110 | 10   | .6772 | 27 | 2.0977 | 266 | 76.3  | 5.6  | 4.9  | 4.2  | 3.5  |
| 20   | .5203 | 25 | 1.0846 | 111 | 20   | .6799 | 27 | 2.1244 | 271 | 87.2  | 6.4  | 5.6  | 4.8  | 4.0  |
| 30   | .5228 | 25 | 1.0957 | 113 | 30   | .6827 | 27 | 2.1515 | 276 | 98.1  | 7.2  | 6.3  | 5.4  | 4.5  |
| 40   | .5254 | 25 | 1.1070 | 114 | 40   | .6854 | 27 | 2.1792 | 281 |       |      |      |      |      |
| 50   | .5279 | 25 | 1.1184 | 116 | 50   | .6882 | 27 | 2.2073 | 287 | 4     | 3    | 2    | 1    | 9    |
| 62 0 | .5305 | 25 | 1.1300 | 117 | 72 0 | .6910 | 27 | 2.2360 | 292 | 10.4  | 0.3  | 0.2  | 0.1  | 0.9  |
| 10   | .5331 | 25 | 1.1418 | 118 | 10   | .6937 | 27 | 2.2653 | 298 | 20.8  | 0.6  | 0.4  | 0.2  | 1.9  |
| 20   | .5356 | 25 | 1.1536 | 120 | 20   | .6965 | 27 | 2.2951 | 304 | 31.2  | 0.9  | 0.6  | 0.3  | 2.8  |
| 30   | .5382 | 26 | 1.1657 | 121 | 30   | .6993 | 28 | 2.3255 | 310 | 41.6  | 1.2  | 0.8  | 0.4  | 3.8  |
| 40   | .5408 | 26 | 1.1778 | 123 | 40   | .7020 | 28 | 2.3565 | 316 | 52.0  | 1.5  | 1.0  | 0.5  | 4.7  |
| 50   | .5434 | 26 | 1.1902 | 125 | 50   | .7048 | 28 | 2.3881 | 322 | 62.4  | 1.8  | 1.2  | 0.6  | 5.7  |
| 63 0 | .5460 | 26 | 1.2027 | 126 | 73 0 | .7076 | 28 | 2.4203 | 328 | 72.8  | 2.1  | 1.4  | 0.7  | 6.6  |
| 10   | .5486 | 26 | 1.2153 | 128 | 10   | .7104 | 28 | 2.4531 | 335 | 83.2  | 2.4  | 1.6  | 0.8  | 7.6  |
| 20   | .5512 | 26 | 1.2281 | 130 | 20   | .7132 | 28 | 2.4867 | 342 | 93.6  | 2.7  | 1.8  | 0.9  | 8.5  |
| 30   | .5538 | 26 | 1.2411 | 131 | 30   | .7160 | 28 | 2.5209 | 349 |       |      |      |      |      |
| 40   | .5564 | 26 | 1.2543 | 133 | 40   | .7187 | 28 | 2.5558 | 356 | 8     | 7    | 6    | 5    | 4    |
| 50   | .5590 | 26 | 1.2676 | 135 | 50   | .7215 | 28 | 2.5915 | 364 | 10.8  | 0.7  | 0.6  | 0.5  | 0.4  |
| 64 0 | .5616 | 26 | 1.2811 | 137 | 74 0 | .7243 | 28 | 2.6279 | 372 | 21.7  | 1.5  | 1.3  | 1.1  | 0.9  |
| 10   | .5642 | 26 | 1.2948 | 139 | 10   | .7271 | 28 | 2.6651 | 380 | 32.5  | 2.2  | 1.9  | 1.6  | 1.3  |
| 20   | .5668 | 26 | 1.3087 | 140 | 20   | .7299 | 28 | 2.7031 | 388 | 43.4  | 3.0  | 2.6  | 2.2  | 1.8  |
| 30   | .5695 | 26 | 1.3228 | 143 | 30   | .7327 | 28 | 2.7420 | 396 | 54.2  | 3.7  | 3.2  | 2.7  | 2.2  |
| 40   | .5721 | 26 | 1.3371 | 144 | 40   | .7355 | 28 | 2.7816 | 406 | 65.1  | 4.5  | 3.9  | 3.3  | 2.7  |
| 50   | .5747 | 26 | 1.3515 | 146 | 50   | .7383 | 28 | 2.8222 | 414 | 75.9  | 5.2  | 4.5  | 3.8  | 3.1  |
| 65 0 | .5774 | 26 | 1.3662 | 148 | 75 0 | .7412 | 28 | 2.8637 | 424 | 86.8  | 6.0  | 5.2  | 4.4  | 3.6  |
| 10   | .5800 | 26 | 1.3810 | 151 | 10   | .7440 | 28 | 2.9061 | 434 | 97.6  | 6.7  | 5.8  | 4.9  | 4.0  |
| 20   | .5826 | 26 | 1.3961 | 152 | 20   | .7468 | 28 | 2.9495 | 444 |       |      |      |      |      |
| 30   | .5853 | 26 | 1.4114 | 155 | 30   | .7496 | 28 | 2.9939 | 454 | 10.8  | 0.7  | 0.6  | 0.5  | 0.4  |
| 40   | .5879 | 26 | 1.4269 | 157 | 40   | .7524 | 28 | 3.0394 | 465 | 20.7  | 0.5  | 0.3  | 0.2  | 0.1  |
| 50   | .5906 | 26 | 1.4426 | 159 | 50   | .7552 | 28 | 3.0859 | 476 | 31.0  | 0.7  | 0.4  | 0.3  | 0.2  |
| 66 0 | .5932 | 26 | 1.4586 | 161 | 76 0 | .7581 | 28 | 3.1335 | 488 | 41.4  | 1.0  | 0.6  | 0.5  | 0.4  |
| 10   | .5959 | 27 | 1.4747 | 164 | 10   | .7609 | 28 | 3.1824 | 500 | 51.7  | 1.7  | 1.1  | 0.7  | 0.5  |
| 20   | .5986 | 27 | 1.4912 | 166 | 20   | .7637 | 28 | 3.2324 | 512 | 62.1  | 2.4  | 1.5  | 0.9  | 0.7  |
| 30   | .6012 | 27 | 1.5078 | 169 | 30   | .7665 | 28 | 3.2836 | 525 | 72.4  | 3.1  | 2.0  | 1.2  | 0.8  |
| 40   | .6039 | 27 | 1.5247 | 171 | 40   | .7694 | 28 | 3.3362 | 539 | 82.8  | 3.8  | 2.4  | 1.4  | 0.9  |
| 50   | .6066 | 27 | 1.5419 | 174 | 50   | .7722 | 28 | 3.3901 | 553 | 93.1  | 4.5  | 2.9  | 1.6  | 1.0  |
| 67 0 | .6092 | 27 | 1.5593 | 177 | 77 0 | .7750 | 28 | 3.4454 | 567 |       |      |      |      |      |
| 10   | .6119 | 27 | 1.5770 | 179 | 10   | .7779 | 28 | 3.5021 | 582 | 2.9   | 2.8  | 2.8  | 2.7  | 2.7  |
| 20   | .6146 | 27 | 1.5949 | 182 | 20   | .7807 | 28 | 3.5604 | 598 | 5.8   | 5.7  | 5.6  | 5.5  | 5.5  |
| 30   | .6173 | 27 | 1.6131 | 184 | 30   | .7835 | 28 | 3.6202 | 614 | 8.7   | 8.5  | 8.4  | 8.2  | 8.2  |
| 40   | .6200 | 27 | 1.6316 | 188 | 40   | .7864 | 28 | 3.6816 | 631 | 11.6  | 11.4 | 11.2 | 11.0 | 11.0 |
| 50   | .6227 | 27 | 1.6504 | 190 | 50   | .7892 | 28 | 3.7448 | 649 | 14.5  | 14.2 | 14.0 | 13.7 | 13.7 |
| 68 0 | .6254 | 27 | 1.6694 | 194 | 78 0 | .7921 | 28 | 3.8097 | 667 | 17.4  | 17.1 | 16.8 | 16.5 | 16.5 |
| 10   | .6281 | 27 | 1.6888 | 196 | 10   | .7949 | 28 | 3.8765 | 686 | 20.3  | 19.9 | 19.6 | 19.2 | 19.2 |
| 20   | .6308 | 27 | 1.7085 | 200 | 20   | .7978 | 28 | 3.9451 | 707 | 23.2  | 22.8 | 22.4 | 22.0 | 22.0 |
| 30   | .6335 | 27 | 1.7285 | 203 | 30   | .8006 | 28 | 4.0158 | 728 | 26.1  | 25.6 | 25.2 | 24.7 | 24.7 |
| 40   | .6362 | 27 | 1.7488 | 206 | 40   | .8035 | 28 | 4.0886 | 749 |       |      |      |      |      |
| 50   | .6389 | 27 | 1.7694 | 210 | 50   | .8063 | 28 | 4.1638 | 772 | 27    | 26   | 26   | 25   | 25   |
| 69 0 | .6416 | 27 | 1.7904 | 213 | 79 0 | .8092 | 28 | 4.2408 | 796 | 2.7   | 2.6  | 2.6  | 2.5  | 2.5  |
| 10   | .6443 | 27 | 1.8117 | 216 | 10   | .8120 | 28 | 4.3205 | 821 | 5.4   | 5.3  | 5.2  | 5.1  | 5.1  |
| 20   | .6470 | 27 | 1.8334 | 220 | 20   | .8149 | 28 | 4.4026 | 847 | 8.1   | 7.9  | 7.8  | 7.6  | 7.6  |
| 30   | .6498 | 27 | 1.8554 | 224 | 30   | .8177 | 29 | 4.4874 | 875 | 10.8  | 10.6 | 10.4 | 10.2 | 10.2 |
| 40   | .6525 | 27 | 1.8778 | 228 | 40   | .8206 | 28 | 4.5749 | 904 | 13.5  | 13.2 | 13.0 | 12.7 | 12.7 |
| 50   | .6552 | 27 | 1.9006 | 231 | 50   | .8235 | 28 | 4.6653 | 934 | 16.2  | 15.9 | 15.6 | 15.3 | 15.3 |
| 70 0 | .6580 | 27 | 1.9238 | 231 | 80 0 | .8263 | 28 | 4.7587 |     | 18.9  | 18.5 | 18.2 | 17.8 | 17.8 |
| °    | Vers. | d. | Exsec. | d.  | °    | Vers. | d. | Exsec. | d.  | P. P. |      |      |      |      |

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS

80°-85°

85°-90°

| ° ' / | Vers. | d. | Exsec.  | d.   | ° ' / | Vers.  | d. | Exsec.   | d.     | P. P.  |
|-------|-------|----|---------|------|-------|--------|----|----------|--------|--------|
| 80 0  | .8263 | 28 | 4.7587  | 966  | 85 0  | .9128  | 29 | 10.4737  | .3946  |        |
| 10    | .8292 | 29 | 4.8554  | 999  | 10    | .9157  | 29 | 10.8683  | .4229  |        |
| 20    | .8321 | 28 | 4.9553  | 1035 | 20    | .9186  | 29 | 11.2912  | .4542  |        |
| 30    | .8349 | 28 | 5.0588  | 1072 | 30    | .9215  | 29 | 11.7455  | .4892  |        |
| 40    | .8378 | 29 | 5.1660  | 1111 | 40    | .9244  | 29 | 12.2347  | .5284  |        |
| 50    | .8407 | 28 | 5.2772  | 1152 | 50    | .9273  | 29 | 12.7631  | .5725  | 29 28  |
| 81 0  | .8435 | 29 | 5.3924  | 1196 | 86 0  | .9302  | 29 | 13.3356  | .6223  | 1 2.9  |
| 10    | .8464 | 28 | 5.5121  | 1242 | 10    | .9331  | 29 | 13.9579  | .6789  | 2 5.9  |
| 20    | .8493 | 29 | 5.6363  | 1291 | 20    | .9360  | 29 | 14.6368  | .7436  | 3 8.8  |
| 30    | .8522 | 28 | 5.7654  | 1343 | 30    | .9389  | 29 | 15.3804  | .8180  | 4 11.8 |
| 40    | .8550 | 29 | 5.8998  | 1398 | 40    | .9418  | 29 | 16.1984  | .9041  | 5 14.7 |
| 50    | .8579 | 29 | 6.0396  | 1456 | 50    | .9447  | 29 | 17.1026  | 1.0047 | 6 17.7 |
| 82 0  | .8608 | 28 | 6.1853  | 1519 | 87 0  | .9476  | 29 | 18.1073  | 1.1230 | 7 20.6 |
| 10    | .8637 | 29 | 6.3372  | 1585 | 10    | .9505  | 29 | 19.2303  | 1.2634 | 8 23.6 |
| 20    | .8666 | 28 | 6.4957  | 1656 | 20    | .9534  | 29 | 20.4937  | 1.4319 | 9 26.5 |
| 30    | .8694 | 29 | 6.6613  | 1731 | 30    | .9564  | 29 | 21.9256  | 1.6365 | 28     |
| 40    | .8723 | 29 | 6.8344  | 1812 | 40    | .9593  | 29 | 23.5621  | 1.8884 | 1 2.8  |
| 50    | .8752 | 29 | 7.0156  | 1898 | 50    | .9622  | 29 | 25.4505  | 2.2032 | 2 5.7  |
| 83 0  | .8781 | 28 | 7.2055  | 1991 | 88 0  | .9651  | 29 | 27.6537  | 2.6039 | 3 8.5  |
| 10    | .8810 | 29 | 7.4046  | 2091 | 10    | .9680  | 29 | 30.2576  | 3.1247 | 4 11.4 |
| 20    | .8839 | 29 | 7.6138  | 2198 | 20    | .9709  | 29 | 33.3823  | 3.8192 | 5 14.2 |
| 30    | .8868 | 29 | 7.8336  | 2315 | 30    | .9738  | 29 | 37.2015  | 4.7741 | 6 17.1 |
| 40    | .8897 | 29 | 8.0651  | 2440 | 40    | .9767  | 29 | 41.9757  | 6.1383 | 7 19.9 |
| 50    | .8926 | 28 | 8.3091  | 2576 | 50    | .9796  | 29 | 48.1140  | 8.1846 | 8 22.8 |
| 84 0  | .8954 | 29 | 8.5667  | 2723 | 89 0  | .9825  | 29 | 56.2987  |        | 9 25.6 |
| 10    | .8983 | 29 | 8.8391  | 2884 | 10    | .9854  | 29 | 67.7573  |        |        |
| 20    | .9012 | 29 | 9.1275  | 3059 | 20    | .9883  | 29 | 84.9456  |        |        |
| 30    | .9041 | 29 | 9.4334  | 3250 | 30    | .9912  | 29 | 113.5930 |        |        |
| 40    | .9070 | 29 | 9.7585  | 3460 | 40    | .9942  | 29 | 170.8883 |        |        |
| 50    | .9099 | 29 | 10.1045 | 3691 | 50    | .9971  | 29 | 342.7752 |        |        |
| 85 0  | .9128 | 29 | 10.4737 | 3961 | 90 0  | 1.0000 | 29 | ∞        |        |        |
| ° ' / | Vers. | d. | Exsec.  | d.   | ° ' / | Vers.  | d. | Exsec.   | d.     | P. P.  |



TABLE XI.—REDUCTION OF BAROMETER READING TO 32° F.

| Temp.<br>°<br>Fahr. | Inches. |       |       |       |       |       |       |       |       |       |       |
|---------------------|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                     | 26.0    | 26.5  | 27.0  | 27.5  | 28.0  | 28.5  | 29.0  | 29.5  | 30.0  | 30.5  | 31.0  |
| 45                  | -.039   | -.039 | -.040 | -.041 | -.042 | -.042 | -.043 | -.044 | -.045 | -.045 | -.046 |
| 46                  | .041    | .042  | .043  | .043  | .044  | .045  | .046  | .046  | .047  | .048  | .049  |
| 47                  | .043    | .044  | .045  | .046  | .047  | .048  | .048  | .049  | .050  | .051  | .052  |
| 48                  | .046    | .047  | .047  | .048  | .049  | .050  | .051  | .052  | .053  | .053  | .054  |
| 49                  | .048    | .049  | .050  | .051  | .052  | .052  | .054  | .054  | .055  | .056  | .057  |
| 50                  | .050    | .051  | .052  | .053  | .054  | .055  | .056  | .057  | .058  | .059  | .060  |
| 51                  | .053    | .054  | .055  | .056  | .057  | .058  | .059  | .060  | .061  | .062  | .063  |
| 52                  | .055    | .056  | .057  | .058  | .059  | .060  | .061  | .062  | .064  | .065  | .066  |
| 53                  | .057    | .058  | .060  | .061  | .062  | .063  | .064  | .065  | .066  | .067  | .068  |
| 54                  | .060    | .061  | .062  | .063  | .064  | .065  | .067  | .068  | .069  | .070  | .071  |
| 55                  | .062    | .063  | .064  | .065  | .066  | .068  | .069  | .070  | .071  | .073  | .074  |
| 56                  | .064    | .065  | .067  | .068  | .069  | .070  | .072  | .073  | .074  | .075  | .077  |
| 57                  | .067    | .068  | .069  | .070  | .072  | .073  | .075  | .076  | .077  | .078  | .080  |
| 58                  | .068    | .070  | .071  | .073  | .074  | .076  | .077  | .078  | .080  | .081  | .082  |
| 59                  | .072    | .073  | .074  | .075  | .077  | .078  | .080  | .081  | .083  | .084  | .085  |
| 60                  | .074    | .076  | .077  | .078  | .079  | .081  | .082  | .084  | .085  | .086  | .088  |
| 61                  | .076    | .077  | .079  | .080  | .082  | .083  | .085  | .086  | .088  | .089  | .091  |
| 62                  | .079    | .080  | .082  | .083  | .085  | .086  | .088  | .089  | .091  | .092  | .094  |
| 63                  | .081    | .082  | .084  | .085  | .087  | .088  | .090  | .091  | .093  | .095  | .096  |
| 64                  | .083    | .085  | .086  | .088  | .090  | .091  | .093  | .094  | .096  | .097  | .099  |
| 65                  | .086    | .087  | .089  | .090  | .092  | .093  | .095  | .097  | .099  | .100  | .102  |
| 66                  | .088    | .089  | .091  | .093  | .095  | .096  | .098  | .099  | .101  | .103  | .105  |
| 67                  | .090    | .092  | .094  | .095  | .097  | .099  | .101  | .102  | .104  | .106  | .108  |
| 68                  | .093    | .094  | .096  | .098  | .100  | .101  | .103  | .105  | .107  | .108  | .110  |
| 69                  | .095    | .097  | .099  | .100  | .102  | .104  | .106  | .107  | .110  | .111  | .113  |
| 70                  | .097    | .099  | .101  | .103  | .105  | .106  | .109  | .110  | .112  | .114  | .116  |
| 71                  | .100    | .101  | .103  | .105  | .107  | .109  | .111  | .113  | .115  | .117  | .119  |
| 72                  | .102    | .104  | .106  | .108  | .110  | .112  | .114  | .116  | .118  | .120  | .122  |
| 73                  | .104    | .106  | .108  | .110  | .112  | .114  | .116  | .118  | .120  | .122  | .124  |
| 74                  | .107    | .109  | .111  | .113  | .115  | .117  | .119  | .121  | .123  | .125  | .127  |
| 75                  | .109    | .111  | .113  | .115  | .117  | .119  | .122  | .124  | .126  | .128  | .130  |
| 76                  | .111    | .113  | .116  | .118  | .120  | .122  | .124  | .126  | .128  | .130  | .133  |
| 77                  | .114    | .116  | .118  | .120  | .122  | .124  | .127  | .129  | .131  | .133  | .136  |
| 78                  | .116    | .118  | .120  | .122  | .125  | .127  | .129  | .131  | .134  | .136  | .138  |
| 79                  | .118    | .120  | .123  | .125  | .127  | .129  | .132  | .134  | .137  | .139  | .141  |
| 80                  | .121    | .123  | .125  | .127  | .130  | .132  | .135  | .137  | .139  | .141  | .144  |
| 81                  | .123    | .125  | .128  | .130  | .132  | .134  | .137  | .139  | .142  | .144  | .147  |
| 82                  | .125    | .128  | .130  | .132  | .135  | .137  | .140  | .142  | .145  | .147  | .149  |
| 83                  | .128    | .130  | .133  | .135  | .138  | .140  | .142  | .145  | .147  | .149  | .152  |
| 84                  | .130    | .132  | .135  | .138  | .140  | .142  | .145  | .147  | .150  | .152  | .155  |
| 85                  | .132    | .134  | .137  | .140  | .143  | .145  | .148  | .150  | .153  | .155  | .158  |
| 86                  | .135    | .137  | .140  | .142  | .145  | .148  | .150  | .153  | .155  | .158  | .161  |
| 87                  | .137    | .139  | .142  | .144  | .148  | .150  | .153  | .155  | .158  | .161  | .163  |
| 88                  | .139    | .142  | .145  | .147  | .150  | .152  | .155  | .158  | .161  | .163  | .166  |
| 89                  | .142    | .144  | .147  | .150  | .153  | .155  | .158  | .161  | .164  | .166  | .169  |
| 90                  | .144    | .147  | .150  | .153  | .155  | .158  | .161  | .164  | .166  | .169  | .172  |
| 91                  | -.146   | -.149 | -.152 | -.155 | -.158 | -.160 | -.163 | -.166 | -.169 | -.172 | -.175 |

TABLE XII.—BAROMETRIC ELEVATIONS.\*

| <i>B</i> | <i>A</i> | Diff. for<br>.01. | <i>B</i> | <i>A</i> | Diff. for<br>.01. | <i>B</i> | <i>A</i> | Diff.<br>.01. |
|----------|----------|-------------------|----------|----------|-------------------|----------|----------|---------------|
| Inches.  | Feet.    | Feet.             | Inches.  | Feet.    | Feet.             | Inches.  | Feet.    | Feet.         |
| 20.0     | 11.047   | —13.6             | 23.7     | 6.423    | —11.5             | 27.4     | 2.470    | —9.           |
| 20.1     | 10.911   | 13.5              | 23.8     | 6.308    | 11.4              | 27.5     | 2.371    | 9.            |
| 20.2     | 10.776   | 13.4              | 23.9     | 6.194    | 11.4              | 27.6     | 2.272    | 9.            |
| 20.3     | 10.642   | 13.4              | 24.0     | 6.080    | 11.3              | 27.7     | 2.173    | 9.            |
| 20.4     | 10.508   | 13.3              | 24.1     | 5.967    | 11.3              | 27.8     | 2.075    | 9.            |
| 20.5     | 10.375   | 13.3              | 24.2     | 5.854    | 11.3              | 27.9     | 1.977    | 9.            |
| 20.6     | 10.242   | 13.2              | 24.3     | 5.741    | 11.2              | 28.0     | 1.880    | 9.            |
| 20.7     | 10.110   | 13.1              | 24.4     | 5.629    | 11.1              | 28.1     | 1.783    | 9.            |
| 20.8     | 9.979    | 13.1              | 24.5     | 5.518    | 11.1              | 28.2     | 1.686    | 9.            |
| 20.9     | 9.848    | 13.0              | 24.6     | 5.407    | 11.1              | 28.3     | 1.589    | 9.            |
| 21.0     | 9.718    | 12.9              | 24.7     | 5.296    | 11.0              | 28.4     | 1.493    | 9.            |
| 21.1     | 9.589    | 12.9              | 24.8     | 5.186    | 10.9              | 28.5     | 1.397    | 9.            |
| 21.2     | 9.460    | 12.8              | 24.9     | 5.077    | 10.9              | 28.6     | 1.302    | 9.            |
| 21.3     | 9.332    | 12.8              | 25.0     | 4.968    | 10.9              | 28.7     | 1.207    | 9.            |
| 21.4     | 9.204    | 12.7              | 25.1     | 4.859    | 10.8              | 28.8     | 1.112    | 9.            |
| 21.5     | 9.077    | 12.6              | 25.2     | 4.751    | 10.8              | 28.9     | 1.018    | 9.            |
| 21.6     | 8.951    | 12.6              | 25.3     | 4.643    | 10.8              | 29.0     | 924      | 9.            |
| 21.7     | 8.825    | 12.5              | 25.4     | 4.535    | 10.7              | 29.1     | 830      | 9.            |
| 21.8     | 8.700    | 12.5              | 25.5     | 4.428    | 10.7              | 29.2     | 736      | 9.            |
| 21.9     | 8.575    | 12.4              | 25.6     | 4.321    | 10.6              | 29.3     | 643      | 9.            |
| 22.0     | 8.451    | 12.4              | 25.7     | 4.215    | 10.6              | 29.4     | 550      | 9.            |
| 22.1     | 8.327    | 12.3              | 25.8     | 4.109    | 10.5              | 29.5     | 458      | 9.            |
| 22.2     | 8.204    | 12.2              | 25.9     | 4.004    | 10.5              | 29.6     | 366      | 9.            |
| 22.3     | 8.082    | 12.2              | 26.0     | 3.899    | 10.4              | 29.7     | 274      | 9.            |
| 22.4     | 7.960    | 12.2              | 26.1     | 3.794    | 10.4              | 29.8     | 182      | 9.            |
| 22.5     | 7.838    | 12.1              | 26.2     | 3.690    | 10.3              | 29.9     | 91       | 9.            |
| 22.6     | 7.717    | 12.0              | 26.3     | 3.586    | 10.3              | 30.0     | 0        | 9.            |
| 22.7     | 7.597    | 12.0              | 26.4     | 3.483    | 10.3              | 30.1     | —91      | 9.            |
| 22.8     | 7.477    | 11.9              | 26.5     | 3.380    | 10.3              | 30.2     | 181      | 9.            |
| 22.9     | 7.358    | 11.9              | 26.6     | 3.277    | 10.2              | 30.3     | 271      | 9.            |
| 23.0     | 7.239    | 11.8              | 26.7     | 3.175    | 10.2              | 30.4     | 361      | 9.            |
| 23.1     | 7.121    | 11.7              | 26.8     | 3.073    | 10.1              | 30.5     | 451      | 8.            |
| 23.2     | 7.004    | 11.7              | 26.9     | 2.972    | 10.1              | 30.6     | 540      | 8.            |
| 23.3     | 6.887    | 11.7              | 27.0     | 2.871    | 10.1              | 30.7     | 629      | 8.            |
| 23.4     | 6.770    | 11.6              | 27.1     | 2.770    | 10.0              | 30.8     | 717      | 8.            |
| 23.5     | 6.554    | 11.6              | 27.2     | 2.670    | 10.0              | 30.9     | 805      | 8.            |
| 23.6     | 6.538    | —11.5             | 27.3     | 2.570    | —10.0             | 31.0     | —893     | —8.           |
| 23.7     | 6.423    |                   | 27.4     | 2.470    |                   |          |          |               |

\* Compiled from Report of U. S. C. &amp; G. Survey for 1881, App. 10 Table

TABLE XIII.—COEFFICIENTS FOR CORRECTIONS FOR TEMPERATURE AND HUMIDITY.\*

| <i>t+t'</i> | <i>C</i> | Diff. for<br>1°. | <i>t+t'</i> | <i>C</i> | Diff. for<br>1°. | <i>t+t'</i> | <i>C</i> | Diff.<br>1°. |
|-------------|----------|------------------|-------------|----------|------------------|-------------|----------|--------------|
| 0°          | — .1024  | 10.9             | 60°         | — .0380  |                  | 120°        | + .0262  | 10.          |
| 10          | .0915    | 10.9             | 70          | .0273    | 10.7             | 130         | .0368    | 10.          |
| 20          | .0806    | 10.8             | 80          | .0166    | 10.7             | 140         | .0472    | 10.          |
| 30          | .0698    | 10.6             | 90          | — .0058  | 10.8             | 150         | .0575    | 10.          |
| 40          | .0592    | 10.6             | 100         | + .0049  | 10.7             | 160         | .0677    | 10.          |
| 50          | .0486    | 10.6             | 110         | .0156    | 10.7             | 170         | .0779    | 10.          |
| 60          | — .0380  |                  | 120         | + .0262  | 10.6             | 180         | + .0879  | 10.          |

\* Compiled from Report of U. S. C. &amp; G. Survey for 1881, App. 10, Tables I, I

$$1 \quad \sin a = \frac{1}{\operatorname{cosec} a} = \frac{\tan a}{\sqrt{1+\tan^2 a}} = \sqrt{\frac{1-\cos 2a}{2}} = \frac{1}{\sqrt{1+\cot^2 a}}$$

$$= \cos a \tan a = \sqrt{1-\cos^2 a} = 2 \sin \frac{1}{2}a \cos \frac{1}{2}a$$

$$= \frac{1+\cos a}{\cot \frac{1}{2}a} = \frac{2 \tan \frac{1}{2}a}{1+\tan^2 \frac{1}{2}a} = \operatorname{vers} a \cot \frac{1}{2}a.$$

$$2 \quad \cos a = \frac{1}{\sec a} = \frac{\cot a}{\sqrt{1+\cot^2 a}} = \frac{1}{\sqrt{1+\tan^2 a}}$$

$$= 1 - \operatorname{vers} a = \sin a \cot a = \sqrt{1-\sin^2 a} = 2 \cos^2 \frac{1}{2}a - 1$$

$$= \sin a \cot \frac{1}{2}a - 1 = \cos^2 \frac{1}{2}a - \sin^2 \frac{1}{2}a = 1 - 2 \sin^2 \frac{1}{2}a.$$

$$3 \quad \tan a = \frac{1}{\cot a} = \frac{\sin a}{\cos a} = \frac{\sec a}{\operatorname{cosec} a} = \frac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}$$

$$= \operatorname{vers} 2a \operatorname{cosec} 2a = \cot a - 2 \cot 2a = \sin a \sec a$$

$$= \frac{\sin 2a}{1+\cos 2a} = \operatorname{exsec} a \cot \frac{1}{2}a = \operatorname{exsec} 2a \cot 2a.$$

$$4 \quad \cot a = \frac{1}{\tan a} = \frac{\cos a}{\sin a} = \frac{\sin 2a}{1-\cos 2a} = \frac{1+\cos 2a}{\sin 2a}$$

$$= \sqrt{\operatorname{cosec}^2 a - 1} = \cot \frac{1}{2}a - \operatorname{cosec} a.$$

$$5 \quad \operatorname{vers} a = 1 - \cos a = \sin a \tan \frac{1}{2}a = 2 \sin^2 \frac{1}{2}a = \cos a \operatorname{exsec} a.$$

$$6 \quad \operatorname{exsec} a = \sec a - 1 = \tan a \tan \frac{1}{2}a = \operatorname{vers} a \sec a.$$

$$7 \quad \sin \frac{1}{2}a = \sqrt{\frac{\operatorname{vers} a}{2}} = \frac{\sin a}{2 \cos \frac{1}{2}a} = \frac{\operatorname{vers} a \cos \frac{1}{2}a}{\sin a}.$$

$$8 \quad \cos \frac{1}{2}a = \sqrt{\frac{1+\cos a}{2}} = \frac{\sin a}{2 \sin \frac{1}{2}a} = \frac{\sin a \sin \frac{1}{2}a}{\operatorname{vers} a}.$$

$$9 \quad \tan \frac{1}{2}a = \operatorname{vers} a \operatorname{cosec} a = \operatorname{cosec} a - \cot a = \frac{\tan a}{1+\sec a}.$$

$$10 \quad \cot \frac{1}{2}a = \frac{1+\cos a}{\sin a} = \operatorname{cosec} a + \cot a = \frac{\tan a}{\operatorname{exsec} a} = \frac{1}{\operatorname{cosec} a - \cot a}.$$

$$11 \quad \operatorname{vers} \frac{1}{2}a = 1 - \sqrt{\frac{1}{2}(1+\cos a)}.$$

$$12 \quad \operatorname{exsec} \frac{1}{2}a = \frac{1}{\sqrt{\frac{1}{2}(1+\cos a)}} - 1.$$

- 13  $\sin 2a = 2 \sin a \cos a = \frac{2 \tan a}{1 + \tan^2 a}.$
- 14  $\cos 2a = \cos^2 a - \sin^2 a = 1 - 2 \sin^2 a = 2 \cos^2 a - 1$   
 $= \frac{1 - \tan^2 a}{1 + \tan^2 a}.$
- 15  $\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}.$
- 16  $\cot 2a = \frac{1}{2} \cot a - \frac{1}{2} \tan a = \frac{\cot^2 a - 1}{2 \cot a} = \frac{1 - \tan^2 a}{2 \tan a}.$
- 17  $\text{vers } 2a = 2 \sin^2 a = 1 - \cos 2a = 2 \sin a \cos a \tan a.$
- 18  $\text{exsec } 2a = \frac{\tan 2a}{\cot a} = \frac{2 \tan^2 a}{1 - \tan^2 a} = \frac{2 \sin^2 a}{1 - 2 \sin^2 a}.$
- 19  $\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b.$
- 20  $\cos (a \pm b) = \cos a \cos b \mp \sin a \sin b.$
- 21  $\sin a + \sin b = 2 \sin \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b).$
- 22  $\sin a - \sin b = 2 \sin \frac{1}{2}(a-b) \cos \frac{1}{2}(a+b).$
- 23  $\cos a + \cos b = 2 \cos \frac{1}{2}(a+b) \cos \frac{1}{2}(a-b).$
- 24  $\cos a - \cos b = -2 \sin \frac{1}{2}(a+b) \sin \frac{1}{2}(a-b).$

Call the sides of any triangle  $A, B, C$ , and the opposite angles  $a, b,$  and  $c$ . Call  $s = \frac{1}{2}(A+B+C)$ .

- 25  $\tan \frac{1}{2}(a-b) = \frac{A-B}{A+B} \tan \frac{1}{2}(a+b) = \frac{A-B}{A+B} \cot \frac{1}{2}c.$
- 26  $C = (A+B) \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = (A-B) \frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)}.$
- 27  $\sin \frac{1}{2}a = \sqrt{\frac{(s-B)(s-C)}{BC}}.$
- 28  $\cos \frac{1}{2}a = \sqrt{\frac{s(s-A)}{BC}}.$
- 29  $\text{vers } a = \frac{2(s-B)(s-C)}{BC}.$
- 30  $\text{Area} = \sqrt{s(s-A)(s-B)(s-C)} = A^2 \frac{\sin b \sin c}{2 \sin a}.$

TABLE XXXI. —USEFUL FORMULE AND CONSTANTS.

|                                                                                         | Logarithm.  |                |
|-----------------------------------------------------------------------------------------|-------------|----------------|
| Circumference of a circle (radius = $r$ ) = $2\pi r$ .                                  |             |                |
| Area of a circle = $\pi r^2$ .                                                          |             |                |
| Area of sector (length of arc = $l$ ) = $\frac{1}{2}lr$ .                               |             |                |
| “ “ “ (angle of arc = $\alpha^\circ$ ) = $\frac{\alpha}{360}\pi r^2$ .                  |             |                |
| Area of segment (chord = $c$ , mid. ord. = $m$ ) = $\frac{2}{3}cm$ (approx.).           |             |                |
| Area of a circle to radius 1                                                            | } = $\pi =$ | 3.1415927      |
| Circumference of a circle to diameter 1                                                 |             |                |
| Surface of a sphere to diameter 1                                                       |             |                |
| Volume of a sphere to radius 1 = $4\pi \div 3 =$                                        |             | 4.1887902      |
| Arc equal to radius expressed in {                                                      | degrees =   | 57.2957795     |
|                                                                                         | minutes =   | 3437 7467708   |
|                                                                                         | seconds =   | 206264.8062471 |
| Length of arc of $1^\circ$ , radius unity.....                                          |             | 0.1745329      |
| Sine of one second = 0.0000048481.....                                                  |             | 4.6855749      |
| Cubic inches in United States standard gallon = 231.....                                |             | 2.3636120      |
| Weight of one cubic foot of water at maximum density (therm.<br>39°.8 F., barom. 30''). |             | 62.379         |
| Weight of one cubic foot of water at maximum density (therm.<br>62° F.).                |             | 62.321         |
| Acceleration due to gravity at latitude of New York in feet per<br>square second.....   |             | 32.15945       |
| Feet in one metre.....                                                                  |             | 3.280869       |
| Metres in one foot.....                                                                 |             | 0.304797       |





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